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Recontextualizing Kalinga's *batek* and *laga* into an ethnomathematical teaching resource: An application of the second generation of didactical engineering

Julius Ceasar Hortelano*, Minie Rose Lapinid

Department of Science Education, De La Salle University, Manila, Philippines
*Correspondence: julius_hortelano@dlsu.edu.ph

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Abstract

Instructional materials that integrate students' cultures make teaching and learning relevant to students. However, there is a lack of culture-based and research-based teaching resources in mathematics that are deemed pertinent to Kalinga's full-modular learning and cognizant of the framework for Philippine mathematics teacher education's principle on realistic mathematics. As such, this study aims to recontextualize the Kalinga tribe's *batek* and *laga* into an Ethnomathematical Teaching Resource (ETR) through the Second Generation of Didactical Engineering (SGDE). Nine elementary teachers and two teacher-trainers participated in the preliminary phase, which was arranged into an ethnomathematical study, first teacher interview, and pretest. These were the basis of the design and a priori assumptions organized into Teaching Strategies (TS), Mathematical Content Knowledge (MCK), and appropriateness test. Subsequently, the realization phase tested the assumptions by administering the ETR, second interview, Communication Index (CI), and post-test. As a result, the a posteriori analysis and validation showed that teacher participants' exposure to the ETR provided potential TS, improved their MCK, and made appropriate to their level. Consequently, the recontextualization process of the cultures described in the study strengthens the awareness and responsibility of the Kalinga teachers to their cultural community. This also contributes to the preservation of the practices.

Keywords: Ethnomathematics, Modular Learning, Second Generation of Didactical Engineering, Teacher Development, Teaching Resources

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Kalinga, one of the recognized ethnolinguistic groups in the Philippines, is home to the traditional tattoo, *batek*, and native textile, *laga*, which identify the Kalinga people as distinct from other ethnolinguistic groups in the country. Primarily, *batek* is practiced after successful headhunting in commemoration of a warrior's bravery and serves as a symbol of economic status, a talisman against evil spirits, and a rite of passage, specifically in the Butbut tribe of Northern Philippines (Calimag et al., 2016; Krutak, 2010; Salvador-Amores, 2011). *Laga* is the traditional clothing worn by the ethnolinguistic group during burials and special occasions such as festivities, courtships, and *bodong* (peace pact) across the homeland (De Las Peñas & Salvador-Amores, 2016). Among the cultural practices of Kalinga, these two have been a growing research interest in ethnography, aimed at reliving these practices from near extinction and unraveling the intricacies present in them. *Batek* is one of the earliest practices in the country dating back to the Spanish colonization, and for over three centuries, its popularity has been declining (Demeterio III, 2017). Research initiatives were conducted to recalibrate this practice for the new generations and take on a new perspective on traditional tattooing. For example, De Las Peñas and Salvador-Amores (2019) studied the mathematics behind the tattoo designs, preceded by Salvador-Amores (2011), who described them as "geometrical." Particularly, they contain motifs that are indicative of points, lines, angles, symmetry, and tessellations. On that same note, De Las Peñas and Salvador-Amores (2016) offered anthropological and mathematical perspectives on *laga* textiles and found that symmetry and frieze patterns are among the features of the design. While these mathematical explorations of *batek* and *laga* are essential and offer a different perspective on the practices, there is a strong recommendation regarding their direct application and recontextualization in education to contribute to their preservation (Demeterio III, 2017). As such, ethnomathematics plays a vital role (D'Ambrosio, 1985).

Ethnomathematics is defined by D'Ambrosio (1985) as the "mathematics practiced among identifiable cultural groups such as national-tribe societies, labor groups, children, and professional classes" (Barton, 1999; Bishop, 1991). It is a growing research interest in Asia, implying that cultural communities are capable of performing mathematical activities even in the absence of formal education (Prahmana & D'Ambrosio, 2020; Rafiepour & Moradalizadeh, 2022). Later on, it developed as a movement against Eurocentrism and the dominance of Western mathematicians, which enabled African and Asian mathematics to be recognized and studied as well (Knijnik, 1993; Powell & Frankenstein, 1997). It is also a sustainable approach in mathematics pedagogy that integrates cultural practices as practical applications of abstract mathematical concepts (Pais, 2013; Shulman, 1986) and a constructive way of designing culturally relevant lesson materials that could bolster students' motivation (Alangui, 2017; Gerdes, 1994; Verner et al., 2019) and situate their learned knowledge (Lave & Wenger, 1991; Renkl, 2001). On the other hand, recontextualization seeks to preserve these cultures for a long time by reframing them in a different context, such as in education. This is an essential task given primarily to teachers in preparing lessons and developing teaching materials that are culturally responsive to the students' context (Science Education Institute- Department of Science and Technology [SEI-DOST] & Philippine Council of Mathematics Teacher Education [MATHTED], 2011; Utami et al., 2021).

Since the pandemic, teachers are compelled to design self-made and contextualized modules that interrelate the current lesson with students' local context, such as their culture. In mathematics, for example, word problem scenarios should integrate students' lived experiences for them to visualize questions and arrive at solutions. This is espoused in the Philippine mathematics teacher education framework by the SEI-DOST and MATHTED (2011), wherein teachers are to include building mathematical connections to their lessons and materials among their responsibilities. Such type of teaching task is challenging for teachers, especially if they do not have ample Pedagogical Content Knowledge (PCK) for Teaching Strategies (TS) and Mathematical Content Knowledge (MCK), specifically for elementary teachers who have limited MCK (Copur-Gencturk & Tolar, 2022; Hill, 2010; Verner et al., 2019). In addition, limited technological resources on the school's end made it more troublesome to execute mathematics instruction while adhering to the said duty. These problems brought about the need for training and needs assessment regarding the design of TS that are reflective of students' contexts. In doing so, the Second Generation of Didactical Engineering (SGDE) can be utilized as a method as it focuses on such teacher problems, particularly on the production of teaching resources out of their needs and developments (Artigue, 2014; Laborde & Perrin-Glorian, 2005; Tempier, 2016). This opens opportunities for researchers and teacher-trainers to collaborate and discuss professional development for teachers that could provide support in executing the specific task given to them (Jones & Pepin, 2016).

The SGDE process starts with phase 1, the preliminary analysis, which considers two levels of questioning: epistemological and institutional. The epistemological level determines the problem to be addressed in didactics (e.g., students' mathematical performance), investigates related histories, and engineers' appropriate solutions. The institutional analysis looks deeper into the context of school and teachers and investigates ordinary practices, points for support, and their possibilities for development, all of which are related to the problem determined. The findings from the first phase are crucial for the second phase, which is the design and a priori analysis. The design focuses on devising remediations based on the problem (e.g., creating resources to assist teachers and students), whereas the a priori analysis generates assumptions on the effects of the remediation to the problem before actual observations. The third phase, realization, tests the assumptions generated through a series of data collection and appropriate measures depending on their nature and how they are best addressed. Lastly, a posteriori analysis and validation interpret the results of the data collected from the previous phase, and internal validation is conducted to compare the a priori and a posteriori analyses (Alves et al., 2020; Artigue, 2014; Margolinas & Drijvers, 2015). This rigorous process is determined to be appropriate to the identified problems of producing a teaching resource and evaluating the needs of teachers in terms of their PCK and MCK. According to SEI-DOST and MATHTED (2011), improving these types of knowledge among teachers allows them to deliver lessons using appropriate TS while having content mastery of the subject matter. More specifically, integrating culture into mathematics pedagogy can improve teachers' teaching strategies and knowledge about mathematics (Alangui, 2017).

While previous studies, such as De Las Peñas and Salvador-Amores (2016; 2019), are consistent with the goal of exploring the mathematical features of *batek* and *laga*, there is still a gap in research on how these cultural practices can be maximized to improve the mathematics teachers of Kalinga and their potentialities in designing instructional materials that are culture-based. In addition, as SGDE is mainly associated with developing resources for teachers (Artigue, 2014; Margolinas & Drijvers, 2015), exploring an ethnomathematical study by utilizing its four phases has yet to be conducted. As such, this paper describes the recontextualization of the *batek* and *laga* into an Ethnomathematical Teaching Resource (ETR) through the SGDE. The study's interdependent twin goals are (1) to uncover the mathematical features of the two cultural practices and (2) to improve Kalinga elementary teachers' PCK and MCK on the selected topics in Geometry through the development of the ETR. These permit a new perspective on the practice of traditional tattooing and wearing of the traditional textile by the Kalinga people, emphasizing the value of culture in a different context. By recontextualizing it into the ETR, the study contributes to the cultural preservation of *laga* and *batek* through their integration into mathematics education. It also serves as a reference point for other ethnolinguistic groups on how to utilize unique traditional practices in developing teaching strategies for teachers around the globe and designing instructional materials using their own culture, especially during precarious situations such as a pandemic. This study also offers an alternative viewpoint for researchers on how to conduct an ethnomathematical study through the process of SGDE. Moreover, the following research questions are addressed in the study:

1. What are the mathematical features of *batek* and *laga*?
2. What is Kalinga elementary teachers' professional development needs as regards their TS and MCK?
3. What assumptions can be made based on the preliminary analysis and design of the ETR in terms of (i)TS, (ii) MCK, and (iii) appropriateness of the ETR for the teachers?
4. How can the assumptions be tested?
5. How do the analyses support the assumptions?

METHODS

Research Design

The SGDE is utilized in the study, which is comparable to design-based research that aims to improve teaching and develop teaching resources (Alves et al., 2020; Artigue, 2014; Barquero & Bosch, 2015; Godino et al., 2012;). Moreover, a mixed method was used. Qualitative data consist of the findings from the ethnomathematical study and structured preliminary interview and post-interview. On the other hand, quantitative data comprise the value for the Communication Index (CI), which is a readability formula in determining the appropriateness of science and mathematics materials (Talisayon, 1983), participants' evaluation of the ETR, and the pretest and posttest scores.

Research Locale and Participants

The study was conducted in the rural district of Northern Pinukpuk, province of Kalinga, Philippines, where the two cultures are situated and practiced. Out of the 11 elementary schools in the district, nine accepted the request for participation, while 2 of the schools declined due to the unavailability of the teachers. Subsequently, nine elementary teachers, one from each school, and two teacher-trainers were chosen from the district through purposive sampling.

'Participants' of the study refer to teachers and teacher-trainers who have undergone the material development process that required their involvement – preliminary phase, design, and realization – arranged in three separate sessions. The teacher participants are local to the province and knowledgeable about the nature of the *batek* and *laga*, while the two teacher-trainers are culture experts, mathematics majors, master teachers, and experienced in training teachers.

Data Collection

Data were collected according to the phases of SGDE. The preliminary phase starts with the epistemological analysis where issues concerning teacher education and fundamental principles were identified through a review of related literature to lay out the grounds for the study. In addition, visitations were done to historical sites, such as the Buscalan village in the Butbut tribe, where the researchers observed directly the practice of tattooing; Kalinga's tourism office to document and record the designs of *laga* as well as the cultural meanings behind the motifs; and archival works using offline and online books to validate results (Cimen, 2014; Demeterio III, 2017; Krutak, 2010; Salvador-Amores, 2011). In this phase, it is where the ethnomathematical study was conducted, and an observation note was utilized to document the qualitative findings. Proper consent forms were prepared for the different research sites, seeking permission, and outlining the purpose of the visit. The researchers also sought the permission of the local elders of the community in their mother tongue for ease of communication. After this, the institutional analysis was carried out by choosing the teacher participants, describing the geographical site of the schools, determining teaching needs through the preliminary structured interviews with the participants, and measuring their baseline MCK in geometry through a pretest.

Based on these preliminary findings, the design phase started the construction of the ETR, and a priori assumptions were generated according to TS, MCK, and its appropriateness for the participants. After generating assumptions, the ETR was distributed among nine elementary teachers, and they were given seven days to examine and study the content. Consequently, the assumptions were tested in the realization phase through different research instruments: a post-structured interview, an evaluation questionnaire of the ETR (see Table 1) adapted from Torrefranca (2017), the CI of each section of the ETR based on Talisayon (1983), and the posttest. Both the pretest and posttest were constructed by the researchers, and these were content-validated by the two teacher-trainers based on the Department of Education's (DepEd) Most Essential Learning Competencies (MELCs).

Table 1. Evaluation Questionnaire of the ETR by the participants

Aspects of the ETR	Items	*M*	*SD*
1. Format of the ETR	1.1 The layout of the teaching resource is arranged in a logical and sequential order, that is, from simple to complex.	4.82	.40
	1.2 The illustrations (e.g., geometrical shapes, *batek* and *laga* designs, tables, etc.) used in the teaching resource are properly laid out for easy reference.	5.00	.00
	1.3 The font size and font style of the teaching resource are readable.	4.73	.47
	1.4 Key points and key concepts are well highlighted to focus attention while reading.	4.36	.67
	1.5 The instructions of the ETR are easy to follow.	4.36	.67
	1.6 The local terms in the teaching resource are properly used.	4.54	.52
	1.7 The meanings of the *batek* and *laga* are correct.	4.55	.69
2. Content of the ETR	2.1 The connection of the *batek* and *laga* to the geometry lessons is well defined.	4.64	.67
	2.2 The integration of *batek* and *laga* is appropriate in teaching the geometry lessons.	4.82	.40
	2.3 The concepts of *batek* and *laga* make it easier to understand the geometry lessons.	4.64	.50
	2.4 The use of *batek* and *laga* designs in the teaching resource clearly represents the ideas of the geometry lessons.	4.73	.65
	2.5 The objectives of the teaching resource are in line with the learning competencies of the geometry lessons prescribed by the Department of Education (DepEd).	4.64	.50
	2.6 Adequate examples are given to each topic in the ETR.	4.55	.69
	2.7 The meanings of the *batek* and *laga* in the teaching resource are well utilized.	4.73	.47
Overall		**4.65**	**.17**

Note: 4.5-5.0- excellent; 3.5-4.49- very good; 2.5-3.49- good; 1.5-2.49- fair; 1.0-1.49- poor

Both tests have thirty items, each having fifteen items on undefined terms, parallel lines, and perpendicular lines, five items on symmetry, and ten items on tessellations. Results gathered to this point were the basis for the interpretation in the a posteriori analysis. Finally, a comparison was made between the a priori and a posteriori analysis in the internal validation (Artigue, 2014; Laborde & Perrin-Glorian, 2005; Margolinas & Drijvers, 2015). Figure 1 summarizes the flow of the data collection method.

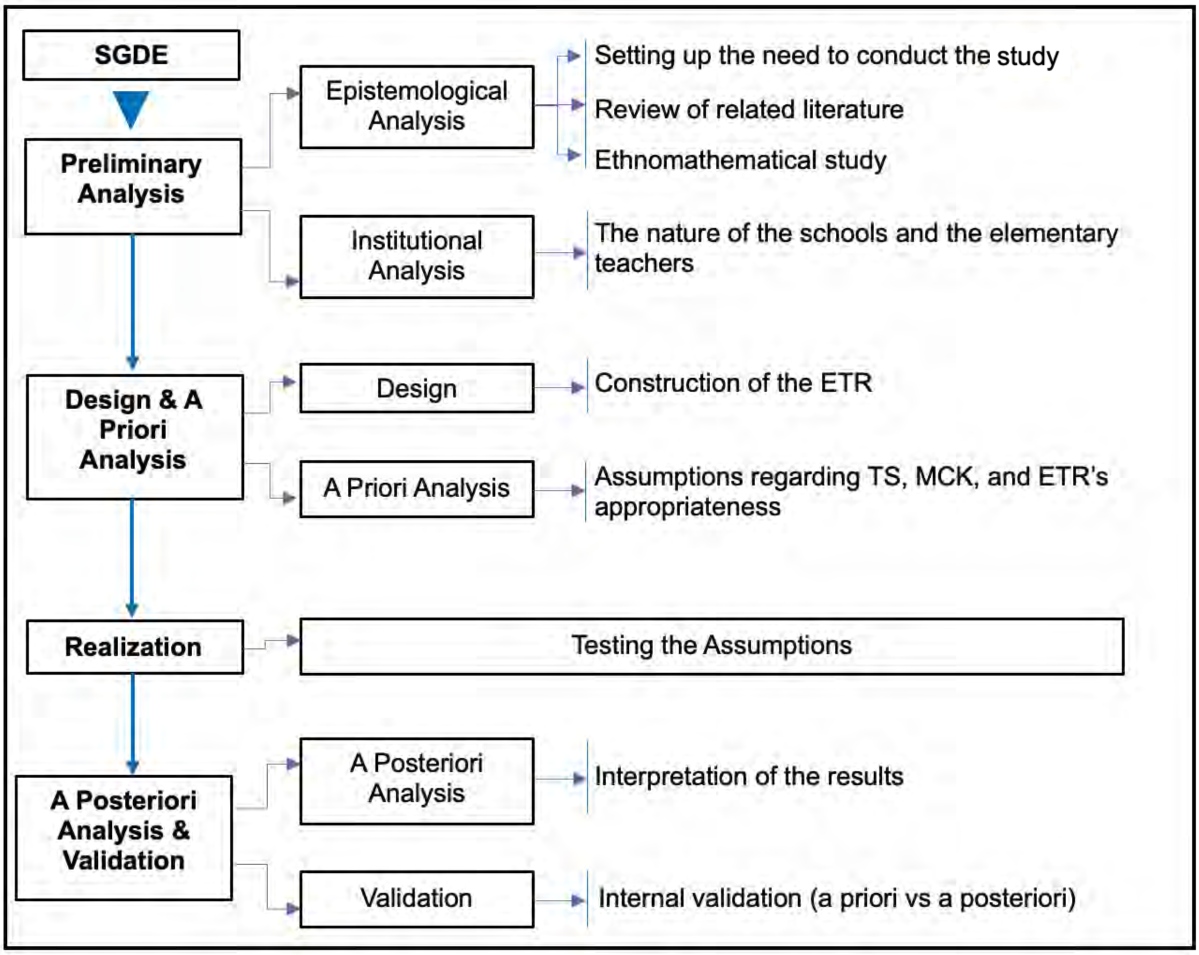

Figure 1. The SGDE process of the study

Data Analysis

Qualitative data from the preliminary interview and post-interview were analyzed using thematic analysis (Nowell et al., 2017) to determine TS arising from the use of the ETR. Their responses were recorded and reviewed by the researchers to get an overall view of the data. Then, coding the responses was performed by highlighting sentences that are useful in answering the research questions and coming up with short labels. Themes were then generated according to the similarities and central ideas of the different labels, and these were reviewed on how they addressed the research questions. To ensure the validity of the results, the analysis was checked and corroborated by the second author. Furthermore, member checking was also carried out to ensure that the result of the analysis was agreed upon among the participants. Consequently, no participant contested the results. Moreover, the findings from the ethnomathematical study were analyzed by listing the presence of mathematical features present in the *batek* and *laga* (Knijnik, 1993). For the pretest and posttest scores, the Wilcoxon signed-rank test was used to compare the scores because of the small sample size. The result of the evaluation questionnaire was analyzed using descriptive statistics, particularly mean (*M*) and standard deviation (*SD*). Lastly, the CI was interpreted using the range of acceptable value, $0 \leq \mathrm{CI} \leq .01$, from Talisayon (1983), with the formula (1):

$$\mathrm{CI} = \frac{\Sigma(\mathrm{FX})}{(\mathrm{Nr})(\mathrm{Nx})} \quad (1)$$

where F is the number of readers indicating a given unclear word; X is the number of times the word appeared in the samples; Nr is the total number of readers; Nx is the total word sample.

RESULTS AND DISCUSSION

Preliminary Analysis

Epistemological analysis: One of the principles in the *framework for Philippine mathematics teacher education* underscores teachers' responsibility to integrate lived experiences and local context:

> "Principle 2: Mathematics must be real to students and therefore, mathematics teachers should be mindful of students' context when teaching mathematics." (SEI-DOST & MATHTED, 2011, p.5)

This principle is best applied to schools where there is an abundance of 'real-life' mathematical connections useful in teaching mathematics and situating knowledge. In Kalinga, cultural practices can produce concrete mathematical representations by studying the cultures closely through ethnomathematics.

Ethnomathematics of batek and laga. (What are the mathematical features present in *batek* and *laga*?) The *batek* of Kalinga has been mainly associated with *kayaw* (headhunting) since 1600 (Demeterio III, 2017). It is done by piercing the skin using the *gisi* (a local tattooing tool), which is composed of a *pat-ik* (a wooden stick) and a *parakuk id lubfan* (a citrus thorn). The sharper end of the thorn is soaked into a wet mixture of pulverized charcoal and water by the *mambabatok* (tattoo artist) for a darker and more durable result. Payment of such tattoos is based on a *changan*, a traditional way of measuring using the tip of the thumb to the tip of the middle finger, which is about four inches. The *laga* is the clothing worn by the Kalinga people decorated with beads and stripes. Using a traditional weaving tool, four colors are used to weave the textile: red, yellow, green, and black, which symbolize loyalty, joy, nature, and bravery, respectively. To this date, there are 23 designs of Kalinga textiles categorized as *kain* (women's skirt), *ba-ag* or *be-e* (men's g-string), and *kagoy* (blanket), each having different applications and meanings.

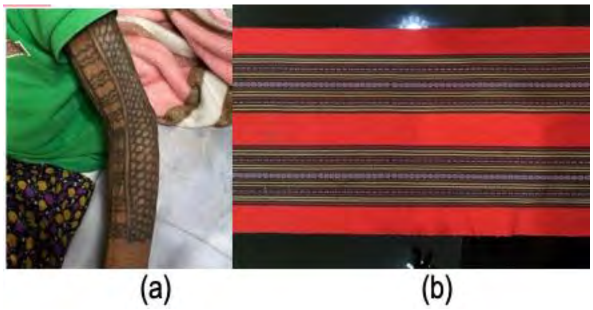

Figure 2. (a) *Inob-obok*, a *batek* design tattooed on an elder's arm; (b) a *laga* used as a blanket

Notwithstanding that both practices are said to be cultural, these are also geometrical. Through archival works, previous ethnographic studies such as De Las Peñas and Salvador-Amores (2016, 2019) and Krutak (2010) concluded that *batek* contains geometric elements such as points, lines, symmetry, and tessellation, and both have provided significant information about the meanings of the tattoos necessary in the design phase. Similarly, *laga* contains perpendicular lines, parallel lines, geometric shapes, and symmetrical motifs (see Figures 2, 3, and 4 for example), which is perceivable by the eye.

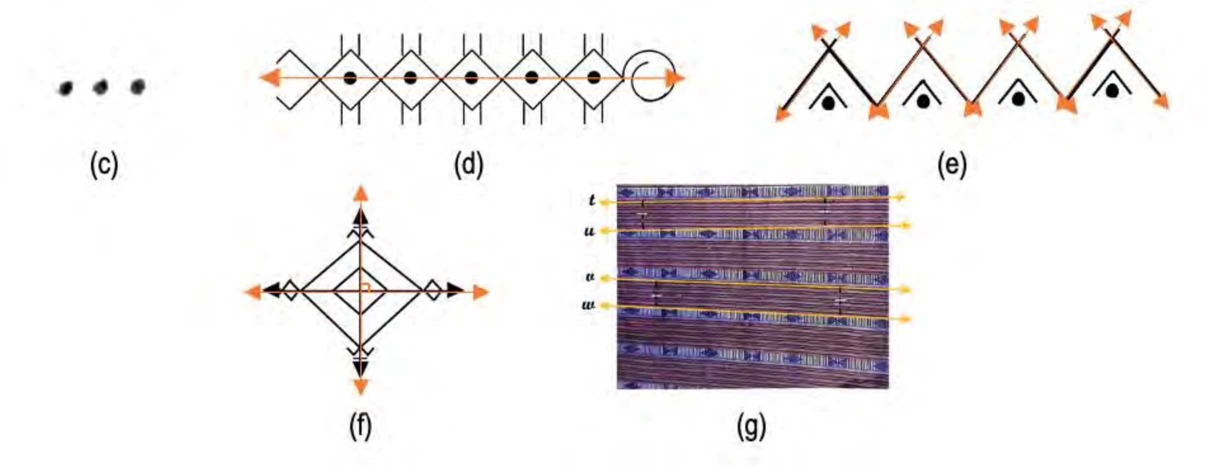

Figure 3. (c) Whang-od's signature, (d) *urog* (snake), (e) *kararag* (prayer), (f) traditional compass, and (g) *lilus-lusong* (mortar)

Apo Whang-od Oggay is the oldest and most famous traditional tattoo artist in the Philippines, known for her signature design. It is composed of three collinear points representing herself, her apprentices, and the next generation of *mambabatok*. The *urog* symbolizes health and protection, characterized by five consecutive rhombuses, each containing a point where a line can be drawn. The *Kararag* symbolizes faith among the Kalinga people depicting hands crossing together and forming intersecting lines. The traditional compass design comprises two perpendicular lines, each extending in opposite directions, portraying the four cardinal directions (North, South, West, and East). The lines of *lilus-lusong*, a type of *laga*, are said to be parallel to each other with a fixed number of threads between the thicker lines, making the distance between lines equal.

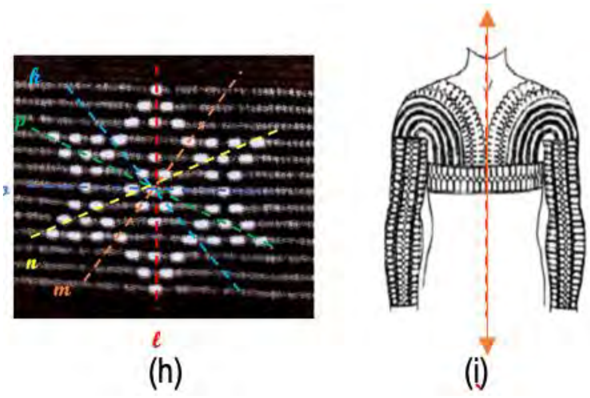

Figure 4. (h) *Bituwon* (star) and (i) *whiing* tattoo

From the *silambituwon*, another type of *laga*, the *bituwon* is a six-pointed star or a hexagram that is symmetrical through six lines. The *whiing* (H. Meyer's drawing in Calimag et al., 2016), often called *bikking*, is one of the most common tattoo designs that is considered the mark of a true warrior during the Japanese occupation in the country. It is characterized as symmetrical through the line between the chests.

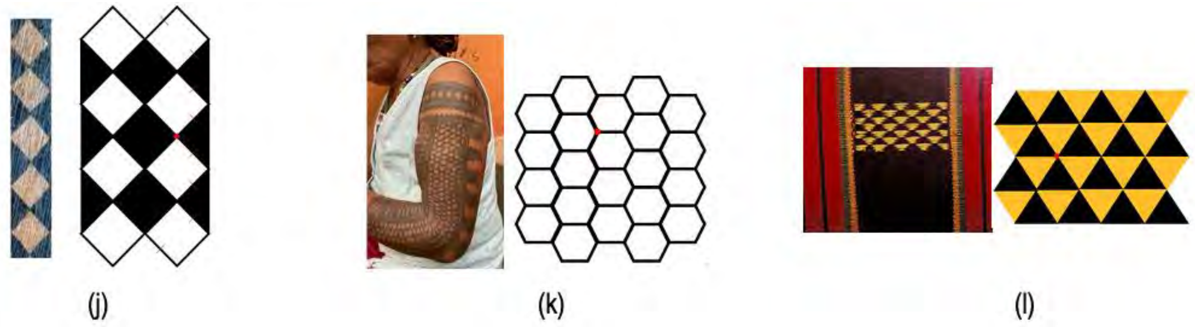

Figure 5. (j) *Tinay'talu* (rice bundles), (k) *Inob-obok* (mat and snake skin), and (l) *Bilallektad*

The *tinay'talu* is a tattoo design that depicts rice as the lifeblood of Kalinga province, and its design depicts a "4.4.4.4" tessellation with a square motif. Similar to the *whiing*, the *inob-obok* is a typical tattoo design of the arm among men and women and it depicts a "6.6.6" tessellation with a hexagon motif (see Figure 5). The *Bilallektad*, which means inverted, is a *laga* design with a "3.3.3.3.3.3" tessellation with a motif of a triangle. Accordingly, "4.4.4.4", "6.6.6", and "3.3.3.3.3.3" are the regular tessellations that are named based on the number of regular polygons being tessellated around a common vertex (red dot) and the number of sides of that polygon. Aside from the works of De Las Peñas and Salvador-Amores (2016; 2019), Krutak (2010), and the data gathered in Buscalan Village, the meanings were validated by Kalinga's tourism office to ensure the correctness of the gathered information.

These mathematical perspectives behind the practice of *batek* and *laga* have clear educational implications. It is also necessary to note their social and cultural implications as to why Kalinga people get tattoos and wear traditional textiles. For example, various meanings are hidden behind the designs of *batek*, such as the *whiing* and *inob-obok*, which corresponds to bravery and fortitude, respectively. *Laga* textiles are also worn for several reasons. The *lilus-lusong* is given as a community token during peace pacts between neighboring tribes, and the *bilallektad* is used as work clothing (see Figure 6). These implications are considered essential for the design of the ETR as they help students and teachers develop the value and appreciation of these practices and strengthen their identity as Kalinga people because they see the application of their culture in the context of mathematics education (D'Ambrosio, 1985). In addition, when these practices are integrated into their lesson, it is likely that they will be able to remember the mathematical concepts learned in the classroom as they see *batek* and *laga* designs in their surroundings and as they deal with their own lives. Although these ethnomathematical findings are essential for the design of the ETR, the current practices and needs of the teacher participants should also be identified through institutional analysis.

Institutional analysis (What are the needs and development of the Kalinga elementary teacher participants in terms of TS and MCK?): Geographically, the participant schools are situated in the rural areas of Kalinga with limited access to internet connection and technological resources. During the provincial lockdown caused by the COVID-19 pandemic, access to school was impossible, and the division office of DepEd Kalinga opted for full-modular learning. The sudden change of situations challenged school administrators and teachers on how to adapt to the current policies of the new normal of education, particularly designing modules in modular learning. Given the lack of internet and technological resources, the rich culture of Kalinga offers an alternative tool for mathematics teachers, but only possible if they are aware of the connections between mathematics and culture. In a social context, the *batek* and *laga* are only conferred during the Indigenous Peoples' (IP) month celebration, October of each year, which affects the awareness of teachers and students on the use of both cultural practices in the pedagogy of mathematics. Thus, discussing the result of the ethnomathematics among the teachers was necessary before proceeding to the design phase. This was done by distributing pamphlets that contain such discussions and preparing them for the succeeding phases. They are also reminded of the cultural and historical implications of the *batek* and *laga* embedded in the ETR rooted in the ethnomathematical study. Teacher Lily, one of the teacher-trainers, gave her first thought about the benefits of ethnomathematics:

> "This will surely make learning mathematics enjoyable in the province, especially since there is a lack of teaching resources in mathematics, and there is a challenge for the teachers to develop strategies wherein culture is integrated into the lesson."

Subsequently, the pretest was administered to assess teachers' initial MCK and there were two points of weakness observed. First is the use of technical terms in the definitions and descriptions of geometric elements wherein teachers stated the following:

> "Lines are made up of points that are close together."
> "Congruent segments are lines that have two equal endpoints."
> "A ray is a line that has an endpoint and an arrowhead that extends."

And the second is the use of proper symbols in geometric notations, wherein there is a common error among teachers' answers (see Table 2 for the responses). These weaknesses were believed to affect the capabilities of teachers to deliver lessons effectively and hinder their achievement of the framework's recommended MCK for elementary teachers.

Table 2. Teachers' Common Errors in Geometric Notations

Naming a…	Teachers' response	Correct answer
line	A to C	$\overleftrightarrow{AC}$
Line segment	H to B	$\overline{HB}$
Ray	B to G	$\overrightarrow{BG}$
Congruent segments	A – C	$\overline{AB} \cong \overline{BC}$
Parallel lines	A C	$\overleftrightarrow{AC} \parallel \overleftrightarrow{DF}$
Perpendicular lines	A to C	$\overleftrightarrow{HG} \perp \overleftrightarrow{AC}$

Design and A Priori Analysis

Design: This phase started the construction of the ETR based on the preliminary findings arranged three-fold: (i) ETR's design for TS, (ii) ETR's design for MCK, and (iii) ETR's appropriateness for the participants.

ETR's design for TS: Embedded in the ETR are 27 figures from the tattoo designs of *batek,* eight figures from the motifs of *laga*, and four supplementary geometric figures, which are all essential in visualizing abstract mathematical concepts. Aside from these mathematics concepts, cultural and teaching implications were also integrated so that the essence of the cultures being recontextualized is not compromised.

ETR's design for MCK: The content of the ETR is about the lessons on undefined terms, symmetry, and tessellation. To give teachers a deeper understanding of the geometry lessons, technical definitions and proper symbols and notations were highlighted with the aid of the researchers' and teacher-trainers' expertise on the subject matter.

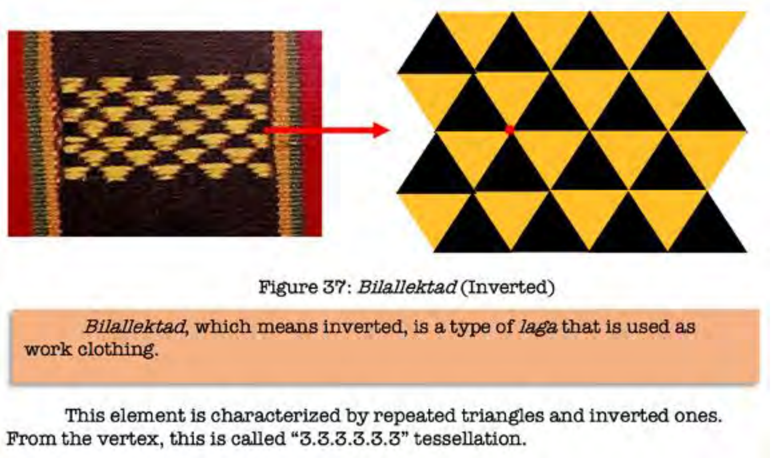

Figure 6. An example of a geometric concept emphasized in the ETR with cultural implications

ETR's appropriateness for the teacher participants: The content of the ETR is based on DepEd's MELCs and the K to 12 mathematics curricula. A combination of English and Kalinga terms (e.g., *gilamat*, *lilus-lusong, bilallektad*) were used to ensure understanding of the texts and to follow the mother tongue teaching policy from kindergarten to grade 3.

A priori analysis: (What assumptions can be made based on the preliminary analysis in terms of (i)TS, (ii) MCK, and (iii) appropriateness of the ETR for the teachers?)

Three general assumptions (Artigue, 2014) were generated by the researchers after the conduct of the preliminary analysis and the design phase:

***Assumption 1*:** The ETR is beneficial for the participants as it provides additional TS in teaching geometry lessons. This contributes to D'Ambrosio (1985) and Pais (2013) realization regarding ethnomathematics as a pedagogy tool and as a reference for teachers in designing culturally relevant mathematics lessons.

***Assumption 2*:** The content of the ETR addresses the points of weakness of the participants, which is necessary for the achievement of SEI-DOST and MATHTED's standards. After the dissemination and thorough review of the ETR, scores in the posttest are higher, or at least with minimal errors, compared to the pretest as evidence of an improved MCK.

***Assumption 3*:** The ETR's format and content are appropriate to the level of the participants as measured through the evaluation questionnaire. In addition, the CI is within the range of acceptable values as it is designed to fit the level of elementary teachers' comprehension of geometry.

Realization, a Posteriori Analysis, and Validation

Of the nine elementary teachers and two teacher-trainers who completed the preliminary analysis of the study, a complete number of participants remained for the realization phase. Each was given a hard copy of the ETR for them to read and review for seven days. Through data collection, which answers the fourth research question of the study, the three assumptions were tested followed by discussions:

Testing Assumption 1: Elementary teachers' post-interview responses were analyzed. All the participants agreed on the importance of the ETR in teaching undefined terms, symmetry, and tessellation. Specifically, four themes were generated as the benefits of the ETR for TS:

1. Situated learning - The use of the ETR assisted elementary teachers in building mathematical connections by designing geometric tasks related to the local context of Kalinga. Teachers Lily and Roger, both teacher-trainers and mathematics majors, agreed that the tattoo designs of *batek* and the motifs of *laga* involve geometric concepts appropriate to be used as concrete examples for the Kalinga learners and said:

   "This (ETR) will, consequently, upgrade the culture and input meaningful learning as they see the importance of mathematics in real-life…teachers will have enough knowledge on how to go about culturally relevant mathematics and to provide support to students as they try to connect math in real life."

2. Systematic teaching - The design of the ETR guided the elementary teachers in the flow of the lesson. In each stage of lesson development, they are provided with figures suitable for motivational activities, lesson proper, and assessment that are also appropriate according to the objectives and competencies found in MELCs. Since the figures are geometrically labeled, concrete figures are easier to integrate into the lessons. For instance, teacher Karen, one of the participants, asserted that:

   "I would use the *lilus-lusong* to let my students recognize a line; the *whiing* or *biking* to visualize symmetry; and the *inob-obok* to visualize tessellation. This is very appropriate as the verbs 'recognize' and 'visualize' are the cognitive verbs used in MELCs and the DepEd's K to 12 mathematics curriculums for the geometry lessons."

3. Comprehension of the lesson - The teacher's mastery and understanding of the lessons are essential for effective teaching. According to the participants, the ETR gave them additional knowledge and an in-depth review of the lessons necessary for them to disseminate knowledge effectively. The teacher-trainers have asserted that this would enable elementary teachers to attain content expertise as recommended by the *framework.* Teacher Teresita, one of the participants, agreed with the teacher-trainers' assertion and stated:

   "Elementary teachers are not given in-depth mathematics lessons in their pre-service years, unlike secondary pre-service teachers. So, having this ETR would help me as it is for my additional knowledge in geometry."

4. Development of culture-based material – Teacher-trainers added that the ETR could be used as a reference in developing culture-based materials in modular learning and typical classroom setup. They were given ideas on how to recontextualize other forms of culture, aside from *batek* and *laga,* in teaching so that students are more inclined to learn. Furthermore, teacher-trainer Lily supported the claim of the ETR towards cultural preservation through education and expressed:

"This material will contribute to the Indigenous Knowledge, Skills, and Practices (IKSP) of Kalinga, and it has the power to teach the teachers how to construct lesson materials like modules that contain our identity as an ethnic group."

Testing assumption 2: *A* Wilcoxon Signed-Rank test indicated that posttest scores (Mdn= 20) were statistically significantly higher than the pretest scores (Mdn= 15), z = -2.54, p < .011. This further implies that the ETR was effective in improving the knowledge of the participants regarding the geometry lessons, and proper use of symbols and notations, or at least minimal errors, was observed in the participants' individual responses in the posttest.

Testing assumption 3: The evaluation questionnaire was distributed to all the participants after the posttest. Both aspects were evaluated, the format and the content, with an overall rating of excellent (M= 4.65, SD= .17). For the CI, the 9 teacher participants were regarded as readers, and all three chapters of the ETR were considered: chapter 1 on undefined terms has 1768 words, chapter 2 on symmetry with 948 words, and chapter 3 on tessellation with 559 words. Table 3 summarizes the result of the CI.

Table 3. CI computation

Unclear words	X_1	X_2	X_3	F	FX_1	FX_2	FX_3
Laga	8	7	4	1	8	7	4
Tessellation	0	0	18	2	0	0	36
Symmetry	0	27	0	2	0	54	0
Batek	7	5	4	1	7	5	4
Hashmark	1	0	0	1	1	0	0
Perpendicular	12	0	0	1	12	0	0
Regular tessellation	0	0	4	1	0	0	4
Reflectional symmetry	0	3	0	1	0	3	0
Translational symmetry	0	3	0	1	0	3	0
Total					**28**	**72**	**48**

Note: $0 \leq \mathrm{CI} \leq .01$; X_1 indicates the number of word appearances in chapter 1; X_2 in chapter 2; X_3 in chapter 3; F indicates the number of readers indicating the unclear word.

Hence, the CI for chapters 1, 2, and 3 are as follows:

$$\mathrm{CI}_1 = \frac{28}{(9)(1768)} = .001 \tag{2}$$

$$\mathrm{CI}_2 = \frac{72}{(9)(948)} = .008 \tag{3}$$

$$\mathrm{CI}_3 = \frac{48}{(9)(559)} = .0095 \tag{4}$$

Hence, all the values of CI are acceptable (Talisayon, 1983) and need no further revision in terms of language and vocabulary. The small indices are caused by a small percentage, 3.1% to be exact, of the appearance of the unclear words in relation to the ETR's total word count. While the CI considers the readability of tables and figures, no tables and figures were listed as unclear for the participants. Lastly, as teachers' and teacher-trainers' involvement were about to be completed, the researchers reminded them of the proper execution of the mathematical and cultural contents of the ETR. For example, as they elaborate the lesson on symmetry using *laga*, it is also necessary to integrate the cultural meaning of the chosen *laga* design. This is to maintain the cultural value and implications of these practices.

In terms of validation and to address the last research question, 'general assumptions' (Artigue, 2014) made in the a priori analysis was all supported in the realization phase and there were no contradicted results. However, the researchers did not anticipate specific data such as the IKSP and systematic teaching as these can only be gathered through a series of data collection. As explicated by one of the teacher trainers, the IKSP, or the Indigenous Knowledge Systems and Practices is one of the programs by the DepEd to strengthen the cultural awareness of teachers and students. By utilizing the ethnomathematics found in *batek* and *laga*, the result of the study contributes to the collection of indigenous knowledge and practices already present in the IKSP domain. There were also unclear words that were not anticipated as seen in Table 2. Nonetheless, these were clarified among the participants as the ETR utilizes widely used terms based on Krutak (2010), Salvador-Amores (2011), and Calimag et al. (2016). Moreover, the results further support previous studies that have conducted ethnomathematical studies in the context of mathematics education, such as Alangui (2017) and Prahmana and D'Ambrosio (2020). It is evident that the findings from the ethnomathematical study and the recontextualization process are effective in addressing the needs of the elementary teachers of Kalinga, specifically improving their MCK and devising potential TS that are reflective of their students' culture. These findings also contribute to the research gaps, expounding the findings of De Las Peñas and Salvador-Amores (2016; 2019) regarding the potentialities of the cultural practices in developing teaching resources and the practice of mathematics teachers of Kalinga.

CONCLUSION

The development of the ETR is beneficial to the elementary teachers of Kalinga as a resource in designing culturally relevant lessons. It is also evident in improving their PCK and MCK by knowing how to teach culturally relevant mathematics while having mastery of the subject matter. Subsequently, this supports them in complying with the national standard set by SEI-DOST and MATHTED regarding the principle of realistic mathematics and the level of MCK they should display as mathematics teachers. Through the interaction of the researchers, teacher-trainers, and the elementary teachers of Kalinga, the recontextualization of *batek* and *laga* into the ETR was initiated. Interactions among these three actors were imperative in research to properly assess elementary teachers' needs and their possible ways of remediation. Other local schools can reproduce the ETR as a contribution to their learning resources and be utilized whenever needed.

With the schools' location in the rural areas of Kalinga, the lack of teaching resources is a challenge. The absence of an internet connection and less conferred cultures limit their ability to design creative instructional materials and think of culture-based and context-based TS. It has been apparent through the study's results, particularly the ethnomathematical findings, that the culture of Kalinga is a rich source of ideas for teaching mathematics and designing self-made modules in modular learning. This could pave the way for teachers to integrate traditional practices in teaching mathematics that could support their instruction in delivering authentic activities and accentuate appreciation and awareness of local practices in honor of the cultural community. The recontextualization of *batek* and *laga* into the ETR is a means to preserve cultures that have existed for several years and a way for elementary teachers to fill the gap between the practical application of mathematics and real-life context. Training teachers to embody these responsibilities and be consistent with the practice may take time, but the development of the ETR signifies a milestone in IP education and Kalinga's cultural preservation. The division of Kalinga should opt for wider teacher training in collaboration with researchers who are experts in the field of ethnomathematics, and the developed ETR can be a reference point for more ethnomathematical resources. As a reference for teachers, they would have the sufficient cultural and mathematical knowledge to teach the lessons in authentic ways, such as bringing actual *laga* designs into the classroom, printouts of *batek* designs, or even using other cultural practices found in the community. It is envisioned that there will be no lapse in the social and historical significance of the traditional practices as teachers use the material in teaching mathematics because these are also embedded in the ETR.

As a limitation of the study, the development has only been centered on the views of teachers and teacher-trainers as the pandemic happened. Accounting for students' perspectives is imperative as it can yield new insights about the nature of the ETR once implemented in a classroom. This is also to test the potential TS gathered from the participants as benefits of using the ETR in a classroom. In addition, considering other local teachers of Kalinga and teacher-trainers to increase the sample size could bring new ideas and validate the findings. Consequently, the researchers recommend future research studies to address these limitations. Other cultural communities around the globe are encouraged to reproduce the study to recontextualize their unique practices around their context through ethnomathematics and SGDE. This can be done by developing a teaching resource and the practice of mathematics teachers or extending its application in developing students' mathematics learning and improving textbooks and the curriculum. Lastly, the division of Kalinga or other researchers should conduct more teacher trainings, focusing on how to develop teaching resources based on other cultural practices of the province that are beneficial in achieving the principle of realistic mathematics.

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