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Research Article

The touch of individuals with visual impairments to geometry: Tactile materials vs origami

Fatma Nur Aktaş

Kahramanmaras Sutcu Imam University, Faculty of Education, Türkiye (ORCID: 0000-0002-3804-3650)

This study aims to examine the roles of tactile materials and origami models in the perception of concept definitions and representations in geometry applications of individuals with visual impairments. This case study draws on data from eight individuals with visual impairments selected according to the purposive sampling method in a mathematics village. Video recordings of focus group interviews conducted with tactile materials and origami models were analyzed by content analysis. Tactile materials made it possible for them to perceive concept definitions and visual representations, while the construction of origami models made every component designable, such as the relationship between concepts, visual prototypes and sub-concepts. Individuals with visual impairments have been quite successful in designing different representations and strategies for geometric concepts with perfect folding. Thus, it has been determined that origami is an accessible material for individuals with visual impairments to do geometry with creases and models.

Keywords: Individuals with visual impairments; Origami; Tactile material; Geometry

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1. Introduction

In concept teaching practices to individuals with special educational needs, materials and arrangements that appeal to the senses that they use actively and effectively should be included (Edwards & Stevens, 1994; Heller et al., 2005). Individuals with visual impairments are classified as blind and low vision, according to using their vision in educational practices. Therefore, individuals with blindness keep their educational practices by using braille and listening to audiobooks. On the other hand, individuals with low vision use different size fonts or standard size fonts with various tools (Heward, 2003). While adjusting font sizes and utilizing various tools is essential for individuals with low vision, it is equally important to reinforce teaching practices. Thus, adaptations in teaching practices for individuals with visual impairments should be supported by tactile and auditory materials (Argyropoulos, 2002; Heller et al., 2005). However, individuals can also use tactile materials and perceive the abstract or concrete concepts represented by these materials, even though they cannot actively benefit from the sight (Agrawal, 2004; Aktaş, 2022; Aktaş & Argün, 2020; Cowan, 2011; Edwards & Stevens, 1994; Horzum &

Address of Corresponding Author

Fatma Nur Aktaş, PhD, Kahramanmaras Sutcu Imam University, Faculty of Education, E Bloc, Avsar Campus, Onikisubat, Kahramanmaras, Türkiye.

fatmanuraktas@ksu.edu.tr

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Arıkan, 2019; Sze, 2005b). Through tactile experience in everyday life, these conceptual structures can be perceived. In others words, it is inevitable for them to perceive the concepts of geometry, because geometry is an experience and interpretation of the space in which the individual lives and moves (Freudenthal, 1973). Therefore, one of the purposes of teaching geometry is to provide students with experiences that enhance their understanding of the world they live. On the other hand, when the spatial perceptions of individuals with visual impairments are examined in daily life, it is known that they are successful in determining reference points and these points are found in their long-term memories (Haber et al., 1993; Toker, 2020). For this reason, teaching geometry to individuals with visual impairments should not be limited to geometry concepts that can be represented by three-dimensional geometric objects that they frequently encounter in their environment. The basic geometrical concepts that individuals perceive randomly in daily life can be perceived through planned and designed practices (Agrawal, 2004; Dede, 2012). In fact, teaching two-dimensional geometric concepts is possible by random folding of a simple piece of paper (Aktaş, 2021; Sze, 2005a). Therefore, origami can be used in teaching practices, such as description and touch, for individuals with disabilities by using different senses (see Sze, 2005b; Aktaş, 2021; Aktaş et al., 2020). However, the effectiveness of simple tactile materials and origami models in the concept learning of individuals with visual impairments is worth examining. Indeed, although both support training tools include visuality, the paper folding steps are surrounded by the skills that use the active sights with visual differences of coloured papers and folding strategies (Hull, 2012).

1.1. Teaching Geometry with Origami

The individuals with blindness reach information about the outside world through their haptic senses, in a way they see by touch (Jansson, 2008). At these touches, they make inferences about the geometric figures, directions and spatial relationships between objects by referring to their bodies, arms or fingers (Klingenberg, 2012). Thus, individuals try to recognize figures by holding them with both hands or in one hand and manipulating them with the other hand. They also continuously measure the figures of objects by touching them in terms of geometric properties such as length, area, and volume as part of their visual path. Thus, visual impairment does not inhibit the ability to process and transform mental images (see Cattaneo et al., 2008). In fact, individuals who are congenitally blind and have low vision have superior tactile recognition abilities compared to those who are sighted or adventitiously blind (Heller et al., 2003). Thus, since paper folding is based on individuals' ability to use their hands (Patkin & Kanner, 2010), it is a suitable support educational tool in inclusive classrooms with individuals with disabilities regardless of disability type (Sze, 2005b). Indeed, Pontes (2010) determined that after the modular paper folding activities in the inclusive classroom, contrary to expectations, students with visual impairments implemented activities based on spatial skills as accurately and quickly as their peers without disabilities, and concept learning was achieved.

The edges, corners and creases of the paper are used for creating figures with origami (Sze, 2005a). Crease is the mark that occurs when the paper is opened by applying pressure with a hard object or fingers to the edge where the paper is folded. The construction of various geometric figures with creases is possible with the axiomatic structure of paper folding that represents the axiomatic structure of geometry (Krier, 2007). Indeed, paper folding includes models created by combining the basic concepts of Euclidean geometry such as plane, point, line segment, polygon and angle (Haga, 2008). The prototype three-dimensional model designed as a result of each folding step and steps is a concrete material for abstract geometry concepts. Therefore, teaching geometry with detailed descriptive paper folding steps turns into economic, easily accessible and tactile-supported enriched practices for individuals with visual impairments. For example; Models obtained by folding paper, such as weather vane, are the product of representing geometry concepts such as line segments, polygons and interior angles on paper (Haga, 2008; Hull, 2012). Moreover, the study of these basic concepts through paper folding is based on active learning through practice and experience and has an advantage over passive learning by simply listening to words. The progression of the process of managing individuals' sense of touch with descriptions is significant in learning with paper folding (Aktaş, 2021).

Diodato (1976) drew attention to the necessity of using paper of appropriate texture to understand the creases and the intersection of creases in paper folding practices that offer the opportunity to teach geometry to individuals with visual impairments via embossed lines. Tinsley (1972), on the other hand, emphasized that unlimited practices can be done for teaching geometry with paper folding regardless of paper type in the research focusing on determining the paper type that gives the best results in the educational practices of individuals with visual impairments. Pinho et al. (2016) presented different paper types to students while creating models through geometry concepts in paper folding practices. The conceptual learning was tested by asking students questions about geometry and algebra concepts (triangle, angle, area, fraction, etc.) while these models were being created. In the result, it was explored that while students with visual impairments create creases more easily with waxed and sulphite papers (a type of paper is derived from wood treated with sulphite), they feel the creases created with craft paper more easily. However, it is interesting that standard A4 paper (familiar/typical paper form) was determined to be sufficient to sense for individuals with visual impairments (Aktaş et al., 2020). Aktaş et al. (2020) interpret this inference as the product of the success of individuals with visual impairments via sensing developed nerve endings based on active touch.

1.2. Basic Geometrical Concepts and Tactile Materials

The concepts of line, line segment and angle are considered basic concepts not only in paper folding steps but also in the construction of geometry (Euclid, 2007). Indeed, frequently studied geometrical concepts such as polygon and coordinate system are defined through these concepts (see Argün et al., 2014). Therefore, the elementary point for teaching geometry is the examination of basic concepts (Unal et al., 2009). However, geometrical concepts are considered a limitation and difficulty for individuals with visual impairments because they contain visual elements (Krahe, 2020). But, empirical evidence suggests that the comprehension of individuals improves with teaching practices supported by tactile materials (see Argyropoulos, 2002). For this reason, in the current research, the basic geometric concepts of line, line segment, angle and angle measure were discussed using tactile materials and origami as the interactive material. Hence, a framework has been presented for instructing geometry to students with visual impairments, offering an alternative viewpoint for researchers to explore advanced geometry concepts.

In addition to the definition of the concept of line based on vector, it can be explained according to student levels as an unbending curve that has no thickness and depth in Euclidean geometry and extends from both directions without interruption. The line is an abstract concept informally as the straight line and is an undefined or primitive basic concept in the axiomatic system (Argün et al., 2014). The line segment is a part of the line determined by the endpoints considering that the line is a set of points on the plane. Intuitively, it can be possible to take a piece of the rope representing the line concept and identify the line segment by drawing attention to the endpoints. Thus, the line segment can be described as a straight line segment bounded by its endpoints (see Aktaş, 2020). Indeed, students with visual impairments can benefit from the rope, cable, metal rods and embossed lines for line and line segment representation (Aktaş, 2020; Bülbül, 2013; Dick & Kubiak, 1997; Horzum & Arıkan, 2019). Thus, it is possible for individuals with visual impairments to understand the definitions of line and line segment concepts and to design their graphic representations symbolically. On the other hand, examining the line concept as a set of points is essential to explain that its two parts, which accept a point as the starting point, constitute the ray concept. Thus, the union of two rays with a common starting point can be defined as an angle. In addition, the concept of angle can be explained by the extension of the line segments, which are the arms of the angle, in geometric figures whose sides are line segments (Kay, 2001). Horzum and Arıkan (2019) determined that students with visual impairments comprehend interior and exterior angles and line segments when designing polygons using magnetic materials.

Although the results of concept learning with tactile materials have been clearly demonstrated, limited reviews of origami practices have been encountered (see Aktaş, 2021; Aktaş et al., 2020 Khare, 2020; Pinho et al., 2016; Pontes, 2010). Khare (2020) determined that it is possible to construct basic geometric concepts with origami by creating a series of rules with axiomatic structures for individuals with blindness. Further, it has been demonstrated that individuals with blindness successfully proved the trisection of an angle problem. Pontes (2010) determined that polyhedron and related sub-concepts are comprehended by individuals with blindness through a didactic sequence that uses the auditory system, speech and haptic in origami practices. Pinho et al. (2016) determined that students with visual impairments were able to distinguish basic geometric concepts on paper while folding origami models and their comprehension improved. Aktas et al. (2020) examined the positive contributions of geometry teaching through origami in the practices within the large framework. Aktaş (2021), on the other hand, revealed the criteria that individuals with visual impairments have difficulty and easy comprehension in various paper folding practices from geometric proof to origami models. For this reason, the current study aims to compare tactile materials and origami in the context of their reflections on the comprehension of individuals with visual impairments to provide the basis for origami practices and advanced geometry teaching. Thus, it should be possible to design alternative support training materials for enriched geometry practices. In this regard, this research sought to answer the following research questions:

RQ 1) What are the roles of tactile materials and origami in the comprehension of individuals with visual impairments for basic geometric concepts?

RQ 2) What is the comparison between tactile materials and origami as support educational materials?

RQ 3) What is the comparison between using tactile materials and origami for concept learning?

2. Method

Since this research aims to examine the role of tactile materials and origami in the comprehension of basic geometry concepts by students with visual impairments, it was designed as a case study. Indeed, a case study is a design that presents the current phenomenon or event in its usual course with a holistic interpretation and allows depth analysis. In addition, research designed in a case study focuses on the process and conditions (Merriam, 1998). Since the research focuses on the different materials and the comprehension of individuals with visual impairments with different qualities, it was designed as a multiple case-embedded design (Yin, 2003).

2.1. Participants

This research was carried out on the first day of a two-day event held in a math village. Participation was limited by various criteria in this event. The criteria were chosen to enable the individual to mobility and comfortably continue the activity in the mathematics village and to reveal the role of materials in their comprehension of geometric concepts. Accordingly, the criteria for the participants determined considering the criterion sampling method were as follows: (i) having no other disability accompanying the visual impairment, (ii) having mobility skill, (iii) having an education level of high school or above, (iv) participating in the organized math village event. It was assumed that students possessed enough pre-knowledge to evaluate the concepts to determine the educational level criteria. However, this pre-knowledge is limited to knowing the names of the concepts, having a general idea of what the prototypes look like, and having examined application examples that do not include visual elements. This limitation was not a criterion for selecting participants. Because it is less likely to determine individuals with visual impairments who have more conceptual knowledge than this limitation (see Aktaş & Argün, 2021). In addition, the maximum variation sampling method was adopted for the participants selected. Thus, individuals with different visual impairments qualifications were determined.

Accordingly, it was possible to make detailed analyzes on the level of using vision, such as folding paper, following creases, and comprehension materials.

The application form with the criteria for participation in the math village event was announced through social media and various societies. The twenty one individuals who met the criteria were determined among the applications and eighteen of them participated in the event. The eight individuals constituted the participants of the research voluntarily. Two of the participants were low vision (P7 and P8), one was congenitally blind and had light perception (P5), and the others were congenitally totally blind. P2, P5, P7 and P8 were students preparing for the university entrance exam. P1 was an undergraduate student, and P3 and P6 were Turkish teachers. All participants were trained in verbal or Turkish-math programs in high schools. The participants used braille except for P7 and P8. Also, the participants had no prior experience in creating origami figures before participating in this research.

2.2. Research Process and Data Collection Tools

The research was carried out in three sessions. Session and break times were determined according to the attention and focus times of individuals with visual impairments (Hadjikakou & Hartas, 2008). Accordingly, the first and second sessions lasted one hour and a one-hour break was given between the two sessions. There was a two-hour break after the second session and the last session lasted approximately two hours. The last session was extended as the participants continued to keep the discussion. The sessions were video recorded in a way to focus on the hand movements of the participants. The research data was obtained with these recordings. In addition, data were obtained through the focus group interview, as the participants could communicate interactively, constantly thinking aloud and interpreting each other's ideas.

The participants were asked concept-oriented (concept definition, representation types and qualities/characteristics) questions and probe questions to examine their thinking in the sessions: 'How would you define the concept of line?', 'what does the line segment look like?', 'can you explain how you fold it?', 'why do you think these two sides are congruent?', 'what comes to mind when you say angle?', 'how to represent the line with this material?', and 'can you represent a 45º angle with this material?'. In addition, they were asked to represent respectively 45º, 90º and 60º angles with origami to examine their process and understanding of working with origami. In the first session, geometric concepts were examined with tactile materials. The rope, wood carving and TV antenna in Figure 1 were preferred. In addition, rulers and protractors were offered to the participants to use if they wished. These materials were preferred since their effectiveness for the concepts discussed in the current research has been proven in the learning processes of students with visual impairments (see Aktaş, 2020). In other sessions, origami practices were carried out. The basic A4 white paper was preferred for origami paper because it is a simple, accessible and perceptible material (Aktaş et al., 2020). Sze (2005a) recommended that individuals with visual impairments fold papers on a smooth, solid and clean surface. Therefore, 50x50 cm chipboard pieces were given to the participants as there were no desks in the math village.

Figure 1
Tactile materials

The researcher assumed the role of an external researcher by directing the questions in the focus group interviews to detail the ideas during the data collection process. In addition, at the end of each session, the researcher assumed the facilitator role by taking care of the participants individually, examining their ideas and holding their hands to help them to examine the material or origami model and follow the creases when necessary. However, explanations and practices that would affect the participants' comprehension were avoided.

2.3. Data Analysis

The research data were analyzed with the content analysis method by transcribing the video recordings of the focus group interviews and examining the discourses and hand gestures of the participants (see Danişman & Tanışlı, 2023 for gesture analysis details). Thus, the reflective notes were taken by marking the discourses and hand gestures, and categories were obtained. The categories were analyzed under two themes related to tactile materials and related to origami. The categories are presented in Table 1 with code examples.

2.4. Validity and Reliability

To increase the credibility of the research, strategies such as prolonged engagement, persistent observation, expert review, and member checking were employed (Patton, 2014). Thus, the participants were given time to tour the math village and get used to the environment before the focus group discussions. Participants were allowed to determine the break times between sessions for the rest. The expert opinion was obtained from a high school mathematics teacher who had a student with visual impairments and was doing a master's degree in this field about the interview questions and the analysis of data. Accordingly, the basic concepts were discussed with tactile materials before designing origami models since the students did not have any experience working with origami. In data analysis, some categories were gathered under a single category and a consensus was reached. The category of informal definition was formed for the first analyzes of tactile materials. However, after consensus, it was decided that these definitions coincide with the intuitive definitions of the participants and should be merged with the category of intuitive definition. The modelling category, such as the roll representation, was obtained for the origami theme. However, it was decided that all models were conceptual representations and were grouped under one category. Thus, conceptual confusion is prevented with origami models. In addition, before the mathematics village activity, a pilot study was conducted with the low vision student of the expert mathematics teacher and the data collection tools were evaluated. Accordingly, it was decided to include two different wooden line (see Figure 1) representations and to discuss the line representation with origami. Thus, the credibility, reliability and consistency of the research were increased. The transferability of the research was supported by including the purposeful sampling method.

3. Results

3.1. Results for Tactile Materials

When the participants were asked the question 'what is the line?', most of them answered 'something like a straight line' and P1 replied 'a straight line that goes to infinity'. However, the participants did not indicate that the line should not undergo any curve or break. In addition, they were also unaware of what they meant by the expression 'goes to infinity'. So they were able to explain the concept with the *intuitive definitions*. Then, the participants sitting side by side in a single file were given the end of a ball of rope and handed it to their friend next to them and asked to stretch the rope as much as they wanted (see Figure 2).

Table 1
Examples for data analyses and categories

Themes and categories	*Codes*	*Data*
Tactile materials		
Intuitive definition	Describing the representation of the concept; exemplifying the concept; representing with tactile material	P5: the line segment is the part between my fingers where I'm holding the rope (he stretched the rope as a straight line segment from the endpoints with his index and thumb fingers)
Inability to perceive sub-concepts	Inability to comprehend tactile material; inability to show an example with tactile material for the concept	P6: I am not sure if the 45º angle will be like this (she had done an acute angle representation with the TV antenna)
Need for explanation	Inability to express the definition of the concept clearly; inability to give an example for the concept	P4: That is a straight line too, but what is that? (he created the line representations with different directions with the rope)
Perceiving to visual prototype	Tactile perception of the prototype for the visual representation of the concept	P3: The arrows of the line indicate that it is infinite. We are drawing the arrows here (pointing at wood carving material)
Ability to perceive abstract concept	Making sense of the concept that cannot be perceived by touching or description	P2: It is like going forever. We are stretched it again and again, but it is not over.
Ability to comprehension the relationship between concepts	Ability to recognize the concepts that are included in the definition of the concept or that it is related	P4: These are rays. For example, the angle forms a right angle like this (he created an angle representation with TV antenna)
Origami		
Intuitive representations	Defining the concept and representing it visually with origami models	Roll and paper streamer representations for the line concept of participants
Ability to design different representations	Representing a concept with various origami models	P2 tried to divide the angle with his fingers and then used the 30-60-90 triangle for the 60º angle representation
Making use of sub-concepts	Benefiting from various geometry concepts and properties involved in the construction of this concept when designing a model for concept representation	P4: I need to fold an equilateral triangle. Each angle should be 60º.
Need for explanation	Description of the origami model s/he designed	P1: I tried to divide the angle into three congruent parts. Because I will get 30º. I folded equally.
Designing the visual prototype	Designing prototypes for visual representations of the concept	P7: I folded an isosceles triangle. I will unfold the paper. I will get a 60-degree angle with the creases.
Making use of the relationship between concepts	Utilizing concepts that are not included in the definition of the concept but are related to geometric properties	P2: I got a rectangle and a square. Now when I unfold the paper I will take advantage of these right angles with creases. A 30-60-90 triangle will appear
Proving	Ability to argumentation the accuracy of geometric ideas with origami models designed for concept representation	P3: Now I am going to get the midpoint by folding it over. Then, I will obtain an isosceles triangle by joining it with the midpoint. I did it right. I will prove it to you.

Figure 2
Representation of the line

After examining the rope representation, although the participants could express the concept of line as a 'straight line', they explained the rope representations that were presented with different directions as 'curves'. Therefore, they could not recognize the *sub-concepts* of the line concept without *explaining* or intervening. Therefore, it was stated to the participants that the points where they hold the rope represent the points on the line. Then, they were asked to raise their arms in different directions as up and down considering these points. But this time P2 explained as 'this is a line segment' and P3 explained as 'it doesn't go to infinity, I guess it is a line segment'. The following dialogue took place when the participants were asked to imagine that the rope could be stretched as much as they wanted:

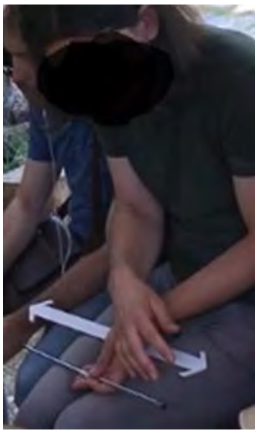
Figure 3
P4 examines the concept of direction

> P4: If it continues like this (raises hand more above) and like this (raises hand below), it may be the line (see Figure 3).
> P3: But in a different direction.

Although they could not express the concept of direction as a term after the *explanation,* they could explain it *intuitively* with the help of rope. Thus, since the concept of vector was not included in their pre-knowledge, the formal definition was briefly stated. As a result, the rope as tactile material has given the *abstract concept* that the line can be stretched as much as desired.

The wood carving materials were presented to participants for the representations of line. Thus, they could visualize the prototype of the line concept in their minds. Participants examined the representations of line in order. The representation of arrows was discussed (see Figure 4). Participants easily stated that 'it can be stretched' and 'it is infinite'. Thus, the wooden tactile materials enabled them to *perceive the visual prototype* as a graphic representation of the line.

Figure 4
Representations for the line



Secondly, they were asked to explain the line segment. P2 sought similarity for the line segment concept by referring to the arrow signs considering the line prototype he obtained from wooden tactile materials:

P2: Arrows will point the inside.
Researcher: Do you want to stretch it inward?
P2: Hmm.
P5: Could they be points?
P4: Point, yes! So it could not be stretch.

The need for explanation was met with a leading question so that the wooden material does not cause misconceptions. However, they were able to perceive the visual prototype of the line segment concept because it was previously explained that each of their finger taps on the rope tactile material represents a point. Then, it was explained to the participants with the intuitive definition that the piece of rope that they hold with their index finger is the line segment representation and that the index fingers represent the endpoints of the line segment. Then, the relationship between the concepts was questioned through the rope tactile material. Because P7 sought a representation relationship between the line and the line segment:

P7: Is the line also expressed with endpoints?
P6: There are no points that we can represent the line. Infinity.
P5: Exactly!
P6: No initial and no end.
P3: But we stretched the rope and held it with our fingers like this.
P6: If it has initial and end points, it will be a line segment.

After this discussion, although the participants answered correctly, there was a need to explain the symbolic representations of the line segment and line.

Finally, when the concept of angle was questioned, P2 replied with ‘the intersection of two lines’ and P4 replied with ‘were not they the ray?’. But, they were not sure about their answers. So, the TV antenna was given to the participants. Participants took turns representing various angles with the antenna tactile material. While examining the material, almost all of the participants first formed the right angle (see Figure 5a) and then acute or obtuse angle representations. P4 and P7 tried to find a relationship between the line and the angle by opening the arms of the antenna to form a 180^{o} angle representation (see Figure 5b). P7 also wondered about the symbolic representation of the angle, ‘how do we call it the way we call the truth, how is it?’. Thus, the concept of linear angle and the symbolic representation of the angle were explained. Thus, the TV antenna tactile material made it possible to perceive visual prototypes and the comprehension of the relationship between concepts.

Figure 5
Examination of TV antenna tactile material

a b c

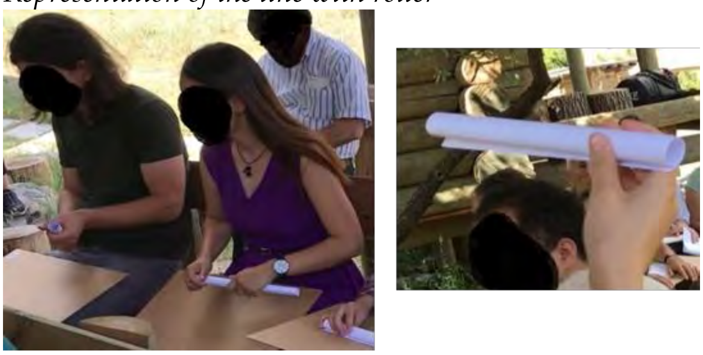
However, although they were able to form acute angles with the TV antenna tactile material, they were not able to represent a 45º angle accurately. They *intuitively* created a 45º representation of an angle (see Figure 5c). Therefore, the participants could *not perceive the sub-concepts* related to the concept of angle.

3.2. Results for Origami

The A4 size papers were given to participants to examine their representations of the line concept with origami. Participants were asked the question ‘How do you create a line or line segment with the help of this piece of paper?’. Participants expressed the following intuitive explanations and representations:

> P6: It is formed the line by folding it forever (laughed)
> P8: Let’s cut a streamer?
> P2: Let’s roll it?

Figure 6
Representation of the line with roller



It was observed that before P2 expressed the idea of the roller, three more participants rolled the paper (see Figure 6). Also, the three participants folded the paper one after the other, while P4 folded the paper in half with the long sides overlapping. Therefore, origami offered the opportunity for designing different representations. Discussion was initiated for the roller representation of P2 to examine the argumentations for the participants’ idea of ‘streamer’ and ‘roller’:

> P6: Since there is a hole here (pointing to the inside of the right circular cylinder), it can create a perception that the line ends here.
> P3: The line with wooden material looked like this.
> P1: For this, we can say that it is not a line because there are no arrows in this.
> Researcher: Please consider the example of the rope.
> P3: We have not thinned the paper yet.
> P5: No matter how much we fold it, it becomes a rectangle, it becomes a cylinder (laughed)
> P2: I made something like spaghetti, flat spaghetti.

Here, the participants tried to describe to each other their concept of the line representations with the origami they designed. P2 was asked to unfold the line representative paper (see Figure 7a), which he folded and called spaghetti, to unfold it for its original state.

> P2: there are lots of line segments (without unfolding the papers).
> P6: we will use them! (with an expression of surprise)

Although the other participants needed for explanation, P4 showed the representation that he doubled the paper (see Figure 7b) at the beginning of the session and he said for confirmation that 'I did it this way'. P4 made a fold by joining the two long sides of the paper and joining the midpoints of the short sides. Thus, it was determined that the participants were able to *design visual prototypes* and tried to seek the *intuitive representations*.

Figure 7
Representations of the line through paper folding

a b

It was stated to the participants that the creases formed by paper folding represent the line or line segments. It was stated that origami figures were made with creases. Next, they were asked to create a 45^{o} angle representation through origami. P1 created a crease by joining one corner of the paper with a point on the opposite edge (see Figure 8a). He unfolded the paper and examined the angles formed by the crease (see Figure 8b). P3, on the other hand, first folded to meet at the long sides, then was meet one corner of the newly obtained rectangle with a point on the long side to form a crease (see Figure 8c and d). While P1 used the chipboard while creating creases, P4 preferred to support it with his mouth. Thus, since *intuitive explanations* were made by *designing different representations*, the participants *perceived the visual prototype* of the line better as a straight line on paper.

Figure 8
Experiences for paper folding

a b c d

P2, on the other hand, tried to obtain an angle of 45^{o} with the help of a right triangle. (see Figure 9a). However, the group discussion started when the lengths of the right triangle's edges he designed were questioned:

> P2: (refolded from the creases to form the triangle) these were congruent (he intuitively tried to feel it with his fingers)
> P5: for example, I folded it in half, then would not the edges be congruent?

Figure 9
P2's model for the angle of 45º

a b c

P2: I folded it like this but I do not know what it is (he folded an isosceles right triangle, see Figure 9b). When you unfold this, I know that they are equal to each other (he unfolded it based on the creases) I know that this is 90º. They were 45º and 45º. I bisected an angle (see Figure 9c). The sum of the measures of the interior angles is 180º.

P2 was able to reason by *perceiving sub-concepts* such as the bisector of the concept, abstract concepts and its visual prototype. However, P2 could not explain how he determined the measure of the right angle. Then, when the participants were asked to obtain an angle of 90º in the inside of paper without using the corner points of the paper, P2 could easily point to an angle of the square he had formed by folding it. In fact, for this, P2 felt the creases and pointed to a single square on the rectangular paper consisting of two squares (see Figure 10a). P4, on the other hand, unfolded the paper and pointed out the right angles between the creases one by one (see Figure 10b). Thus, origami allowed participants to benefit from the relationship *between concepts* and *design different representations*. In this application, participants with blindness were more successful than participants with low vision in meeting the edges of the paper, creating creases as the straight line, and following the creases.

Figure 10
Representations for the angle of 90º

a b

In the last session, the participants were asked to obtain an angle of 60º. Participants shared their ideas by thinking aloud:

P8: we should divide 90 by 3 and get its two parts.
P5: we need to find the 30-60-90 triangle.
P3: right!
P7: I think we should fold an equilateral triangle.

P4 again tried to fold a triangle representing the equilateral triangle by supporting it with his mouth (see Figure 11a). Meanwhile, P2 was trying to explain while creating the creases:

Figure 11
Representations for the angle of 60º

a b c

P2: This is the square, this is the second and this is the third (putting his fingers together, he tried to evenly divide the angle, see Figure 11c) this is half of that (he pointed to the angle it made by folding and the angle made by its bisector)
P8: How do you know it is half?
P2: Hmm. I will find it soon.
P8: we need to fold 180 by 3, but how?
P4: I did. Since the sum of the measures of the interior angles is 180, the angles are equal, 60 degrees when divided by 3.
Researcher: how did you prove that the lengths of edges are equal?
P4: do not you see it is equal (laughs) if we measure it with a ruler, it will be equal (see Figure 11b).
Researcher: can you explain how you fold it?
P4: (he re-unfolded the paper first) if we name the corners of the rectangle A, B, C, and D, I have met A and C. There were excesses on the edges. I folded them.
Researcher: Ok. Even if I am convinced that the two edges are equal because of the folding, why is the length of the other side equal to the length of these sides?
P4: I cannot explain it theoretically. But let's measure it, it will definitely be equal.

Since P4 insisted, the lengths of the edges were determined by comparison on the chipboard (see Figure 12a). It was determined as a result of the measurement that it is an isosceles triangle (see Figure 12b and c). The statements of P2 and P8 in this discussion enabled them to arrive at the idea of dividing an angle into equal parts.

Figure 12
P4's model for the angle of 60º

a b c

P2 *designed a visual prototype* for the bisector concept and tried to benefit from t*he relationship between the concepts*. Trying to measure angles with his fingers also satisfies *the need for intuitive explanations*. P4, on the other hand, tried to *intuitively* obtain equilateral triangles and was able to *design a visual prototype* by making use of *the relationship between concepts*. Meanwhile, P2 had designed a new model (see Figure 13).

P2: For the 30-60-90 triangle, the edges were a and 2a, right? This is a, this is 2a. This is the square and I doubled it. These edges are equal. 2a and a are the sides. Then when you meet these two corners, somehow, this angle will be 60 degrees.

Figure 13
P2's model for the angle of 60°

P2 indicated the relationship between the lengths by following the creases. In order to prove that the lengths are equal, he folded the creases on top of each other and compared them. Thus, the indicators of *designing different representations*, *relationships between concepts* and *benefiting from sub-concepts* were encountered. P3 continued to focus on the earlier idea of P2 and P8 dividing the angle into equilateral angles:

> P3: Miss! I folded the edges to overlap (see Figure 14a). It was equal, so I cut it in half. If I fold them up to this point they will be of equal length (see Figure 14b). I can prove it. I fold again the lengths are equal (see Figure 14c). This is 90°. Since I used the square, the point I mapped to is the midpoint. This angles are 45° and 45°. So it was not 60 (laughs).

Figure 14
P3's proof for his model

a b c

P3 presented a *different representation* to the concept of bisector and tried to *prove* his idea by making use of *the relationship between the concepts*. Similarly, P4 created creases representing the diagonals and obtained the angles of 45° and then $22,5^{\circ}$. Thus he pointed out the angle of $67,5^{\circ}$. The discussion was ended here and the strategy for obtaining an equilateral triangle and an angle of 60° through origami was explained.

4. Discussion

The comparison of the use of tactile materials and origami models in geometry teaching was discussed under two titles as support educational materials and their roles in concept learning.

4.1. The Comparison as the Support Educational Materials

In examining basic geometric concepts with the help of tactile materials and origami, the differences in the thoughts of individuals with visual impairments can be discussed in three contexts. The most striking context is that while examining geometric concepts with tactile materials, the concept definitions, representation types and visual elements of the concepts remained at the level of perception. On the other hand, different representations, models and visual prototypes of geometric concepts could be designed in origami activities. In other words, while origami inherently provides the individual with the opportunity to construct geometric concepts through folding (Krier, 2007), tactile materials make abstract geometric concepts perceptible. Just as sighted individuals represent and implement concepts by drawing geometric figures on paper with a pencil, individuals with visual impairments can also do geometry with the help of creases. These active geometric processes have revealed the difference in concept definition and construction of representations as the second context. While tactile materials have made abstract concepts perceptible, they have only provided informal explanations of verbal definitions. So, the power of tactile materials to represent the sub-concepts of the concept without any supporting educational tools or explanations has been weak. On the other hand, origami has revealed the designing of different representations and deep geometric thinking through models for abstract concepts. Therefore, origami offers rich opportunities for teachers and researchers to examine the perceptions, prototypes and understandings of individuals with visually impaired for abstract and visual concepts.

The last different context that emerged in the comparison of tactile materials and origami models is that origami representations lead the individual to think about the relationship between conceptual and procedural learning, as paper folding steps involve proof and argumentation processes. Because the ideas put forward in the folding steps need to be proved by sub-concepts and using the relationship between concepts. Thus, origami leads individuals with visual impairments to geometric thinking, reasoning and argumentation. The results point to a remarkable difference in this process. While the tactile materials need to be explained as the representative of the geometric concept and the relationship between the concepts by the educator, the learner tries to explain their ideas and steps in origami models. In other words, origami offers opportunities for active learning and practice of concepts to the learner.

The differences in the results do not lead to the conclusion that tactile materials are not partially effective in teaching geometry to individuals with visual impairments. On the contrary, long-term tactile exploration of materials is essential for individuals with visual impairments to create mental representations (Argyropoulos, 2002). Likewise, while designing origami models, individuals with visual impairments may need to unfold and refold the paper to examine the creases (Aktaş, 2021). However, individuals with visual impairments had easily able to create creases and master the folding steps while designing origami models in the current research. In addition, they tended not only to represent geometric concepts with origami but also to prove their ideas by looking for different types of representation. This remarkable result, which strengthens origami applications, first emerged as a result of the disposition of concept images and prototypes with tactile materials. Thus, presenting tactile materials as final representations of origami models facilitates reasoning processes with paper folding.

Individuals with visual impairments utilized not only their fingers but also their arms and even their mouths to create creases while designing origami models. These various sensory data have increased their perceptions of conceptual understanding. Thus, origami materials offer kinesthetic sensors as well as tactile sensors that tactile materials offer. This rich perceptual difference is due to the haptic nature of origami (Pontes, 2010). The sensory diversity, the excitement for designing different models, and the happiness because of the perception have decreased the cognitive efforts of individuals with visual impairments in geometry teaching with origami and prolonged their periods of mental fatigue. Therefore, origami teaching practices have replaced the usual requests for breaks (see Hadjikakou & Hartas, 2008) with long-term group discussions. In addition, contrary to the suggestions in the literature such as the paper type (Pinho et al., 2016) or the need for a solid and smooth floor (Sze, 2005a), the texture of a simple A4 paper was sufficient for individuals to follow the creases and the individuals were able to create the creases with their fingertips. It is also remarkable that individuals with blindness were more successful in creating sharp, clear and straight creases than individuals with low vision. Moreover, they could easily do that. The results of the material and its folding processes are clear evidence that individuals with visual impairments have strong nerve endings based on active touch (Aktaş et al., 2020). However, it was determined that individuals needed to name the points in order to describe the corner points and creases they felt while proving their ideas. Indeed, as the number of corner points and creases that individuals need to examine increases as the folding steps increase, it is essential to name the points for mental mapping (Aktaş, 2021; Krahe, 2020).

4.2. The Comparison for Concept Learning

The rope and wooden tactile materials for the representations of the line concept can be included in geometry practices for individuals with visual impairments to perceive the concept definition and graphical representation (Aktaş, 2020; Bülbül, 2013). The rope is a useful material in the context of utilising the idea of stretching to allow them to perceive that the line is a set of points with an infinite number of elements. However, it must be emphasized that the line is a set of points (Aktaş, 2020). Indeed, individuals with visual impairments have tended to represent the endpoints of the line segment, which they perceive as part of the line, with their fingers. The rope is more utilitarian than wooden materials to allow them to perceive the graphical representation of the line on paper. Because it has been determined that the rope tactile material is effective in order to prevent the perception of thickness and depth. When the participants created the line representations by the paper folding, it should not be thought that the wooden materials have negative reflections because they make roller or streamer. Here, individuals with visual impairments used their perceptions of wooden tactile materials to describe their origami models. However, the rope for the tactile material may be preferred since it is a similar representation of the line on paper. Thus, when individuals with visual impairments are expected to represent the line concept through paper folding, they can notice the crease representation similar to the straight line drawn with a pencil by sighted individuals. The crease makes it possible not only to perceive the line segment representation on the plane but also to prove the geometry propositions by drawing auxiliary lines. Thus, individuals with visual impairments can easily do geometry practices. For example; they can design an angle of 45º and determine the angle bisector. They can represent the angle of 67.5º with the angle of 22.5º they obtained. Moreover, they can make use of the relationship between the square and the right triangle, as well as make use of the triangle properties.

The TV antenna is a creative idea for the perception of visual prototypes of the angle concept (Aktaş, 2020). It facilitates comprehension for individuals with visual impairments to perceive sub-concepts such as ray and angle types. However, this tactile material is an under-representation when the concept of angle measures must be considered. Here, understanding that the ray and the line are a set of points offers the opportunity to create arguments with geometric shapes whose angle measures are known through paper folding. Indeed, participants excelled in reasoning and geometry when representing angles of 45º, 90º, and 60º. It is also noteworthy that they benefited from the relationships between geometric concepts as well as their skills in paper folding and interpretation. Therefore, if simple origami models are to be studied, the practice can be started with paper folding. However, it should be started with tactile materials or simple origami models before discussing advanced geometric ideas. Because the pre-knowledge of related geometric concepts may be required to realize the paper folding steps. For example; the pre-knowledge about the concepts of the equilateral triangle, 30-60-90 triangle, or bisector may be required to design a representation for the angle of 60º. Therefore, origami can used be as a geometry material that should progress in the light of pre-knowledge. Indeed, Pinho et al. (2016) first examined the pre-knowledge of individuals so that they could design familiar origami models. However, it is interesting that participants were able to come up with the idea of dividing an angle into equal parts, despite their lack of pre-knowledge in the current study. However, dividing an angle into two or three equal parts is an advanced geometry concept (Kay, 2001). So, this advanced reasoning is an advantage offered by the construction of paper folding.

It is not surprising that individuals with visual impairments used their fingers to measure length and angle on paper folding steps. Because Klingenberg (2012) clearly stated that individuals with visual impairments use their fingers for shape and direction perception. However, the results are surprising that the individual with visual impairments created a perfect isosceles triangle representation and measured the edges of the triangle model using the side of the chipboard as a ruler without scale. Also, other surprising results are that they could not simply use rulers or protractors to indicate that the lengths or angles are equal, and instead of these they found the ratios such as a and 2a by doubling or determining the midpoints or creating unit squares through the creases. Therefore, individuals with visual impairments can master geometry with appropriate support training material and basic pre-knowledge. So, origami is a material that enables individuals with visual impairments to do geometry with paper, just like sighted individuals.

5. Limitations and Suggestions

The limitations of this research are that the participants were determined at least at the high school level and focused on basic geometric concepts. These limitations are due to the necessity of starting with the basic concepts to evaluate the geometry practices with origami. Thus, this basic research has been put forward for the selection of tactile materials and the design of origami practices when the geometry concepts will be taught to students with visual impairments who have no pre-knowledge. The participants had certain pre-knowledge -they were only familiar with the concepts and did not have conceptual knowledge- it enabled them to reason in the paper folding steps. In addition, it aimed to eliminate the pre-knowledge variable in comparing tactile materials and origami representations. Thus, an overall evaluation has been tried to be obtained by minimizing individual differences. Of course, in further research, the conceptual learning process should be examined with in-depth clinical interviews with individuals with visual impairments at the prior class level. Thus, the qualities that origami models should have as a material and the steps of origami supported practices for individuals with visual impairments will be determined.

The selected tactile materials and examination of these tactile materials before origami practices are not the limitations of this research. Because the tactile materials provided the background or pre-knowledge for the images and prototypes of the concepts as detailed in the discussion. However, the priority relationships of materials and even origami models for the concept learning process or learning trajectories may be investigated. In addition, origami models, which are the combination of concepts, may be examined after the construction of the pre-knowledge is completed, and a guide may be presented for teaching geometry with origami to individuals with visual impairments. Therefore, further research is needed to design geometry practices enriched with paper folding activities and to determine the criteria for these designs to develop and implement.

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