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Mathematics belief impact on metacognition in solving geometry: Middle school students

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ABSTRACT

Mathematical beliefs and metacognitive knowledge play significant roles in solving mathematical problems; thus, this study aims to investigate the influence of middle school students' beliefs on their metacognitive knowledge when solving geometry problems. This study utilizes both quantitative and qualitative research methods. A linear regression test was used to determine the effect of middle school students' beliefs on their metacognitive knowledge. The results of the quantitative research analysis were followed up with a qualitative research approach to describe the metacognitive knowledge of students who have high and low confidence in solving geometric problems. This research involved 352 middle school students in the Tarakan area. Based on the results of linear regression, it is known that the beliefs of middle school students have a positive effect on their metacognitive knowledge when solving geometric problems. In addition, it was found that students with different beliefs could solve a given geometry problem, but the approach to solving it varied among subjects. Middle school students have diverse beliefs, but these variations do not affect their capacity to apply their metacognitive knowledge at every stage of solving mathematical problems.

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1. INTRODUCTION

One of the main goals in the process of learning mathematics is for students to be able to solve a given problem. However, there are still many students who find it difficult and feel anxious when learning mathematics or solving math problems [1]. Many factors can influence students' ability to solve problems, such as working memory skills, cognitive awareness, beliefs and anxiety about mathematics [2]–[4]. Cognitive awareness and understanding of the problems faced by students are necessary in the process of solving mathematical problems. Through awareness, students can improve their abilities [5]. However, the results of a study conducted by Setyawati and Indrasari [6] show that students have not maximized their awareness when solving mathematical problems even though it is essential. The principles of metacognition involve being aware of knowledge and knowing how to apply it to solve problems. The results of the research by Sutama *et al.* [7] shows that there are differences in the metacognitive activity of junior high school students with field independent and field dependent cognitive styles in solving mathematical problems. In addition, the results of research conducted by Suliani *et al.* [8] state that both female and male junior high school students were able to utilize their metacognition in solving math problems.

The concept of metacognition was introduced to describe and explain how a person can control his thinking during learning and problem-solving, especially when a person experiences cognitive failure and encounter difficulties in information processing and problem-solving [9]–[11]. The metacognitive aspect is related to students' ability to organize their own thoughts. Lioe *et al*. [12] also stated that metacognition is one of the main components of solving math problems. This emphasizes students' ability to monitor their own thinking. This is in line with the concept of metacognition developed by Flavell [13]. Therefore, students who excel at problem-solving always monitor their thinking process and evaluate the results they achieve. These students know when to employ an effective strategy and when to change this strategy to make a decision that aligns with a certain goal. There are three categories of metacognition: metacognitive knowledge, metacognitive skills and metacognitive experiences [13], [14].

In this study, the researchers focused solely on aspects of metacognitive knowledge. This is based on the idea that students who utilize their metacognition can understand how to solve tasks or problems. Metacognitive knowledge can influence someone in solving mathematical problems, particularly those related to geometry. According to Zulyanty [15] metacognitive knowledge helps individuals recognize the truth and identify mistakes made when solving problems. In addition, it can help identify where the error lies in problem-solving [13]. Based on this, metacognitive knowledge is an understanding of the process of thinking about what to think about, how, and when to approach certain tasks.

Metacognitive knowledge serves as the foundation for utilizing cognitive and metacognitive strategies in problem-solving process. It is developed through metacognitive skills, which involve managing cognitive processes to achieve cognitive goals during problem-solving. Additionally, emotions play a role in differentiating between metacognitive knowledge and metacognitive experiences. The presence of feeling implies that the metacognitive experience involves a sense that the current subjective experience is a result of the cognitive activity that occurred during the cognitive process. Metacognitive knowledge involves three components: the learning process and one's beliefs about how they learn and others learn; learning tasks and how to process information effectively; and determining which strategies to use and when to utilize it [13].

Apart from the cognitive aspect, students solving math problems must also pay attention to the affective aspects, namely aspects that influence students' tendencies to solve problems [10], [11]. One of the affective aspects that students must possess when learning mathematics is a belief in the subject and problem-solving. The results of research conducted by Ozturk and Guven [16] concluded that beliefs not only affect the process of problem-solving but also influence personal factors such as life experiences.

Solving problems related to affective aspects (beliefs) leads to the conclusion that students who struggle to solve problem often feel frustrated. The results of research conducted by McLeod [17] show that students who are unable to solve problems often panic. The same thing was done by Schoenfeld [18] who showed that there was a strong relationship between the mathematics test results expected by students and students' beliefs related to their abilities. Furthermore, belief can be divided into two categories: belief in mathematics [19] and belief in solving mathematical problems [20]. Furthermore, Ozturk and Guven [16] classify beliefs into two categories: high beliefs and low beliefs.

Geometry itself is a challenging subject for learners, particularly when it is being learned remotely. In addition to space and form, geometry encompasses the concepts of distance, scale and relative position of figures. Moreover, numerous occupations, such as architects, mechanical engineers, technicians and draughts men utilize geometry. Due to that, geometry is an essential branch of mathematics. The majority of learners find geometry difficult to study and have no desire to do so. This is due to the fact that learners frequently feel unsure of themselves about what they have learned, experience anxiety when studying it and are unable to use geometric theory to solve their problems [21].

So far, experts have conducted research in order to provide solutions to problems related to metacognitive activity. For example, the role of metacognition is used to explain the relationship between initial difficulties, students' understanding of reading, and the process of conjecturing [22], [23]. However, research related to the relationship between metacognitive knowledge and junior high school students' beliefs in solving geometric problems is still very limited in general [24], [25]. Even though there is a very important essence when it involves students' beliefs in solving geometric problems to identify the metacognitive knowledge of junior high school students, it is important to carry out further research, namely to investigate and explore the metacognitive knowledge of junior high school students who have high and low belief in solving geometric problems.

2. METHOD

Explaining this study aims to provide a comprehensive understanding of the metacognitive knowledge of students who have high and low confidence in solving geometric problems, as well as their relationships. The research approaches used are descriptive quantitative and qualitative. The design refers to the collection, analysis, and integration of both qualitative and quantitative data at multiple stages of a research [26]. The quantitative approach was intended to examine the correlation between metacognition and students' beliefs in solving mathematical problems using the t-test statistical test. On the other hand, the descriptive qualitative approach aimed to explore and investigate further regarding the metacognitive knowledge of research subjects while solving geometric problems.

The hypothesis formulated in this study was that there is a functional relationship between metacognitive knowledge and junior high school students' beliefs about solving geometric problems. The population in this study included all students of a state middle school in Tarakan. The sample for this study included 352 students of the middle school who were selected using a simple random sampling technique that considered the homogeneity of the population. This is in line with Roscoe [27] who stated that an appropriate sample size in research is between 30 and 500. The time for conducting research was in the even semester of the 2021-2022 academic year.

Data regarding the students' beliefs was collected using the Indiana mathematics belief (IMB) scale questionnaire instrument, and the metacognitive knowledge questionnaire was used to collect the students' metacognitive knowledge. After that, the correlation between metacognitive knowledge and students' beliefs in solving mathematical problems was analyzed. Math tests were administered to assess the subjects' mathematical ability. In the interview stage, a geometry problem solving assignment was given to the selected subjects to evaluate their metacognitive knowledge. Furthermore, the IMB scale questionnaire consisted of 30 statement items [20] the metacognitive knowledge questionnaire consisted of 14 statement items [14], [28] related to middle school students' metacognitive abilities in solving mathematical problems, they were found to be valid and reliable ($\text{Cronbach's Alpha} = 0.934$); the mathematics test consisted of 5 questions covering various subjects that students have studied. The problem to be solved by the selected subjects for interviews was as follows: it is known that Firman will make a square photo frame with an outer diagonal of the frame measuring $80\sqrt{2}$ cm. Subjects were asked to calculate the total length of wood that Firman would use and the minimum cost that Firman would incur if the price per meter of wood was IDR 40,000.

Data analysis techniques in this study used descriptive statistics, data reduction, data presentation, triangulation, analysis and conclusion. To test the correlation between metacognition and students' beliefs in solving mathematical problems, the study utilized the statistical t-test and to analyze the subject's metacognitive knowledge in solving geometric problems, Polya stages [29] consisting of understanding the problem, making plans, carrying out problem solving and evaluating for each stage, were utilized. The subject's metacognitive knowledge was analyzed by looking at how the subject carried out metacognitive knowledge activities that involve his knowledge of strategies which affect the direction and results of his cognitive endeavors.

3. RESULTS AND DISCUSSION

A total of 352 middle school students participated in this study. All respondents completed a series of tests, namely basic mathematics tests and then a questionnaire probing their mathematical belief in solving geometric problems. After that, the respondents were asked to complete a metacognition knowledge questionnaire. Respondents who took part in the study consisted of 167 male respondents and 185 female respondents, with an age range of 12 to 14 years. Furthermore, 145 respondents had high belief in solving geometric problems, consisting of 60 male respondents and 85 female respondents and 207 respondents had low belief in solving geometric problems, consisting of 107 male respondents and 100 female respondents. Description of the test results is shown in Table 1.

Table 1. High/low belief and metacognitive knowledge

	High/low beliefs	Metacognitive knowledge
Mean	86.91	42.25
Standard deviation	7.23	7.36
Min	61.40	19.71
Max	106.17	62.35

From the Table 1, it can be seen that the average student's belief is 86.91 and the average metacognitive knowledge is 42.25. This shows that students have a low level of mathematical belief in solving geometric problems, which has an impact on their metacognitive knowledge. The researchers used the SPSS application to test the normality of metacognitive knowledge data and students' beliefs about solving geometric problems. The data used were scores of students' beliefs in solving math problems and scores of students' metacognitive knowledge. Residual regression must follow a normal distribution and this condition can be achieved by using the normal predicted probability (PP) plot, as shown in Figure 1.

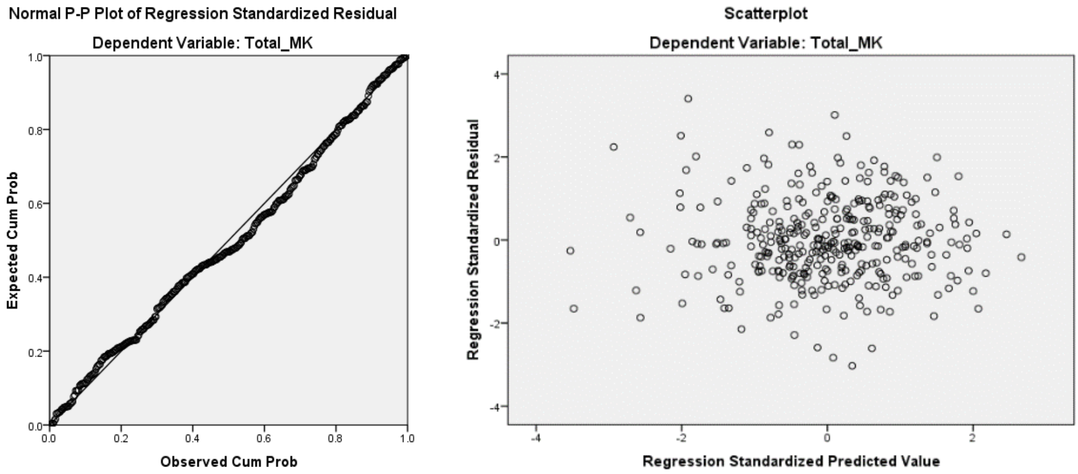

Figure 1. Normal prediction probability plot (left) and scatterplot of residuals (right)

Figure 1 (left), shows a plot of metacognitive knowledge score points that correspond to the normality diagonal line. This indicates that the normal conditions are met. Furthermore, homoscedasticity refers to whether these residuals are evenly distributed and this condition can be checked by the distribution of the residuals. The scatter plot of the residuals in Figure 1 (right) shows no particular pattern and the points are evenly distributed above and below zero on the X axis. Furthermore, the points are also distributed evenly to the left and right of zero on the Y axis. Therefore, it can be concluded that the homoscedasticity requirements are met. The residuals are normally distributed and homoscedastic, so the student's belief variable in the regression has a linear relationship with the student's metacognitive knowledge variable in solving geometric problems. Output regression analysis of students' metacognition and a belief in solving a geometry problem displayed in Table 2.

The results of the summary results using SPSS from the regression analysis on students' metacognitive beliefs and knowledge in solving geometry problems yield an R-squared value of 0.208 as shown in Table 2, which indicates that the effect of the independent variable (belief) on the dependent variable (metacognitive knowledge) is 20.8%, while other variables explain the rest. The next step is to determine the regression model, test the suitability of the model and investigate the variables that affect metacognitive knowledge. Table 3 and Table 4 show the coefficients and significance values of linear regression analysis.

Table 2. Regression analysis of students' metacognition and belief in solving a geometry problem

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	0.457[a]	0.208	0.206	6.56

Note: a. Predictor: (Constant), Total belief

Table 3. Analysis of variance (ANOVA) from the regression analysis out students' metacognition and belief in solving a geometry problem

Model	Sum of squares	df	Mean square	F	Nilai p sig
Regression	3,965.769	1	3,965.769	92.181	0.000
Residual	15,057.514	350	43.021		
Total	19,023.283	351			

Table 4. The output coefficients of regression analysis out students' metacognition and belief in solving a geometry problem

Model	Unstandardized coefficients B	Unstandardized coefficients Std. Error	Standardized coefficients Beta	t	p sig. value
Constant	1.833	4.224		0.434	0.665
High/low belief	0.465	0.048	0.457	9.601	0.000

Note: Dependent variable; metacognitive knowledge; Tolerance: 1.000; VIF: 1.000

This is shown from the results of the ANOVA table test. It was found that F = 92.181 with p value Sig. = 0.000 < 0.05. Therefore, the overall regression model fits the data. The regression model is: y = 1.833 + 0.465x, where the variable y is defined as the dependent variable, namely students' metacognitive knowledge in solving geometric problems, while the variable x is defined as the independent variable, namely students' beliefs in solving mathematical problems. Thus, middle school students' beliefs affect their metacognitive knowledge when solving geometric problems. This means if there are students who have high confidence in solving geometric problems, it is predicted that these students will have high metacognitive knowledge of solving geometric problems. This is in accordance with the research of Setyawati and Indrasari [30] which shows that students who have more belief in their mathematical abilities use better metacognitive strategies. It can be said that the determinants of students' success in solving mathematical problems do not only depend on their perception of thought processes but also on their beliefs about solving mathematical problems. When students have good beliefs, they can improve their cognitive skills [16], [18].

To find out the effect of mathematical beliefs on metacognitive knowledge, a qualitative study was conducted by giving assignments to 57 respondents. Selected participants consisted of 19 subjects with high confidence in solving geometric problems and 38 subjects with low confidence in solving geometric problems. Based on the results of the answers given by the participants, two students were selected, consisting of one student with high belief and one student with low belief, who were identified through the random sampling technique as research subjects who had unique problem-solving strategies and were able to communicate their ideas when solving geometry problems.

Subjects who have high belief in solving geometry problems with an IMB score of 114 are labeled with high mathematical belief (HMB), and subjects who have low belief in solving geometry problems with an IMB score of 77 are labeled with low mathematical belief (LMB). The two subjects were female students and had relatively balanced mathematical abilities, as shown in the TPMM score; the subject with high belief earned a score of 100, and the subject with low belief earned a score of 95. The researchers also conducted interviews with the two subjects, then presented in full the results of the analysis of knowledge data. metacognitive subject in solving geometry problems.

In general, both subjects can solve geometry problems correctly, but there were differences in how they answer the given geometry problem. Both HMB and LMB utilized metacognitive knowledge when solving a mathematical problem. The two subjects reconsidered their own understanding on the task at hand and decided on an effective strategy for solving the problem. Both subjects were able to assess their respective abilities by mentioning their cognitive weaknesses and strengths when facing a task. Both subjects were also aware of the steps that can be taken when faced with a particular task and can decide which strategy to use. HMB was aware of her own knowledge in understanding the problem, specifically the knowledge of how to select information to find important sentences in the problem. This is in line with the results of a research conducted by Margono *et al.* [31] which stated that subjects who answered consistently at the stage of understanding the problem had knowledge of themselves, knowledge of cognitive tasks and knowledge of strategies.

On the other hand, LMB was aware of her own knowledge when facing certain cognitive tasks by knowing the formula to use. Although LMB realized that understanding the problem was difficult, this realization was further reinforced when LMB successfully solved a mathematical problem. She demonstrated her understanding by knowing the purpose of the given problem. This is in line with the opinion of Tobias and Everson [32], which states that students who have a lower understanding of what they know and do not know may have greater difficulty retrieving previous lessons. In addition, LMB realized that solving a problem required accuracy and the ability to understand the problem.

HMB and LMB systematically completed tasks based on the stages of Polya's problem-solving process. At the stage of understanding the problem, both subjects were aware of their own knowledge regarding cognitive abilities, the tasks that must be carried out, and the cognitive strategies that should be employed to comprehend the problem. The two subjects, in order to understand the problem, first read and identified every word or sentence in the problem, marked it as known, and asked for information in the problem. The two subjects also decided on the formula to be used by utilizing their initial knowledge. This knowledge was obtained from the learning experiences of the two subjects. Over time, it has been proven to change students' beliefs regarding solving problems [33]–[36]. Thus, students can utilize their metacognitive knowledge to solve a mathematical problem.

When understanding a problem, HMB could understand how to select information to find important sentences in the problem. In addition, steps taken to understand the problem were reading, understanding, and remembering the topic being discussed. HMB was also aware of deciding on effective strategies to understand problems based on his learning experience. This finding is in line with the results of research conducted by Riani *et al.* [37] which states that junior high school students can recall whether they have solved problems like this before, think about whether previous knowledge can help solve problems, and relate what is known and asked about problems with previous knowledge. On the other hand, LMB, in understanding the problem, utilized the following steps: to record or mark important things in geometry problems, determine the formulas used, and be aware of strategies that can be used by making sketches of drawings and setting formulas to answer these problems.

When preparing the problem-solving plan, HMB was aware of her own knowledge regarding his ability to identify and select important sentences and determine topics that were appropriate to the problem. In addition, HMB was also aware of her abilities in terms of being able to show the keywords in the questions and knowing the topics being discussed. The steps that HMB took were to re-read the problem, mark important information, write down important information in their own language, determine the formula and determine the most effective strategy to be able to plan problem solving based on their learning experience. Meanwhile, LMB did not make plans to solve geometric problems, so LMB metacognition in preparing problem-solving plans could not be described.

At the stage of applying or carrying out the problem-solving plan, both HMB and LMB knew how to properly carry out the formula that had been established based on their understanding. Furthermore, the steps used by HMB and LMB are carried out in accordance with the important information and questions provided. The two subjects also applied algebraic principles and operations in a systematic manner. This is in line with the results of research conducted by Nicolaou and Philippou [38] which states that students' beliefs can improve their problem-solving and problem-posing skills in just a few weeks.

HMB reconsidered the knowledge required to implement its formulated plans, specifically by understanding how to apply relevant formulas to the topic under discussion. In addition, HMB was also aware of her ability apply the formula based on the topic being discussed. The steps that HMB took were to complete them sequentially according to the questions on the problem and determine the most effective strategy to apply the formula that had been determined based on the learning experience shown in Figure 2. This is in line with the opinion of Alzahrani [39] who stated that metacognitive awareness can improve students' learning process by rethinking the knowledge they have. Mokos and Kafoussi [40] describe a metacognitive activity that is often used in solving open problems as the meta of procedural knowledge.

$$1:1:\sqrt{2}$$
$$1:\sqrt{2} = a:c$$
$$\frac{1}{\sqrt{2}} = \frac{a}{80\sqrt{2}}$$
$$1 \times 80\sqrt{2} = \sqrt{2} \times a$$
$$a = \frac{80\sqrt{2}}{\sqrt{2}}$$
$$a = 80$$

$$a = b$$
$$80 = 80$$
$$\text{length of wood} = 4 \times a = 4 \times 80 = 320$$
$$= 320 \times 0.01 = 3.2 \text{ m}$$
$$\text{charges} = \text{IDR } 40{,}000 \times 3.2 \text{ m} = \text{IDR } 128{,}000$$

Figure 2. Results of HMB (subject with high belief) written work

On the other hand, LMB utilized her metacognitive knowledge by realizing the ability to apply things that are known to the formula to use. This was done with the ability the subject had. Dunlosky and Bjork [41] explained that metacognition is the mind's ability to monitor and regulate itself, or, in other words, the ability be aware of one's own knowledge. Further steps that were taken by LMB were to apply the information contained in the problem to the formula to be used and to be aware of the strategies that can be implemented using the principles and operations of algebraic calculations, as illustrated in the example shown in Figure 3.

Is Know: Diagonal = $80\sqrt{2}$ cm, Price of wood per meter IDR 40,000.

Asked: Total length of timber and minimum costs incurred?
Answer: $b^2 = c^2 - a^2$

$x^2 = (80\sqrt{2})^2 - x^2$

$(x^2 + x^2) = 12,800$

$2x^2 = 12,800$

$x^2 = \frac{12,800}{2} = 6,400$

$x = \sqrt{6,400} = 80$ cm

If the length of one side of the wood is 80cm, then the length of the four sides of the wood is = $80 \times 4 = 320$ cm.
320 cm = …. converted to meters
= 3.2 m
minimum costs incurred:
= 3.2 × Price of wood per meter
= 3.2 × IDR 40,000 = IDR 128,000

Figure 3. Results of LMB (subject with low belief) written work

At the re-examining stage, both subjects recognized the appropriateness of their actions based on their knowledge. The two subjects used different alternative methods to ensure that the calculations carried out were appropriate for the problems they were facing. HMB utilized her metacognitive knowledge when re-examining the solutions obtained, namely by realizing her ability to see the results obtained and knowledge related to the topic being discussed. The steps taken were to adjust the results obtained and if the solutions obtained were uncertain, it would be recalculated in a different way. In line with the findings of a study by Juniati and Budayasa [42] which states that students feel more confident and competent when they can use the theory they have learned effectively.

On the other hand, LMB thought differently in utilizing her metacognitive knowledge, namely by realizing her ability to find out in advance the sides and angles of a square if the diagonal that divides the square was known. The subject's awareness at each stage of problem-solving could increase the subject's ability to solve problems properly and correctly. This is supported by the findings of a study Kozikoğlu [43] which states that metacognition has a close relationship with mathematical problem-solving abilities. In addition, Vissariou and Desli [28] argues that when students involve their metacognition in solving a problem, they are able to represent and solve mathematical problems correctly, evaluate the effectiveness of strategies, and recognize mistakes they have made. In addition, involving metacognitive knowledge in solving problems can increase students' confidence so that they do not feel anxious when facing similar problems at a later time. The following briefly presents the differences in HMB and LMB metacognition in solving geometry problems, as show in Table 5 (see Appendix).

4. CONCLUSION

In conclusion, students' mathematical beliefs significantly impact their metacognitive knowledge in solving geometric problems. This means if there are students who have high mathematical belief, it is predicted that these students will also have high scores in metacognitive knowledge when solving geometric problems. Furthermore, the metacognitive knowledge of the subject with high belief in their understanding mathematical problems involves being aware of their own knowledge about the skills required to solve problems and the ability to identify problems by specifying the known and requested information. Furthermore, the subject is aware of the steps that can be taken to solve the problem. These steps include reading the problem repeatedly, selecting the relevant information and connecting it with prior knowledge. This allows them to determine the appropriate concept or formula to use in order to solve the problem. Every problem-solving exercise undertaken by the subject with high belief always utilizes the metacognitive knowledge. On the other hand, the subject with low belief in utilizing their metacognitive knowledge experienced differences in their cognitive tasks. They lack awareness of their own knowledge and struggle to identify the appropriate formula to us, although the subject realized the difficulty of understand the problem.

In scientific terms, this research findings can serve as a reference for educators and teachers in designing more effective learning strategies to enhance students' mathematical belief and metacognitive knowledge in mathematics thereby assisting in improving their learning achievements.

APPENDIX

Table 5. Differences and similarities of HMB and LMB metacognitive knowledge in solving geometry problems

Polya's problem solving stages	HMB (subject with high belief)	LMB (subject with low belief)
Identify known information, information being asked, check the adequacy of information.	– Understanding the problem by reading and identifying every word or sentence in the problem and marking it as known and asked information in the problem. – Deciding the formula to be used from the subject's prior knowledge. – Realizing the knowledge in understanding the problem regarding how to select information to find important sentences in the problem. – Be aware of the steps that can be taken in understanding the problem by first reading, understanding, and remembering the topic being discussed. – Realizing to decide on an effective strategy to understand the problem based on the learning experience it has.	– Understanding the problem by reading and identifying every word or sentence in the problem and marking it as known and asked information in the problem. – Deciding the formula to be used from the subject's prior knowledge. – Knowing the steps to be used, namely noting/marking important things in the problem and determining the formula used. – Making decision by sketching a picture and setting a formula to answer the given problem.
Identifying operations and strategies in designing a problem-solving plan at hand.	– Able to identify and select important sentences and determine topics that are appropriate to the problem. – Planning problem solving, namely in the form of the ability to show the keywords in the problem and know the topic being discussed. – Identifying the steps that can be taken in planning problem solving by re-reading the problem, marking important information, writing down important information in their own language, and determining formulas and establishing the most effective strategies to be able to plan problem solving based on their learning experience.	
Finding a solution, checking every step of the strategy that has been set to prove the strategy chosen is correct.	– Recognizing and knowing how to apply formulas that are appropriate to the topic being discussed. – Realizing his ability to operate the formula according to the topic being discussed. – Completing sequentially according to the questions on the problem and establishing the most effective strategy to be able to apply the formula he has set based on his learning experience.	– Realizing the ability to apply the things that are known and the formulas to be used. – Knowing the steps that can be taken by applying the information contained in the problem to the formula to be used. – Be aware of making decisions using algebraic principles and arithmetic operations.
Check the overall effectiveness of the problem approach and assess the solutions obtained	– Looking at the results obtained and knowledge related to the topic being discussed. – Checking the suitability of the solution. – Be aware of the steps that had been taken by adjusting the results obtained and re-calculate in an alternative way if necessary. – Using different alternatives to ensure that the calculations carried out were in accordance with the problem at hand.	– Be aware of the ability to find out in advance the sides and angles of a square. – Checking the suitability of the solution. – Knowing the steps to be taken by matching the results obtained with the information listed in the problem. – Using different alternatives to ensure that the calculations carried out were in accordance with the problem at hand.

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