A Case Study on Students' Critical Thinking in Online Learning: Epistemological Obstacle in Proof, Generalization, Alternative Answer, and Problem Solving

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Abstract: Critical thinking is a key transversal competency of the 21st century, but some students have difficulty, especially during the transition to online learning due to the COVID-19 pandemic. This study aims to identify epistemological obstacles in critical thinking related to proof, generalization, alternative answers, and problem-solving. This online learning involved 30 prospective mathematics teachers through video conferences. An exploratory case study was conducted on 9 mathematics teacher candidates with the highest exam scores. Data were collected from the results of 4 mathematical critical thinking questions. The data were analyzed and described based on a predetermined framework. The results show various epistemological obstacles in critical thinking, namely difficulty in proving the relationship between two concepts, generalizing the relationship, finding multiple alternative solutions, and solving problems. The epistemological obstacles found can be the focus of lecturers in creating more structured online learning. Online learning needs to be well-planned in terms of the use of learning resources, learning media, and integration with technology. Learning should pay more attention to understanding the relationship between concepts, the flexibility of concepts and procedures, as well as the habit of drawing in geometry learning..

Keywords: epistemological obstacle, critical thinking, proof, generalization, alternative answer, problem-solving

INTRODUCTION

The COVID-19 pandemic has led to the widespread closure of face-to-face learning activities globally (Chertoff et al., 2020; Ferrell & Ryan, 2020; Toquero, 2020). As a result, emergency instructions were issued to shift to online learning, which is a model of education that utilizes several network technologies (Moore et al., 2011), including applications such as WhatsApp, Telegram, YouTube, and video conferencing.

The adjustment to online learning implementation has led to several issues, including institutions having limited time to prepare for online learning, leading to anxiety among educators, students, and parents (Daniel, 2020). Moreover, problems arise due to the lack of experience using e-learning applications (Daniel, 2020; Zaharah & Kirilova, 2020), causing some schools to be ill-prepared to effectively implement e-learning in the learning process, particularly at the student level (Mailizar et al., 2020).

The source of problems in the learning process is related to didactic design (Suryadi, 2019), which can create epistemological obstacles when the limitations of the context used in didactic design are encountered (Sulastri et al., 2022). Epistemological obstacles can occur when students misunderstand tasks or questions given to them (Brousseau, 1997), arising from the limitation of one's knowledge in a specific context, which cannot be applied in the current situation (Suryadi, 2013).

Based on preliminary studies, several problems often occur in proof, generalization, determining alternative answers, and problem-solving (Luritawaty & Prabawanto, 2020), which are some indicators of critical thinking (Daniel & Auriac, 2011; Ennis, 2011; Sanders, 2016; Facione, 2011). This finding is in line with Safrida et al. (2018), who reported that the mathematical critical thinking abilities of pre-service mathematics teachers in one Indonesian university were still low. Moreover, students often exhibit limited critical thinking skills and slow development (Karandinou, 2012; Pascarella et al., 2011).

The critical thinking skill, which has not been optimally achieved, needs further attention. Critical thinking is a key transversal competency in the 21st century that should be owned and developed by the education system, particularly at the secondary and tertiary levels (UNESCO, 2016 ; Moore, 2013). Furthermore, it is a complex process that requires high-level reasoning to achieve desired outcomes (Da et al., 2011; Halpern, 2014). It also involves reflective and logical thinking, which focuses on deciding what to believe and do to make wise decisions through ideas or actions (Dominguez et al., 2015; Ennis, 1993; Ennis, 2015).

Critical thinking is important because it is relevant to academic fields, particularly in higher education (Wechsler et al., 2018). It also helps students to effectively engage with social events, knowledge, and practical problems in planning, managing, monitoring, and evaluating academic tasks (Peter 2012). Critical thinking skills are expected to prevent errors in decision-making (Butler, 2012). Students with these skills can engage in positive and productive activities daily. Critical thinking also significantly correlates with Grade Points (GP) (Facione, 2011).

The issues arising from online learning implementation and the importance of critical thinking skills that do not align with current achievements make this study important. Studies on epistemological obstacles need to be conducted to understand the problems that arise in the series of critical thinking processes. Some related studies on epistemological obstacles have been carried out (Obreque & Andalon, 2020; Siagian et al., 2022; Sulastri et al., 2022; Sunariah & Mulyana, 2020). However, studies on epistemological obstacles in online learning have not been widely conducted in critical thinking skills, especially in proof, generalization, determination of alternative answers, and problem-solving. Therefore, this study examined the epistemological obstacles in online learning, particularly to identify possible obstacles when students perform cognitive actions in critical thinking related to proof, generalization, determination of alternative answers, and problem-solving. This study used geometry material because previous studies had revealed that the material was relatively difficult for students (Fujita et al., 2017; Brunheira & Ponte, 2019 ; Vasilyeva et al., 2013).

METHOD

Critical thinking skills are one of the crucial goals in the curriculum. The importance of critical thinking skills is detailed in Figure 1.

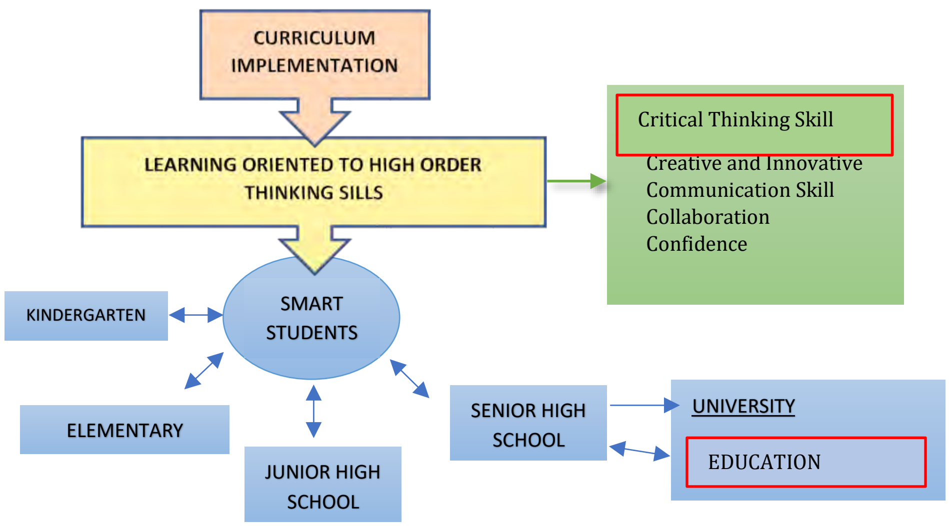


Figure 1: HOTS-oriented learning grand design (Ariyana et al., 2018)

The critical thinking skills in this study were marked by various achievement indicators, consisting of proof, generalization, determination of alternative answers, and problem-solving. The material studied was related to these indicators, such as geometry in college for the Basic Education Capita II selection course. Cube, cuboid, prism, and pyramid were used as objects. Furthermore, the concepts were taught in online learning. Learning is conducted through Zoom meetings with adapted learning steps from Arends (2012), as described in Table 1.

Table 1. Steps to Implementation of Online Learning

Steps	Activity
Introduction	- Presenting the problem through Zoom meeting - Grouping students through a Zoom meeting
Core Activities	- Delivery of materials and exploration through a Zoom meeting - Presentation and discussion through Zoom meeting
Closing	- Analysis and evaluation of the problem-solving process through a Zoom meeting

The exploration case study design was used because of its ability to manage various evidence and explore situations (Yin, 2018). Additionally, it can identify and describe epistemological obstacles in students' critical thinking processes during online learning. The preparation began with selecting the sample description, which is prospective mathematics teacher students. The selection was based on several related studies stating that their critical thinking achievement is still low.

The preparation continued with selecting the location and obtaining permission. The chosen location was one of Indonesia's educational institutions with a mathematics education program aligned with the targeted subject. Based on preliminary studies, there were still issues with developing critical thinking skills in this location. Permission was obtained from the head of the program at the institute. The study involved 30 second-year prospective mathematics teacher students aged 18-20 years old with the application of online learning. Moreover, nine students with the highest test scores were chosen as samples for further investigation. They voluntarily agreed to participate in this study. They consisted of 1 male and 8 female with a total average score of 10.5 out of a maximum score of 24.

The data in this study were collected from the mathematical critical thinking instrument test results, which were used as a reference in exploring epistemological obstacles that influenced the sample. The test was conducted at the end of the learning process after studying all the material. The essay questions form was used to test the student's ability. Unlike multiple-choice questions, essay questions allow responses to state the reasons for choosing an answer (Ennis, 1993). This also opens up opportunities for exploring the answers given. The test questions were validated by two experts, mathematics education lecturers at one of the universities in Bandung, Indonesia, before being used to measure mathematical critical thinking ability. The instrument comprised four questions representing these skills indicators (Daniel & Auriac, 2011; Ennis, 2011; Sanders, 2016; Facione, 2011).

The first question was related to the ability to prove. The subjects were given a situation about two prisms with certain volumes and conditions, and then they were asked to prove whether the three sides of the base of the first prism were always larger than the second prism or not. The second question was related to the ability to generalize. The subjects were given a situation about a cuboid and a triangular prism, and then they were asked to generalize the relationship between the two shapes. The third question was related to alternative answers. A problem was presented about fitting cube-shaped boxes into a cuboid cabinet. The subjects were asked to calculate the maximum number of boxes fitted into the cabinet using two methods. Finally, the fourth question related to the subjects' problem-solving ability. The subjects were given a picture of a pyramid-shaped building with certain conditions described. They were asked to calculate the area of material needed to construct the building according to the given conditions. The answers to all four questions are then collected as data. The data was also supplemented with interview results to identify and confirm the subjects' difficulties in critical mathematical thinking.

The data were analyzed by focusing on students' misconceptions and errors (Tan Sisman & Aksu, 2016). The first stage of the analysis was the scoring key creation. One point was given for a correct answer, and zero points were given for an incorrect answer. The second stage of data analysis was the development of an assessment framework based on indicators of mathematical critical thinking ability (Daniel & Auriac, 2011; Ennis, 2011; Sanders, 2016; Facione, 2011) with the Frisco theory, consisting of focus, reason, inference, situation, clarity, and overview (Ennis, 1991; Ennis, 1996). The Frisco theory helps demonstrate critical analysis and improve critical thinking ability (Ennis, 1996; (Dominguez et al., 2015). The third stage of data analysis was identifying student errors in their answers. This was performed by transforming the answers into Microsoft Word and categorizing them sequentially according to the framework in the second stage. The fourth data analysis stage investigated the relationship between student difficulties and the problem-solving theory framework. Next, in the fifth stage, the reasons for identifying epistemological obstacles were explained, while in the sixth stage, the results of identifying epistemological obstacles were discussed. In this final step, a descriptive analysis of student errors was conducted in relation to online learning implementation. An overview of the data analysis in this study is presented in Figure 2.

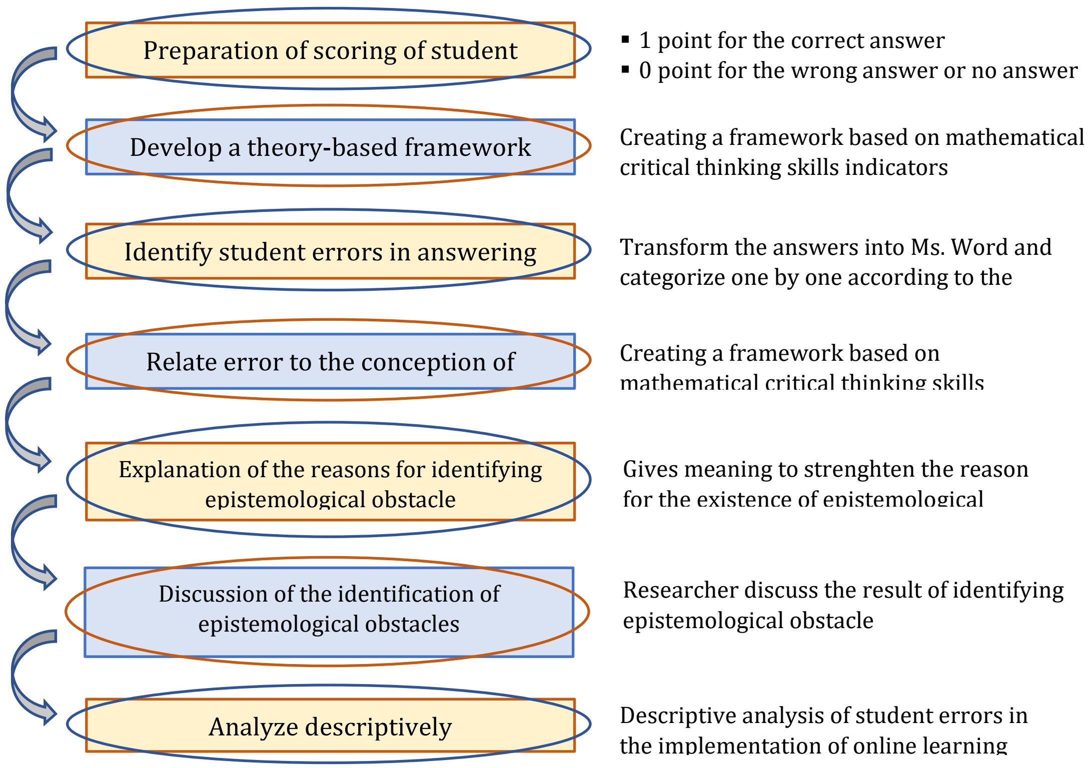


Figure 2: Data Analysis Framework

RESULTS

The analysis results of the mathematical critical thinking ability test of the nine samples in this study are empirically presented in Table 1.

Table 1. The result of critical thinking skills test

	Proof	**Generalization**	**Alternative answer**	**Problem Solving**
	Question 1	Question 2	Question 3	Question 4
N	9.00	9.00	9.00	9.00
Average	2.55	3.00	1.83	3.11

The maximum score for each question was 6. Based on Table 1, the average score for the first question was 2.55, which indicates that students have difficulty. Furthermore, the respondents found it difficult to prove the inference, situation, clarity, and overview sections. In the second question, an average value of 3.00 was obtained, indicating that they have difficulty generalizing in the situation, clarity, and overview sections.

Furthermore, the average scores obtained in the third and fourth questions were 1.83 and 3.11, respectively. This shows that students have difficulty in determining alternative answers and solving problems. They were only able to answer questions in one way and have problems in the situation, clarity, and overview sections. They also struggled with problem-solving, specifically in inference, situation, clarity, and overview.

The findings in Table 1 were strengthened by the calculation results of the percentage of students who cannot answer the questions correctly, as shown in Table 2.

Table 2. Percentage of students who cannot give the correct answer

No	**Question Indicator**	**Total Student**	**Percentage of students who cannot give the correct answer**	
			Total	(%)
1	Proving that the length of the prism base is related to the prism volume	9.00	8.00	88.89
2	Generalizing the relationship between cuboids and prisms	9.00	8.00	88.89
3	Counting the number of cubes that can be accommodated by the cuboids with at least two alternative answers	9.00	9.00	100.00
4	Calculating the surface area of a pyramid with certain conditions	9.00	7.00	77.78

Table 2 shows that the majority of students gave incorrect answers to the questions. For the first and second questions, only 1 student answered correctly, while for the third question, all answers were incorrect. Furthermore, two students answered the fourth question correctly. Epistemological obstacles in the critical thinking process occurred in the areas of inference, situation, clarity, and overview. The epistemological obstacles for each question are presented in Figure 3.

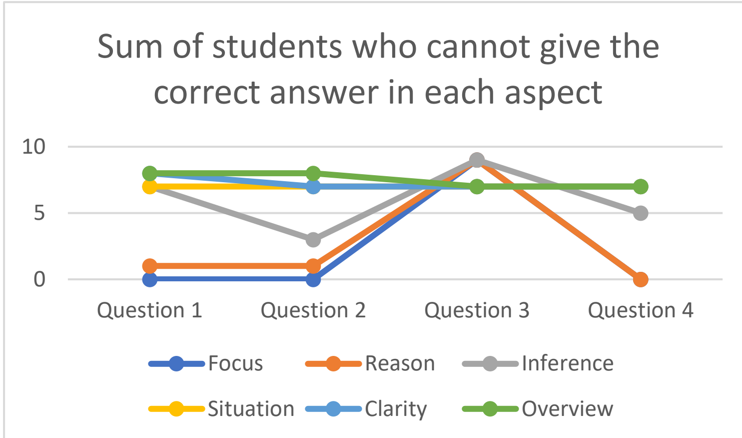


Figure 3: Epistemological obstacle in each question

Figure 3 shows five epistemological obstacles for proof in the first question. The first obstacle occurred in the reason stage, where 11.11% of students had difficulty explaining the idea or making a diagram for analysis. They directly calculated the volume of the prism without analyzing the idea. Therefore, they had difficulty continuing the problem-solving idea because the other parts were unknown. Furthermore, the second, third, and fourth types of obstacles occurred in the inference stage. In the second type, 33.33% of students failed to manipulate algebra related to the substitution of the basic elements of the prism. They only utilized the properties of an isosceles right triangle with two sides of the same length and wrote the formula for the volume ratio of the prism correctly. Students did not try to replace the area of the prism base with its details. In the third type, 11.11% had difficulty comparing volumes. They could not utilize the properties of an isosceles right triangle, where two of its sides have the same length. In the fourth type of obstacle, 22.22% had difficulty determining the formula for the prism's volume. The written formula was exchanged with the volume formula. The fifth epistemological obstacle occurred in the inference, situation, clarity, and overview stages, where 11.11%, 77.78%, 88.89%, and 88.89% of students could not answer the question. They could not understand the idea, adapt to the given situation, clarify the assumed or symbolized elements, and review the problem-solving process.

In question two, there are 5 types of epistemological obstacles to generalization. There were 11.11% of students that had difficulty in the reasoning stage. This is because they could not express the idea of analyzing the prism and the properties of the cube to determine the relationship between them. Furthermore, 22.22% of students experienced epistemological obstacles in the inference stage and made mistakes in analyzing some prism's properties. The third type of obstacle was encountered at the situation stage. Students could not determine the generalization that a cube is a four-sided prism because it does not have the properties of both a prism and a cube. The fourth obstacle was experienced by 22.22% of students at the overview stage, such as hastily drawing conclusions without reviewing the problem-solving process. The fifth type of question involved inference, situation, clarity, and overview, in which 11.11%, 77.78%, and 66.67% of students could not answer the given questions.

In question three, there were 6 types of epistemological obstacles in determining alternative answers. The first type occurred at the focus stage, where 100% of the students could not focus on identifying a problem related to at least two types of solutions. The second obstacle type was experienced at the reasoning stage, and all samples could only provide one idea related to problem-solving. Two students suggested analyzing a cube and the size of a cuboid, while seven students proposed using a formula. The third obstacle occurred at the inference stage, where 22.22% of students could only place and arrange a cube's size in a cuboid, while 77.78% performed calculations on the test material. The fourth type was experienced at the situation stage, where 77.78% of students could not connect the volume found with the problem situation. They directly divided the cube's volume by the cube's volume without considering that the latter was not water or sand, which could occupy all space. The cube was a solid object with certain dimensions, causing empty space inside the cuboid that could not be filled. The fifth obstacle occurred during the overview, where 77.78% of students drew the wrong conclusion. In the sixth type, 77.78% could not answer at the clarity stage. Additionally, they could not clarify the elements assumed or symbolized.

In the fourth question, there were four types of epistemological obstacles in problem-solving. The first type occurred at the inference stage, where 55.56% of students could draw the net of a pyramid and determine the part covered with an equilateral triangle glass. However, they could not calculate its area because the height was unknown. The second type of obstacle occurred at the situation stage, and 22.22% could not connect the area of the triangle with the desired condition, such as the required glass area is equal to three times the area of the equilateral triangle. The required glass area was assumed to be equal to the area of the triangle. The third type was at the overview stage, where 22.22% of students made conclusions without double-checking the problem-solving process. The fourth obstacle occurred at the situation, clarity, and image stages, where 22.22%, 77.78%, and 55.56% of students, respectively, could not answer the questions. Similar problems were also found in previous questions.

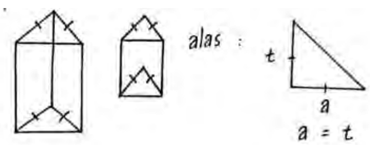
DISCUSSION

Epistemological obstacles in the proofing dimension

The findings of this study reveal that a significant percentage of students, approximately 88.89%, struggle with providing proof for a given statement. Specifically, the research explains that students face difficulties in proving the relationship between the base of a prism and its corresponding volume. To further illustrate this issue, Figure 4 provides an example of common errors made by students in their attempts to prove this relationship.

1. Two prisms are right-angled isosceles triangles, and the height of the volume of prism 1 is greater than that of prism 2. Are the lengths of the three sides of the base of Prism 1 greater than Prism 2?

Answer:



$V_1 > V_2$

$La_1 t_1 > La_2 t_2$

$\frac{1}{2} a_1 t_1 t > \frac{1}{2} a_2 t_2 t$

$\frac{1}{2} t . t_1 . t > \frac{1}{2} t . t_2 . t$

$\frac{1}{2} t^3 > \frac{1}{2} t^3$

Wrong

Figure 4: Epistemological obstacle in proofing

In general, students demonstrate understanding of the concept of a right-angled triangle as the base of a prism, the formula for calculating volume, and the ability to write comparative formulas. However, they often make mistakes when attempting to combine these concepts with proof. For example, students may substitute algebraic expressions with equations without understanding how to integrate formulas to fit the desired situation. These errors are consistent with common mistakes made in mathematics, such as misunderstanding algebraic expressions and variable concepts (Jupri et al., 2014), substituting expressions into equations, solving and rearranging them, as well as solving equations and formulas (Rushton, 2014).

The errors made by students in combining concepts from related subjects suggest that their connection skills in mathematics may be weak. To strengthen critical thinking skills, it is necessary to establish connections to the real world, other disciplines, and other concepts (García-García & Dolores-Flores, 2021). The constraints found in the study suggest that students struggle to link one concept to another, indicating knowledge compartmentalization. This situation is a common concern where only one type of knowledge is developed separately from others (Yao et al., 2021).

Epistemological obstacles in the generalization dimension

The findings on the dimension of generalization revealed that a significant percentage of students (88.89%) could not generalize the relationship between a cuboid and a rectangular parallelepiped. While 77.78% of students understood the properties of both shapes, they had difficulty determining the relationship between them. To illustrate this point, Figure 5 shows an example of a student's incorrect answer.

2. Given: Objects with cuboid and prism shapes Question: Is every object with a cuboid shape also a prism? Answer:	
Characteristics of a cuboid: • Has a base and top with the same shape • Congruent Vertical edges • V= l x w x h • Base and top can have various shapes.	Characteristics of a prism: • Has a base and a top with the same shape • Congruent vertical edges • V = Base area x height • Base and top have only one shape
Cuboid and prism are different	

Figure 5: Epistemological obstacle in generalization

The students tend to memorize the properties of concepts without fully understanding them. Prioritizing memorization over understanding can limit their ability to think critically (Firdaus & Kailani, 2015), making it difficult for them to find connections between different concepts. This finding is consistent with the previous results regarding the weakness of students in mathematical connections, which can result in a lack of understanding in critical thinking due to the positive correlation between the two (García-García & Dolores-Flores, 2021; Cai & Ding, 2017). A student with a strong foundation in mathematical knowledge can connect ideas, concepts, procedures, representations, and meanings (García-García & Dolores-Flores, 2021).

Epistemological obstacles in the alternative answer dimension

The findings on the alternative answer dimension reveal that 100% of the students had difficulty calculating the number of cubes that a cuboid can accommodate with at least two alternative answers. Furthermore, 22.22% could find the answer but only with one method, while the remaining 77.78% who failed still used only one method. This shows that students have difficulty creating flexibility to solve a problem. Their focus is limited to one method they master without considering other alternatives. Figure 6 shows an example of a student's incorrect answer.

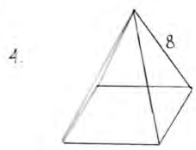
3. Method 1:
A cuboid-shaped cabinet, l = 50cm, w = 25 cm, h = 100cm
A cube-shaped souvenir box, V = 1 dm^3
What is the maximum number of souvenir boxes that can fit in the cabinet?
Answer:
V cuboid = l x w x h = 50 x 25 x 100 = 125000 cm^3
V cube = 1 dm^3 = 1000 cm^3
Since the units are the same, the maximum number of boxes is:
V cuboid: V cube = 125000 :1000 = 125
Method 2:

Figure 6: Epistemological obstacle in alternative answer

The concept of volume formulas for cubes and cuboids is the most common way to solve maximum capacity problems. The rigidity of conceptual knowledge in this subject is evident from the mistakes made. The volume concept is directly applied without considering the problem situation. Emphasis on theory without variation in problems is a potential source of student failure (Cai & Ding, 2017). The sample did not consider the dimensions of the binding cube, so the likelihood of not achieving the desired result increases. All procedures and rules for volume problems are directly applied regardless of the situation. Emphasis should be placed on the importance of flexibility while providing solutions. In-depth procedural knowledge, which refers to flexibility in using procedures and rules, needs further attention (Star, 2015), especially in volume. Determining the area and volume of objects is crucial for students to succeed in mathematics and science (Vasilyeva et al., 2013).

Epistemological obstacles in the problem-solving dimension

The findings on problem-solving dimensions reveal that 77.78% of students have difficulty calculating the surface area of a pyramid under certain conditions. These tasks are presented in the form of story, namely in the context of building a pyramid-shaped building where each side is covered with glass except for the front and bottom. Previous studies have revealed that difficulties often arise when solving contextual problems in narrative form (Wawan & Retnawati, 2022). This is evident from the students' challenges in understanding the problem. Figure 6 shows an example of students' errors in their answers.



All sides except the front side and equilateral triangle base will be covered in glass. What is the required glass area?

The glass will be installed on the vertical side of the triangle.

$$L = \frac{1}{2} \cdot a \cdot t$$
$$= \frac{1}{2} \cdot 8 \cdot t$$
$$= 4\ t$$
$$L =$$

Figure 7: Epistemological obstacle in problem-solving

Figure 7 shows that the students misunderstood the required glass area as the triangle area on the pyramid side. Furthermore, 66.67% did not break down the image in the problem into pyramid nets to simplify their work. The use of images or symbols for problem-solving can help students understand mathematics (Yao et al., 2021). There is often a tendency to approach challenges when learning geometry by expressing mathematical solutions and drawing (Sumaji et al., 2019). Additionally, 55.56% of the students had difficulty calculating the area of an equilateral triangle.

The commonly used formula to calculate the area of a triangle is (1/2) x base x height. Therefore, some students make mistakes by first calculating the triangle's height using the Pythagorean theorem. Errors in determining the base of the triangle also occur when confusing it with the length of the base when calculating the height of the triangle. Only one student used another formula to find the area, for example, $\sqrt{s(s-a)(s-b)(s-c)}$, with s being $(\frac{1}{2})(a+b+c)$.

Epistemological Obstacles Causes in Online Learning

Several factors cause epistemological obstacles encountered in online learning implementation. Problems start to arise at the learning core stage, namely during the delivery of materials, exploration, presentation, and discussion. Based on interviews with prospective teachers, it is known that there are often obstacles in delivering materials in online learning due to the unpreparedness of some supporting facilities, such as unstable signals. This makes the delivery of materials often interrupted, resulting in incomplete acceptance. As a result, students only memorize concepts without a deeper understanding, which opens misconceptions possibility. Special attention is needed to overcome difficulties and misconceptions in learning materials (Sebsibe, et al., 2019). Supporting tools for online learning need to be prepared carefully as an anticipation form for possible obstacles, such as complete electronic guidebooks that can be accessed anytime and anywhere.

Another issue that arises in the exploration stage is suboptimal learning conditions. The lack of interesting and interactive online learning media introductions and presentations can decrease students' motivation to explore. Therefore, a comprehensive study is needed to identify various media or applications that can be used in online learning. The lack of experience among teachers in using online applications can also create difficulties in online learning (Zaharah & Kirilova, 2020).

To make mathematics learning more interesting, teachers can utilize various technologies in online learning. Technological advancements can create interesting stimuli to promote active and creative learning activities (Papadakis et al., 2016). The use of technology can also help overcome obstacles in using images or symbols for problem-solving by making mathematics more concrete. Technology-based mathematics applications can help students create, analyze, and clarify images. However, teachers need to choose the appropriate tool and ensure its accuracy in representing mathematical concepts in the classroom (Sulastri et al., 2022).

In addition to using technology, teachers can also provide stimuli for students to explore deeply using various online learning sources. This can help students overcome epistemological obstacles by creating flexibility in problem-solving. Students will not only rely on one method they have mastered but will also consider other alternatives through exploration and knowledge-building.

The next problem occurs during the discussion phase, which can be quite challenging. Minimal interaction becomes a challenge in online learning. Basically, online learning is very flexible and provides opportunities for more interaction (Chertoff et al., 2020). Therefore, it is essential to educate prospective teachers about the benefits of online learning and stimulate their participation in discussions through engaging questions or interesting games. This approach aims to hasten the adaptation process to online learning.

CONCLUSION

In conclusion, the epistemological obstacles related to critical thinking were discovered in online learning, with students struggling to prove the relationship between two concepts, generalize relationships, seek alternative solutions, and solve problems. These difficulties arose partly because the steps of online learning had not been fully optimized. At the time, online learning emphasized virtual learning through live video conferencing, which became a hindrance for students when they encountered signal issues. As a result, they missed some parts of their learning, which caused confusion when they could rejoin the class but with new material. Additionally, the lack of information regarding the use of interactive, technology-based online learning media made the exploration activity, which is at the core of learning, less effective. Students tended to be passive, not exploring much information by browsing online, but only using available learning resources. However, in-depth exploration activities could stimulate the development of critical thinking skills. Students also appeared to be perplexed when interacting with other students. The difficulty in adapting to online learning was still apparent.

To foster the development of critical thinking skills, various enhancements are necessary in online learning. For this purpose, online learning should be well-prepared and structured, which can be initiated through good planning. To achieve this, lecturers can prepare comprehensive learning guidelines in e-modules or other materials that focus on understanding the relationship between concepts, the flexibility of concepts and procedures, and the habit of drawing in geometry learning. Furthermore, lecturers can provide ample information about the use of learning sources, learning media, and appropriate technology to support online learning.

Well-prepared online learning, combined with appropriate technology and correctly delivered to students, can make learning mathematics meaningful. Students' exploration of their abilities can be maximized by the opportunities provided by online learning, both from unlimited learning resources and technology that can be integrated. Students can also learn without spatial and temporal limitations. These things can certainly stimulate students to improve their critical thinking skills.

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APPENDIX

MATHEMATICAL CRITICAL THINKING TEST QUESTIONS

1. Two upright prisms of the same height have a base in the form of an isosceles triangle. If the volume of the first prism is greater than the second prism, is it certain that the three sides of the base of the first prism are larger than the second prism? Prove it!
2. There are objects in the form of cuboid and prisms with a volume of 60 cm^3 each. Must every cuboid-shaped object be a prism? Explain your reasons in detail!
3. A storage cupboard and a souvenir box are shown as follows.

Volume = 125000 cm^3

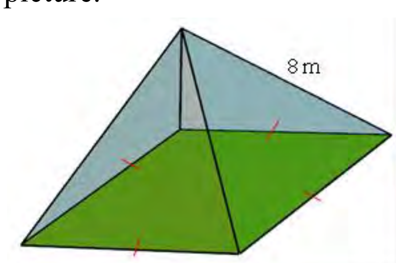
Cupboard

Each volume = 1 dm^3

Souvenir box

If the cupboard will be used to store several souvenir boxes, count in at least two different ways how many maximum souvenir boxes the cupboard can accomodate!

4. Look at the following picture.



Garden Building Design

A company wants to build an indoor park according to the design of a garden building. All the vertical sides will be made in the form of equilateral triangles. Furthermore, the building will be completely covered with glass, except for the front which is left open. Flowers and grass will be planted at the base. Calculate the area of glass needed to make the building according to the desired design!