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Mathematical Semantics Representation Obstacles of Preservice Mathematics Teachers to Solve Geometry Problems

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Abstract

This study aimed to describe students' semantic representation obstacles in solving geometric problems. There are three types of obstacles, that are ontogeny, epistemology, and didactics obstacles. This study was carried out on three subjects with different obstacles and focused on math semantic representations in solving geometry problems. The research subjects were 65 students, consisting of 20 male students and 45 female students. Subjects with different obstacles are called subject ontogeny (SO), subject epistemology (SE), and subject didactic (SD). The purpose of the subject is to receive descriptions of the three types of obstacles. The instrument used in this research was a geometry problem-solving test which was validated by two expert validators. The instrument test on problem-solving questions was about Euclidean geometry course material. The representations that surface for SO Subjects navigating ontogenic obstacles include verbal, symbolic, and visual representations. In the identification stage, the SO showed symbolic and verbal representation. In the verification stage, the SO showed symbolic and verbal representations. Meanwhile, SE dealt with an epistemological obstacle consisting of all types of symbolic and verbal representations in all stages. The last SD subject dealing with didactic obstacles showed only a visual representation.

Keywords: Pbstacles, Representation, Mathematical semantics.

Introduction

Mathematics is a compulsory subject from primary to university levels. This subject aims to develop students' mathematics skills, such as reasoning, problem-solving, communicating, making connections, and representation by National Council of Teachers of Mathematics (NCTM, 2000). Mathematics skill is key for students in learning.

Representation is one of the mathematical abilities that are important for students. It is supported by NCTM (2000), mentioning that representation is one of the standards in learning math. Representation is a tool to enhance student's comprehension in learning math (Ball, 2015), building a strong connection with math (Mhlolo et al., 2012); (Adu-Gyamfi et al., 2016), and solving math problems (Jao, 2013). The use of different mathematical representations is an important aspect of mathematics. This learning can be derived both from a process perspective and from the evaluation of learning mathematics (Afriyani et al., 2018).

Representation could be seen from two perspectives, namely internal and external representations. The internal perspective of representation views representation as a cognitive process of the individual. At the same time, the external perspective of representation is defined as a perspective in seeing physically observable representations (sternberg, 2012). External representations are certainly inseparable from internal representations since what a person presents in their external representation is the embodiment of a person's internal representation of their mind. Representation is in the form of numbers, algebraic equations, graphs, tables, and diagrams, which are external forms of mathematical concepts. The ability of representation plays an important role in solving problems of story problems (Maulyda et al., 2020). With various forms of representation students need to be able to associate various forms of representation to strengthen students in constructing concepts and solving mathematical problems (NCTM, 2000); (Pape & Tchoshanov, 2001).

Preservice mathematics teachers representation abilities are shown through their semantic processes when faced with

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problems to be solved (Brizuela et al., 2015). Meanwhile, semantics is a branch of linguistics that studies the meaning of a form of representation. In particular, semantics is defined as the study of the meaning of words (Chaer, 2009). The term semantics is used for several different things, such as semantic process, semantic structure, semantic knowledge, and semantic strategy. All of these are different things but still closely related to meaning.

The semantic process is the interpretation of a form of representation in the form of words, symbols, and graphics. Additionally, translating a form of representation is also a semantic process that aims to maintain meaning even in different representations (Alcock & Inglis, 2009); (Bassok, 2003). In the semantic process, students are trained to communicate their ideas both in spoken and written forms so that the ideas can be used as a basis for conclusions. Basically, the semantic process is inseparable from the semantic structure that belongs to the problem-solver.

The semantic structure faced by the students that belong to problem solver, in studying mathematics and experiencing daily life, always faces problems. Likewise, when students are exposed to mathematical problem situations in class, they will strive to process and solve the problem in a way they already know. Problems encountered by students are the learning obstacles in the classroom. In addition, during the lecture activities, students sometimes experience problems or obstacles that arise during the teaching and learning process.

The learning aim cannot be optimally attained due to the frequent obstacles that arise during the learning process. A lecturer needs to be aware of and examine how students interact with their environment and acquire mathematics, whether they are having success or difficulty, according to (Ignacio et al., 2006) By ignoring the follow-up methods of the hurdles during the learning process, teachers frequently miss the obstacles that students face when solving each arithmetic issue. In addition, insufficient attention has been paid in the study and literature on mathematics education to the lecturers' opinions of the causes of students' learning obstacles (Bingolbali et al., 2011). As a result, there might be additional geometry-related roadblocks that are the problem. In this sense, instructors should assist students in acquiring persistence and a wider perspective on mathematics.

Geometry deals with abstract concepts that are symbolized. In geometry, there are also definitions, axioms, and theorems to indicate the truth of a statement to be proven, as (Chazan & Lueke, 2009) stated. In mathematics, evidence is very fundamental, as stated by (Varghese, 2009). Yet, many students have not succeeded in making the proof. Some researchers (Alcock & Inglis, 2009); (Starvrou, 2014); & (Imamoğlu & Srivastava, 2015) reported that students have difficulty constructing the evidence. According to (Imamoğlu & Srivastava, 2015) even final-year students still face several obstacles in constructing and evaluating the proof. (Starvrou, 2014) described that students often repeated the same mistakes when completing evidence. Some researchers (Oflaz et al., 2016); (Magajna, 2013);(Cirillo & Herbst, 2012) have also investigated students' difficulties in constructing proof on Euclid's geometry materials. (Oflaz et al., 2016) mentioned that preservice teachers experience obstacles in solving geometry problems.

Problem-solving is an ability that must be possessed to solve mathematical problems (Marsitin et al., 2022). Numerous studies on problem-solving have been conducted as a result of the significance of problem solving in the learning of mathematics (Gurat, 2018; Intaros et al., 2014; Sa'Dijah et al., 2020; Schoenfeld, 2016). Problem solving techniques used in teacher-student interactions were examined in (Gurat, 2018) Additionally, it was noted by (Sa'Dijah et al., 2020) that solving problems is a key component of learning mathematics. Since a result, developing problem-solving skills becomes the main focus of mathematics education. (Schoenfeld, 2016) claimed that when using the proper strategy to discover a solution is difficult or unknown, problem-solving is a learning process to finish new and unfamiliar jobs. Contested mathematical questions require the ability to solve problems.

Previous studies focused on learning obstacles mentioned that students were facing learning obstacles, including 1) ontogenic barrier, the inability of students to understand the purpose of the problem presented; 2) epistemological barrier, the inability of students to understand the concept of fractions as part of a whole and the inability of students to add and subtract fractions; and, 3) didactic barrier caused by the material presentation and the teacher's informal teaching method. Hence, it is necessary to overcome the obstacles that arise from obstacles that come from students and teachers (Fauzi, 2020).

This study is a continuation of research that was first presented at the International Conference on Mathematics and its Applications (ICOMATHAPP) and focuses on three types of obstacles: ontogeny, epistemology, and didactics with different levels of difficulty. Ontogeny and didactic obstacles were shown to be present in subjects with medium and low abilities in this investigation. Epistemological obstacles were discovered in persons with high ability. Ontogeny obstacles take the shape of students' conceptual ignorance and their struggles with the representation of both symbolic and visual representations. This is demonstrated by the subject's inability to draw rectangular chords, lack of conceptual comprehension, errors in multiplying angles, and failure to depict incorrect angles. The epistemological obstacles challenges are manifested in the students' failure to comprehend geometric ideas. This is evident from the students who were able to draw quadrilaterals accurately but who missed in calculating the angles and demonstrating angular congruences. Learning approaches and tactics are what obstruct didactic process. It is challenging to express the didactic material visually.

Several studies have discussed obstacles (Fauzi, 2020); (Kurniawan et al., 2017). In this study, we observed three types of obstacles, namely ontogeny, epistemology, and didactics, which were affected by internal and external factors. Research on the semantics of (Chiu et al., 2014) focused on extracting mathematical relationships of explicit problems in arithmetic and geometry problems. Additionally, in this study, semantics are portrayed through four components: parsing and annotation, entity identification, relation extraction, and equation instantiation. The results of those previous studies provided three essential contributions to semantics, namely (1) a new approach to understanding problems that extract equivalent relations representing problems in finding solutions, (2) a semantic syntax model, and (3) a proposal for extracting explicit relations from explicit word problems of arithmetic and geometry problems. Research on representation (Ernaningsih & Wicasari, 2017) reported three types of representations, namely icon representation, verbal representation, and visual representation. And research on the cognitive obstacles (Murniasih et al., 2020) verbal representations, a tendency to generalize, a tendency to rely on intuition, strategy, and less meaningful learning all of which defined as obstacles. However, no research is investigating semantic math representation obstacles in solving geometry problems. Our preliminary study showed that mathematical semantic representation obstacles in solving geometry problems. Based on the previous review and preliminary study, this research was conducted to discover the barriers to mathematical semantic representation in solving mathematical geometry. The results of this study are expected to provide awareness for the lecturers that students may obtain different understandings of the materials delivered by the lecturers. Therefore, it is important to trace back the causes of mathematical semantic representation barriers among university students. At this point, this study's results can provide input for lecturers and students in developing mathematics learning models concerned with the barriers to the mathematical semantic representation of the future success of mathematics learning.

Theoretical Review

Obstacles

Obtaining new knowledge is the essence of the learning process is all about. When learning anything new, a person frequently runs across obstacles or problems. Obstacle according to the cambride dictionary has the meaning of everything that hinders or hinders us against the act of progress. Meanwhile, according to (Moru, 2017), an obstacle is something that prevents individuals from achieving a goal. (Mulyadi, 2010) states that the learning process experiences difficulties, which is a certain condition characterized by obstacles in activities to achieve goals, so that it requires more effort to overcome them. These obstacles may be realized and may not be realized by the person experiencing them.

In the learning process, students often experience obstacles in learning, which are called learning barriers. Learning barriers greatly affect the learning process. In practice, the analysis of learning barriers is included in the series of analyzing the intellectual framework of didactic research and learning trajectories and the relationship gap between educators and learners (learning gap) (Yusuf et al., 2017).

The four types of obstacles and learning constraints identified by Cornu (1991) are cognitive, ontogenic, didactic, and epistemological obstacles. According to Cornu (1991), ontogenic obstacles happen in accordance with the child's mental development stages, didactic obstacles happen as a result of the teacher's less meaningful instruction, and epistemological obstacles are brought on by difficult math concepts. Cognitive obstacles happen when a difficulty arises in the learning process. Brousseau (2002) questions Cornu's descriptions of obstacles. According to him, students, teachers, and the knowledge system interact in a complicated way for the purpose of acquiring new knowledge (Brousseau, 2002). It is challenging to categorize obstacles to learning into different sorts using this paradigm. Ontogenetic growth, an impossible lesson, and teachers who conduct less effective instruction can all be obstacles. Cognitive obstacles can be ontogenic, didactic, or epistemological, according to (Brousseau, 2002) (Table 1).

Representation

Mathematical ideas are communicated through various forms, such as words, pictures, symbols, objects, or actions. The diversity of conveying mathematical ideas is not only related to communication skills but also representation skills. Representation is needed by students because it has

Fig 1: The difference in obstacles classification

an important role in learning during lectures. According to (Goldin & Kaput, 1996) that representation refers to an arrangement of characters, images, or concrete objects that symbolize abstract ideas and may include manipulative materials (physical objects), drawings or diagrams, real-life situations, spoken language, or written symbols. In general, Goldin & Shteingold (2001) mentioned that representation can be interpreted as a configuration (form) that can describe or represent something in another form. In line with the opinion of (Hwang, 2007) explained that mathematical representation is the process of modeling something from the real world into abstract concepts and symbols.

Representation is defined in this study as a manifestation or way of expressing mathematical ideas visually, verbally, or symbolically (Table 1).

Table 1: Types of Mathematical Representations

Visual Representation	*Verbal Representation*	*Symbolic Representation*
A manifestation or way of expressing mathematical ideas using images including: diagrams, lines, numbers, graphs, schemes and other images.	A manifestation or way of expressing mathematical ideas using spoken language or written text.	A manifestation or way of expressing mathematical ideas using formal mathematical language, including: numbers or numeric, variables, or other symbols.

Semantic

In mathematics learning, semantics has an important role in the problem solving process. The problem presented must be translated into another form of representation known by the problem solver while maintaining its meaning (Rahmawati et al., 2021). The goal is to help problem solvers find the final solution to the problem presented. Mathematical problems can be presented with different structures but have the same meaning.

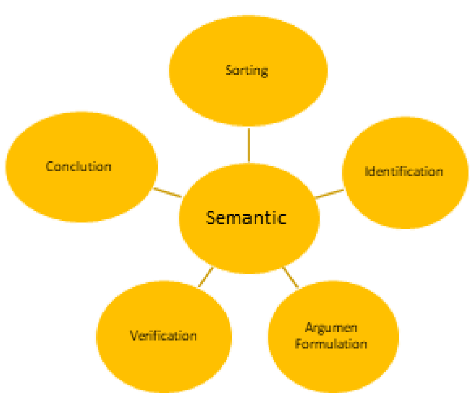


Fig 2. Semantic indicators

Therefore, it is important for problem solvers to consider the particular form of representation that helps them in solving the problem. The process of translating representations is known as the semantic process (Alcock & Inglis, 2009) In problem solving, the semantic process cannot stand alone as it also involves the semantic structure that the problem solver has (Figure 2).

Method

The research conducted aims to study and describe the obstacles to represent students' mathematical semantics in solving geometric problems. To get an overview of the process of representing students' mathematical semantic representation obstacles in solving problems, the researcher tried to do an in-depth analysis. Researchers tell students about what they think, do, write, draw, express and body gestures when solving problems. Preservice mathematics teachers' obstacles related were analysed based on the accomplishment of a written test and an interview.

Participants

This research was administered bachelor degree 2nd semester at the Mathematics Education Study Program, IKIP Budi Utomo Malang, class of 2021, involving students who were taking Euclid Geometry courses. The research subjects were 65 students, consisting of 20 male students and 45 female students. Subjects with different obstacles are called subject ontogeny (SO), subject epistemology (SE), and subject didactic (SD). The geometry problem-solving test was designed and developed, as well as validated by two expert validators. The selected subjects were facing the existing types of obstacles and the new types as well. The selection criteria included students who had (1) ontogeny, epistemology, and didactic obstacles; and (2) the ability to communicate their opinions or ideas both verbally and in written forms. To determine this, we asked for advice from the lecturer who taught the class.

The selection of each student with the aforementioned criteria was because we aimed to see the obstacles that emerged in solving the problem. Thus, in this study, the selection was intended only for data comparison and triangulation because we assumed that there was no gender role in the students' obstacles in mathematical semantic representation, specifically in solving problems.

Suppose there are similarities between male and female students in the obstacles of semantic representation of mathematics in solving problems. In that case, the obstacles of semantic representation of mathematics in solving problems show differences if seen from gender. It can be used as a reference or improvement for further research to consider gender in selecting research subjects.

Geometry Problem Solving Test

Inside a circle, there is a quadrilateral. **PQRS** is a cyclic quadrilateral and **T** is the intersection point of its diagonals. Prove that $PT \times TR = ST \times TQ$!

Findings

Based on the obstacles students faced from the analysis of problem-solving tasks, it referred to the indicators of obstacles in students' semantic representation of mathematics in solving geometry problems. The analysis was conducted after the interview process had been done. The indicators of obstacles in the semantic representation of mathematics are presented in Table 2.

TQ on the geometry problem-solving test. Then, their answers showed barriers to the semantic representation of mathematics. The barriers found are ontogeny, epistemology, and didactic barriers. Thus, in this study, we focused on three

Table 2: The Indicators of Obstacles in Semantic Representation of Mathematics

No	*Types of Mathematical Representations*	*Mathematical Semantics*	*Indicators of Mathematical Semantic Representation*
1.	Visual	Sorting	Representing data and sorting out information from a problem into figures, diagrams, graphs, or tables from the problem text. Interpreting the situation of the problem in their language into figures, diagrams,-graphs, or tables. Providing notes or verbal expressions that support the problem situation into figures, diagrams, graphs, or tables.
		Identification	Determining the keywords of the problem in the form of figures, diagrams, graphs, or tables. Interpreting each keyword determined from the problem into figures, diagrams, graphs, or tables.
		Arguments Formulation	Determining the unknown variable in the figure, diagram, graph, or table. Make a representation of an unknown variable in a figure, diagram, graph, or table. Constructing relationships between figures, diagrams, graphs, or tables. Interpreting the relationship between figures, diagrams, graphs, or tables.
		Verification	Providing proof of the calculation process in figures, diagrams, graphs, or tables. Providing an explanation of each calculation process in the figure, diagram, graph, or table. Interpreting the results of calculations made in figures, diagrams, graphs, or tables.
		Conclusion	Providing the final answer according to the context of the problem with figures, diagrams, graphs, or tables. Interpreting the final answer given with pictures, diagrams, graphs, or tables.
2.	Symbolic	Sorting	Creating and sorting out equations or mathematical models from the given problem. Solving and annotating problems using mathematical expressions.
		Identification	Determining keywords in the mathematical equation or model. Interpreting each keyword determined from the problem in the mathematical equation or model.
		Arguments Formulation	Defining the unknown variable. Representing the unknown variable. Building relationships between mathematical equations or models. Interpreting relations between mathematical equations or models.
		Verification	Providing proof of the calculation process between mathematical equations or models. Providing an explanation of each calculation process in a mathematical equation or model. Interpreting the results of calculations performed on mathematical equations or models.
		Conclusion	Providing the final answer according to the context of the problem in the mathematical equation or model. Interpreting the final answer given in the mathematical equation or model.

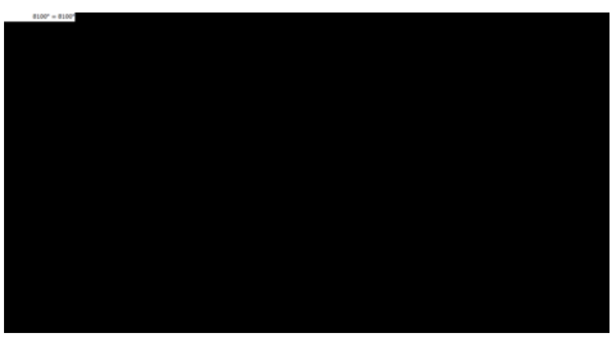
No	*Types of Mathematical Representations*	*Mathematical Semantics*	*Indicators of Mathematical Semantic Representation*
3.	Verbal	Sorting	Composing and sorting stories that match the representations presented. Answering and notetaking problems in written words or texts.
		Identification	Determining and constructing a story that corresponds to the representation presented. Interpreting each defined keyword and creating a story that corresponds to the presented representation.
		Arguments Formulation	Determining the unknown variable with a story that matches the presented representation. Make a representation of the unknown variable with the story that corresponds to the presented representation. Building relationships between stories that match the presented representations. Interpreting relations between stories that correspond to the presented representations.
		Verification	Providing proof of the calculation process between stories in accordance with the presented representation. Providing an explanation of each calculation process in the story in accordance with the presented representation. Interpreting the results of calculations performed on the story in accordance with the presented representation.
		Conclusion	Providing the final answer according to the context of the problem in the story in accordance with the presented representation. Interpreting the final answer given to the story in accordance with the presented representation.

subjects, namely ontogeny barriers (SO), didactic barriers (SD), and epistemological barriers (SE). From students' answers, we focused on three students who showed various kinds of mathematical semantic representation obstacles.

The following is a description of the obstacles to students' semantic representation of mathematics in solving geometry problems:

a. ***Sorting***

Figure 2 mentioned T as the intersection point and a quadrilateral PQRS. Students were able to draw a quadrilateral PQRS, but it was not formed from the diagonals of the chords. It can be seen from students' answers that students were confused and did not understand the questions. According to the results of the interview with the students, their focus is on the angles inside the circle, even though the drawing is wrong.



Translated Version:

$So, the\ size\ of\ each\ corner\ \angle P = 90°, \angle Q = 90°, \angle R = 90°\ and\ \angle S = 90°$

$thus, PT\ x\ TR = ST\ x\ TQ$

$90°\ x\ 90° = 90°\ x\ 90°$

$8100° = 8100°$

Fig 3. Answer From Subject With Ontogeny Obstacle (SO)

The followings are the interview excerpts of Subject 1.

Mathematical Semantics		*Interview Excerpt*	*Types of Mathematical Representation*
Sorting	*P*	: After reading the problem, what do you understand from the problem presented?	*Symbolic* *Verbal*
	SO	*: After I read the question, there is something that I got, which is the quadrilateral of the chords inside the circle with T as the center point.*	
	P	: Which information in the problem led you to this understanding?	*Symbolic* *Verbal*
	SO	*: PQRS is a quadrilateral, and T is the center point.*	
	P	: What are the next things you think or will do after reading the problem?	*Visual* *Symbolic* *Verbal*
	S	*: What I think first, I will draw the circle, then I will draw the quadrilateral, and after that, I will find the angles.*	

Based on the interview results at the sorting stage, Subject 1 completed the sorting stage by restating the data and sorting information from a statement into a picture (P1) and informing which ones were understood (P2). In addition, Subject 1 thought or would do after reading the problem (P3). Thus, subject 1 has symbolic and verbal representation types (SO1 and SO2); and visual, symbolic, and verbal representation types (SO3).

b. Identification

Mathematical Semantics		*Interview Excerpt*	*Types of Mathematical Representation*
Identification	*P*	: Is there any keyword that you determined from the problem?	*Symbolic* *Verbal*
	SO	*: Yes, ma'am, the quadrilateral of PQRS and T is the intersection point.*	
	P	: Why did you choose those keywords?	*Symbolic* *Verbal*
	SO	*: Because the information is very clear from the question*	
	P	: What does the keyword mean?	*Symbolic* *Verbal*
	S	*: It means that I have to draw the quadrilateral first, with T as the center point, to know the angles.*	

The picture above shows that students understand the information and orders given by the researcher. Students understand that the picture is part of the whole and can be partitioned into 5 parts. Next, students redraw the whole circle to determine the number of partitions of the whole circle and the 3/8 part in question. Concepts constructed by students

a. Argument Formulation

Mathematical Semantics			*Interview Excerpt*	*Types of Mathematical Representation*
Argument Formulation	*P*	*:* What will you do to prove the statement?		*Symbolic* *Verbal*
	SE	*: seen from the definition of angles in triangles one by one, then prove its congruence.*		

Mathematical Semantics			*Interview Excerpt*	*Types of Mathematical Representation*
	P	*:* Why did you do that?		*Symbolic Verbal*
	SE	*: Because I'm going to prove the congruent sides.*		
	P	*:* Can you explain the steps for the proof?		*Symbolic Verbal*
	SE	*: Starting from proving the angles of a right triangle, then proving its properties, and then followed by the congruent sides.*		
	P	*:* Which information did you use to do this?		*Symbolic Verbal*
	SE	*: There is a right angle (perpendicular).*		

According to the interview results at the argument formulation stage, subject 2 proved the statement in their answer (P1) and informed the reason for making the statement (P2) and the information used to answer the question. Therefore, subject 2 has symbolic and verbal representation types (SE1, SE2, and SE3)

d. Verification

Mathematical Semantics		*Interview Excerpt*	*Types of Mathematical Representation*
Verification	*P*	:How did you get the new conclusion/statement from the given statement?	*Symbolic Verbal*
	SO	*: The conclusion I made was simply by looking at the picture and the angles, ma'am.*	
	P	*:* Is there any mathematical concept you used to support your conclusion or statement? Mention it!	*Symbolic Verbal*
	SO	*: The concept I know is that if it is perpendicular, then the angle must be .*	
	P	*:* Describe the process of algebraic operations that you performed!	*Symbolic Verbal*
	S	*: PT x TR = ST x TQ*	

Based on the result of the interview at the verification stage, subject 1 determined new conclusions or statements from the statements given (P1) and informed the mathematical concepts used to support the conclusions or statements (P2). In addition, explaining the process of algebraic operations performed (P3). Thus, subject 1 has symbolic and verbal representation types (SO1, SO2, and S3).

e. Conclusion

Mathematical Semantics		*Interview Excerpt*	*Types of Mathematical Representation*
Verification	P	: What conclusions can you make to answer the problem presented?	Symbolic Verbal
	SO	: That the quadrilateral of the chords has the same angle.	
	P	: Are you sure if the answer is correct?	Symbolic Verbal

Mathematical Semantics		*Interview Excerpt*	*Types of Mathematical Representation*
	SO	: Sure, ma'am, seen from the angle 90	
	P	: Does your answer reflect the question of the problem?	Symbolic Verbal
	SO	: Yes	
	P	: What is the meaning of your conclusion?	Symbolic Verbal
	SO	: Meaning that all the angles are 90 so PT x TR = ST x TQ	

From the interview results at the conclusion stage, Subject 1 determined the conclusions drawn to answer the problem presented (P1), and the subject felt sure if the answer was correct (P2). As well as explaining whether the answer reflects the problem question presented (P3). Moreover, it explains the meaning of the conclusion that has been made (P4). Consequently, subject 1 has symbolic and verbal representation types (SO1, SO2, and S3).

Translated Version:

Explanation

- PQRS is a chords quadrilateral
- $\angle PQR, \angle QRS, \angle PSR, and\ \angle SPQ\ is\ 90^{\circ}$
- $\angle PTS \cong \angle QTR\ or\ \angle PQT \cong \angle SRT$
- $\angle PQR + \angle PSR = 180°$
- $\angle PTS\ and\ \angle RTS\ are\ right\ angles$
- $\angle PTQ\ and\ \angle RTQ\ are\ right\ angles$
- $\overline{PT} \cong \overline{RT}$ or $\overline{ST} \cong \overline{TQ}$
- PT × TR = ST × TQ

Reason

Known:

- Definition of right angle
- Definition of congruent angles
- The sum property of two opposite angles
- The definition of two straight lines
- The definition of two straight lines
- The definition of center point
- Proven

Fig. 4: Answer From Subject With Epistemological Obstacle (SE)

Epistemological obstacles regarding students' inability to understand geometric concepts, namely PTQ and STR, will be congruent. Students must draw a rectangle correctly first to prove that PTQ and STR are congruent. They have never experienced this geometry problem before, so this experience leads to obstacles for students in solving the problem. Then, based on the results of interviews conducted by students to prove that PTQ and STR are congruent, they made mistakes in determining the angles.

The followings are the interview excerpts of Subject 2.

a. Sorting

Mathematical Semantics			Interview Excerpt	Types of Mathematical Representation
Sorting	P	: After reading the problem, what do you understand from the problem presented?		Visual
	SD	: I have no idea on how to draw quadrilateral chords.		
	P	: Which information in the problem led you to this understanding?		Visual
	SD	: PQRS quadrilateral and T intersection point.		
	P	: What are the next things you will do or think about after reading the problem?		Visual
	SD	: I still do not understand what to do. I am still confused.	s	

Based on the interview results at the sorting stage, subject 3 completed the sorting stage because they represented the data, sorted the information from a statement into picture form (P1), and informed which ones were understood (P2). In addition, he/she thought or would do it after reading the problem (P3). Therefore, Subject 3 has a visual representation type (SO1, SO2, and SO3).

b. Identification

Mathematical Semantics		*Interview Excerpt*	*Types of Mathematical Representation*
Identification	P	: Is there any keyword that you determined from the problem??	Simbolik Verbal
	SE	: There is a quadrilateral chords PQRS and T intersection point	
	P	: Why did you choose those keywords?	Simbolik Verbal
	SE	: Because the information is very clear from the question	
	P	: What is the meaning of those keywords?	Simbolik Verbal
	SE	: The meaning is that it can be proven to be congruent later because it intersects.	

Based on the interview results at the identification stage, subject 3 determined the keywords obtained in the problem (P1) and informed the meaning of the keywords (P2). Therefore, it implies that Subject 3 has a visual representation type (SO1).

c. Argument Formulation

Mathematical Semantics			*Interview Excerpt*	*Types of Mathematical Representation*
Argument Formulation	P	: What will you do to prove the statement?		Symbolic Verbal
	SE	: seen from the definition of angles in triangles one by one, then prove its congruence.		
	P	: Why did you do that?		Symbolic Verbal
	SE	: Because I'm going to prove the congruent sides.		
	P	: Can you explain the steps for the proof?		Symbolic Verbal

Mathematical Semantics		*Interview Excerpt*	*Types of Mathematical Representation*
	SE	: Starting from proving the angles of a right triangle, then proving its properties, and then followed by the congruent sides.	
	P	: Which information did you use to do this?	Symbolic Verbal
	SE	: There is a right angle (perpendicular).	

According to the interview results at the argument formulation stage, subject 2 proved the statement in their answer (P1) and informed the reason for making the statement (P2) and the information used to answer the question. Therefore, subject 2 has symbolic and verbal representation types (SE1, SE2, and SE3).

d. Verification

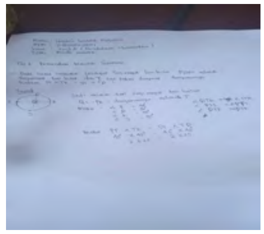
Mathematical Semantics		*Interview Excerpt*	*Types of Mathematical Representation*
Verification	P	: How did you get the new conclusion/statement from the given statement?	Symbolic Verbal
	SE	: Starting from observing the angle then proving congruent and proving the sides	
	P	: Is there any mathematical concept that you used to support the conclusion or statement? Mention it!	Symbolic Verbal
	SE	: Using the concept of right triangle	
	P	: Describe the process of algebraic operations that you performed!	Symbolic Verbal
	SE	: PT x TR = ST x TQ	

From the results of the interview at the verification stage, subject 2 determined the conclusion or new statement from the given statement (P1) and informed the mathematical concepts used to support the conclusion or statement (P2). As well as explaining the process of algebraic operations performed (P3). Thus, subject 2 has symbolic and verbal representation types (SE1, SE2, and SE3).

e. Conclussion

Mathematical Semantics		*Interview Excerpt*	*Types of Mathematical Representation*
Verifikasi	P	: What conclusions did you draw to answer the problem presented?	Symbolic Verbal
	SE	: The conclusion that quadrilateral chords has the exactly the same angles,	
	P	: Are you sure that your answer is correct?	Symbolic Verbal
	SE	: Because the angles facing each other in 90.	
	P	: Does your answer reflect the question from the problem?	Symbolic Verbal
	SE	: Yes	
	P	: What is the meaning of your conclusion?	Symbolic Verbal
	SE	: Proven that PT x TR = ST x TQ	

The interview results at the conclusion stage with subject 2 showed the conclusions drawn to answer the problem presented (P1), and the subject felt confident with the answer (P2). It also explains whether the answer reflects the problem question (P3). Lastly, it also explains the meaning of the conclusion that has been made (P4). Thus, subject 2 has symbolic and verbal representation types (SE1, SE2, and SE3).



Translated Version:
Answer

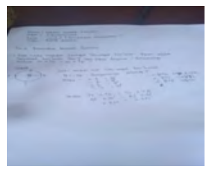


Fig 5: Answer From Subject Didactic Obstacle (SD)

The followings are the interview excerpts of Subject 3.

a. Sorting

Mathematical Semantics		*Interview Excerpt*	*Types of Mathematical Representation*
Sorting	P	: After reading the problem, what do you understand from the problem presented?	Visual
	SD	: I have no idea on how to draw quadrilateral chords.	
	P	: Which information in the problem led you to this understanding?	Visual
	SD	: PQRS quadrilateral and T intersection point.	
	P	: What are the next things you will do or think about after reading the problem?	Visual
	SD	: I still do not understand what to do. I am still confused.	

Based on the interview results at the sorting stage, subject 3 completed the sorting stage because they represented the data, sorted the information from a statement into picture form (P1), and informed which ones were understood (P2). In addition, he/she thought or would do it after reading the problem (P3). Therefore, Subject 3 has a visual representation type (SO1, SO2, and SO3).

b. Identification

Mathematical Semantics		*Interview Excerpt*	*Types of Mathematical Representation*
Identification	P	: Is there any keyword that you determined from the problem??	Simbolik Verbal

Mathematical Semantics		*Interview Excerpt*	*Types of Mathematical Representation*
	SE	: There is a quadrilateral chords PQRS and T intersection point	
	P	: Why did you choose those keywords?	Simbolik Verbal
	SE	: Because the information is very clear from the question	
	P	: What is the meaning of those keywords?	Simbolik Verbal
	SE	: The meaning is that it can be proven to be congruent later because it intersects.	

Based on the interview results at the identification stage, subject 3 determined the keywords obtained in the problem (P1) and informed the meaning of the keywords (P2). Therefore, it implies that Subject 3 has a visual representation type (SO1).

c. Argument Formulation

Mathematical Semantics		*Interview Excerpt*	*Types of Mathematical Representation*
Argument Formulation	P	: What will you do to prove the statement?	Symbolic Verbal
	SE	: seen from the definition of angles in triangles one by one, then prove its congruence.	
	P	: Why did you do that?	Symbolic Verbal
	SE	: Because I'm going to prove the congruent sides.	
	P	: Can you explain the steps for the proof?	Symbolic Verbal

According to the interview results at the argument formulation stage, subject 2 proved the statement in their answer (P1) and informed the reason for making the statement (P2) and the information used to answer the question. Therefore, subject 2 has symbolic and verbal representation types (SE1, SE2, and SE3

d. Verification

Mathematical Semantics		*Interview Excerpt*	*Types of Mathematical Representation*
Verification	P	: How did you get the new conclusion/statement from the given statement?	Symbolic Verbal
	SE	: Starting from observing the angle then proving congruent and proving the sides	
	P	: Is there any mathematical concept that you used to support the conclusion or statement? Mention it!	Symbolic Verbal
	SE	: Using the concept of right triangle	
	P	: Describe the process of algebraic operations that you performed!	Symbolic Verbal
	SE	: PT x TR = ST x TQ	

From the results of the interview at the verification stage, subject 2 determined the conclusion or new statement from the given statement (P1) and informed the mathematical concepts used to support the conclusion or statement (P2). As well as explaining the process of algebraic operations performed (P3). Thus, subject 2 has symbolic and verbal representation types (SE1, SE2, and SE3).

e. Conclussion

Mathematical Semantics				*Interview Excerpt*	*Types of Mathematical Representation*
Verifikasi	P		: What conclusions did you draw to answer the problem presented?		Symbolic Verbal
	SE		: The conclusion that quadrilateral chords has the exactly the same angles,		
	P		: Are you sure that your answer is correct?		Symbolic Verbal
	SE		: Because the angles facing each other in 90.		
	P		: Does your answer reflect the question from the problem?		Symbolic Verbal
	SE		: Yes		
	P		: What is the meaning of your conclusion?		Symbolic Verbal
	SE		: Proven that PT x TR = ST x TQ		

Based on the interview results at the conclusion stage, subject 3 determined the conclusions drawn to answer the problem presented (P1), and the subject was confident if the answer was correct (P2). The subject also explained the answer to the problem presented (P3) and described the meaning of the conclusion that has been made (P4). Thus, subject 3 has a visual representation type because SO cannot make a conclusion (SO1, SO2, and SO3).

Learning obstacles can be caused by learning strategies and methods. In addition to the presented material, this is also the basis for the emergence of obstacles that students have. Regarding didactic obstacles, it can be seen from students who have difficulty representing an image's shape as a rectangle.

In the books used by students, there are only simple examples of questions, with No. Open-ended questions like the ones used in this study. Our presented question is not only creating didactic obstacles related to mathematical semantic representation in geometric problems but also causing other didactic obstacles, namely students' difficulties in solving geometric problems.

Discussion

In order to solve geometry difficulties, this study tries to describe the barriers that students face when using mathematical semantic representation. The act of thinking entails taking in information both internally and externally, storing it in long-term memory, and retrieving it when necessary. Therefore, the thinking process in this study includes the following steps: (a) giving the problem meaning through information gathering, storage, and expression through representation; (b) arguing through the use of length and width changes, translations, and calculation proof; and (c) drawing conclusions through the use of the provided final solution. The three subjects presented their semantic representations of mathematics, namely visual, symbolic, and verbal representations, based on the outcomes of their performance and the description above. The college pupils demonstrated ontogenic, epistemology, and didactic obstacles.

The reason for this is that by understanding the problem situation and giving meaning to the realized problem students' understanding of problems is strongly correlated with their background knowledge and experience (Lodge, 2018). Similar to the first discovery, Results from previous studies presented at the presented at the International Conference on Mathematics and its Applications (ICOMATHAPP) found subjects with medium and low abilities had ontogeny and didactic obstacles. Epistemological obstacles, meantime, were discovered in high-ability subjects. Due to ontogeny obstacles, students struggle to comprehend topics when presented with both symbolic and visual representations. By multiplying the angles and displaying the incorrect angles, the student failed to draw a rectangular chord and couldn't understand the notion. The epistemological challenge is that students can draw chord quadrilaterals accurately yet make errors while establishing congruences and finding angles. Hurdles to didactic learning, where subjects have trouble communicating their teacher-teaching experiences using visuals.

The second finding revealed that the subjects represented the issue symbolically in order to give it meaning. The ability to translate mathematical issues into representations of arithmetic formulas is what distinguishes symbolic representation skills (Gagatsis & Shiakalli, 2004; Goldin, 2002; Lesh, Posting, & Behr, 1987; Supandi et al., 2018). This symbol is used by the subject to solve problems effectively because doing so is made easier by its use. Schoenfeld's hypothesis, which contends that representation is a tool for comprehending information from the topic at hand, is supported by the study's findings (Stylianou, 2010) The pupils' representations are undoubtedly influenced by the issue solver's expertise and experience. The basis for selecting the best problem-solving approach is the breadth of the students' knowledge and experience. The students' depth of knowledge and experience is the foundation for determining the appropriate problem-solving strategy (Clement, 2008; Hegarty et al., 1995). Besides, connecting concepts, rules, and principles is essential in finding solutions to an unstructured problem (Hong, 1998; Jonassen, 1997; Shin et al., 2003).

One subject answered the problem but could not draw the quadrilateral chords in solving unstructured problems. It is not essential to start from a certain point because individuals solving the problem are given the freedom to start (Jonassen, 1997; Shin et al., 2003). Students are free to express their opinions with their existing knowledge to solve the given problem. Each subject came up with ideas by adding information to help them solve the problem, as reported in the research of Abdillah et al. (2016) that giving initials at length and width.

Conclusion

The obstacles students face from the analysis of problem-solving tasks refer to the indicators of obstacles in students' semantic representation of mathematics in solving geometry problems. Barriers that are found are ontogeny, epistemology, and didactic barriers. Further, as mentioned previously, there are three types of obstacles found in this study, namely ontogeny obstacles (SO), didactic obstacles (SD), and epistemological obstacles (SE). The three students with those barriers showed various mathematical semantic representation obstacles that emerged in this study.

In the subject with SO, who has ontogeny obstacles, the types of representations that appear are visual, symbolic, and verbal representations. While at the identification stage, SO has symbolic and verbal representation types. At the argument formulation stage, SO has symbolic and verbal representation types. It is the same as in the verification stage which SO has symbolic and verbal representation types. At the conclusion stage, SO has symbolic and verbal representation types. Meanwhile, SE subject with epistemology obstacles has symbolic and verbal representation types at all stages. Lastly, SD subject with didactic barriers has only visual representation type.

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