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Research Article

Exploring transdisciplinary, technology-assisted, and architectural modelling STEAM practices through a cultural lens

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ABSTRACT

In this paper, we propose novel transdisciplinary STEAM practices to enable students and teachers to model architecture using technology. Architectural modelling can foster students' mathematical knowledge and computational thinking while connecting them to other disciplines such as culture and history. Our study focuses on enabling architectural, cultural, and historical diversity in educational practices. Moreover, the study tries to foster participants' modelling skills and innovative technology use as augmented reality and 3D printing. Thus, this paper will describe three case studies from Austria, Libya, and Indonesia and how these STEAM practices were used in different ways to allow participants to express their diversities through modelling diverse architectural constructions cross-culturally. We followed a qualitative data analysis approach for the participants' interviews, questionnaires and artefacts including architectural modelling, disciplines connections and lesson plans. The data analysis resulted in emerging themes emphasizing STEAM practices' possibilities to connect architecture to culture and history and highlighting the participants' cultural diversities in each of the three case studies.

Keywords: STEAM, architecture, culture, history, diversity, transdisciplinary, technology

INTRODUCTION

Architecture is a real-life construction that offers various functions since the beginning of human existence. Because architecture can serve as a shelter, religious, cultural, or artistic element. Since architecture is perceived differently due to its multiple purposes, this reflects the diverse nature of architecture (Martin & Casault, 2005).

The aim of the paper is to utilize architecture in a transdisciplinary science, technology, engineering, arts, and mathematics (STEAM) (Belbase et al., 2021) education practice, highlight its diverse nature, and encourage teachers to use it in their teaching. In this paper, we will employ architecture for mathematical modelling and technology in relation to STEAM education practices. These proposed STEAM practices allow cultural diversity expression to learners through architectural relationships, by analyzing architectural components and forms and remodeling them based on mathematical operations with a connection to learners' culture and history.

Architecture types range from ancient to modern to post-modern (Van Den Arend, 1909). These STEAM practices allow teachers and students to benefit from these architectural diverse types by mathematically modelling them using GeoGebra. GeoGebra (https://www.geogebra.org/, accessed on 14 June 2023) provides mathematical functions to construct models in both 2D and 3D views. GeoGebra modelling could help participants learn about diverse architectural types and try to simulate the architectural models' structures.

The culturally focused curriculums hinder the learners' international cultural exposure to learning about other cultures extensively compared to their own (Williams, 1992). In order to overcome such a challenge, we proposed a cultural lens, which is a metaphor for a virtual tool that we proposed to introduce architecture to participants during STEAM practices from different cultures than their own. We use this tool by modelling architecture by analyzing its components mathematically and simulating them using GeoGebra tools and functions, then we collect cultural and historical information related to these architectures. Afterwards, we evaluate cultural lens effect on participants' architectural choices from their final STEAM practice outcomes.

These STEAM practices may allow the use of a cultural lens that would zoom in on each participant's architectural model choice, connect to the participant cultures and maintain cultural architectural diversity. This paper highlights how these STEAM practices promote cultural knowledge through architectural modelling by emphasizing the participants' cultural and historical connections. Therefore, we will discuss three case studies in Austria, Libya and Indonesia and we will show the diverse architectural outcomes and the participants' motivations behind their architectural choices.

LITERATURE REVIEW

Introducing architecture in mathematics education is an opportunity to foster diversity (Martin & Casault, 2005). Diversity is described as "long-standing intra-state cultural differences in societies with differing ethnic, racial, linguistic and religious groups or to new forms of cultural diversity brought about by demographic changes and migratory movements" (Luciak, 2010, p. 44). Diversity in education is a challenge to teachers, as they have to consider which approach would foster their students' learning requirements and overcome all the social differences (Luciak, 2010). Alternatively, it could be perceived as an opportunity "for self-reflection and reflection on one's own cultural background" (Luciak, 2010, p. 58). Diversity in education can allow students to get exposed to different cultures and languages, which could lead to better personal development, interactive awareness, and a connection between students. Culture can be interpreted as a "system of specific symbols such as patterns of interpretation, expression, and orientation. The members of a culture internalize these orientations, interpretations and activity patterns" (Luciak 2010, p. 43).

The true meaning and definition of cultural diversity greatly vary, especially in teacher education (Fylkesnes, 2018). In this study, we emphasize the bond between architecture, regions, cultures, and history. We focus on diversity as cultural diversity, historical diversity and architectural diversity. Architecture and culture are strongly connected, and we make use of these connections in educational practices. Architecture is "a work, which contains all the history, culture, thinking and art of a society can improve the expression of architect and show the main message of architecture in this stage according to realities and necessities of building" (Hessam & Sotoue, 2016, p. 23). With reference to the cultural connection to architecture and that it is most significant in the architectural form (Hessam & Sotoue, 2016). They state that architecture is affected by regions that are also affected by the cultural environment. Architecture considers culture as the main guiding factor that affects buildings' creations in terms of the "ethnicity, people and new operational methods of beliefs, religion, climate, geography, economy, life and social structure of a region" (Hessam & Sotoue, 2016, p. 23). Furthermore, form plays a very important role in architectural definitions and its shapes define the ideas and interpretations of these buildings and structures (Hessam & Sotoue, 2016). Moreover, this helps emphasizing the mathematical forms in architecture as symmetrical functions, which shows how architectural geometrical shapes are all based on mathematics (Gyulai & Katona, 2020; Katona, 2020). With respect to the discussion on how culture played a huge role in the formation of the shapes and forms of architecture

(Katona, 2020). Therefore, these ideas and interpretations of forms are important connections in these STEAM practices as they connect participants to mathematics and culture, which allows them to get exposed to diverse ideas, methods, and innovations.

Culture and history are well connected and blended to appear in an Architectural form (Katona, 2020). “Architecture is like a viewpoint, which changes during time, has roots in culture, which will never get a fixed form” (Hessam & Sotoue, 2016, p. 22). “The architectural form is the most obvious index of cultural effect on architecture” (Hessam & Sotoue, 2016, p. 23). Thus, we are encapsulating architecture in STEAM practices to foster mathematical modelling by analyzing its forms and components by remodeling them using GeoGebra.

Teachers should respect cultural diversity in their classrooms especially if they teach multicultural learners and provide those learners with a room to express themselves and their cultures. One of the reasons for multicultural learners in a classroom is due to the immigration process that made diversity in students a significant feature in today’s classrooms (Guo, 2023). Following literature recommendations schools can promote specific programs for language learning, and this could improve the syllabus taught with multicultural subjects (Luciak, 2010). Moreover, this approach could make the teaching process more “culturally responsive” to suit the requirements of immigrants and multicultural participants (Luciak, 2010). However, this approach is challenging for teachers to take into consideration all the students’ cultural variations as well as the diverse ethnic and migrant aspects that are continuously changing (Luciak, 2010). Other study tried to address this rapid raise of cultural and ethnic diversities in the classrooms by conducting cultural diversity trainings for pre-service teachers to evaluate their motivation, behavior, and beliefs (Civitillo et al., 2018). Civitillo et al. (2018) found out that teachers need training programs during their teacher education to help foster their awareness and beliefs towards culturally diverse education. Furthermore, teachers and educators need to be trained to build connections between students with cultural variations and aim for a culturally responsive education (Gay, 2013). Therefore, in this paper we try to address this gap by proposing STEAM practices for enabling teachers and learners with cultural expression opportunities for addressing culturally diverse education.

Developing transdisciplinary STEAM practices is considered a challenging task for teachers, especially those who are learning and teaching focused disciplines. Those teachers find difficulty in integrating several disciplines together or designing learning practices that fosters several knowledge areas (English, 2016; Soublis, 2017). Therefore, in these proposed transdisciplinary STEAM practices we offer teachers ways to integrate several disciplines such as architecture, culture, history to mathematics education through mathematical modelling.

Mathematical modelling is considered a challenging process to teachers that has to be learned in order to be taught (Blum, 1993; Blum & Ferri, 2009). Therefore, it is recommended that teachers receive trainings to get them aware with the mathematical modelling process that allows them to transform a real-world problem or situation into a mathematical problem, try to solve it using mathematical tools and specific technologies. Afterwards, teachers are encouraged to transform these mathematical findings and map them to deliver a solution addressing initial real-world problems (Niss & Blum, 2020). Therefore, literature suggests that teachers need to receive sufficient training in order to adopt the mathematical modelling in their teaching and especially if they will use specific technologies to help them apply the mathematical modelling process (Blum, 1993; Niss & Blum, 2020). Hence, in these STEAM practices we provide teachers with professional development (PD) workshops to get them acquainted with using mathematical modelling using GeoGebra.

We will discuss further the theoretical framework that guided this study in the coming section.

THEORETICAL FRAMEWORK

This paper is focusing on the proposed STEAM practices that use architecture for mathematics education (Gyulai & Katona, 2020) while promoting cultural and historical connections. STEAM education is a transdisciplinary approach, that connects disciplines and capitalizes on various methodologies to address challenging problems (Costantino, 2018; Herro & Quigley, 2017; Piro, 2010). Therefore, we are extending STEAM transdisciplinary nature and connecting it to architecture, culture, and history to foster learners’ mathematical modelling skills, technology awareness and mathematical knowledge encapsulation in these

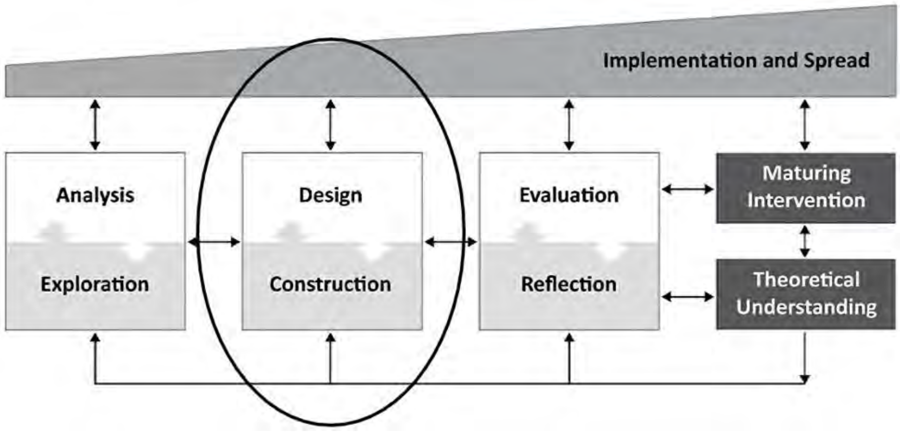


Figure 1. Generic model for performing design research in educational contexts (McKenney & Reeves, 2018)

STEAM practices. These connections to other disciplines in designed unified practices could increase learners' awareness of other methodologies and inquiry forms (Costantino, 2018; Piro, 2010).

We are presenting three case studies that took place in Austria, Libya and Indonesia. Due to the diversity of the cultural backgrounds between the three countries, we decided to adopt the adaptive, meaningful, organic, and environmental-based architecture (AMOEBA) (Gunawardena et al., 2003) framework as a theoretical basis for the proposed STEAM practices.

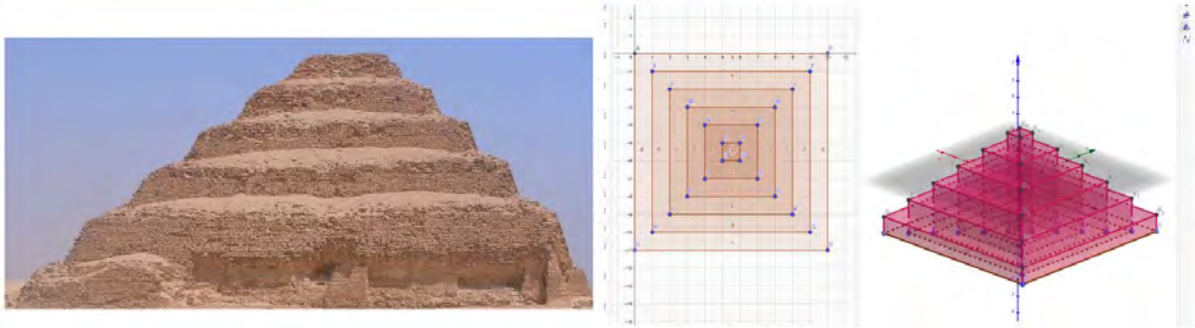
AMOEBA emerged from the research of "intercultural fields" for example the cross-cultural mindset and computer-mediated interaction. AMOEBA meaning is "a single-cell organism that can live in hybrid environments, yet it manages to survive without having a unique shape but a defined structure" (Gunawardena et al., 2003). The authors used AMOEBA vocabulary for the Online course design framework, which intends to assist the development and the learning processes between the mentors and the persons engaged in the learning course of action. AMOEBA was adopted to regulate the interventions and PD workshops in different cultural settings, for example by taking care of the variables that could be affected by cultural changes such as language, format, communication channel, activities and tasks, teaching-learning methods, technologies used, and knowledge used during the interventions. Accepting AMOEBA in these three case studies allowed us to restructure the practices' design to fit and adapt to the addressed culture. In terms of taking care of the language changes, architectural examples used during the interventions are the learning content and the selected learning environments as well as the data collection methods.

AMOEBA contributed to guiding the design of each case study, the language spoken and written in the learning materials such as the GeoGebra book and classroom and the culturally relevant architectural examples were all adapted to meet cultural variations. Hence, AMOEBA framework assisted the design phase of these STEAM practices cross-culturally during the interventions and PD workshops and the data collection methods to adapt to the participants' cultural diversity to reach better learning outcomes. We will address the methodology used in this study.

METHODOLOGY

In this study, we follow a design-based research (DBR) methodology, where we link theory to design and practice (Bakker, 2018). The design phase included the development of the learning materials of STEAM practices that were affected by cultural diversity and adapted using AMOEBA model in each cycle in the three different countries. **Figure 1** shows DBR diagram, where we focus on the design and construction and how the participants' cultural diversity had a huge influence on DBR phases.

DBR methodology is flexible, which allowed this study to be implemented cross-culturally to meet various alterations in each cycle to redesign and adapt to cultural diversity. Moreover, DBR allowed us to redesign the learning environments and update the design of the learning materials in some of the cycles. In this paper, we discuss three cycles, an Austrian cycle that included a teacher and students and two other cycles with mathematics teachers from Libya and Indonesia.



A B

Figure 2. (A) Saqqara step pyramid from ancient Egyptian architecture (https://pixabay.com/photos/saqqara-stepped-pyramid-ancient-1344249/); (B) Saqqara pyramid modelling using GeoGebra (Source: Authors' own elaboration)

STEAM Practices Cross-Cultural Design

This study follows DBR methodology and part of its' design heuristics is a web tool to aid teachers in constructing their lesson plans based on the proposed STEAM practices. The web tool is referred to as a dynamic lesson plan (DLP), which consists of four components for the teachers to choose from such as the student's age, the architectural type, the learning environment, and finally the technology for the architecture modelling and visualization (El Bedewy et al., 2021). The architectural choices proposed to teachers through this DLP tool are categorized, as follows: Ancient architecture, modern architecture, based on mathematical concepts architecture (connected to the mathematical curricula they are teaching), inventing their own (to allow their learners to create their own imaginary architecture) or free choice (to enable their students the freedom of choice of any architecture) (El Bedewy et al., 2021). DLP tool's aim is to provide teachers with a wide variety of options under each category, which may result in diverse ways to implement the proposed STEAM practices.

STEAM practices workshops were designed and guided by the theoretical frameworks as AMOEBA framework and were based on engineering-design-based learning (EDL) and problem-based learning to design the transdisciplinary tasks and modelling problems (Grubbs & Strimel, 2015). The modelling problems are offered to the workshop participants based on easy tasks through to harder ones and therefore, we offer scaffolding to participants. EDL helps in offering solutions to transdisciplinary problems by following the engineering design and scientific inquiry in the form of the 5E instructional learning model (engage, explore, explain, engineer, and evaluate) (Bybee, 1997; Bybee & Landes, 1990; Grubbs & Strimel, 2015).

The researcher who conducted these workshops in the three case studies is one of the authors of this paper. Hence, we started the three case studies with an introduction to engage and show participants how to model architecture using GeoGebra. The introduction started by exploring the main ideas behind these STEAM practices and the possible modelling and visualization technologies that can be used to simulate architectural constructions. We used the cultural lens idea by introducing multiple architectural modelling examples inspired by ancient Egyptian architectures such as the Saqqara step pyramid and Obelisk to allow cultural diversity.

Hence, the cultural lens was used to introduce participants to ancient Egyptian architecture, explore these examples and connect to historical relationships by providing information about these architectures. Moreover, the reason behind choosing these two ancient Egyptian architectural examples is their simple forms, which allowed us to explain how to model them and introduce participants to GeoGebra's main modelling tools and functions.

Saqqara step pyramid is a famous ancient Egyptian architecture in Egypt. We start by explaining the modelling steps to the workshop participants using GeoGebra. First, by analyzing the pyramid components, drawing the base of the pyramid as a polygon in the 2D view, and afterwards using the extrude to prism tool in the GeoGebra 3D view, to give the pyramid base different equal step height (**Figure 2**). These analyses of architectural models is part of engineering a solution to the modelling problem.

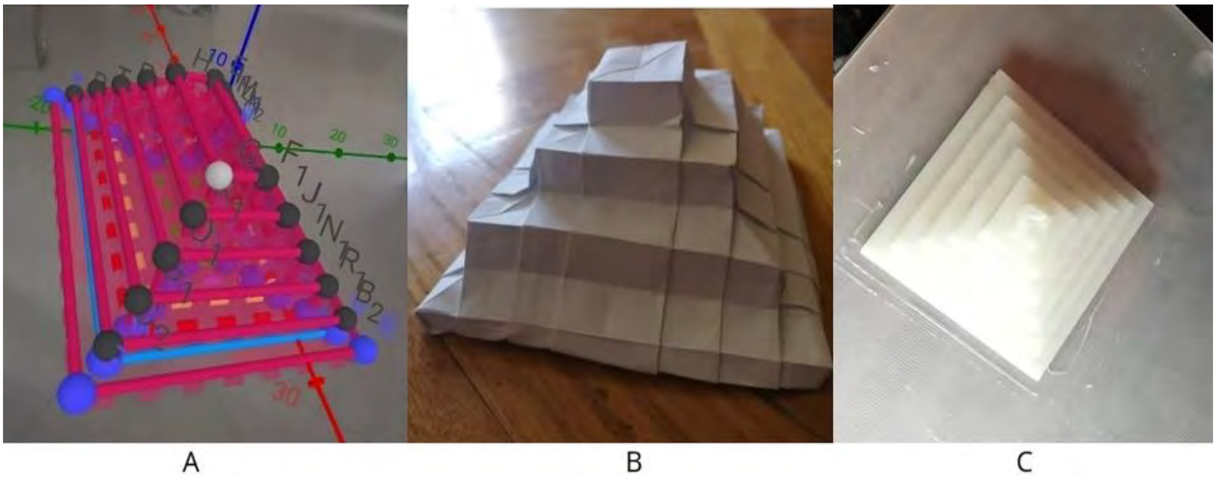


Figure 3. (A) 3D transformation of Saqqara pyramid using GeoGebra AR (Source: Authors' own elaboration); (B) Origami (Source: Field study, 8 year old student of Saqqara Pyramid, reprinted with permission); & (C) 3D printing (Source: Authors' own elaboration)

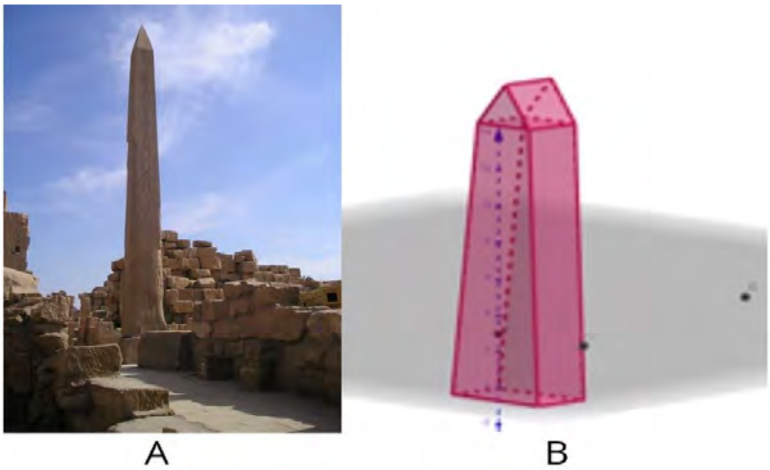


Figure 4. (A) Obelisk from ancient Egyptian architecture (https://pixabay.com/photos/egypt-luxor-karnak-obelisk-487/); & (B) Obelisk modelling using GeoGebra (Source: Authors' own elaboration)

We introduce the workshop participants to the 3D transformation idea by representing the same architectural model in various ways physical and digital, which is a key concept of these STEAM practices in utilizing technology. Therefore, we allow participants to *explore* various technology visualization options ranging from physical representations using 4D frames, 3D printing and Origami technologies, or digital representations using augmented reality (AR), virtual reality and 3D scanning technologies. Accordingly, we provide participants with images of the Saqqara step pyramid represented in two ways using GeoGebra AR, Origami, and 3D printing in order to motivate participants to visualize their architectural choices in digital and physical representations (**Figure 3**).

The second cultural lens architectural example used during the workshop's introduction is the Obelisk from ancient Egyptian architecture, to support cultural diversity. We provide historical relationships to the Obelisk architecture by exploring historical information. Afterwards, we explain the mathematical relationship by engineering a solution to this architectural model, such as by modelling the Obelisk base in GeoGebra 2D view using a polygon tool and extruding it using prism and pyramid extrusion tool in GeoGebra 3D view (**Figure 4**).

Part of the engineering process is to use tools and technologies; therefore, we provide a physical representation of the Obelisks using 3D printing technology and a digital representation using GeoGebra AR technology (**Figure 5**).

We allow participants to engineer their own solutions by modelling these two architectural examples during the first workshop sessions, we follow their modelling progress in real-time using the GeoGebra

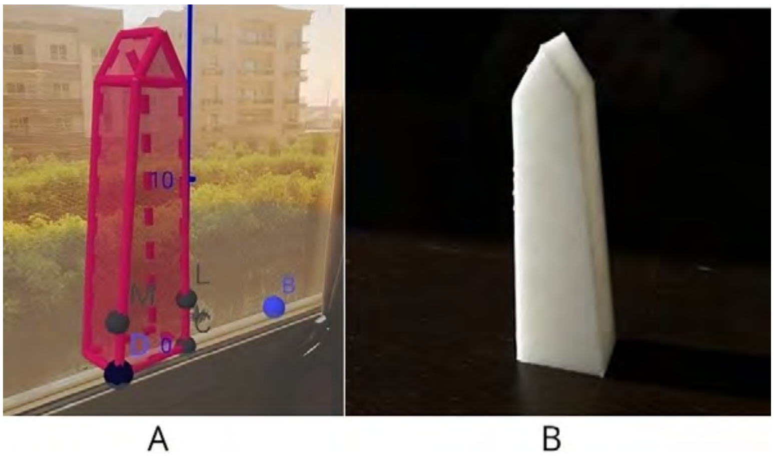


Figure 5. (A) Obelisk using GeoGebra AR; & (B) Obelisk in 3D printing visualization (Source: Authors' own elaboration)

classroom, which allows us to provide scaffolding for participants (Astafieva et al., 2021). This was the workshop introductory part that took place in the three case studies, where we start by engaging participants in STEAM practices learning goals then we explain the basics of the GeoGebra architectural modelling, and how to use GeoGebra functions and tools in 2D and 3D views to engineer a solution while exploring the architecture's cultural and historical relationships.

The rest of the workshop in each case study, was to use culturally relevant architectural modelling examples from each country as Austria, Libya (MENA Region) and Indonesia. Afterwards, we provide participants with unguided tasks in the form of final projects to model architectures, this allows them to engineer their own solutions to their architectural modelling. These STEAM practices allow participants to analyze architectures from mathematical, cultural, and historical perspectives while utilizing technology to stress on transdisciplinary learning experiences. The cultural and historical connection takes place by allowing the participants to collect some historical facts about their chosen architecture and mention their cultural motivation in choosing these architectures, this is considered part of their *engineered* solution towards these STEAM practices outcomes. For example, some participants would choose an architecture that is related to their country, origins, or backgrounds, others would choose an architecture that they think looks nice, they visited in another country or based on mathematical equations that they would like to practice using GeoGebra modelling. There are motivational reasons for choosing an architecture, and this reflects the participants' cultural connection and diversity preferences they possess during STEAM practices utilization. Moreover, these practices allow participants to express their cultural diversity through architectural modelling and to tell a story behind their architectural choices. We follow the aforementioned steps with an evaluation process that is culturally driven. Therefore, we will discuss the three case studies and outcomes in the coming sections.

METHODS

We will discuss three case studies taking place in the form of workshops, where all of the participants participated on a purposeful sampling basis.

The Austrian workshop was conducted with an Austrian in-service middle-aged female teacher and 35 students in an Austrian vocational high school, their ages ranging from 15 to 16 years old. The Libyan workshop took place with 30 in-service mathematics teachers and was hosted by a Libyan organization conducting a GeoGebra introductory course online. The participants who provided outcomes were 16 teachers from Libya, and other participants from other countries in the MENA region: two from Egypt, three from Palestine, and one teacher from Algeria, Tunisia, and Saudi Arabia. In this paper we focus on the Libyan teachers' outcomes from these STEAM practices workshops as they were the majority of participants in these workshops, moreover, this paper draws attention to a country-based structure for equal sampling purposes in the case studies. The Indonesian workshop was held as part of a STEAM PD master's university course that was led by a Mathematics professor, seven in-service mathematics teachers and three pre-service mathematics teachers.

The data collection methods we used were semi-structured interviews, questionnaires and GeoGebra classroom questions. The three data collection methods were divided into sets of questions that focus on capturing the participants' teaching methods, the technology uses, STEAM education knowledge, lesson planning process, DLP tool reflections and opinions about these transdisciplinary STEAM practices, how the participants connected to architecture, culture and history, the reasons behind the participants' architectural choices and how the participants will use these STEAM practices. Moreover, we collected the participants' artefacts, which consisted of architectural modelling GeoGebra files, PDF files or word documents that included the historical and cultural relationship to the architectures and mathematical relationship that they used during modelling architecture. Moreover, these documents contained information about the visualization technology they used, which could be AR or 3D printing, this was dependent on technology affordance to participants, and it contained lesson plans. We followed an inductive-deductive coding approach to analyze the semi-structured interview and questionnaire data and document analysis for the participants' artefact analysis, which resulted in emerging themes (Weber, 1990). We used a tree map form to help in visualizing some of the frequencies for specific codes. Moreover, we followed that with data triangulation to see the relationship between the codes and themes across the data sources. The coding process has been evaluated with other researchers from the field to increase the reliability and validity of the findings presented in this paper. Furthermore, we will highlight how the workshops were structured taking place in different countries and the participants' outcomes to analyze how these STEAM practices promote cultural diversity.

THREE CASE STUDIES

In this section of the paper, we will discuss the structure of each case study that took place in Austria, Libya and Indonesia and the participants' practices outcomes.

Austria Case Study

In the Austrian case study, the language used during the workshop and in the learning materials was presented in English. The teacher used German as the native speaking language when communicating with her students and she translated to us the students' questions and inquiries during the workshop. We established a teacher-researcher collaboration methodological recommendation to design the workshop content introduced to students, technology use and assessment possibilities for these STEAM practices implementation. The workshop was held only once and took place in a hybrid form, half of the students were online, and the other half attended from the school. The cycle lasted for four months, during which we conducted four semi-structured interviews with the teacher to capture her reflection on STEAM practices and on her students' progress and inquiries, which we will explain further.

In our first meeting with the Austrian teacher, we introduced STEAM practices concepts and GeoGebra modelling tools and functions. In the second meeting we designed the workshop with the Austrian teacher to be taught to her students, we agreed to include the workshop introductory part that includes ancient Egyptian architecture examples such as the Saqqara step pyramid and the Obelisk. Afterwards, the teacher decided to choose an Austrian ancient architectural example, the Carnuntum from Upper Austria, to be introduced to the students during the workshop (**Figure 6**).

The Austrian teacher chose the Carnuntum because she knew that students learned it in their history classes, it connected to their culture and would establish a historical relationship and discussion about the Romans who used to live in Austria. We provided the Carnuntum modelling steps using the polygon tool and transformation functions to reflect and rotate the model in GeoGebra, this was followed by the teacher's recommendation on the technology preference to use GeoGebra AR visualization (**Figure 7**).

The Austrian teacher assigned the students a final project task at the end of the workshop, to model architecture and connect to culture, history, and technology and to mention the motivations behind their architectural choices. The teacher gave her students the architectural choice freedom to allow them to model any architecture they like without any cultural, historical, geographical, or language restrictions as she said:

Figure 6. The Carnuntum: An ancient Austrian architecture (https://de.wikipedia.org/wiki/Heidentor_(Carnuntum)#/media/Datei:Heidentor_Carnuntum_4487.jpg)

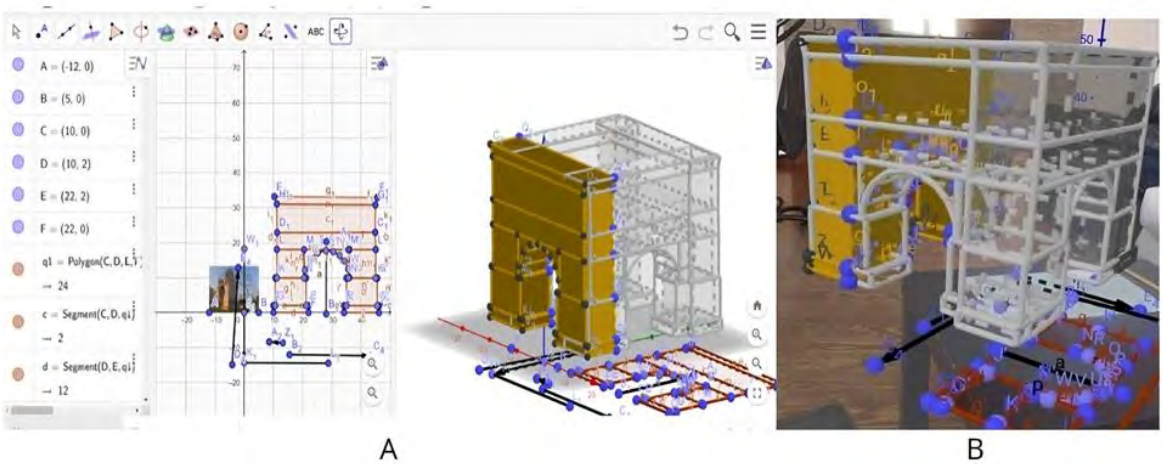


Figure 7. (A) The Carnuntum modelling in GeoGebra; & (B) The Carnuntum visualization using GeoGebra AR (Source: Authors' own elaboration)

> "I would give them, you know, the freedom that they can choose. They can choose anything does not have to be from Austria. (International) is also a positive thing" (interview).

The student's final projects including the motivation and historical facts were translated into English as initially some of them were represented in German.

Austria case study results

After the data analysis of the Austrian case study from the teachers' interviews and from the students' artefacts, several themes emerged, which we will discuss in this section. ***The majority of the students chose architecture from different places than their original countries*.** The summary of the students' architectural choices on a cultural and geographical basis can be visualized in **Figure 8**. We provide a tree form map as a result of our analysis showing which architectural choice belongs to a certain defined category from the analyzed codes, which have a geographical region classification. Therefore, either the students chose an architecture from their own country, from another country or another object to model that is not an architectural model. The first level of the tree map concludes the codes corresponding frequencies from 22 participants, eight students chose architectures from the same country, 10 represented architectures from different countries, and one student modelled something different than an architectural model. The second level of the tree map shows the names of the architectures the students modelled during their final project and their corresponding geographical regions and in which countries they are located (**Figure 8**).

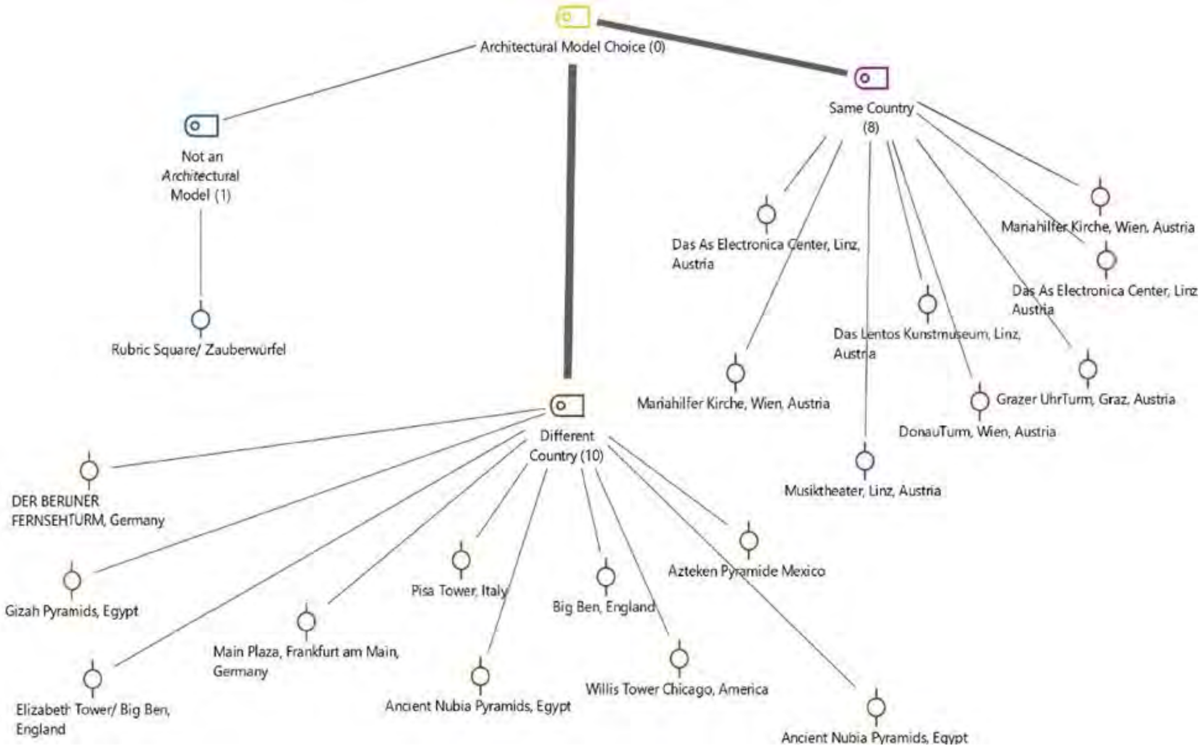


Figure 8. A tree-map showing three categories of students' architectural choices & their corresponding frequencies, width of line showing highest frequency (n=22) (Source: Authors' own elaboration, using MaxQDA software)

The frequencies denote that most of the students chose architecture outside their own country. These findings contribute to the fact that the cultural boundaries that we assumed and expected to find are diminished in one way or another, and one cannot assume that students would prefer architecture from a certain country to another. However, we found out that the architectural location is not as relevant as the reasons behind choosing these architectures, which was obvious from the motivations students provided to justify their architectural choices. Hence, these findings lead to a second theme that appeared from the teacher interview: ***The Austrian teacher showed a positive attitude towards the architectural, cultural, historical and language diversity she received from the students' final projects***. As she reported,

> "I do not care what the students choose as an architecture, because it could also be that they were on a holiday somewhere else than Austria, which they liked and developed a connection to another place outside Austria. And I have another student, they want to choose something from their home country only. So, it did not matter if they did not connect to the culture of their country, it does not matter if they are interested in another culture from another country" (interview).

This theme revealed the motivation behind choosing architecture is important to provide a relevant justification along with historical facts, which can show the student's interests in other architectures and cultures not necessarily their own. The students' architectural preferences could allow them to tell a story about cultural, historical, technological, and mathematical relationships. Another final theme emerged that ***some Students represented their final projects in the English language while others used the German language***, this also reveals the freedom of choice the teacher gave to her students as she did not specify any language preference. Therefore, we will show four of the students' architectural choices with their corresponding justifications/motivations, mathematical modelling using GeoGebra and historical and cultural relationships from the students' final projects, as evidence for the previously discussed emerging themes (**Appendix A**).

The teacher reflected on the overall STEAM practices learning goals positively as she stated that it connected her students to culture and history during the architectural modelling, which was also based on the student's mathematical knowledge and allowed them to explore technologies such as GeoGebra AR. We

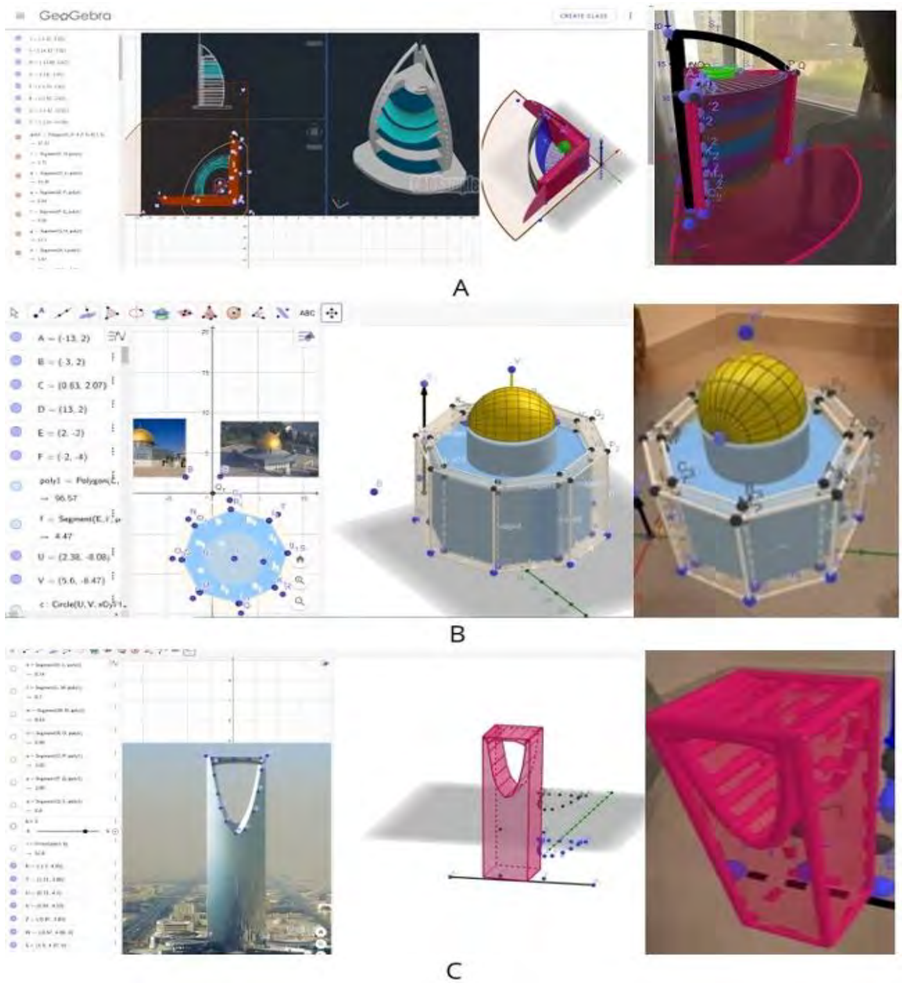


Figure 9. Three selected architectures from MENA Region during PD workshop showing GeoGebra 3D modelling & GeoGebra AR: (A) Burj Kalifa in Dubai, (B) Dome of Rock Mosque in Palestine, & (C) Kingdom Center Tower in Riyadh Saudi Arabia (Source: Authors' own elaboration)

will now shed some light on another case study that took place in Libya to explore the cultural connection to these STEAM practices from another region.

Libya Case Study

In the Libyan case study, the language used during the whole workshop, the learning materials and the questionnaire questions for the data collection were translated into Arabic. The architectural examples presented during the workshop were diverse architectural examples from MENA Region to promote architectural and cultural diversity.

The Libyan case study took place online in the form of four workshop sessions as part of an ongoing PD GeoGebra course for teachers from MENA Region, these courses were divided into two parts the first part took place with 6 participants and lasted for three months and the second part took place with 24 participants and lasted for four months to implement the proposed STEAM practices. The participating teachers at the time of the workshop already knew how to use GeoGebra in terms of interface, tools and functions familiarity. The workshop was conducted in Arabic and during the first workshop session, the aim was to engage participants into STEAM practices idea of architectural modelling by introducing the 3D mathematical modelling tools while connecting to culture and history. We used a cultural lens by exploring with the teachers some mathematical modelling of ancient Egyptian architectural examples such as the Saqqara step pyramid and the Obelisk. These architectural examples allowed us to explain to teachers the basic GeoGebra modelling tools such as 2D and 3D polygons, extrusions and transformations. During the second workshop session, we explored a collection of ancient and modern architecture from MENA Region countries to foster cultural diversity and to establish a historical relationship by presenting some historical information about these

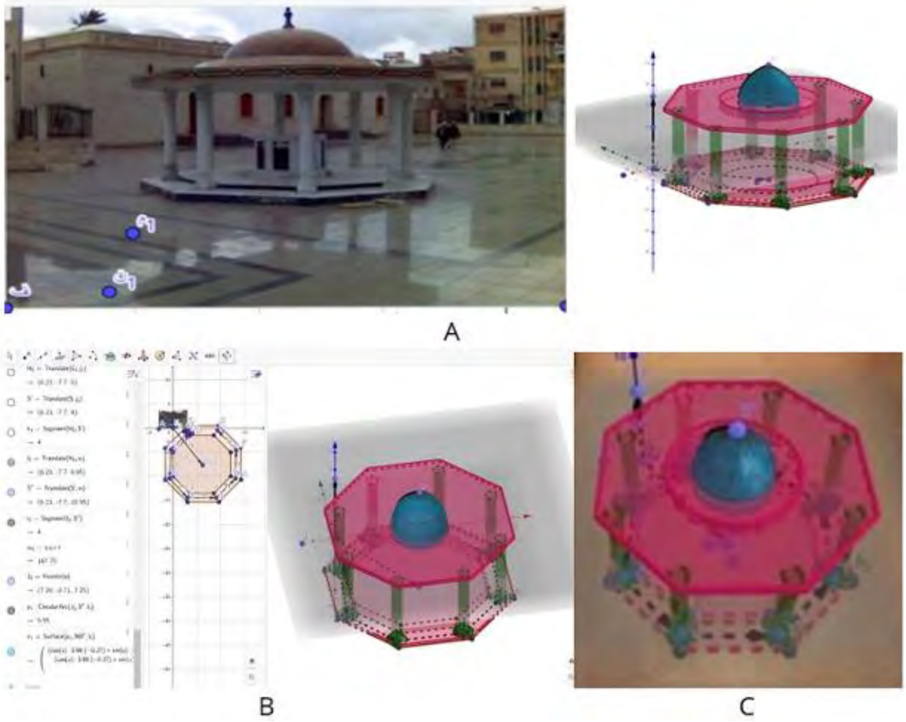


Figure 10. (A) Part of a Libyan Mosque; (B) GeoGebra 3D modelling of the Mosque; & (C) The Mosque in AR (Source: Authors' own elaboration)

architectures. We followed that by explaining a solution to these architectural models, by engineering a modelling solution using GeoGebra and by introducing technologies using GeoGebra AR (**Figure 9**).

The third workshop session was dedicated to scaffolding the teachers during their modelling attempts that we were tracking from the GeoGebra classroom (Astafieva et al., 2021). One of these modelling attempts was part of a mosque in Libya by a Libyan teacher, we used this architectural example and explained to participants how to model (engineer) a solution and how to use the surface of revolution function in GeoGebra (**Figure 10**) (El Bedewy et al., 2021).

The third workshop session was to further scaffold teachers in their modelling and engineering solutions to their architectural model choices. Moreover, to respond to their inquiries and introduce technologies to help visualize the modelled architecture in physical and digital forms. Since the workshop took place online, we chose GeoGebra AR, therefore, we selected some architectural examples the teachers modelled and showed them the steps to visualize them using GeoGebra AR. In the fourth workshop session, we collected the teachers' artefacts that were presented as GeoGebra files and collective videos of their work during the workshop, which included images of their architectural models' choices, and cultural and historical information about these architectures. All the collected data were translated from Arabic to English. We will highlight the results that emerged from the teachers' artefacts analysis.

Libyan case study results

In this section, we will highlight the emerging themes that were a result of the Libyan teachers' artefact analysis of PD course. The first emerging theme was from the analysis of the GeoGebra classroom of teachers' initial architectural choices and modelling attempts that took place before the final project submissions, ***two Teachers showed cultural diversity during their learning by modelling architectures from places other not Libya.*** One example from a Libyan teacher was the pyramids of Giza models from Egypt, the teacher mentioned in her interview that they are great architectural constructions and that was the reason behind her choice (**Figure 11**).

Another example from a Libyan teacher was the Ibiza tower from Italy, the teacher stated the reason behind her architectural choice as

> "It is a mystery and so interesting to try to model using GeoGebra" (interview) (**Figure 12**).

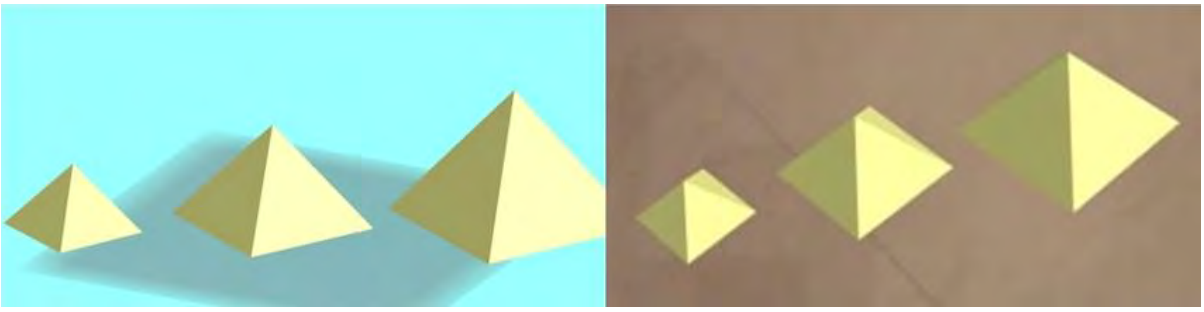


Figure 11. Pyramids of Egypt modelled by a Libyan teacher using GeoGebra & we provided them visualization using GeoGebra AR (Source: Field study, created by participant, using GeoGebra 3D and GeoGebra AR, reprinted with permission)

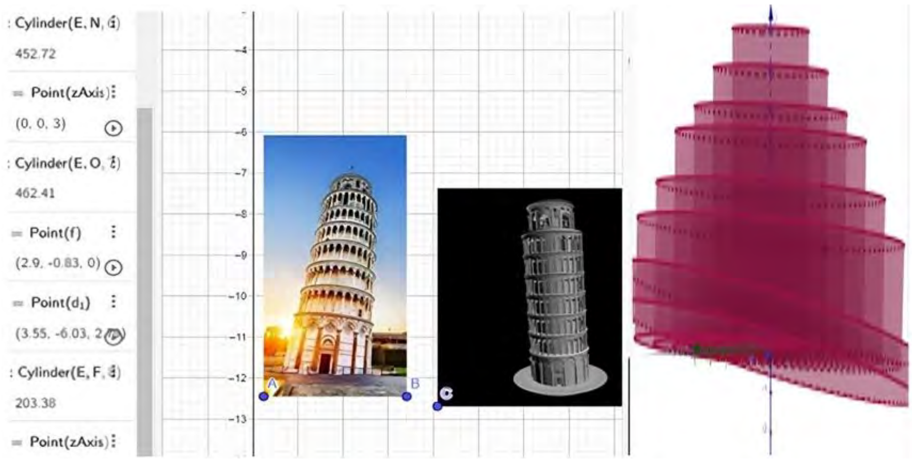


Figure 12. The Ibiza tower, Italy 3D modelling by a Libyan teacher (Source: Field study, created by participant, using GeoGebra 3D and GeoGebra AR, reprinted with permission)

On the contrary, the second theme that emerged from the teachers' final projects analysis and from the interviews and questionnaire analysis was different from that which emerged from the GeoGebra classroom analysis. The teachers' final project analysis that included the teachers from the first theme shows that ***all the teachers did not show any cultural diversity in their final projects, and they all chose Libyan architectural models from their own country to connect to their own culture and history.*** The teachers' architectural reasons were different, but they all wanted to connect their students to cultural and historical reasons. As one of the teachers stated,

> "I would like to connect my students to the culture of our country and city and integrate history as well" (interview).

Other teachers wanted to connect to real-life examples such as architecture that all students saw in reality and is well known to all the students, as a Libyan teacher stated:

> "I will take architectural models, for students, everyone knows, not only that the students saw this architecture on a TV, in a book, magazine or through the Internet, even not a video of this architecture because all of this is not like in reality. I will choose something from the city we live in. This is how I think about architecture. We would not get out of the city, where the students live, because if the student were to see the real building, the aim is to model architectures that they see with their own eyes. I would choose simple architecture, not a difficult one from the city to start with and knowing its history. A building that would exist in the city, simple, and at the same time well known to everyone in the classroom" (interview).

Another Libyan teacher said, choosing an architecture from my city would allow students to tell a story about it when they pass by it in reality and feel proud that they modelled it during their mathematics class:

> "I will start with my students, with a model from my city, the city we live in. are there. So that if the students happened by chance that they pass in front of this architectural building in my city**,** they would say and tell a story that they accomplished to model this architecture in the classroom" (interview).

The third emerging theme shows that ***teachers' architectural choices were also based on teaching preferences and were related to mathematics behind architectures, providing teaching examples to their students***. Teachers described the mathematics that they used during modelling these architectural examples in GeoGebra and how it could help them as teaching examples and they described the mathematical tools and functions used, such as 3D polygons, lines, curves, and transformations as translation, rotation and reflections. Moreover, one of the Libyan teachers said she would start with a simple architectural example from her city that includes simple geometric shapes, as follows:

> "I will start with architecture from my city with the students with a simple long cube-shaped building. And maybe we can add any other simple form to this one as a start maybe a dome. To introduce students to different architectures with simple forms and simple architectural principles" (interview).

Another Libyan teacher's approach in explaining architectural mathematical modelling to students by scaffolding them and preparing them to use the forms of the architecture first, then to construct these architectures using the trained forms:

> "I will prepare the students on the forms that will be used in these architectural models before we start as a preparation for this modelling task. So, I will prepare them on a form for tomorrow's lesson for example use the shapes first then use the architectural structure" (interview).

The last emerging theme shows that ***all teachers represented their work in the Arabic language and did not show language diversity***. This could be due to the fact that the spoken and written language used during the course was Arabic. In order to show these emerging themes, we provide some examples that include modelled Libyan architectural examples, the mathematical concepts behind these architectures, the reason behind the teachers' architectural choices and historical information about these architectures, that were captured from their final projects. These examples are evidence of the emerging themes and show cultural, historical and mathematical relationships to the teachers' architectural choices as captured from their artefacts (**Appendix B**).

Indonesia Case Study

In the Indonesian case study, the language used was English, while there was a real-time translation into the Indonesian language through the workshop sessions, and the participants' inquiries were translated into English, moreover, the architectural examples used were from Indonesia and the questionnaire and the final project requirements were translated into the Indonesian language.

The Indonesian PD workshop took place in the form of eight online weekly sessions that were held by a mathematics education department in an Indonesian university for three months. The course was designed by the researcher of this study and one of the paper authors in collaboration with a mathematics education professor and the mathematics education department dean, where they both approved the workshop content. All the workshop materials were provided in English while some of the data collection methods were translated into the Indonesian language. We used the English language during the workshop and the mathematics education professor was providing translation to the participating teachers to the Indonesian language because not all the participants had the same English language proficiency to ensure their understanding and the participants' questions asked in Indonesian language were translated to the researcher. The participants were modelling step by step in the GeoGebra classroom all the architectural examples that we provided through the eight sessions. For each architectural model, we explored and explained to participants cultural and historical information, and asked participants to collect further cultural and historical information about these architectural examples and to write them in GeoGebra classroom.

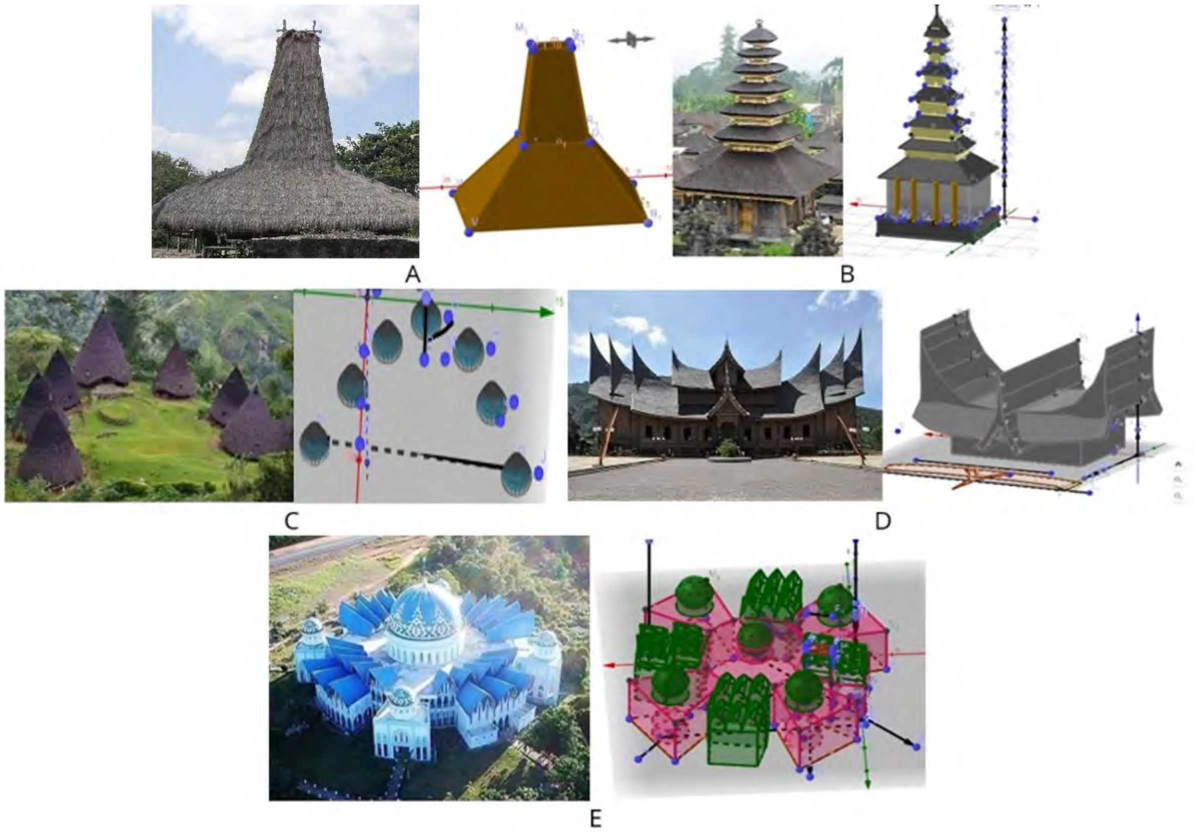


Figure 13. Indonesia architectural examples real images & 3D modelling using GeoGebra: (A) Sumba Indonesian House (https://en.wikipedia.org/wiki/Sumbanese_traditional_house#/media/File:House-skulls.jpg); (B) The Mother Temple of Besakih or Pura Besakih, Bali (https://en.wikipedia.org/wiki/Besakih_Temple#/media/File:Pura_Besakih.JPG); (C) Mbaru Niang House, East Nusa Tenggara (https://en.wikipedia.org/wiki/Wikipedia:Featured_picture_candidates/Rumah_Adat_Mbaru_Niang#/media/File:Rumah_Adat_Mbaru_Niang.jpg); (D) Rumah Gadang (https://id.wikipedia.org/wiki/Rumah_Gadang#/media/Berkas:Istano_Pagaruyuang.jpg); & (E) Nurussalam Great Mosque Batulicin, Kab (https://x.com/RagamMasjid/status/1362310342357884934?s=20) (Source: Field study, models are created participant, using GeoGebra 3D)

During the first session, we engaged participants into STEAM practices concepts and some literature on mathematical modelling and assigned participants tasks to collect literature on STEAM practices that connect to culture, history, and architectural mathematical modelling. Afterwards, we utilized cultural lens architectural examples from ancient Egyptian architecture, Saqqara step pyramid modelling using GeoGebra. In the following six workshop sessions, we used famous Indonesian architectural modelling collections that we chose based on the complexity of the models in order to scaffold the teachers from simple examples to more complex ones (**Figure 13**). We used to explain to the participants how to model these architectural examples, moreover, we explored different modelling approaches. Also, for some of architectural examples, we provided more than one engineered modelling approach to foster participants' problem-solving skills.

One of the later workshop sessions was dedicated to scaffolding teachers in their architectural modelling and responding to their inquiries, to ensure that all participants are on the same level of understanding. Moreover, during the later sessions, we introduced the use of technologies such as AR, which was recommended in the case of online learning by the mathematics education professor. The mathematics education professor was planning to facilitate a 3D printer for the participants to visualize their work in a physical form; this can be investigated in future publications. Later, the teachers were assigned an unguided task to provide their own engineered solution to these STEAM practices that connect architecture, culture, and history. The last workshop session was to collect participants' artifacts, which were submitted in English language. Evaluation process was in form of a presentation of teachers implementing artifacts to their peers.

Indonesian case study results

The first theme that emerged from the participants' final projects and questionnaire analysis is that ***all the participants chose Indonesian architecture in their final projects***. Some of the teachers stated several reasons for choosing architecture from their own country as for the architecture's beauty or aesthetic look, for its cultural or historical significance in their area and to their people, for its religious relationships to their people or for the architecture's touristic significance in their city. A second theme appeared that ***most of the participants used these STEAM practices to model architecture and highlight cultural and historical relationships as an opportunity to tell stories behind their cultures they originally belong to***. One of the participants mentioned that culture plays an important role in their country as a unique aspect that differentiates a tribe from another and they consider it their treasure

> "There are also various meanings behind all the richness of tribes, customs, cultures, traditions, languages, tastes in Indonesia. Each region with its characteristics deviates from the meaning of the wealth it has. This has become the great treasure of Indonesia" (final project PDF document).

> "I was motivated to choose this architecture because I come from the Dayak tribe. This architecture is also in the location, where I live now, Central Kalimantan. there is a lot of history and stories in this building for the Dayak people" (final project PDF document).

All the participants' motivations were culturally focused on their cities and their origins and one of the teachers said, these STEAM practices can help me tell the story of my unique culture to everyone and it can go international

> "I am so happy because I can introduce my culture to be international. I can introduce mathematics to my culture, which is Rumah Betang. I am too proud and too thankful" (questionnaire).

The third theme that emerged from the workshop observations and from the final projects' analysis is that ***all participants used English language to represent their final projects and presented their work in English during their last sessions In Front of their peers***. This was surprising to the university professor because she stated she was proud of her students that they were talking in English, expressing themselves and presenting their work. A fourth theme emerged that shows ***the majority of the participants changed their attitudes towards architectural modelling using GeoGebra and believed it was not hard to achieve***. This was also obvious from the GeoGebra classroom that their mathematical modelling proficiency increased during the workshop sessions. Moreover, a final theme shows that ***some participants provided several ways to model the same architectural model using GeoGebra, which reflects their problem-solving skills proficiency***. Therefore, we show some of the participants' architectural modelling outcomes from their final projects as evidence for the emerging themes and to show readers how they represented their architectural relationship by utilizing these STEAM practices (**Appendix C**).

Lesson Plans Comparisons

In this section, we selected three lesson plans created by three in-service participating teachers from the workshops that took place in Austria, Libya, and Indonesia, to find similarities and differences between them. The lesson plans were selected based on their availability in the case of Austria and Libya and based on their length and richness of information in the case of Indonesia workshops. Moreover, the available lesson plans were analyzed to check for the participant's understanding of the transdisciplinary STEAM practices and how will they implement them and use them with their students. Therefore, we used the Austrian lesson plan that was created by the teacher when she implemented these STEAM practices with her students. The Libyan lesson plan was created and delivered after a year after the Libyan teacher finished PD workshop. The Indonesian lesson plan was created by the Indonesian teacher at the end of their PD workshops and was submitted as part of her final project. We will discuss the emerging themes between the three lesson plans.

The three lesson plans' learning aims included GeoGebra learning and mathematical learning through the use of these STEAM practices. The Austrian and Indonesian lesson plans included architectural learning, in the form of mathematical knowledge or cultural and historical knowledge behind these architectures.

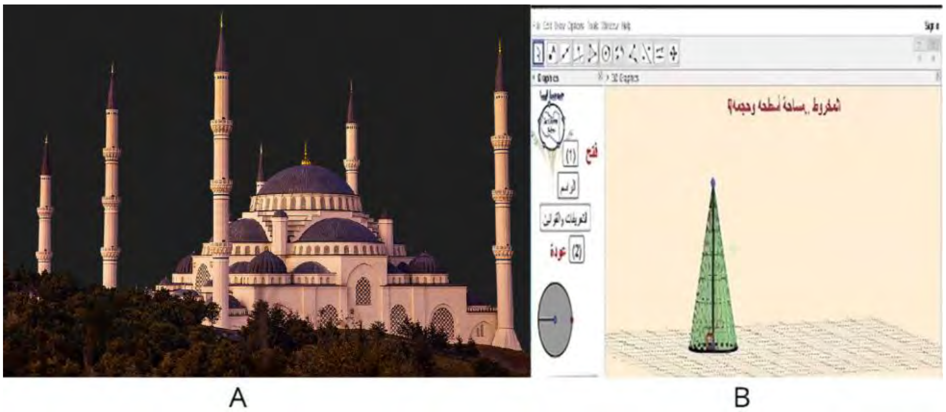


Figure 14. (A) Mosque minaret cone shape (https://pixabay.com/illustrations/mosque-turkey-sultan-ahmed-4338926/); & (B) cone shape using GeoGebra (Source: Field study, created by participant, using GeoGebra 3D, reprinted with permission)

The Libyan lesson plan learning aim was based on mathematical preferences only that focuses to teach students about the mathematical 3D cone shape and the teacher used GeoGebra and real architectural pictures to illustrate her mathematical learning intentions (**Figure 14**).

The Austrian lesson plan learning aim was to learn about GeoGebra, mathematical functions used in 3D programs, architectural information and GeoGebra AR. The Indonesian lesson plan learning aim was about shapes behind real architectures and how to model them using GeoGebra.

The lesson plans referred to three types of learning environments for applying these STEAM practices, the Indonesian lesson plan referred to the online learning environment and the Austrian lesson plan referred to a hybrid learning environment because of the pandemic in both cases while the Libyan lesson plan referred to classroom learning environments that took place after the pandemic.

The Austrian and Indonesian lesson plans indicated the use of technology such as GeoGebra 2D and 3D for modelling purposes, while the Libyan lesson plans referred to GeoGebra 3D use for visualization purposes. Moreover, the Austrian and Libyan lesson plans included GeoGebra AR technology visualization for applying these STEAM practices. The Indonesian lesson plan included other technology-learning platforms to be used by students such as Google meet and classroom while they referred to WhatsApp for students' communication.

The teaching methods used in the three lesson plans all shared similar concepts to foster students' cognitive skills such as collaboration, exploration, inquiry, critical thinking and problem-solving skills. The Indonesian lesson plan included evaluation based on the problem-solving process followed by students and based on "4C-Critical Thinking and Communication". The Indonesian lesson plan included information on students' collaboration and working in groups of three students to model architecture together and apply these STEAM practices. The Austrian lesson plan included points in the assessment that reflects students' inquiry skills evaluation in collecting architectural knowledge and solving architectural modelling unguided tasks.

Scaffolding is needed for applying these STEAM practices. The Indonesian lesson plan included instructions to scaffold the students while learning these STEAM practices, by guiding them through using GeoGebra modelling "the teacher provides guidance to students". The Libyan lesson plan included an unspecified number of sessions to finish the learning aims according to the student's understanding, which denotes scaffolding. Although the Austrian lesson plan did not mention scaffolding explicitly, the Austrian teacher reported in her interview that she provided scaffolding to her students during and after the workshop in using GeoGebra.

The lesson plans were created in diverse languages, the Libyan lesson plan was delivered in the Arabic language while the Austrian and Indonesian lesson plans were delivered in the English language, this shows that PD workshop used languages could have affected the lesson plan language later, which could denote language diversity for these STEAM practices.

STEAM practices three lesson plans showed transdisciplinary features as a strong explicit connection of mathematics to architecture, while it showed an implicit cultural and historical connection. The Austrian lesson plan was the only one to mention a collection of information related to the architecture selected by students as one of the learning aims, but the teacher did not specify any specific disciplines in connection to this required information. Although it was not written in the lesson plan, the Austrian teacher communicated this orally in the interview "the students have to do some research on their architectural building and write a few sentences or add some links or photos". This shows an explicit connection to architecture and an implicit connection to culture and history. The Indonesian lesson plan included only information about the architectures to be used in applying these STEAM practices as ancient Indonesian architectures. While the Libyan lesson plan did not mention anything related to other disciplines' connections, they used architectural examples to foster their mathematics teaching in an implicit way. Therefore, the three lesson plans defined the transdisciplinary connections for these STEAM practices, in diverse ways linking between architecture, culture and history.

The Austrian and Indonesian lesson plans provided information on assessing and evaluating the outcomes of applying these STEAM practices including students' attitude, knowledge and skills assessments. The Indonesian lesson plan referred to the assessments by observing learners' attitudes, skills and knowledge that could be reflected from their inquiries during these practices implementation or from the presentation and discussion of results. The Austrian lesson plan included a rubric in the form of a table with distributed weights to assess if students' outcomes include the file name format, motivation, information, GeoGebra 2D/3D and GeoGebra AR architectural modelling screenshots and extra points for complicated architectural modelling designs as they mentioned in their interview "If a student has chosen a more advanced architectural type (with advanced shapes), it is possible to gain additional points".

DISCUSSION

We discussed in this paper three case studies that took place in Austria, Libya and Indonesia in the form of online workshops, with participants coming from diverse cultures. From the data analysis and emerging themes from the use cases, we noticed some similarities and differences in the way participants utilized these STEAM practices that showed architectural, historical and cultural diversity (Martin & Casault, 2005).

The Austrian use case was different than that took place in Libya and Indonesia because the Austrian teacher introduced these STEAM practices to her students. The teacher established an Austrian cultural connection during the workshop as she used an ancient Austrian architectural example that was inspired by the students' history curriculum. But the teacher used another international approach as she referred to it, by providing her students with cultural freedom by choosing any architecture from any place or culture they like in their final projects, hence, she removed any cultural or regional boundaries in the way she utilized these STEAM practices. Therefore, we ended up with a diverse data set of modelled architectures, with diverse historical and cultural relationships, and with diverse stories describing the students' motivations about their architectural choices (Fylkesnes, 2018). Hence, the Austrian teacher praised her students' diverse outcomes, and we noticed that this architectural diversity may have exposed students to new cultures and new historical information through the usage of these diverse architectural models in their STEAM practices implementation. Which is a clear application of cultural exposure using the cultural lens concept and promoting ways to integrate culture into educational practices (Luciak, 2010).

In the Libyan case study, we noticed that only two teachers chose architecture from outside their countries during their learning of these STEAM practices, but this changed by the end of the workshop and appeared in their final projects' architectural choices when all the teachers chose architecture only from Libya. We believe this may be because the Libyan teachers focused on their teaching process for their students during their STEAM practice learning. Hence, their architectural choices were based on mathematical concept relationships, which they described with the architectural forms, the mathematical operations, and functions in relation to their architectural models (Hessam & Sotoue, 2016). Therefore, the Libyan teachers were trying to connect architectural modelling to the mathematics they are teaching, which was obvious from their interview and questionnaire data. Moreover, the Libyan teachers' cultural and historical facts as well as their language use in their final projects' representation were all culturally based and strongly bound to their own origins and cultures. As the Libyan teachers emphasized that this is how they will utilize these STEAM practices in their teaching to allow students to connect to culturally and historically Libyan architectures that are inspired by various Libyan cities (Luciak, 2010).

The Indonesian use case showed that participants had a strong relationship to their cultures throughout the workshops, none of the participants represented any diversity except in the languages used for their final project representation. The participants' justified their choices as in Indonesia there are multiple cultures and therefore, these STEAM practices gave them the opportunity to visualize and spread their cultures, which they believed is a treasure (Luciak, 2010). The Indonesian teachers showed several approaches towards solving mathematical modelling problems, which fostered their problem-solving skills through these STEAM practices implementation.

Therefore, we noticed the three case studies highlighted the teachers' and students' perspectives about utilizing these STEAM practices were different in terms of mathematical emphasis and cultural and historical relationship. Students would choose architectures that differ from teachers who are continuously looking for learning goals to use in their teaching practices. Therefore, it is up to the teacher to decide in which direction they will utilize these STEAM practices by allowing cultural diversity through architectural diverse options or by focusing on their own cultural connections only with their students. Moreover, it is also important to understand the reasons behind the architectural choices teachers or students choose because this reflects their cultural lens acquisition. As some of the Libyan teachers noted that they prefer architecture that all the students saw in real life, architecture that they can go and visit with the school on a field trip, or architecture that the students learned on a cultural or historical basis, and they want to strengthen these connections by establishing collaborations with history teachers to apply these STEAM practices as the Austrian teacher did during the workshop utilizing architecture that the students learned in their history class.

The lesson plans findings show that the teachers used these STEAM practices in different ways including aims, transdisciplinary connections, teaching methods, technology use, learning environments and assessment ways. The Austrian and Indonesian lesson plans show that the teacher's lesson aim was focused on implementing these transdisciplinary STEAM practices. While the Libyan lesson plan, showed that the lesson aim was purely mathematical and the teacher used the architectural modelling idea to emphasize the mathematical aim, so she provided pictures of architecture that support the mathematical lesson she is teaching. She said in her interview that she did so to connect mathematics to real life and overcome shortages from deviating from the specified curriculum. Therefore, these findings highlight that these transdisciplinary STEAM practices may be considered flexible and could be used in different ways to serve diverse learning aims.

Teachers in the three case studies received PD and training to help teachers apply transdisciplinary practices and integrate disciplines that supported them in using these STEAM practices (Civitillo et al., 2018). Teachers utilized the transdisciplinary nature of these STEAM practices and managed to integrate several disciplines together (English, 2016; Soublis T., 2017). Furthermore, these STEAM practices trainings introduced teachers to mathematical modelling and provided them with the opportunity to practice it, which is a crucial thing for teachers training and development (Blum, 1993; Blum & Ferri, 2009; Niss & Blum, 2020).

In comparing and contrasting the three case studies, we are discussing the diverse options these STEAM practices can offer. These transdisciplinary STEAM practices can offer various ranges of diversity ranging from architectural, cultural, historical, technological and language that are all connected to each other and affect each other in one way or another. Moreover, they affect the mathematical modelling approaches that are connected to the architectural choices' forms (Niss & Blum, 2020). In addition, teachers can use these transdisciplinary STEAM practices in different ways by deciding on which diversities to adopt when they introduce these STEAM practices to their students. The Austrian teacher encouraged architectural diversity to her students and the students' outputs reflected this diversity, by choosing architectures from different places around the world and connecting to cultural and historical diversity. On the contrary we did not find cultural diversities among the other two use cases taking place in Libya and Indonesia when it comes to architectural choices. These variations could be due to several reasons as it could be that teachers prefer to connect to something the students for sure see and know as in the Libyan case, or teachers already have enough diversity among their country and region that they prefer to utilize these STEAM practices as an opportunity to show these cultural diversities. Hence, these reflections on the study outcomes emphasize that the architectural, cultural and historical diversities are established through these STEAM practices utilization. These findings show that these transdisciplinary STEAM practices trained and allowed teachers to apply practices that are culturally responsive (Gay, 2013). These practices could be used in teaching and learning situations aiding the transformation of education to culturally responsive education (Gay, 2013). The three case studies' diverse views may allow us to promote these transdisciplinary STEAM practices from a cultural lens as they open room for cultural diversity through the architectural diversity nature (Fylkesnes, 2018; Martin & Casault, 2005).

CONCLUSIONS

In this paper, we propose transdisciplinary STEAM practices to aid in solving problems such as immigration and cross-cultural movements between countries and regions that result in multicultural participants, and if we keep providing standardized curriculums this may affect participants' cultural identity. These problems are sometimes neglected by educational authorities or hard to maintain to offer an inclusive education for cross-cultural students (Luciak, 2010). Therefore, these STEAM practices fill the gap of introducing cultural diversities and overcoming cross-cultural adaptations in teaching and learning situations. Instead of updating curriculums to achieve inclusive education goals, we can introduce practices that can offer a range of diverse expressions to multicultural students and for culturally responsive education (Gay, 2013). Hence, these STEAM practices could allow multicultural students to connect to their origins and tell motivational stories about their cultures by providing students with the opportunity to model any architecture and connect to its culture and history. The students' architectural choices could connect them to a memory or a place by telling motivational story behind their choices. Therefore, we believe these transdisciplinary STEAM practices promote diversity, which allows participants to express themselves, connect to their cultures, or explore new ones while learning mathematics. Hence, these STEAM practices may help address the immigration complexities and allow teachers and students to connect to their culture and provide teachers with a tool to handle multicultural students (Gay, 2013; Guo, 2023; Luciak, 2010). Furthermore, these transdisciplinary STEAM practices overcame transdisciplinary challenges teachers usually meet by allowing them to implement and design transdisciplinary learning practices through PD workshops. Moreover, these PD workshops offered teachers an opportunity to learn and apply mathematical modelling using GeoGebra.

In the future, this study could be exploited by other teachers and mentors to explore other ways of promoting various diversities and how teachers promote these transdisciplinary STEAM practices with their students. For example, by encouraging teachers to possibly use these STEAM practices allowing students to share architectural cultural stories with their peers promoting architectural and cultural diversity. Moreover, we can guide teachers to allow students to use their own languages to express these architectures and cultures. Furthermore, these STEAM practices could be utilized by curriculum designers or policy makers to be integrated in educational curriculum to promote learners' cultural diversity and expression.

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Ethics declaration: The authors declared that the research work has fulfilled all relevant ethical guidelines including statements of approval/consent. All necessary permissions were obtained, and guidelines followed for our study. The study was approved by the institutional ethics committee of institution name here on date here (Approval code: code here). Written informed consents were obtained from the participants.

Declaration of interest: The authors declared no competing interest.

Data availability: Data generated or analyzed during this study are available from the authors on request.

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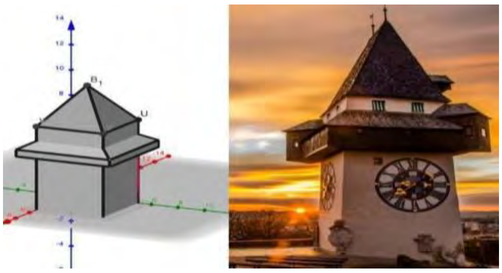
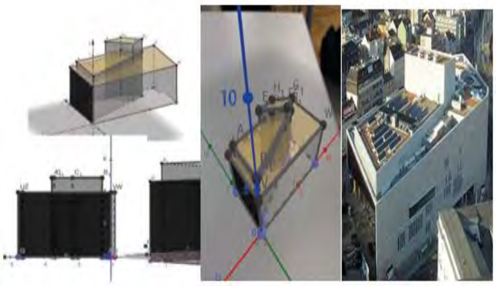
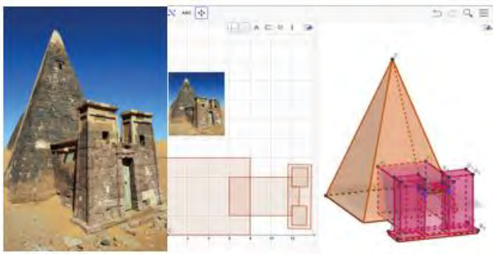
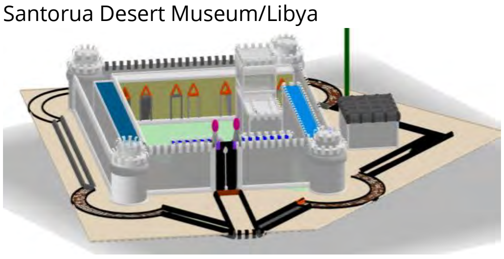
APPENDIX A

Table A1. Samples of student's final projects with their motivations & historical facts

Students' architectural choice	Justification/motivation	Historical facts
Graz Clock Tower/Austria 	I visited this tower when he was little with my parents in Graz, since then I wish to visit again but I never managed.	Graz Clock Tower is 28 meters height. It stands on Schloßberg & is with its clock face over five meters in diameter & gold-plated hands are landmark of Graz.
Music Theater/Linz/Austria 	Building I chose is Music Theatre. It's a big theatre in heart of Linz, & one of my favorite places to be. I have been there many times, & I'm totally fascinated by its architecture & modern style. I chose to rebuild music theater, as I am a dancer myself, & during COVID-19 times have really been missing going to theater & standing on a stage.	It's still relatively new, as it was built from 2008 to 2013 & opened with its first show ("Spuren der Verirrten") on April 12th, 2013. It includes three stages, one of them big main stage "der große Saal".
Berlin TV Tower/Germany 	Berlin television tower came to my mind because I was doing a history presentation about life in GDR (& accordingly also Berlin) & thus immediately thought of well-known television tower.	- 368 meters high - Tallest building in Germany - 26,000 tons weight - Ball has a diameter of 32 meters viewing platform & a café at 203 meter altitude - 100-million-mark construction costs
Pyramid of Ancient Nubia/Egypt 	- It looks beautiful - I like Egypt - Old architecture>new architecture - Easy to build	Pyramids are smaller & steeper than royal ones of previous eras. Hills cannot be assigned with certainty to a king. It consists of actual pyramid & small mortuary temple.
Elizabeth Tower/Big Ben/United Kingdom 	- Looks artistic - I really want to see it in real life in future - It is symmetrical	Today, whole tower is commonly known as Big Ben, although that name is incorrect. Only its bell is called Big Ben. Tower was officially known as clock tower, in September 2012. Tower was renamed in honor of 60th anniversary of throne of Queen Elizabeth II in Elizabeth Tower. Tower has a height of 96.3 meters.

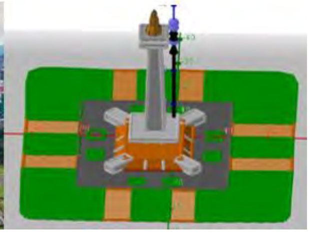
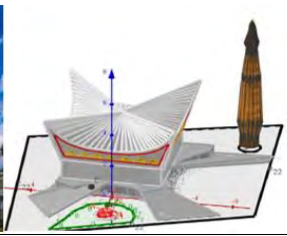
APPENDIX B

Table B1. Results of some of the Libyan teachers captured from their final projects

Teachers' architectural choice	Mathematical concept	Teacher's motivation & facts
Santorua Desert Museum/Libya 	In line with concept, where I made basics of shape, then geometric transformations as simplified reflection of engineering process that provides us with accuracy, speed, & effort. I used cylinders, clear prisms, polygons, columns, parallel lines, triangles, & arcs.	As this model exists until now built by Italians in city of Hun Al-Jufra in Libya. And our ancestors confronted enemies, & we are proud of them towards their example in rejecting humiliation for sake of homeland. Italians established this model as a fortified center for them.
A minaret as a corner of Sidi Abd al-Salam al-Asmar/Libya 	- Stereotactic geometry & geometric transformations. - Stereograms & transformations of ancient Islamic geometries & history	It was founded by Sheikh Abd Al-Salam & set up a corner next to it, & for his success is foundation of a facility to house poor & it has a library that adapts many books. Purpose of this library is to spread Islamic teachings for more than five centuries.
The Marcus Arch/Libya 	- 3D geometric shapes & their relationship to architecture - It consists of a set of stereoscopic geometric shapes	This arch was built to commemorate Roman Emperor Marcus Aurelius, who ruled from 161 to 180. This arch is located in Bab al-Bahr neighborhood north of old city of Tripoli. This model is part of culture of my country & ancient history of Libya.
The Clock Tower/Libya 	Square, cylinder, & semi-circle geometric shapes in decoration	Clock tower/an ancient building. Tower was built in Ottoman era between (1866-1870) & it was ordered to be built by governor, Ali Reda Pasha, with purpose of alerting residents of Tripoli to time, where there is a clock at top of tower on its four sides. It is related to history of Tripoli, & it is still present in square that was named after him, Clock Square in old city of Tripoli.
Tripoli Tower/Libya 	Rotation as well as clear symmetry vertical and horizontal for this building	An administrative complex in days of former era of President Muammar, it was called Al-Fateh Tower, & because this building can be seen from all areas of city & from all directions, & for this reason, site of its location was chosen. It is considered first tower as well as tallest tower built in my country.

APPENDIX C

Table C1. Some samples of Indonesian participants' final projects with their motivations & historical & cultural relationships

Architectural model	Motivations/justification	Historical & cultural relationship
National Monument (Indonesian: Monument Nasional, abbreviated Monas)  	National Monument is an icon of city of Jakarta. Located in center of Jakarta, it becomes an attractive tourist spot & educational center for residents of Jakarta & its surroundings.	President Soekarno said that monument concept is a characteristic of Indonesian culture, which is also shown through concept of building historic temples. President Soekarno, who planned to build a monument similar to Eiffel Tower in field right in front of Merdeka Palace.
Betang House  	I was motivated to choose this architecture because I come from Dayaktribe. This architecture is also in location, where I live now, Central Kalimantan. There is a lot of history & stories in this building for Dayak people..	Betang house is a symbol of local wisdom of indigenous peoples in Indonesia. A million meanings are contained behind Rumah Betang. For construction alone, upstream of house must face rising sun For Dayak, it signifies that yare hard workers. Dayak have to work to survive since rising of sun. As for downstream, house is made in direction of sunset. Philosophically it means that work of Dayak tribe will stop in afternoon & start again tomorrow morning.
Masjid Agung Demak  	Masjid Agung Demak is oldest mosque in Indonesia & as one of centers for spread of Islam on island of Java by Wali Songo.	Masjid Agung Demak is a blend of Hindu & Islamic culture. This proves that there has been cultural acculturation between Islamic & Hindu cultures in Kingdom of Demak. This is intended so that people see a new religious symbol, but without changing something that already exists, so that Islam is widely accepted by community.
Masjid Raya Padang  	I love beauty & unique design, which was a challenge to model it suing GeoGebra.	On roof of mosque there is no dome, but a goujon-shaped roof, like ancient mosques in Java, which is an acculturation of Islamic, Hindu, & Minangkabau cultures.

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