Journal of Education and Learning; Vol. 13, No. 3; 2024
ISSN 1927-5250 E-ISSN 1927-5269
Published by Canadian Center of Science and Education

Bridging Gaps: Pre-Service Mathematics Teachers' Handling the Difficulties in Posing Real-World Mathematical Problems

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Received: December 10, 2023 Accepted: February 20, 2024 Online Published: April 10, 2024

doi:10.5539/jel.v13n3p133 URL: https://doi.org/10.5539/jel.v13n3p133

Abstract

Real-world mathematical problem (RWMP) solving and posing are important aspects of teaching and learning mathematical modelling, as well as developing a mathematization disposition for both teachers and students. Several researchers have explored blockages or difficulties in such modelling processes and in problem posing. However, prior research has identified difficulties that the pre-service mathematics teachers (PSMTs) encountered when they tried to pose a modelling problem by choosing the general topic themselves, as there is little known about possible obstacles that PSMTs can encounter when trying to pose a real-world problem relevant to a given mathematical topic. The current study explored the difficulties encountered by PSMTs in a RWMP-posing activity. The target group was 23 PSMTs with prior experience in mathematical modelling and mathematical problem posing. The findings showed that the PSMTs struggled with: (a) task organization, which involved selecting and understanding mathematical knowledge; (b) specialized content knowledge, which included a lack of real-world knowledge and difficulty in connecting mathematical concepts to real-world contexts; and (c) individual considerations of aptness, which encompassed authenticity, interest, complexity, language, and relevance to task organization. The PSMTs applied various strategies to complete the posing task, such as using problem-posing and solving heuristics, adapting existing problems, sharing and discussing with friends, and considering the perspective of a typical student. The implications of these findings should help in developing preparatory instructional practices for mathematics teachers.

Keywords: real-world mathematical problem, pre-service mathematics teachers, difficulties, handlings, problem posing

1. Introduction

Mathematical problem posing is a challenging and complex task for teachers, preservice teachers, and students (Silver, 2013; Cai & Hwang, 2002; Joaquin, 2024). Problem-posing activities enhance various mathematical abilities for the problem poser, including understanding of mathematical knowledge (both conceptual and procedural), problem-solving skills, mathematical modeling competency, and creativity (Cai et al., 2013; Silver, 1997). Nowadays, the goal of teaching and learning mathematics in various countries focuses on developing the student's ability to apply mathematical knowledge and skills to solve real-world mathematical problems. Hence, the role of the teacher will be transformed from solely focusing on teaching mathematical knowledge and skills to enhancing their students' applied knowledge and skills in various contexts through real-world problems that are connected to mathematics (Kaiser & Grünewald, 2015). Generally, the characteristics of mathematical real-world problems are realistic because they incorporate context, conditions, and data that correspond to real-world situations. These problems involve specific criteria and can be interpreted in various ways, such as modelling problems, application problems, and contextualized problems (Brown, 2019; McGrane, 2020). Therefore, the teacher and preservice teacher must be able to solve and pose the real-world mathematical problems (also known as the modelling problem), which is a necessary tool to enhance students' mathematical learning in the present and future (Blum, 2015; Sevinc & Lesh, 2018; Fukushima, 2021; Brady, Ramírez, & Lesh, 2024). Although the preservice teachers' abilities to pose meaningful problems were impacted by the students' own abilities, research has shown that preservice teachers encountered difficulties when attempting to combine mathematical modeling and problem posing in the classroom, particularly when starting with real-world contexts. The students often posed problems that were irrelevant to mathematics, unsuitable for their level, lacked a sense of ownership, and faced difficulties in connecting the mathematical content to real-world situations (Hansen & Hana, 2015).

Studying the difficulties in mathematical problem posing has various aspects. For example, Albert Mallart, Vicenç Font and Javier Diez (2017) challenged PSMTs to pose a real-world mathematical problem for upper primary school students on measuring the surfaces of two-dimensional shapes. They found that PSMTs found it difficult to pose the problem in terms relevant to everyday life and to adapt it to the school curriculum at a specific educational level. Additionally, Helena Osana and Diana Royea (2011), as well as Jinxia Xie and Joanna O. Masingila (2017), investigated PSMTs to pose word problems involving fractions. Their findings revealed that the PSMTs struggled with conceptual knowledge in mathematics, particularly in understanding fractions and their operations. They faced difficulties in constructing meaningful solutions and representing those solutions symbolically. Other studies on the difficulty in mathematical problem posing have focused primarily on tasks that provide PSMTs with free problem posing, starting from data or conditions in mathematics and contextualizing them in real life. However, there are a few studies that specifically examine the challenges associated with posing problems derived from mathematical knowledge or theorems and connecting them to real-world scenarios to engage problem solvers. The present paper addresses this research gap by using a problem-posing framework adapted from Kontorovich et al. (2011) to explore: (1) the difficulties encountered by PSMTs in real-world problem-posing activities and (2) how they handle these difficulties. The findings from the analysis of the difficulty in posing and teaching real-world mathematical problems posed by PSMTs should provide necessary information for educators and problem-posing task designers. This information is crucial for designing tasks that accommodate the limitations and lack of experience in problem-posing faced by PSMTs. Careful consideration and intentional design of efficient problem-posing tasks are essential as they serve as empowering tools to encourage and motivate PSMTs, allowing them to showcase their full potential in posing meaningful mathematical problems that enhance students' learning in the future. This is particularly important in the context of Thai students, as data from the OECD (Organisation for Economic Cooperation and Development) Assessment Report reveals that approximately one-half of 15-year-old Thai students did not attain the international basic proficiency level in mathematics during PISA 2009 and PISA 2012. Additionally, these students have limited or no experience in applying their mathematical knowledge and skills to solve real-world problems (Klainin 2015). Therefore, findings that reflect the difficulty and handling of real-world mathematical problems posed by PSMTs are crucial for developing the problem-posing abilities of PSMTs and enhancing the quality of student learning in mathematics. Notably, there is almost no research that has focused on the challenges faced by PSMTs when engaging in real-world mathematical problem-posing activities within the context of Thailand.

This research aimed to answer two questions:

1) What are the difficulties encountered by PSMTs in real-world problem-posing activities?

2) How do they handle these difficulties?

The findings of this research will serve as a basis for designing learning activities that aim to develop PSMTs' abilities to pose problems linked to the real world. This is particularly important in the context of the development of preservice teachers in Thailand, where the curriculum for basic mathematics education has been improved. The focus of the curriculum is on applying knowledge, mathematical skills, and processes to solve real-world problems more so than in the previous curriculum.

2. Method

The goal was to understand the difficulties of a PSMT in handling posed real-world mathematical problems.

2.1 Participant

The study involved 23 PSMTs who had prior experience in mathematical modelling activities and problem posing. They were voluntarily enrolled in a selected course for mathematics teacher bachelor's degree program. Based on the decision of the ethics committee, it was determined that there was no ethical violation in this study. Participants voluntarily participated in the study, and it was not expected that they would be exposed to any adverse effects or harm. Privacy of the participants was ensured by presenting the difficulties encountered by them in a collective manner, without disclosing specific names or identities.

2.2 Study Context

This study was a part of a research project at Kasetsart University, Bangkok, Thailand to develop preservice teachers' competencies in real-world mathematical problem solving, with the aim of encouraging middle-school students to engage in mathematization and solving real-world mathematical problems. The PSMTs were divided into six equal groups that were assigned to participate in the real-world mathematical problem-posing activity. Three of these groups participated in a semi-structured problem-posing task, where they had to connect the concept involved in the theorem of the angle at the circumference in a semicircle (Figure 1) with a real-world situation or problem. The other three groups were engaged in a free problem-posing task, where they independently chose a mathematical theorem to create a real-world mathematical problem and then solved it using the chosen theorem.

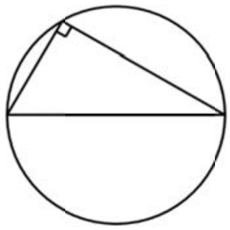

Figure 1. The angle at the circumference in a semicircle is 90 degrees

2.3 Exploring Difficulties of PSMTs in Handling Real-World Problem-Posing

A qualitative approach was developed using a case study. We utilized the difficulties in handling the problem-posing framework adapted from Kontorovich et al. (2012) to identify the difficulties experienced by the PSMTs in a real-world problem-posing activity (Figure 2). Due to the COVID-19 pandemic, the study omitted the component of group dynamics and interactions due to the challenges in effectively exploring these aspects through online learning platforms. The PSMTs' difficulties in handling the problem were collected through observations, semi-structured interviews, and content analysis, which involved examining the PSMTs' problem-posing reports and reflections. To answer the research question, we analyzed the PSMTs' responses in a RWMP posing activity based on a coding scheme to identify the difficulties in handling real-world problem posing that consisted of four components: (1) task organization, which involves assigning tasks to PSMTs, including conditions about the theorem and a real-world problem; (2) knowledge base, which pertains to understanding theorems and having knowledge of the real-world; (3) problem posing heuristics and schemes, which are strategies applied by PSMTs when posing problems; and (4) consideration of aptness, which refers to their awareness while posing problems.

The PSMTs' responses were coded by three independent coders and then discussed with the research teams to obtain a consensus. In this study, we presented three examples of real-world-related problems, namely the *igloo problem*, *the space station*, and *the fireworks festival*, which were designed by the PSMTs (Appendices A–C). These examples aim to illustrate and elaborate on the difficulties encountered by and the handling applied by the PSMTs.

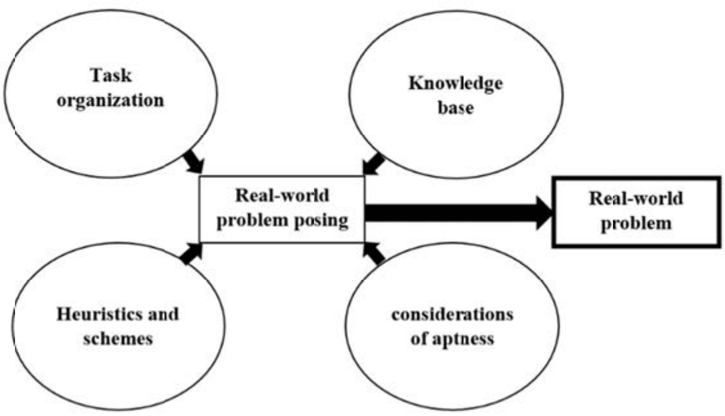

Figure 2. Difficulties and handling in real-world problem posing framework

3. Results

The results showed that PSMTs faced various difficulties while attempting to pose real-world mathematical problems. They spent a considerable amount of time collaborating in groups to satisfactorily handle the challenges and achieve the goals of the mathematical problem-posing tasks. The details of the difficulties and how they were addressed are as follows.

3.1 PSMTs' Difficulties in Real-World Problem-Posing

We observed the PSMTs while they posed the problem, conducted post-posing reflections, and analyzed their real-world problem-posing reports. The results revealed that the difficulties encountered by PSMTs in a real-world problem-posing activity can be categorized into three facets, excluding the difficulties associated with problem of posing heuristics and schemes, as follows:

1) Task organization: PSMTs in both groups (semi-structured problem-posing and free problem-posing) encountered difficulties in organizing the assigned task. They aimed to establish a connection between a mathematical theorem and a real-world context, posing a problem that would be interesting, authentic, and challenging for students. However, the free problem-posing groups faced greater difficulty in this task compared to the semi-structured problem-posing groups. The free problem-posing groups had to independently choose a mathematical theorem and understand it before determining the starting point for problem-posing. They spent substantial time searching for suitable theorems from various sources, such as websites and textbooks, and then selected one as the initial point for posing a problem. Unfortunately, if the chosen theorem could not be effectively connected to a real-world context, they had to change the theorem until an appropriate one was found.

2) Knowledge base: Both groups of PSMTs made efforts to connect mathematics with real-world contexts. The semi-structured problem-posing groups encountered difficulties in connecting the concept of the angle at the circumference in a semicircle theorem to a real-world situation, as they were not familiar with this theorem in real-world scenarios. It was necessary for them to gather more information, explore relevant situations, and find problems in the real world that could be appropriately connected to the theorem. For example, the groups working on the igloo problem and the orbital space station problem attempted to identify objects or situations resembling circles by conducting internet research and engaging in group discussions. Eventually, they came up with the contexts of a snow house and an orbital spaceflight, respectively. On the other hand, the free problem-posing groups had the advantage of independently choosing and changing the theorem. They selected theorems with which they were already familiar in a real-world context, such as the Pythagorean theorem, that they already used to solve real-world problems. Consequently, they were able to establish connections more readily between mathematics and real-world contexts.

3) Considerations of aptness: This facet can be divided into two periods. First, the PSMTs focused on establishing connections between mathematics and real-world contexts. They tried to connect mathematical theorems with real-world situations, considering various factors, such as authenticity, interest, and complexity of the context, as illustrated in Figures 3 and 4.

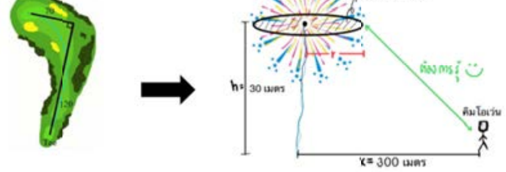

Figure 3. Sequence of changing context in *fireworks festival problem*

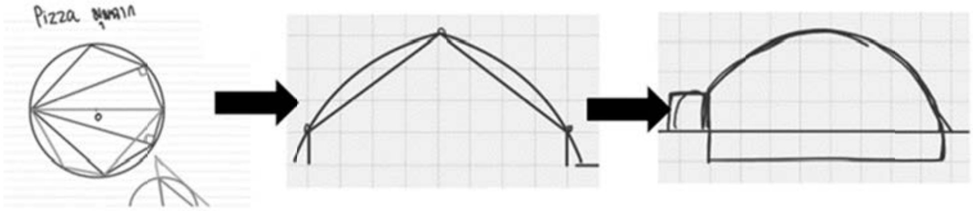

Figure 4. Sequence of changing context in *the igloo problem*

The sequence of changing contexts in the igloo problem (Figure 3) demonstrates their considerations for capturing student interest and ensuring authenticity in the real world. Initially, they thought of using a circle as a pizza, but they assumed students would not be interested, so they changed it to a tent. Although the tent was interesting for the students, it lacked authenticity because its shape did not resemble a circle. Finally, they settled on an igloo, which was both interesting and authentic. Furthermore, the transitions from the context of driving a golf ball to other contexts such as the height of a building, a tree, or of fireworks (Figure 4) illustrated their concem for authenticity by utilizing the Pythagorean theorem to solve the problem. This was relevant to the ball's motion following projectile motion. Additionally, they considered not only interest and authenticity but also complexity. For example, they discussed the context of the orbital space station problem and recognized that it might be too complex for students to grasp the idea of its flight being circular. In the second period, the PSMTs formulated the problem and evaluated it by considering the conditions of authenticity, interest, and complexity that were suitable for students, as depicted in Figures 5 and 6.

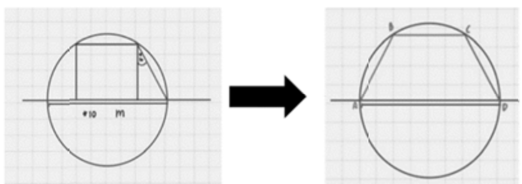

Figure 5. Sequence of changing conditions in *the igloo problem*

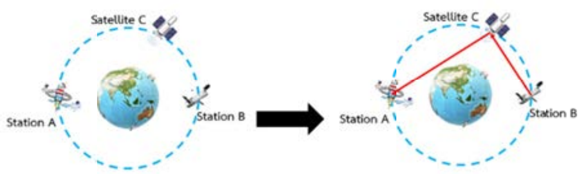

Figure 6. Sequence of changing conditions in *the orbital space station problem*

With *the igloo problem* (Figure 5), the PSMTs modified the condition of the shape inside the igloo from a rectangle to a trapezoid to enhance authenticity and complexity. Conversely, they simplified the conditions of *the orbital space station problem* (Figure 6) by introducing line segments to represent the distances between the spacecraft. Additionally, they endeavored to adapt their problems to more engaging contexts. In relation to the Pythagorean theorem, they shifted the problem's context from the conventional textbook scenarios (such as determining the height of a building, tree, or flagpole) to the Fireworks Festival in Korea.

Although both of the PSMT groups encountered difficulties, there were some differences between the groups. The groups engaged in the free problem posing had the flexibility to change both the theorem and the context until they were satisfied. Conversely, the groups involved in the semi-structured problem-posing could only change the context. However, we did not find any difficulties related to heuristics and schemes in problem posing, as the participants had prior experience in applying various techniques and strategies, including the powerful 'What if Not' strategy proposed by Brown and Walter (2005). This strategy involves five stages: (1) choosing a starting point, (2) listing attributes, (3) "what-if-not-ing", (4) asking questions or posing problems, and (5) analyzing the problem. An example of the application of this strategy will be provided in the next topic.

3.2 PSMTs' Handling of Difficulties in Real-World Problem-Posing

Regarding the three PSMTs' difficulties mentioned, the groups applied teamwork while posing problems. They engaged ideas and used discussions to collectively accomplish the assigned task. They applied three strategies to handle the difficulties in real-world problem posing: 1) utilizing problem-posing heuristics, 2) utilizing problem-posing sources, and 3) considering the thought process of a typical student. These strategies are elaborated on as follows:

1) Utilizing problem-posing heuristics: all PSMT groups applied the What if Not strategy to address the difficulties they encountered. This strategy involved questioning if a certain attribute was not suitable and considering alternative options. For example, when dealing with the considerations of aptness, they initially considered using a pizza as an attribute but deemed it lacking authenticity. Subsequently, they changed it to a tent and eventually settled on an igloo. Furthermore, they applied the What if Not strategy to modify the shape inside the igloo, transitioning from a rectangle to a trapezoid to introduce greater complexity. Additionally, they used this strategy to shift the context from a building's height to the Fireworks Festival in Korea, making the problem more engaging.

Utilizing the What if Not strategy demonstrated its effectiveness in helping PSMTs overcome the challenges posed by the assigned task, particularly in addressing the considerations of aptness. It was observed that the PSMTs did not initially list numerous attributes but focused on a specific attribute and evaluated its suitability. If found unsuitable, they swiftly switched to alternative attributes. This approach differed from the suggestion by Brown and Walter (2005) to list multiple attributes related to the starting point. Overall, the What if Not strategy proved to be a powerful tool that enabled PSMTs to overcome obstacles in problem posing and effectively address the considerations of aptness in their posing process.

2) Using problem-posing sources: we observed that the PSMTs encountered difficulties in three phases during problem posing: (a) selecting a suitable mathematics theorem, (b) establishing a connection between the theorem and the real-world context, and (c) modifying the problem accordingly. To address these challenges, they made use of various sources, including textbooks and websites. For example, in the fireworks festival problem, they relied on their previous experience with problem-solving in textbooks to identify the Pythagorean theorem as the starting point for the problem. Once they established the starting point, they endeavored to change the context from the typical textbook examples (such as heighting buildings, trees, and flagpoles) by researching real-life applications of the Pythagorean theorem on websites. They also assessed the conditions and context of the problem to ensure they aligned with the assigned tasks.

3) Considering the thought process of a typical student: the PSMTs demonstrated their concern for the problem solvers (students) while posing the problem. During the problem-posing process, they followed several steps. First, they reviewed and selected well-known and uncomplicated mathematical theorems that would be suitable for students. Second, they endeavored to find a real-world context that would be interesting, relevant, and relatable to the students. Lastly, they defined the problem and evaluated its appropriateness for the students. They carefully read and considered the problem, considering the students' viewpoint. In addition, they interpreted the problem and created illustrations to simplify it, with some groups adding additional information or modifying the conditions to ensure clarity.

4. Discussion

Several interesting issues arose from the difficulties the PSMTs faced when posing real-world mathematical problems and the approaches they used to handle these difficulties, as follows:

1) Both the semi-problem posing and free problem posing groups encountered difficulty in choosing an appropriate and engaging real-world context for posing the mathematical problem. This process required skill in mathematization, which is a challenging skill to use effectively in solving modelling problems (Bonotto, 2010; Stillman, 2015). The process of connecting a real-world context with mathematical content to pose the problem was complex and challenging. It necessitated an understanding of the real-world situation, exploration, and posing of problems in accordance with the hypothesized process model for modelling-related problem posing as considered by Hartmann, Krawitz and Schukajlow (2022) and shown in Figure 7.

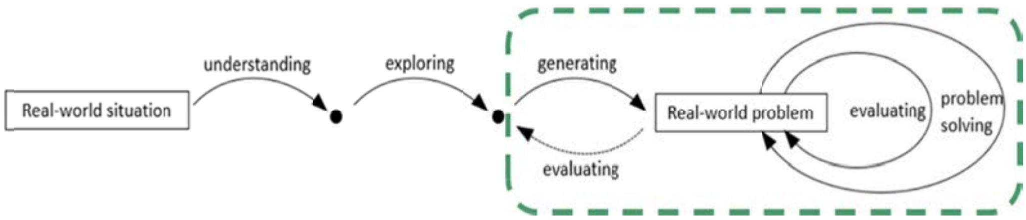

Figure 7. Hypothesized process model for modelling-related problem posing

Furthermore, even though the PSMTs had experience in mathematical modelling and task design, which enhanced their ability to connect real-world contexts with mathematical content, they still encountered difficulties. Our results indicated that the main difficulty for the PSMTs was their concern about the problem solver, particularly the students' interests and the complexity of handling the problem. This observation aligns with the findings of Rellensmann and Schukajlow (2017), with PSMTs encountering difficulties in accurately assessing and predicting students' situational interest, as their judgments often do not match the students' actual interests. Additionally, the tasks assigned to the PSMTs may have contributed to the difficulties they faced. However, they were able to overcome these difficulties and succeed in the problem posing task by utilizing problem sources such as textbooks and websites, as well as drawing from their previous experiences. This approach allowed them to consider various aspects of connecting the real-world context with mathematical content, which related to the resources or starting points for enhancing the mathematizing ability (Stillman, 2015).

2) The challenges faced by both groups involved not only the difficulties in connecting a real-world context with mathematical content, but also the consideration of aptness for students. The PSMTs attempted to pose problems that were appropriate for students, challenging, and provoked student interest. The task provided them with worthwhile mathematical problems that were challenging, realistic, and interesting, thus enhancing students' ability to apply mathematics to solve real-world problems (Borromeo Ferri, 2017). However, this ability needed continuous development by the PSMTs. They overcame this difficulty by using sources to find a real-world context that interested the students and the mathematical content that was appropriate for them. Corresponding with Takahashi (2021), supporting those sources was a necessary tool for posing mathematical problems. Additionally, utilizing problem-posing heuristics, such as the "What if Not" strategy, was a key success in overcoming the difficulty of considering aptness. This strategy helped the PSMTs to simplify the conditions in the posed problems, making them suitable for the students. The "What if Not" strategy helped PSMTs simplify the conditions in the posed problems, making them suitable for the students. It provided PSMTs with various aspects to consider when using it in problem posing (El Sayed, 2002; Lavy & Bershadsky, 2003; Kovács, 2024).

3) Effective collaboration was a key to success in achieving the problem-posing activities. We observed that the PSMTs brainstormed ideas, utilized technology, and engaged in productive group discussions. The collaborative problem-posing activities facilitated the PSMTs in sharing their diverse viewpoints and insights. This approach not only advanced their problem-posing skill but also fostered the development of critical thinking, effective communication, and teamwork skills. This alignment between our observation and the research by Crespo (2020), highlighting the essential role of collaboration in enhancing the problem-posing abilities of PSMTs.

5. Conclusion and Implementation

The findings from this study highlight several challenges faced by the PSMTs in relation to task organization, specialized content knowledge, and individual considerations of aptness. The PSMTs struggled with selecting and understanding mathematical theorems and knowledge, having sufficient ability to connect real-world contexts to mathematical concepts, while they also encountered difficulties in judging what students would enjoy and what would interest students in the problems assigned.

However, despite these challenges, the PSMTs applied various successful strategies to deal with these difficulties, including problem-posing and solving heuristics (especially the What-if-Not strategy), adapting existing problems, and productively discussing and collaborating with peers.

The implications of these findings are important in both research and practice. First, the study contributes to the understanding of PSMTs' struggles and in the preparation of the additional needed support for the development of professional mathematics teachers. In particular, teachers and educators can collaboratively design interventions to enhance their specific abilities related to real-world mathematics problem-posing skill. For example, selecting starting points (mathematical theorems) that are related to the PSMTs' prior experience from the perspective of a non-experienced real-world problem poser, practicing the What-if-Not strategy as a powerful heuristic for PSMTs when posing both mathematical and real-world mathematical problems, and determining the condition of the primary problem-posing activity should not concern individual considerations of student's aptness.

Additionally, the findings reinforce the importance of incorporating real-world contexts and applications in teaching of mathematical principles to both students and preservice teachers. Specifically, educators should provide the PSMTs with opportunities to explore and engage with authentic, real-world mathematical problems to enhance specialized content knowledge, ability in connecting mathematical concepts to practical situations, and effectiveness in teaching mathematics to future students.

Furthermore, the study highlights the value of collaborative approaches in problem posing. Encouraging PSMTs to share and discuss problem-posing strategies with their peers can foster a supportive professional learning community, enabling the PSMTs to learn from their collective experiences and perspectives. This collaborative approach can enhance the quality and diversity of posed problems, leading to improved mathematical learning experiences for students. Thus, educators should consider the productive learning environment and collaboration when designing problem posing activities for PSMTs.

While this study has provided valuable insights, it is essential to acknowledge its limitations. The findings are based on a specific sample of PSMTs and are perhaps not applicable to the broader population. Future research should aim to include a more diverse sample to ensure greater representativeness. Additionally, longitudinal studies can provide a deeper understanding of the impact of learning interventions on PSMTs' real-world mathematics problem-posing abilities over time.

Acknowledgments

Not applicable

Authors' contributions

Not applicable

Funding

Not applicable

Competing interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Informed consent

Obtained.

Ethics approval

The Publication Ethics Committee of the Canadian Center of Science and Education.

The journal's policies adhere to the Core Practices established by the Committee on Publication Ethics (COPE).

Provenance and peer review

Not commissioned; externally double-blind peer reviewed.

Data availability statement

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

Data sharing statement

No additional data are available.

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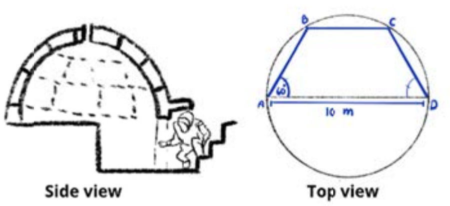
Appendix A

The igloo problem

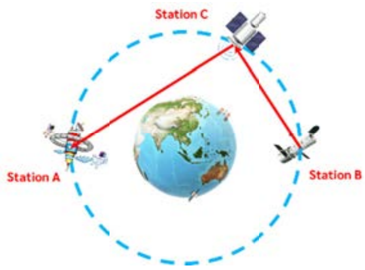
An Eskimo family constructs an igloo (a snow house made of snow bricks). Inside the igloo, they create a sleeping area in the shape of a trapezoid, which is covered with leather, as depicted in the figure below. The question is: What is the minimum amount of leather they need?



Appendix B

The space station problem

Space station B is photographing lunar phenomena by sending a signal via satellite C back to station A on Earth. Find the length of time it takes for the signal from station B to reach station A when stations A, B, and the center of the Earth are aligned on the same plane.



Given:

- Station B is 5,000 kilometers away from Satellite C.
- Stations A and B float 500 kilometers above the Earth's surface.
- The Earth's radius is 6,000 kilometers.
- The signal travels at a speed of 100 km/s.

Appendix C

The fireworks festival problem

At the 2022 International Fireworks festival in Korea, Mr. Kim Owen stood at a viewpoint 300 meters away from the location where the fireworks were set up. The fireworks rose up to a height of 30 meters and spread out in a horizontal plane, covering an area of 154 square meters. Mr. Kim Owen believes that the shortest distance from the fireworks to him is approximately 300 meters. Do you agree or disagree with this statement?