THE EFFECTS OF POLYA'S PROBLEM SOLVING WITH DIGITAL BAR MODEL ON THE ALGEBRAIC THINKING SKILLS OF SEVENTH GRADERS

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Abstract

There is a dearth of empirical data to support the positive effects of problem solving (PS) combined with digital technology in the classroom, despite claims that these activities improve students' algebraic thinking abilities. Therefore, the purpose of this study was to evaluate how the teaching method known as Polya's problem solving with digital bar model (PSDMB) affected the seventh graders' ability to think algebraically. Ralston's framework, which covers Generalised Arithmetic, Function, and Modelling within the topic of Linear Equation, served as the foundation for the evaluation of algebraic thinking abilities. A quasi-experimental pre-test and post-test control group design was employed. A total of 90 seventh graders, aged twelve- to thirteen-year-olds, from a secondary school in Tambunan, Sabah, Malaysia, made up the sample. Three teaching groups were formed out of these randomly chosen students: PSDMB (n = 30), Bar Model (MB) (n = 30), and Conventional Problem Solving (CPS) (n = 30). Both the pre- and post-algebraic thinking tests were taken by students. The post-test results were analysed using MANCOVA with the students' pre-test results acting as covariates. The results indicated that students in the PSDMB group performed notably better in Generalised Arithmetic, Function, and Modelling than those in the MB group, who, in turn, outperformed those in the CPS group. These results imply that incorporating digital bar model into problem-based learning is a successful strategy for improving seventh graders' algebraic thinking abilities.
Keywords: *algebraic thinking skills, digital bar model, Polya's problem solving, seventh graders*

Introduction

For students to be prepared to solve mathematical problems, algebraic thinking abilities are essential. These issues require more than just the simple application of formulas or algorithms; instead, they call for critical and creative thinking. To find solutions, students need to be able to analyse problems, think creatively, and use a variety of problem-solving techniques. Multiple ways to solve non-routine problems are frequently available, which fosters a deeper comprehension of mathematics and improves students' problem-solving skills (Mamat & Wahab, 2022). Additionally, students preparing for college coursework and jobs require algebraic thinking skills. However, the Trends in International Mathematics and Science Study (TIMSS) results show that Malaysian students' proficiency in algebraic thinking skills for problem-solving is still below the intended benchmark (Mullis, 2017). Just 5% of students in Malaysia, which is ranked 29th out of 42 participating countries in TIMSS, are able to apply, reason, and generalise when solving algebraic problems. One of the main obstacles to learning non-routine problem solving, according to Stacey (2005), is that students need a wide

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variety of skills in order to solve these kinds of problems. Consequently, educating students in problem-solving is considered a demanding undertaking (Dendane, 2009). According to Reiss and Renkl (2002) and Novotná (2014), heuristic strategies are deemed valuable for enhancing non-routine problem-solving skills. Heuristic solutions, in Pólya's (1945) view, are concerned with understanding the successive steps that lead to successful resolutions of problem situations, especially the mental processes that are involved in making connections, associations, and comparisons during the problem-solving process. Based on Polya's work, this heuristic approach consists of four distinct stages: understanding the problem, coming up with a plan of action, carrying out the selected strategy, and lastly, assessing and fine-tuning the solutions obtained. The heuristic problem-solving approach in algebra education focuses on teaching and learning algebraic concepts through activities that are closely connected to particular contextual settings. This means tackling real-world issues that might not be immediately obvious or simple due to their complexity.

Literature Review

Learning Environment for Developing Algebraic Thinking Skills

Diversifying teaching strategies for algebraic topics is crucial for fostering students' algebraic thinking skills (Blanton & Kaput, 2003; Blanton et al., 2019a, 2019b). The emphasis on student-centred approaches in 21st-century education has led to the perception that conventional teaching methods are insufficient. Ramsden (1992) and Anthony and Walshaw (2009) suggested that quality teaching requires a systematic approach and the use of assignments that encourage active student participation and collaboration, thus developing students' competency. Problem-solving methods, as proposed by Lesh and Zawojewski (2007), are effective in promoting student-centred learning and cultivating algebraic thinking skills. Radford (2014) and Boaler and Sengupta-Irving (2016) argued that specially designed classroom activities are necessary to support students' development in formal algebra and promote algebraic thinking. Additionally, Gurtner (1992), Godwin and Beswetherick (2003), supported by Hegedus and Kaput (2005), recommended structured tasks, the use of appropriate digital tools, and explicit interventions involving student interactions to cultivate students' algebraic thinking skills in the classroom.

Effects of Problem Solving and Technology on Student's Algebraic Thinking

There is little empirical data supporting the benefits of problem solving combined with technology in Malaysian classrooms, despite assertions regarding the beneficial effects of both on students' algebraic thinking abilities. Mustaffa et al. (2018) asserted in their study that the incorporation of algebraic reasoning into problem solving learning methodologies holds the potential to assist educators in enhancing the teaching and acquisition of algebra. By integrating algebraic reasoning into problem solving context, learners can develop a deeper understanding of algebra and its practical applications. The utilization of this framework is viable in virtual learning environments, allowing students to engage in algebraic reasoning activities with flexibility and adaptability. Research by Vagner Campeão and Carvalho (2021), discussed the development of Algebrizar, an applet designed to promote algebraic thinking in early elementary school. Algebrizar incorporates a playful approach to problem solving, treating it like a game with trails and scores to encourage students to think algebraically. This aligns the programme with the competencies and skills required by the official curriculum for this age group. According to research by Baysal and Sevinc (2021), using the bar model method in mathematics instruction can help students understand algebra better and solve problems more effectively, which in turn can help them develop their algebraic thinking abilities.

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Bar model drawing is a useful tactic for improving elementary students' accuracy in arithmetic word problem solving and their capacity to apply cognitive strategies, according to Morin et al. (2017). The "model method" of teaching mathematics in Singaporean primary schools primarily uses bar diagrams to aid students in visualising the problem structures found in word problems. "AlgeBAR" is a software tool that was created to scaffold the learning of the algebraic process, specifically the formulation of equations, as described by Looi et al. (2007). This intervention pedagogy is technology-enabled, aiming to support students in transitioning from bar diagrams to algebraic problem-solving to develop algebraic thinking skills among students.

Infusion of Digital Bar Model into Polya's Problem Solving Model

Costa and Brandt (2001) contended that instructing students about the process of thinking, which emphasizes thinking as a subject matter, is insufficient in facilitating effective learning of thinking. They contend that using teaching strategies they refer to as "teaching for thinking," it is essential to create a classroom climate that encourages thinking. Thinking-Based Learning is a method that Swartz and his associates suggest (Swartz & Parks, 1994; Swartz et al., 2007). It involves teaching curriculum content and thinking skills at the same time in a lesson. Beyer (1997) posited that the enhancement of thinking-based learning can be achieved through the integration of teaching for thinking and teaching about thinking. The integration of these two approaches results in a more explicit, systematic, clear, and focused thinking-based learning process.

Accordingly, the current investigation implemented an infusion approach, wherein the process of solving algebraic problems, particularly utilizing Polya's problem-solving strategy alongside the digital bar model, is combined with teaching guidance on the development of cognitive and reasoning abilities. Through the utilization of digital bar models, which serve as pedagogical tools for fostering cognitive growth, students receive explicit instruction on enhancing their thinking skills within the framework of this pedagogical approach. Later, they are asked to use the digital bar model to solve algebraic problems using Polya (teaching for thinking). This study used a comprehensive infusion approach, wherein each algebraic problem was solved by concurrently completing the steps of the Polya and digital bar model. In order to better understand how seventh-grade students' algebraic thinking is affected when solving algebraic problems, this study utilized Polya's problem solving with the digital bar model teaching method.

The Framework for Algebraic Thinking Skills

Ralston's Algebraic Thinking Skills served as the foundation for the algebraic thinking framework used in this study. Three primary categories of algebraic thinking skills were identified by Kaput et al. (2008): generalized arithmetic, function, and modelling. More information about the connected aspects of these three constructs has been provided by Ralston (2013). Efficient calculation and generalisation are key components of the generalised arithmetic construct. Number pattern recognition is a component of functional thinking skills, whereas open problem solving, similarity detection, and variable computation are components of modelling.

Generalized Arithmetic

Generalised arithmetic, as described by Kaput et al. (2008), encompasses the study of algebraic systems and structures, incorporating relational and computational laws within mathematics. Fujii and Stephens (2001) suggested that teaching generalized arithmetic can advance students' development in algebraic thinking, transitioning smoothly from basic arithmetic. Algebraic thinking is a thought process defined in the literature as the ability to discern and symbolize patterns in mathematical terms (Mason, 1989; Sfard & Linchevski, 1994). It heavily relies on one's skill to conceptualize and use generalizations and to employ suitable representational methods to convey these generalizations effectively. It is on the groundwork laid by basic arithmetic that these skills are built and further nurture the growth of algebraic thinking capabilities (Carpenter et al., 2005; Jacobs et al., 2007).

Function

According to Blanton and Kaput's (2005) research, lower secondary school students are capable of using a variety of representations when they are performing functional reasoning. This encompasses their capacity to simulate and resolve equations with indeterminate quantities employing symbolic language, as well as their adeptness in articulating covariations, recursive relations, and data matching both verbally and symbolically. The research carried out by Brizuela and Lara-Roth (2001), Carraher et al. (2007), Carraher and Schliemann (2007) and Moss London McNab (2011) corroborate the findings posited by Blanton and Kaput (2004). According to an analysis conducted by Blanton and Kaput (2004) with fifth-grade students, scaffolding can help students develop functional algebraic thinking skills. Students' cognitive processes can be shaped in a way that promotes a deeper comprehension of data and the capacity to interpret functional relationships more effectively by gradually introducing tables, graphs, pictures, words, and symbols.

Modelling

Ralston (2013) emphasized that modelling plays a pivotal role in fostering the development of algebraic thinking skills. This involves carrying out calculations involving variables, comprehending similarities, and solving open-number sentences. Students are indirectly introduced to variables and unknown values through assignments that call for filling in a blank with a value. This helps students understand how different arithmetic operations relate to one another. Additionally, a number of academics stress the significance of equality in helping students develop algebraic thinking abilities (Carpenter et al., 2005; Rittle-Johnson & Alibali, 1999). Riley-Johnson and Alibali (1999) elucidated that the symbol "equal to" encompasses three distinct connotations: the comparability of two magnitudes, a connection, and the existence of two components within an equation. Nonetheless, studies indicate that a lot of students struggle to comprehend variables, particularly when it comes to using them symbolically in algebra (Küchemann, 1978; Usiskin, 1988).

Purpose and Hypothesis

Previous research indicates that the utilization of problem solving in the learning process greatly enhances students' algebraic thinking. Moreover, various studies provide evidence that the integration of problem solving and thinking tools cultivates students' algebraic thinking skills. It has been demonstrated that problem solving promotes self-regulation and critical thinking, thereby facilitating the development of algebraic thinking skills. The utilization of the digital bar model in conjunction with Polya's problem-solving model has been shown to significantly gain proficiency in a wide range of algebraic thinking skills such as generalized arithmetic, functional analysis, and mathematical modelling. This integration enhances the ability to tackle algebraic problems effectively and fosters a deeper understanding of mathematical concepts within the context of problem-solving strategies. Although incorporating this method has shown promise for enhancing key elements of algebraic thinking, the extent of its positive impact on seventh-grade students remains unclear. Consequently, the main objective of this research is to determine the degree to which the integration of Polya's problem-solving with the digital bar model can foster seventh-grade students' algebraic thinking skills.

In order to determine the extent to which these interventions support students' algebraic thinking, this research used the Polya's problem-solving with digital bar model teaching method (PSDMB) and bar model teaching method (MB) to test the "Infusion Approach" hypothesis against the "non-infusion approach" hypothesis. Furthermore, the research assessed the degree to which the PSDMB and MB teaching methods have an effect on students' algebraic thinking skills, in comparison to the Conventional Problem Solving (CPS) method. Therefore, the PSDMB, MB, and CPS were the three teaching methods used in this study. Thus, the following alternative hypothesis was put forth:

> Students taught via the PSDMB teaching method would perform significantly higher than students taught via the MB teaching method, who in turn would perform significantly higher than students taught via the CPS teaching method in algebraic thinking skills.

Therefore, the purpose of this study was to determine how much the PSDMB teaching method could support the development of seventh graders' algebraic thinking abilities in the areas of i) Generalized Arithmetic; ii) Function; and iii) Modelling. In order to ascertain whether another Bar Model teaching method was just as successful in achieving the intended student outcomes, this study compared two distinct Polya with the Bar Model and Polya with non-Bar Model teaching methods. Thus, the purpose of this study was to find out if students who received intervention from three different teaching methods differed significantly in their ability to think algebraically.

Research Methodology

Design

To explore how three different teaching methods affect the teaching and learning of algebraic thinking among seventh graders, this study employed a quasi-experimental pre-test post-test control group design. The independent variables were the three teaching methods: PSDMB and MB (Experimental group) versus CPS (Control group). The dependent variables were the students' algebraic thinking abilities, specifically in Generalised Arithmetic, Function, and Modelling.

Sample

A total of 90 seventh graders from one rural fully government-funded secondary school in Tambunan, Sabah, Malaysia, participated as research sample. The school was selected based on the socio-demographic similarity, such that the students were similar in age and ethnicity. Students comprised of 48 (53.3%) females and 42 (46.7%) males aged 13 years old. The Kadazandusun is the largest ethnic group in the school, comprising both the Kadazan and Dusun tribes. Three classes were selected based on the comparable pre-test mean scores achieved by students in the Algebraic Thinking Test (ATT) and randomly assigned to one of the intervention groups as an intact group: the PSDMB, MB, or the CPS.

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Research Ethics

The researcher obtained the cooperation and permission from the school to carry out the study. The researcher also obtained consent from the parents or guardians of the students involved. In addition, the researcher acquired the respondents' permission for their participation in the research. The permission was obtained through written documents. The researcher clarified the purpose of the research and explained that the data would be used as empirical data for the study. The information collected was only for research matters, and the participant's identity would not be exposed.

Instrument

Researchers developed an assessment tool known as the Algebraic Thinking Test (ATT) to evaluate the experimental treatments (Jahudin & Siew, 2023). Two identical and concurrent ATT were used as the pre-test and post-test. Nine open-ended, subjective questions made up each of the parallel ATT. The questions were created using the content of the Standard Document of Curriculum and Assessment of Seventh Grade Mathematics and the Curriculum Standard of Secondary School, with a particular emphasis on algebra and relations (Curriculum Development Division, 2017).

All items on the Algebraic Thinking Test (ATT) showed positive values, with PTMEA-CORR scores ranging from 0.46 to 0.92, indicating their alignment with the construct being measured. The ATT demonstrated strong reliability, with a Cronbach's alpha coefficient (KR-20) of 0.90. Additionally, the item separation index of 6.29 surpassed the threshold of 3.0, and the item reliability score of 0.98 fell within the excellent range (Sumintono & Widhiarso, 2015). These results suggest that the ATT has a favourable item distribution and is a valid and reliable tool for evaluating algebraic thinking among seventh graders.

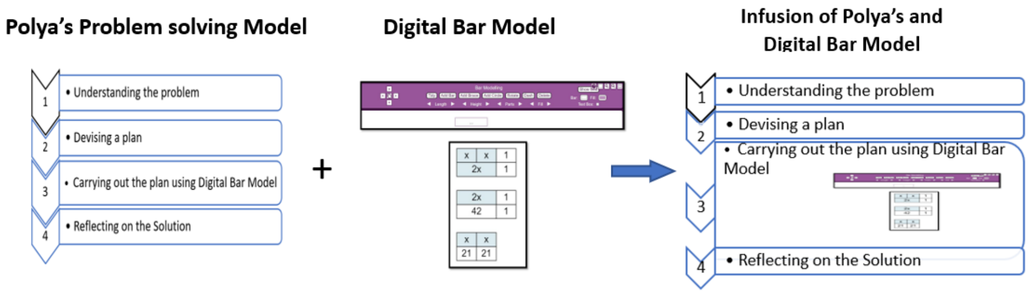
The Implementation of the Teaching Methods

PSDMB

High reliability and validity were found in the Polya and digital bar model learning module, known as the Algebraic Thinking Skills (ATS) module, which was created using Polya's (1945) and digital bar model (Jahudin & Siew, 2023). The main task in the ATS Module is the Polya's Problem-Solving Model (1945), which consists of four stages: (i) understanding the problem, (ii) devising a plan, (iii) carrying out the plan, and (iv) reflecting on the solution.

The infusion of Polya and the digital bar model learning module for the Seventh Graders included six learning activities on linear equations. In this study, the PSDMB method involved using the ATS module and the digital bar model method to help students solve algebraic problem-solving questions that require algebraic thinking skills. The implementation of strategies, the third step in Polya's problem-solving model, incorporated the digital bar model as an additional strategy (Figure 1). The PSDMB method was student-centred, with the teacher acting solely as a facilitator. Adapted from Ng and Lee (2009), this method also considered trial and error, reversal, elimination, and substitution methods in solving algebraic problems, particularly in Linear Equations, as outlined in the Curriculum Standard of Secondary School (CSSS) and the Standard Document of Curriculum and Assessment (SDCA) for Seventh Grade Mathematics.

Figure 1
Infusion of Digital Bar Model into the Polya's Problem-Solving Model



MB

The MB method is a teaching and learning approach that exposes students to algebraic problem-solving questions that require algebraic thinking skills, similar to those found in the ATS module. However, it transforms the digital bar model method into the regular bar model method by having students create their own bar drawings, which are used in Polya's Problem Solving Model at the implementation phase, which is the third step. Similar to PSDMB, this bar model method was also adapted from Ng and Lee (2009); like PSDMB, it is student-centred, with the teacher serving only as a facilitator. As specified in the Curriculum Standard of Secondary School (CSSS) and the Standard Document of Curriculum and Assessment (SDCA) of Seventh Grade Mathematics, the application of the bar model in this method also takes into account alternative approaches to solving algebraic problems, particularly in the topic of linear equations.

CPS

The conventional problem-solving method is a teaching and learning method where students are exposed to algebraic problem-solving questions that involve algebraic thinking skills as found in the ATS module. The conventional problem-solving method placed great emphasis on student-centred learning of algebra as stated in CSSS and the SDCA of Seventh grade Mathematics. Problem-solving methods such as trial and error, reversal, graph method, elimination, and substitution were applied in solving algebraic problems through the Polya Model, especially in the topic of Linear Equations without applying the digital bar model or regular bar model.

The Training of Teachers

The study involved a teacher who completed a two-week specialised training and coaching programme that concentrated on implementing PSDMB, MB, and CPS teaching methods. Before the study ever started, the teacher received a thorough ATS module that included details about Polya's approach to problem-solving, digital bar model, algebraic thinking skills and suggested results for every task. In addition, the teacher was trained in the PSDMB method of group activity facilitation. Through recurring in-person visits, the researchers closely observed the teacher to make sure that the implementation was dependable and consistent. The teacher was chosen on the basis of her readiness and willingness to take part in the study as well as her more than ten years of experience in teaching Mathematics. One teacher was assigned to teach the three classes using PSDMB, MB, and CPS, respectively, in order to eliminate any possibility of bias. The teacher conducted the intervention in three different classes over a 10-week period between October and November 2022, using different teaching and learning methods in each class.

Data Analysis

Preliminary Analysis

Preliminary analysis was undertaken to determine if the prerequisite assumptions for MANOVA and MANCOVA were satisfied. The analysis focused on the following assumptions for the statistical tests: (a) multivariate normal distribution; (b) equality of group population covariance matrices; (c) linear relationship between covariates and dependent variables; (d) absence of multicollinearity; and (e) homogeneity of variance in dependent variables.

Pre-Experimental Research

Pre-experimental research's main goal was to determine whether respondents in the three teaching groups had similar prior knowledge of pre-algebraic thinking (pre-AT), pre-generalized arithmetic (pre-GA), pre-function (pre-F), and pre-modelling (pre-M). A one-way multivariate analysis of variance (MANOVA) was performed to ascertain whether there were significant statistical differences in the mean scores on pre-AT, pre-GA, pre-F, and pre-M between the three groups. A univariate *F*-test (ANOVA) was performed to ascertain whether or not there were significant statistical differences between students in each of the three teaching groups on pre-AT, pre-GA, pre-F, and pre-M in situations where the overall multivariate test (MANOVA) did not produce significant results.

A multivariate analysis of covariance (MANCOVA) was carried out to look into the main effects of the three distinct teaching approaches on students' post-AT, post-GA, post-F, and post-M while controlling for the covariates (pre-AT, pre-GA, pre-F, and pre-M). Any unnecessary differences between the groups can be accounted for by removing the covariates' effects from the dependent variables using the MANCOVA (Hair et al., 1998). A univariate *F*-test (ANCOVA) was performed on the post-test mean scores using the pre-test mean scores as covariates if the overall multivariate test (MANCOVA) produced significant results. This was done in order to determine whether the teaching groups had a statistically significant main effect on the post-AT, post-GA, post-F, and post-M.

With SPSS for Windows (Version 22), the presumptions needed for the MANCOVA/ MANOVA and inferential statistics analyses were examined. The Wilk's Lambda was utilised to evaluate the multivariate differences in this study, with a significance level of .05. This test is frequently used in multivariate analyses to look at differences between the means of identified subject groups on a variety of dependent variables (Everitt & Dunn, 1991). The eta square (η^2) was used to calculate the effect size index (*f*). Based on Cohen's approximate characterization (Cohen 1988, p. 284–288), $0.2 \leq f \leq 0.4$ is considered a small size effect, $0.4 < f \leq 0.7$, a medium size effect, and $0.7 < f \leq 1.0$, a large size effect.

Research Results

Preliminary Analysis

An initial examination showed that there was equal variance among the dependent variables, the data fit a multivariate normal distribution, a linear relationship was present between the covariates and the dependent variables, there was no issue with multicollinearity, and all the single and multiple variable preconditions for MANOVA/MANCOVA were met. However, the study found violations in the assumption of equal covariance matrices across groups in both the Pre ATT (Box's M = 27.592, F(10, 16082.869) = 2.553, p < .01) and Post ATT (Box's M = 47.994, F(12, 36680.538) = 3.795, p < .01) phases. According to Grice and Iwasaki (2007), Pillai's trace is a robust measure to counter the frequent breach of the equal group covariance assumption in MANOVA and MANCOVA. Hence, Pillai's trace was utilized as the multivariate criterion for determining the significance of the model in interpreting the outcomes of the analysis.

Descriptive Statistics

Table 1 provides an overview of the descriptive statistics for students' pre-and post-test scores in algebraic thinking skills, covering Generalized Arithmetic (GA), Function (F), and Modelling (M).

Table 1
Descriptive Statistics of the Dependent Variables

Dependent Variables	Intervention Group	*N*	Pre-Test		Post-Test	
			M	*SD*	*M*	*SD*
Algebraic Thinking (AT)	PSDMB	30	4.43	1.04	24.67	2.45
	MB	30	4.47	1.38	18.67	2.68
	CPS	30	5.93	2.48	13.00	2.98
	Total	90	4.94	1.48	18.78	5.49
Generalized Arithmetic (GA)	PSDMB	30	1.47	0.57	7.30	1.12
	MB	30	1.40	0.56	5.50	1.08
	CPS	30	1.70	0.59	3.83	0.87
	Total	90	1.52	0.58	5.58	1.72
Function (F)	PSDMB	30	1.27	0.45	9.33	1.40
	MB	30	1.30	0.53	6.50	1.20
	CPS	30	1.73	0.58	4.47	1.22
	Total	90	1.43	0.56	6.77	2.37
Modelling (M)	PSDMB	30	1.70	0.53	8.13	1.14
	MB	30	1.77	0.63	6.70	1.58
	CPS	30	2.57	0.73	4.60	1.73
	Total	90	2.01	0.74	6.48	2.08

The Pre-Experimental Research Results

MANOVA Analysis

There were no statistically significant differences between the three groups in Pre-AT, Pre-GA, Pre-F, and Pre-M, according to the results of the MANOVA and ANOVA (Table 2).

Table 2
Summary of Multivariate Analysis of Variance (MANOVA) Results and followed-up ANOVA Results on Pre-test mean Scores

MANOVA effect and dependent variables	Multivariate, *F*	Univariate, *F*
Group Effect	*df* = 8, 162 Pillai's Trace *F* = 12.462, *P* = .05	2, 58.815
Pre- AT		*F* = 2.625, *p* = .078
Pre- GA		*F* = 1.729, *p* = .184
Pre-F		*F* = 2.279, *p* = .108
Pre-M		*F* = 1.029, *p* = .362

The Experimental Research Results

The MANCOVA results showed that teaching methods had significant main effects on dependent variables [Pilai's Trace =.762, $F(8, 162) = 12.462$, $p < .05$]. Subsequent ANCOVA revealed that the teaching methods had significant main effects on AT [$F(2, 110.946) = 824.890$, $p <.05$], $\eta^2 =.721$], GA [$F(2, 82.916) = 86.142$, $p <.05$, $\eta^2 =.659$], F [$F(2, 93.429) = 149.846$, $p <.05$, $\eta^2 =.685$,], and M [$F(2, 29.714) = 69.414$, $p <.05$, $\eta^2 =.409$]. There was a strong correlation found between the teaching methods and the dependent variables, meaning that the teaching methods accounted for 72.1% of the variance in AT, 65.9% in GA, 68.5% in F, and 40.9% in M.

Subsequent analysis using the Post hoc pairwise test showed that seventh-grade students using the PSDMB method performed significantly better in AT, GA, F, and M than their peers in the MB group ($P_{AT} < .05$, $P_{GA} < .05$, $P_F < .05$, and $P_M < .05$, respectively). These students then significantly outperformed their peers in the CPS group ($P_{AT} < .05$, $P_{GA} < .05$, $P_F < .05$, and $P_M < .05$, respectively). As a result, the researchers failed to reject the alternative hypothesis postulated in the research.

Table 3 demonstrates a significant and very big effect size when comparing the PSDMB and CPS teaching methods in AT (4.27), GA (3.36), F (3.70), and M (2.41). In the meantime, the comparison between PSDMB and MB in AT (2.33), GA (1.64), F (2.18), and M (1.04) revealed a significant mix of big and fairly big effect sizes in the analysis. Comparing the MB and CPS in AT (2.03), GA (1.60), F (1.68), and M (1.27), on the other hand, revealed a significant and fairly big effect size.

Table 3
Summary of Post Hoc Pairwise Comparison

	Comparison Group	*MD*	*p*	Effect	Interpretation
Algebraic	PSDMB vs MB	5.996	< .05	2.33	Big
Thinking	PSDMB vs CPS	11.497	< .05	4.27	Big
(AT)	MB vs CPS	5.501	< .05	2.03	Big
Generalized	PSDMB vs MB	1.781	< .05	1.64	Big
Arithmetic	PSDMB vs CPS	3.433	< .05	3.36	Big
(GA)	MB vs CPS	1.652	< .05	1.60	Big
Function	PSDMB vs MB	2.820	< .05	2.18	Big
(F)	PSDMB vs CPS	4.683	< .05	3.70	Big
	MB vs CPS	1.863	< .05	1.68	Big
Modelling	PSDMB vs MB	1.430	< .05	1.04	Big
(M)	PSDMB vs CPS	3.381	< .05	2.41	Big
	MB vs CPS	1.951	< .05	1.27	Big

Discussion

In general, the findings of this study indicate that students who were taught using the PSDMB method achieved significantly higher level of performance compared to those taught using the MB method. Additionally, it was observed that students who were taught using the MB method achieved significantly higher level of performance compared to those taught using the CPS method in algebraic thinking skills, including Generalized Arithmetic, Function, and Modelling. When comparing the PSDMB and CPS methods and the PSDMB and MB methods, respectively, there is a significant effect size of more than one (1), indicating that the PSDMB method is the most successful teaching method for promoting algebraic thinking skills in seventh-grade students, including Generalised Arithmetic, Function and Modelling. With a comparatively large effect size, students who were taught using the MB method performed better overall than those who were taught using the CPS method.

The intervention using the ATS Module has effectively established a structured technology-aided learning environment for students to enhance their proficiency in algebraic thinking. This is consistent with the findings of a previous study conducted by Yerushalmy (2005) and Akcaoglu (2014), students who receive assistance with digital technology are capable of solving problems and illustrating functional connections between variables, thus indirectly bolstering their problem-solving skills. Denner et al. (2012) further supported that digital technologies contribute to the improvement of cognitive abilities and problem-solving aptitude. Additionally, Szabo et al. (2020) asserted that the integration of mathematical content into problem-solving endeavours can enhance 21st-century skills as well as algebraic thinking.

Moreover, exposure to the ATS Module also facilitates the development of students' algebraic thinking skills through engaging in complex problem-solving activities. By emphasizing reasoning over rote memorization, Polya's approach to problem-solving in algebra encourages critical thinking. The ATS Module also facilitates active digital learning, encouraging a constructivist approach that empowers students to generate new knowledge, tackle problems, and engage in critical thinking. The integration of the digital bar model into problem-solving processes enables students to comprehend and execute their problem-solving strategies. The heuristic method, particularly the implementation of the digital bar model aids students in comprehending abstract algebra through visual representation. The digital bar model employed as a visual tool in the PSDMB method enhances students' proficiency in solving algebraic problems and refining their algebraic thinking skills.

Despite the PSDMB method being superior to the MB method in terms of promoting algebraic thinking skills, the MB method still demonstrates a significant impact compared to the CPS method. The CPS method primarily focuses on one-directional learning approaches, which fail to provide students with well-structured cognitive activities, particularly those pertaining to algebraic thinking skills (Rahman et al., 2022; Ryandi et al., 2018). Conventional teaching methods, as highlighted by Beneke and Ostrisky (2008), prove inadequate in promoting active learning and fail to stimulate meaningful classroom discussions. Additionally, the reliance on teachers to regulate answers impedes students' ability to engage in fruitful discussions and develop their ideas. Consequently, the conventional teaching method obstructs the development of students' algebraic thinking skills.

Generalized Arithmetic

The PSDMB method utilizing Polya problem-solving models and digital bar models has effectively provided students with opportunities to enhance their algebraic thinking skills in arithmetic generalizations that involve the efficient manipulation of numbers and generalizations. The utilization of digital visualization, particularly through digital bar models, facilitates the creation of data images that aid in the communication of information. This process requires the conversion of data from one form to another. Consequently, data visualization assists students in developing a straightforward approach to perceiving data, allowing them to quickly identify and showcase uncommon patterns before they can cultivate their thinking abilities. Ziatdinov and Valles (2022) further emphasized the numerous advantages of digital applications, such as GeoGebra, in the fields of algebra, calculus, physics, and linear programming. These applications consistently contribute to the enhancement of students' performance, capabilities, and comprehension due to their inherent characteristics, which enable students to engage in visual learning, particularly for complex and concise topics.

The PSDMB method can expand generalization and formalization involving the articulation and representation of unifying ideas that make the mathematical relationship clearly visible with the presence of digital technology. Digital technology is thought to be extremely significant in the teaching and learning of algebra (Bokhove & Drijvers, 2012; Kieran, 2007), and it has an overall positive impact on mathematical achievement (Li & Ma, 2010; Slavin & Lake, 2008). Furthermore, the results of a study by Kieran (2007) showed that early algebra learning is significantly impacted by technology use. Students' participation in technology-based learning is also boosted by this exposure to the digital world (McCrindle & Fell, 2019). In order to stimulate students' interest and speed up learning, educators have begun integrating technology into the curriculum and the teaching and learning process.

Although the findings demonstrate the proficiency of students in the PSDMB method in mastering the construct of algebraic thinking skills in arithmetic generalization, the results also indicate that the proficiency of arithmetic generalization among MB students is superior to that of the CPS method. This study is supported by the findings of Wong et al. (2020) study, which indicate that the bar model can enhance students' ability to construct mathematical models based on their understanding of arithmetic concepts compared to conventional methods of problem solving. This is because the use of the bar model assists students in solving high-level mathematical problems. The Bar model serves as a tool for students to reinforce understanding and identify relationships between operations and appropriate solutions in order to find reasonable answers. This is further supported by the study of Baoler (2016), which indicates that students are able to interpret problems by drawing rectangular bars to symbolize problem situations with their correct and accurate solutions compared to conventional problem-solving methods.

Function

The students' capacity to represent connections between distinct quantities using symbols, variables, and equations in the PSDMB and MB learning methods was also observed in their mean scores gained in this study. The findings revealed that the Function construct in the PSDMB method resulted in a significantly higher mean score compared to the MB method. This difference can be attributed to the well-planned instruction and utilization of the digital bar model as a supplementary tool in solving algebraic problems in the PSDMB method. According to Cuban (2001), visual aids, particularly those related to the sense of vision, contribute significantly to students' comprehension, accounting for up to 83% of the impact. The integration of bar model applications in the form of digital technology in PSDMB methods enhances students' problem-solving abilities. These findings align with the studies conducted by Shabiralyani et al. (2015) and Asad et al. (2016), which indicate that the use of visual materials such as pictures, animations, videos, and films stimulates thinking, initiates discussion and ultimately improves the quality of learning.

It is found that the achievement of students using the PSDMB method surpasses that of the MB method in the construct of Function. Moreno and Fuentes (2021) supported that teaching and learning in mathematical activities designed within a technological environment stimulate and maintain positive conditions in the classroom, enhance the process of experimentation, visualization, reasoning, and direct communication through student-centred learning. The ATS Module-based learning not only incorporates technology but also emphasizes student-centred instruction, providing a new dimension for students to nurture their creativity while engaging in the formulation of relationships among variables, patterns, and similarities through brainstorming activities. The way students generalize and establish relationships influences their justification of solutions (Ellis, 2007). Implicitly, as students generalize, establish relationships, and provide justification, their communication and creativity are also enhanced, contributing to the cultivation of their algebraic thinking skills in Function.

The results of the study revealed a significant difference in the mean scores between the MB method and the CPS method. These findings indicate that the MB method excels in facilitating relationships and providing justifications, thereby enhancing communication and creativity in the resolution of algebraic problems. A study conducted by Khairiree (2019) also found that lower secondary school students are capable of representing associations and relationships using the bar model. This implies that the utilization of bar models in discussing associations, relationships, and patterns has a notable impact on students' ability to solve algebraic problems.

The results of this study are further corroborated by the research conducted by Hofer (2015), which demonstrates that children have the capacity to comprehend mathematical concepts and appreciate the significance of relationships through tangible experiences utilizing the Bar Model. As such, this method serves to bridge the divide between the concrete and the abstract. Wong et al. (2020) contended that the bar model provides a more efficient and widely accepted approach to the instruction of mathematics. In instances where conventional teaching methods prove ineffective, it becomes imperative to periodically introduce innovative interventions. The systematic process of implementing the bar model strategy can assist in addressing learning disabilities that impede students' proficiency of functional thinking skills, in contrast to the CPS method. This discrepancy emerges due to the fact that the CPS method employs a teaching approach that places emphasis on the Polya problem-solving throughout the teaching and learning process yet lacks opportunities for students to engage in critical thinking in the absence of aids such as bar models.

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Modelling

The final construct in stimulating algebra thinking skills is a skill in modelling thinking, where students should demonstrate their ability to use variables, especially in problem-solving, and understand the concept of equality and the meaning of the "equal to" symbol. In the development of modelling, students who follow PSDMB interventions have progressed through a phase of comprehending problems based on visual bar models (Kriegler, 2008), reasoning algebraically through the context of generalizations and relationships that are formed and represented in various forms such as symbols and diagrams (Blanton et al., 2017), and structuring complex data systematically through heuristic methods (Marshall et al., 2006). Diagrams based on heuristic concepts also elucidate the quantity within the context of the narrative and the relationship that exists between them, aiding in the comprehension of abstract concepts and consequently facilitating the problem-solving process (Bishop, 1989). The PSDMB teaching and learning method encourages students to approach algebraic questions in a more systematic manner, guiding them step by step towards finding solutions and promoting a more open exploration throughout the process of understanding problems, formulating solution strategies, implementing those strategies, and revising selected solutions, with the availability of digital bar model.

While the MB method showed a lower level of performance in the construct of modelling compared to the PSDMB method, the achievement of the post-test mean scores of the MB method showed a significant improvement and a large effect size compared to the CPS method. This is consistent with the findings of the Pratiwi et al. (2020) study, which demonstrated that the achievement of algebraic thinking skills in students who learn to use diagrams and bar modelling is superior to that of students who learn using conventional approaches. This is because the application of tables and diagrams involves a process of translating abstract concepts and utilizing symbols to represent concrete objects, thereby enhancing students' understanding. Consequently, the use of algebraic problem-solving approaches to improve algebraic thinking skills with the assistance of bar model methods is highly recommended in the teaching and learning of algebra. This is because the MB method conveys the structure and relationships within the problem in a meaningful manner (Winn, 1987) while enhancing students' understanding of algebra due to the inclusion of three different modes of representation in the bar drawing, namely text-shaped solution questions, diagrams representing relationships, and symbols and equations (Ng & Lee, 2009).

The difference in mean scores between the MB and CPS methods was also interpreted by Buzan (2002) that the deliberate utilization of visual aids in learning would stimulate the simultaneous utilization of the left brain and right brain, thereby facilitating the development of thinking in the student's cognitive abilities and enabling effective teaching of thinking. This study's findings are also supported by Chan and Foong (2013), who explained that teaching aids like the bar model shift the focus from the final answer to the work process and the relationship between known and unknown quantities. This approach enhances students' abilities and thinking through the systematic compilation of ideas, thus facilitating abstraction in algebraic learning.

Additionally, the CPS teaching and learning method, which had the lowest post-mean scores in modelling compared to the PSDMB and MB methods and decreased by the end of the study, was due to the lack of prioritisation in teaching thinking. Changwong et al. (2018) pointed out that the absence of a clear emphasis on teaching thinking approaches hinders students' proficiency and deters them from fully applying contexts for problem-solving to develop their algebraic thinking skills (Booker & Bond, 2009; Kaput, 2008; Kaput et al., 2008; Lins et al., 2001; Schliemann et al., 2003). Toma and Greca (2018) and Afsari et al. (2021) stressed that conventional teaching and learning methods without any teaching aids fail to create an active and interactive discussion environment, which hinders students' ability to engage in idea exchange and solution formation. Furthermore, Khalaf and Zin (2018) stressed that students place a great deal of trust in their teachers, believing that they will have a significant influence on the learning environment in the classroom. The intended learning objectives are difficult to attain because of the students' passive behaviour and lack of proactive engagement.

Conclusions

The current study addresses the existing gap in the literature by utilizing the Polya's problem solving with digital bar model teaching method (PSDMB) through the use of a self-developed Algebraic Thinking Skill module (ATS). The purpose of this module is to enhance the algebraic thinking abilities of seventh-grade students on the topic of linear equations. The findings of the study indicate that students who were taught using the PSDMB method demonstrated a higher level of algebraic thinking skills compared to those taught using the MB and CPS methods. These results suggest that explicit teaching on thinking skills has a greater impact on students. However, it is important to note that creating a learning environment that promotes algebraic thinking solely through the use of problem-solving techniques and bar models is not sufficient. The inclusion of a digital module that explicitly teaches about thinking is necessary in order to effectively cultivate algebraic thinking and solve algebraic problems. This research highlights the importance of incorporating the teaching of algebraic thinking skills through the infusion of Polya's problem-solving techniques and digital bar model into secondary school mathematics lessons. By doing so, students' algebraic thinking abilities can be enhanced in the aspects of Generalized Arithmetic, Function, and Modelling.

This study offers strong evidence that, in order to promote algebraic thinking in students, primary and secondary school math teachers should include problem-solving strategies and thinking skills instruction (particularly, the use of digital bar models) in their algebra curricula. Additionally, this study encourages more research into the possible benefits of utilising various teaching strategies and thinking aids to encourage seventh-grade students to think algebraically about specific subjects. It is pertinent to recognise the limitations of the PSDMB infusion method, even though the study's findings indicate that it greatly improves seventh-grade students' algebraic thinking. With 30 students per teaching method, the study's sample size was quite small and might not be entirely representative of all secondary school students. Furthermore, following lessons learned in 10 weeks, the research's quantitative data were gathered and examined. Future studies can use a mixed methods approach, with more learning activities, a longer study period, and a larger sample size in order to fully assess the impact of using digital bar models in solving algebraic problems. Additionally, comparing rural and urban schools would shed light on how students' algebraic thinking is impacted by their surroundings in algebra classes.

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Declaration of Interest

The authors declare no competing interest.

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