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Infusing Quantitative Reasoning Skills into a Differential Equation Class in an Urban Public Community College

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Abstract

This research centers on implementing Quantitative Reasoning (QR) within a differential equations course at an urban public community college. As a participant in the Numeracy Infusion for College Educators (NICE) faculty development program, I sought to integrate QR skills into my curriculum. Students in the course were introduced to QR goals using real-world data sets, particularly those related to population growth, which aim to enhance their understanding, sharpen their problem-solving abilities, and cultivate a positive perspective on the real-world relevance of mathematics. Preliminary findings indicate varied levels of QR skill development among students. These results underscore the potential benefits of infusing QR into mathematics courses and provide insights for educators looking to adopt similar strategies in their teaching.

Keywords

quantitative reasoning, differential equations, faculty development, numeracy infusion, real-life data, logistic population growth, problem-solving skills

Cover Page Footnote

Dr. Tanvir Prince holds a Doctorate in Mathematics from Stony Brook University and presently serves as an Associate Professor of Mathematics at Hostos Community College, part of the City University of New York system. With a keen interest in fields such as Topological Quantum Field Theory and Recreational Mathematics, Dr. Prince has recently developed a growing enthusiasm for Mathematics Education. In addition to his research, Dr. Prince is actively engaged in collaborative projects to introduce community college students to mathematical research early in their academic pursuits.

Introduction and Background

Quantitative Reasoning (QR) skills include understanding and analyzing numerical information (Madison & Dingman 2010). Despite QR's growing importance for making informed decisions in various aspects of life, many students in higher education still need help acquiring and applying these skills (Grawe 2011).

Steen (2001) identified the need to develop and strengthen QR skills among undergraduate students, especially in STEM (Science, Technology, Engineering, and Mathematics). Students engaging in mathematics through real-world applications are more likely to develop a deeper understanding and improve their problem-solving abilities (Boaler 1993). One way to facilitate this development is by integrating QR skills into existing curricula, particularly in courses focusing on mathematical modeling and applications such as differential equations (Ganter & Haver 2011).

Differential equations have long been recognized as a powerful tool for modeling and analyzing real-world phenomena in various disciplines, including physics, engineering, and biology (Zill 2012). Students taking a differential equation class are often required to create and analyze models based on first-order differential equations, which can be an ideal platform for developing QR skills (Oehrtman et al. 2008). In recent years, there have been increased efforts to introduce QR-focused instructional strategies in the teaching of differential equations (Sriraman & Lesh 2006). Recent studies have highlighted the importance of QR in undergraduate-level questions in science and mathematics, emphasizing the need for a robust understanding of QR benchmarksBao et al., 2009).

As a participant in the Numeracy Infusion for College Educators (NICE) faculty development program (https://tinyurl.com/343t7cxv), my focus is not on assessing the NICE program as a whole but on the specific implementation of QR within my differential equations course at an urban public community college. Differential equations courses, which delve into topics like rates of change and integration, offer a fertile ground for the infusion of QR, enhancing students' grasp of complex mathematical concepts (Briggs, Cochran, & Gillett, 2010. To gain a deeper understanding of the implementation and assessment of the NICE faculty development program, please see Wang & Wilder (2015).

This study contributes to the broader discourse on QR in mathematics education by assessing its implementation in a specific course setting. I aim to evaluate the effectiveness of QR integration in enhancing students' problem-solving abilities and appreciation of mathematics in real-world contexts. This exploration serves as a valuable addition to the ongoing discussions on the symbiotic relationship between critical thinking, numeracy, and mathematics education. More of this will be explained in the literature review section below.

Literature Review

QR skills are indispensable for undergraduate students, particularly in STEM fields such as differential equations (Association of American Colleges and Universities 2010; Elrod & Park, 2020 Zollman 2012). These skills enable students to analyze and interpret quantitative information, significantly affecting their academic success and future career prospects (Steen 2001; Madison & Steen 2008; Vacher & Chavez, 2009 QR skills encompass various mathematical concepts, including rates, proportions, and relationships (Mayes, Peterson, & Bonilla, 2013 Oehrtman et al. 2008). Developing these skills has been linked to improved problem-solving abilities, critical thinking, and overall academic performance (Hake 1998; Paul & Elder 2006; Bressoud et al. 2015; Rittle-Johnson & Schneider 2015). Moreover, QR self-efficacy, which refers to confidence in applying these skills, has influenced academic outcomes and career readiness (Pajares & Miller 1994; Lightsey, 1999 Steen 2001).

I have employed several innovative methods (please read the research design section to see some of the specific ways I redesigned the course) to enhance QR skills. These include problem-based learning, flipped classrooms, and active strategies such as group work and collaborative learning (Crouch & Mazur 2001; Freeman et al. 2014; Smith 2019; Johnson 2020). These approaches have been effective in helping students grasp complex mathematical concepts and develop a deeper understanding of the subject matter (Tall & Razali 1993; Byerley & Thompson 2017).

In my differential equations class, I have used graphing calculators to integrate technology into QR education. These tools and software platforms, like Maple, Mathematica, and MatLab, are increasingly valuable for dynamic visualizations and computational capabilities (Johnson & Larsen 2012; Zwickl et al. 2012). Although virtual labs and simulation-based learning have enhanced the learning experience (Wilensky & Reisman 2006; Dalgarno et al. 2009), I still need to incorporate these methods into my teaching. The primary reason is the significant time investment required to learn about virtual labs and the associated costs.

Assessment practices, such as formative and authentic assessments, are crucial in developing QR skills (Wiggins 1998; Vacher & Chavez, 2009 Smith 2018). These assessments provide ongoing feedback and measure the application of skills in real-world contexts, thereby supporting students' growth in QR skills (Association of American Colleges and Universities 2010).

Interdisciplinary approaches have also gained attention for their effectiveness in teaching QR skills. Incorporating examples from various disciplines like biology, economics, and physics can significantly improve QR skills (Berlin & White 2012; Small, Doll, Bergman, & Heggestad, 2017. Promoting STEM literacy, including

QR skills, requires an integrated approach that spans multiple disciplines (Zollman 2012).

Quantitative methods such as pre- and post-tests and longitudinal studies offer empirical evidence for the effectiveness of these pedagogical strategies (Williams 2019). Both critical thinking and higher-order reasoning are integral to QR skills and should be explicitly taught and assessed (Gogus, 2012 Facione 1990). I have used pre- and post-tests to evaluate the effectiveness of my QR implementation in my differential equation class.

Lastly, the NICE initiative enhances my instructional practices when implementing QR in my differential class. The program gave me a comprehensive understanding of QR and numeracy, emphasizing their importance in higher education. Research suggests that teacher professional development programs focused on QR skills can help improve instructional practices and student learning outcomes (Kezar & Gehrke 2015). This program exposed me to innovative methods for implementing and assessing QR in my courses, enriching the curriculum, and enhancing student learning outcomes. In my experience, participation in the NICE program led to a deeper understanding of QR, its relevance in higher education, and specific ways to integrate and assess it in a differential equations course. Based on the NICE faculty development initiative, I will explain some particular ways I redesigned the course in the “intervention” subsection of the research design.

These strategies have been instrumental in shaping my approach to teaching QR skills in my differential equations course. By examining these effective strategies for teaching QR skills within this context, this research will encourage instructors who teach higher-level mathematics classes, including but not limited to Calculus, linear algebra, differential equations, etc., to implement similar QR skills in their curriculum.

Research Design

This study employs a quasi-experimental design featuring a pretest-posttest framework. The focus is evaluating the impact of integrating QR skills into a differential equations course at an urban public community college. Specifically, the study examines how implementing QR-focused teaching strategies, informed by my participation in the NICE program, affects students’ QR self-efficacy and skills in my differential equations class.

Pretest-Posttest Measures

Before the course intervention, a pretest survey was administered to gauge students’ self-efficacy and competence in nine critical aspects of statistics and QR. A post-test survey followed the intervention to measure any changes. The survey was designed to align closely with the QR objectives.

Interventions

The intervention in this study was informed by my participation in the NICE program, which provided a comprehensive set of best practices for QR instruction. The following elements were integrated into the differential equations course to enhance QR skills:

1. The course was redesigned to include real-world examples that make QR more relevant and relatable to students. For instance, differential equations were applied to scenarios in fields like biology and economics.
2. Group work and other active learning strategies were employed to foster a more engaging classroom environment. These methods were aimed at helping students internalize QR concepts more effectively.
3. Graphing calculators were emphasized as a teaching aid for complex QR topics. The technology was integrated into the curriculum to complement traditional teaching methods.
4. Quizzes and in-class activities were introduced as formative assessments to gauge students' QR skills continuously. These assessments were designed to provide immediate feedback, allowing for timely instructional adjustments.
5. The course adopted a scaffolded learning approach, gradually introducing students to increasingly complex QR tasks. This method aimed to build a solid foundation before tackling more advanced problems.
6. Projects integrating QR with subjects like biology or economics were introduced. These projects aimed to broaden students' understanding of how QR skills can be applied across disciplines.
7. The course included exercises designed to test mathematical skills and critical thinking, aligning with NICE's emphasis on the holistic development of QR skills.
8. NICE-provided rubrics were adapted to assess students' QR skills in the course. These rubrics offered a standardized approach to evaluation, making it easier to measure learning outcomes.
9. While the NICE program also emphasized the value of reflective practices like journals or discussions, this aspect still needs to be implemented in the current iteration of the differential equations course.

By incorporating these NICE-informed strategies, the intervention aimed to assess whether such an approach would improve QR self-efficacy and skills.

Materials and Instruments

The materials used in this study include course materials, such as textbooks, lectures, homework assignments, and materials from the NICE faculty development program for QR instruction. The pre- and post-survey questionnaire assessed students' confidence levels in nine statistics and QR aspects (Charts and Graphs, Tables, Bar Graph, Pie Chart, Line Graph, Best-fitting line, Model from Data, Future Prediction from Data, Use Technology for Statistics). The project assignment and assessment rubric evaluated students' achievement of QR goals 1-3. The population data table for harbor seals in the Wadden Sea over 1997–2012 was used as a real-life data set for students to apply the QR goals. Graphing calculators (TI-84 and TI-84 Plus) were used to perform logistic regression and graphing. Additional reading materials and videos were provided to supplement students' knowledge of the logistic population growth model. Finally, students used paper and pencil to complete calculations and written responses.

Data Collection Procedure

Pre- and Post-Surveys

Students were surveyed before and after the intervention to measure their self-efficacy in nine specific aspects of QR. These aspects included interpreting Charts and Graphs, Tables, Bar Graphs, Pie Charts, Line Graphs, Best-fitting Lines, Models from Data, Future Predictions from Data, and the Use of Technology for Statistics. The survey used a 10-point Likert scale to gauge student confidence.

Project-Based Assessment

Students were assigned a project that required them to apply their QR skills to a real-world scenario, explicitly involving the logistic growth model. The project was designed to assess three main QR goals:

1. **Knowledge and Conceptual Understanding**: Students were tasked with extracting relevant information from a given data set to construct and solve a first-order differential equation model. Examples of such models included fish harvesting, logistic population growth, and heat transfer.
2. **Thinking and Other Skills**: Post-solution, students were required to conduct an error analysis to assess the model's accuracy about the actual real-world data.
3. **Attitudes, Values, and Habits of Mind**: Students used their models to make future predictions, such as estimating future population growth, thereby demonstrating the applicability of differential equations in real-life scenarios.

Grading Rubric

The project was evaluated using a grading scale that ranged from 0 to 4. The grading criteria were as follows:

- **Score 0**: No relevant information provided
- **Score 1**: Minimal relevant information with little explanation
- **Score 2**: At least 50% correct with appropriate explanation
- **Score 3**: At least 70% correct with appropriate explanation
- **Score 4**: At least 90% correct with appropriate explanation

The assessments focused on the accuracy and the quality of the explanations provided in the project. All data, including pre- and post-surveys and project assessments, were collected anonymously to ensure unbiased results.

Data Analysis

The data collected for this study were analyzed using descriptive and inferential statistics. The analysis focused on comparing the results of the pre-survey and post-survey to evaluate the effectiveness of the NICE program in improving the QR skills of undergraduate students in differential equation classes. In addition, the project assessments were analyzed using a rubric to evaluate student performance.

Pre-Survey Data Analysis

Referring to Table 1, the mean score of the pre-survey was 4.62, with a standard deviation of 3.41. The range of scores was from 1.1 to 9.2, with a median score of 5.5. The data is slightly skewed to the right with a skewness value of 0.05, indicating that students recorded lower scores on the survey compared to the post-survey.

Table 1
Pre-Survey Data

Topic	Score	Descriptive Statistics	
Charts and Graphs	8.3	Mean	4.62
Tables	9.2	Standard Error	1.14
Bar Graph	7.4	Median	5.5
Pie Chart	6.4	Mode	1.1
Line Graph	5.5	Standard Deviation	3.41
Best-fitting line	1.4	Sample Variance	11.63
Model from Data	1.1	Kurtosis	-2.11
Future Prediction from Data	1.2	Skewness	0.05
Use Technology for Statistics	1.1	Range	8.1
		Minimum	1.1
		Maximum	9.2
		Sum	41.6
		Count	9
		Confidence Level (95.0%)	2.62

The pre-survey results showed that the students had varying understandings and knowledge of the topics. For example, the highest mean score was obtained in the table topic with a score of 9.2, charts and graphs with a score of 8.3, and bar graphs with a score of 7.4. On the other hand, the topics with the lowest mean scores were using technology for statistics, with a score of 1.1, and future prediction from data, with a score of 1.2. This is probably because topics such as “tables,” “graphs,” “charts,” etc., are more common knowledge, while the other topics are more specialized topics of QR.

The standard error of the mean score was 1.14, indicating a relatively high variability of scores. The confidence level for the survey results was calculated to be 95%, with a confidence interval of ±2.62. This means we can be 95% confident that the actual population mean lies between 2.00 and 7.24.

In summary, the pre-survey results suggest that the students had a wide range of understanding and knowledge of statistical graphs and analysis, with some topics being better understood than others. The results provide a baseline for evaluating the effectiveness of the project in improving the student's understanding and knowledge of these topics.

Post-Survey Data Analysis

The post-survey data included the same topics as the pre-survey and was conducted after completing the project. Table 2 and Table 3 show an improvement in all the topics compared to the pre-survey. For example, the mean score for charts and graphs increased from 8.3 to 9.7, tables increased from 9.2 to 9.8, bar graphs increased from 7.4 to 9.6, pie charts increased from 6.4 to 9.8, line graphs increased from 5.5 to 8.9, best-fitting line increased from 1.4 to 9.6, model from data increased from 1.1 to 9.8, future prediction from data increased from 1.2 to 9.3, and use technology for statistics increased from 1.1 to 8.9.

Table 2
Post-Survey Data

Topic	Score	Descriptive Statistics	
Charts and Graphs	9.7	Mean	9.49
Tables	9.8	Standard Error	0.12
Bar Graph	9.6	Median	9.6
Pie Chart	9.8	Mode	9.8
Line Graph	8.9	Standard Deviation	0.37
Best-fitting line	9.6	Sample Variance	0.14
Model from Data	9.8	Kurtosis	-0.64
Future Prediction from Data	9.3	Skewness	-1.00
Use Technology for Statistics	8.9	Range	0.9
		Minimum	8.9
		Maximum	9.8
		Sum	85.4
		Count	9
		Confidence Level (95.0%)	0.28

Table 3
Pre- and Post-Survey Comparison
***t*-Test: Paired Two Sample for Means**

	Pre-Survey Score	Post-Survey Score
Mean	4.62	9.49
Variance	11.63	0.14
Observations	9	9
Pearson Correlation	0.38	
Hypothesized Mean Difference	0	
df	8	
t Stat	-4.45	
P(T<=*t*) one-tail	0.008	
t Critical one-tail	1.86	
P(T<=*t*) two-tail	0.002	
t Critical two-tail	2.31	

The increase in mean scores for each topic indicates that the project positively impacted the students' understanding of the topics related to data visualization and statistical analysis. The most significant improvement was observed for the topics of best-fitting line, model from data, and future prediction from data, with an increase of 8.2, 8.7, and 8.1 points, respectively. This suggests that the project improved students' ability to use mathematical models and predict outcomes based on data.

The standard deviation for all topics decreased significantly compared to the pre-survey, indicating less variability in the post-survey scores. This suggests that the project helped to improve the consistency of understanding of the topics among the students.

The *t*-test analysis shows a significant difference between the pre-survey and post-survey mean scores, with a *p*-value of 0.002, indicating that the post-survey scores are significantly higher than the pre-survey scores. Furthermore, the effect size is large, with a Cohen's *d* value of 3.75, indicating that the project substantially impacted the students' understanding of data visualization and statistical analysis.

Overall, the post-survey data suggests that the project successfully improved the student's understanding of the topics related to data visualization and statistical analysis. Furthermore, the improvement was observed across all topics, and the effect size was large, indicating a substantial impact of the project on the students' learning of our QR goals.

QR Assessment

The QR assessment conducted in this study served as a measure to gauge the degree to which the project and all the other interventions facilitated the students' grasp of QR. This evaluation was structured around the three primary goals outlined for QR, with each of three goals assessed through a pair of questions. Therefore, with each question allocated 10 points, the overall score potential totaled 60 points. The scoring rubric ranges from 0 to 4 and aims to comprehensively reflect the students' proficiency levels (please see the grading rubrics in the data collection procedure section).

The results derived were demonstrative of the varying proficiency levels among students. It was observed that the student's scores ranged from 13 at the lowest to 45 at the highest. However, no student achieved the maximum potential score of 60, indicating room for further enhancement in QR understanding and application.

Regarding distribution (Fig. 1), the scores were predominantly concentrated in the middle range. A closer look reveals that 33% of students received a score of 2, while 40% were awarded a score of 3. Further, down the scale, 20% of the students received a score of 1. However, it is noteworthy that no students were assigned a score of 0. This lack of zero scores indicates that all students engaged with the project and earnestly attempted to meet the QR goals to varying degrees of success.

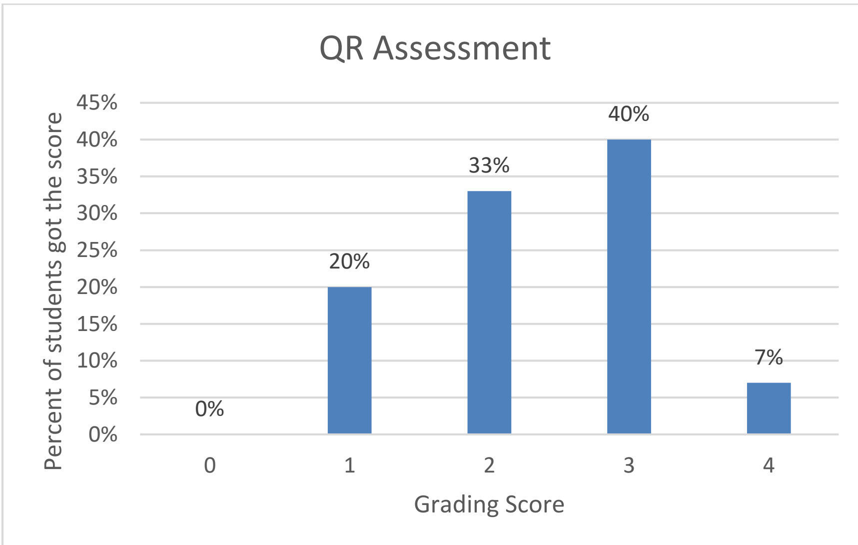

Figure 1. QR assessment of student proficiency levels

Despite these diverging score ranges, it was clear that while most students had made some headway in meeting the QR goals, they still needed to fully achieve all three goals. This indicates that while the project was beneficial, aspects that need improvement in future implementations remain. Nevertheless, the scoring rubrics, in their transparency and objectivity, provided an efficient tool for evaluating the students' proficiency levels, paving the way for identifying areas of improvement in teaching strategies and learning methodologies.

Limitations and Future Direction

One limitation of this study is that the sample size was relatively small. This may limit the generalizability of the results to a larger population. In addition, the study was conducted in a specific setting (an urban public community college in NYC), and the results may not apply to other contexts.

Another limitation is that the QR assessment was based on a subjective rubric, which could introduce bias in the scoring. Future studies could address this limitation by using a more objective assessment method, such as a standardized test.

Additionally, the study did not collect data on the long-term retention of the skills taught in the project. Future studies could track the retention of these skills over a more extended period to determine if the project had a lasting impact on students' statistical literacy.

Also, this study did not include a "self-reflection" component, where students could document their progress in a journal or blog throughout the semester. Such a practice would offer students a comprehensive overview of their development over the term. It serves as an effective method for tracking long-term progress and provides insights into individual strengths and weaknesses.

Finally, this study was conducted in a traditional classroom setting. Future studies could investigate the effectiveness of similar projects in online or hybrid learning environments, which have become more prevalent in recent years.

Conclusion

This paper shows that implementing teaching strategies derived from the NICE program in a differential equations class has yielded noteworthy results. Specifically, the post-survey and QR assessment data reveal a marked improvement in students' quantitative reasoning (QR) skills. While 80% of students scored two or higher in the QR assessment, the absence of maximum scores suggests room for refinement in the teaching approach.

This study significantly contributes to mathematics education by demonstrating the potential of a project-based learning approach inspired by NICE in enhancing QR skills. Unlike traditional pedagogical methods, this approach engages students more practically and interactively. The increase in post-survey scores across various QR topics provides empirical evidence of the effectiveness of this teaching strategy.

For educators and policymakers, the findings offer actionable insights. They suggest a project-based approach to QR can be successfully integrated into higher-level mathematics courses like differential equations. Moreover, the study pinpoints specific QR topics students excelled in and requires further instructional focus. For example, the absence of a perfect score in the QR assessment underscores that more work is needed to ensure that all students can fully grasp and apply these crucial skills. Strategies may need to be adapted to address this, but it is clear that the project's basis and methodology are sound, serving as a promising foundation for improvement. This type of information is invaluable for the development of targeted educational interventions and curricula.

While the study has limitations, such as its small sample size and specific application to a differential equations class in an urban public community college, it opens avenues for future research. Future studies could explore the long-term retention of QR skills and the effectiveness of similar pedagogical strategies in different educational settings, including online or hybrid models.

In summary, the study not only elevates students' QR skills but also provides a framework that educators can adapt to meet the specific needs of their student populations. As such, this research is a significant addition to the growing body of work to enhance quantitative literacy in higher education, particularly in STEM.

In the grand scheme of things, this research enriches the dialog surrounding the importance of quantitative literacy in contemporary education. Moreover, it adds weight to the assertion that QR is not merely a peripheral tool but a core competency that significantly influences a learner's ability to navigate an increasingly quantitative world effectively.

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