The Effect of Computer Supported Collaborative Dynamic Learning Environment on High School Students' Success in Mathematics Classroom

Türkan Berrin Kağızmanlı Köse[1], Enver Tatar[2]

[1]Giresun University, Türkiye,

[2] Ankara, Türkiye

berrin.kagizmanli@giresun.edu.tr , entatar@gmail.com

Abstract: This research discuss and considers possible components in computer supported collaborative dynamic learning environment in mathematics classroom. The aim of the research is to observe the applicability of this environment to the courses and to determine its effect on student success. The research group consisted of 68 high school students and their mathematics teacher. Accordingly, in the learning environment in the teaching of the first objective, it was observed that the collaborative teams were working tangentially. Upon examining the results observed in the teaching of the other three objectives, it was observed the behaviors of the collaborative teams were carried out in accordance with the environment and that the teams and the course teacher were more effective. Teaching through computer supported collaborative dynamic learning had a more positive effect on the success of high school students.

Keywords: computer supported collaborative learning. dynamic learning. GeoGebra. instruction of lines. worksheets

INTRODUCTION

This research how to design the technological environment for collaboration and how teachers and researchers learn in the context of collaborative activities. Teachers are considered to be key figures in ensuring the use of computer-based cognitive tools in mathematics teaching (Umay, 2004). Accordingly, the question of when and how teachers and students use this tool for effective and long-lasting learning becomes crucial. It is possible to ensure students' effective and permanent learning through social interactions in computer-supported collaborative environments. Johnson and Johnson (1994) state in their study that collaborative learning is used not only as a learning process but also as a form of classroom management and that the best way to manage technology-supported instruction is through collaborative learning. Computer Supported Collaborative Learning (CSCL) is a combination of the concepts of computer, support, cooperation, and learning (Jones, Dirckinck-Holmfeld, & Lindström, 2006). It is possible for teachers to integrate computers and computer technologies into their lessons and for students to do their learning activities collaboratively in the classroom environment with learning environments prepared in accordance with the purpose of the lesson.

In the field of CSCL various approaches are applied to explain when and for whom collaborative learning can be benefcial as well as how the design of CSCL can be improved (Schnaubert & Vogel, 2022). CSCL is found in a wide range of fields, from education to sociology, architecture to economics. CSCL, the effects of which have been investigated in different cultures, leads to the emergence of new concepts with each new research (Jones et al., 2006). Although much progress has been made, questions remain about the sequencing of education and curriculum to support learning and collaboration in the classroom, and in particular about the approaches used in these classrooms (Lee, Chan, & Aalst, 2006).

Today, CSCL has emerged as a combination of collaborative learning and the use of computers and computer-related technologies such as the Internet (Stahl, Koschmann, & Suthers, 2006) or software created for many different infrastructures and purposes in the learning environment. Lipponen, Rahikainen, Lallimo, and Hakkarainen (2003) state that collaborative learning is supported by computers and the question of how students who form collaborative learning groups can learn better by working interactively through computers has formed CSCL. According to Adams (2004), collaborative learning has been examined by researchers in terms of different variables affecting learning such as gender, the number of students required to be in the group, talent, collaborative learning skills, and other ability factors. However, with the increasing use of computers in classroom environments, these variables, which have been investigated in collaborative learning, moved to this new field with the combination of computers and collaborative learning. Researchers have begun to answer many of these questions in the field of computer-supported collaborative teaching (Adams, 2004).

Ching Sing et al. (2011) state that the teacher's educational and pedagogical skills play an important role in creating the learning environment, conducting activities, and engaging the students. Accordingly, teachers should support students in deciding with whom and with which educational tools they want to work. This will enable students to interact more dynamically and give them responsibility so that they learn to build knowledge across communities (Ching Sing et al., 2011). Stahl et al. (2006) stated that CSCL supports individual learning, but that learning cannot be reduced to this and that one of the effects of group work is to enable individual learning. From the student's point of view, team building brings with it some problems. These problems can be listed as difficulties in interacting, difficulties in motivation, and the provision of the necessary educational tools (Ching Sing et al., 2011). Thus, it is necessary to provide the technological infrastructure that will enable teachers and groups to work together interactively and enable groups to easily access learning products. In such learning environments, there are debates about how one should comprehend the educational content and the approaches taken to define the analytic structure applied (Arnseth & Ludvigsen, 2006). In the literature, it is possible to come across more studies that examine CSCL's synchronized collaborative teams working together in the internet environment, while CSCL's collaborative studies created in the classroom environment are less common. However, how one handles the computer-aided structure applied during collaborative work in mathematics education is important for students and teachers to adopt this learning style. If the technological structure applied in Computer Supported Collaborative Learning (CSCL) environments is dynamic, this method can be expressed as Computer Supported Collaborative Dynamic Learning (CSCDL). Considering the dynamic structure of mathematics, CSCL is applied as CSCDL in mathematics education. Considering that mathematical expressions contain variables and that students need to learn different situations and properties within each concept, the importance of integrating dynamic learning in CSCL environments becomes apparent. Accordingly, one can define the CSCDL presented in this study as a learning method in which students form collaborative teams and reach generalizations of the definitions and concepts expected from them through the dynamic mathematics software they use on computers, while their teachers guide them with hints. Accordingly, there are a wide variety of studies in the literature on the use of technologies created by GeoGebra as a computer and dynamic software in classroom environments (Adelabu, Marange & Alex, 2022; Joshi & Singh, 2020; Khalil, Khalil & Haq, 2019; Thapa, Dahal, & Pant, 2022; Uwurukundo, Maniraho & Rwibasira, 2022). As a matter of fact, one cannot ignore the suggestions of mathematics educators about the introduction of technology into the classroom environment. However, with the advancement of technology, it is seen that people become lonely and are only interested in technology. In this case, in the coming years when face-to-face interaction will always remain valid, there may be a generation that is not really socialized and that spends time only on the internet through social media. It is possible to reverse this situation by using technology in a useful way in terms of education. In other words, it is possible to raise individuals who both use advanced technologies and truly cooperate and interact with each other.

A core challenge is understanding how the open-ended, ever-evolving process of collaborative inquiry can be organized, regulated, and supported in a manner that leverages students' agency and creative imagination (Tao & Zhang, 2021). According to Mapile & Lapinid (2023), students interaction with each other, more ideas are created, student assistance, encouragement among learners, development of skills, timeefficiency in finishing a task, and better output produced. Before explaining the role of technology in collaborative learning, it is possible to summarize the features of collaborative learning (Ching Sing, et al., 2011). Student Teams-Achievement Divisions (STAD) which is the technique of collaborative learning was developed by Slavin (1990). This technique was used in this study. Determining that STAD is the most researched technique among the results of collaborative learning, mathematical operations and applications are suitable for carrying out concepts in science, language use and mechanics, geography and map coverage (Slavin, 1994). This technique has five elements: teams, presentation, quizzes, individual progress points, and team reward.

In this technique, students are first divided into heterogeneous groups of three, four or five in terms of achievement level, gender, and race. Students then work as a team on learning materials such as worksheets and support each other's learning. Dynamic materials and worksheets were used in this research. A new CSCDL environment was created by combining the dynamic learning environment with collaborative learning. The goal here is for the whole team to work together to ensure that other team members learn the subject thoroughly. Students work for the team and teams work for their members. Finally, students take the exam individually. The individual progress score of each student in the group is determined. Team scores are obtained by averaging the progress scores of the students on each team. Thus, the student competes with himself (Açıkgöz, 1992; Slavin, 1994). It aims to develop learning tools and to be aware of the relationships between concepts in order to effectively support the learning of group members consisting of students (Buder, 2011).

Analytic geometry, which provides spatial thinking and schematic visualization, deals with the nature of two- and three-dimensional shapes, spatial concepts, and planar, deductive, and coordinate geometries (Chinnappan & Lawson, 2005). According to Young (1909), analytical geometry, which is the basic idea of graphically representing the relationship between two or more variables, is embedded in all professional fields. Many people find themselves seriously inadequate due to their lack of knowledge of analytic geometry, and the industrial world states that the solution to difficult problems cannot be overcome without the help of analytic geometry (Young, 1909). Pritt (2010) emphasized that thanks to analytic geometry, mathematicians can work in more dimensions than one can imagine. He also stated that scientists can solve the problems encountered in writing secret coded messages in cryptography, physicists can base their vector and analysis studies on analytical geometry, astronomers can check the point location of telescopes with the help of trigonometry, and biologists can solve equations obtained from theories by using spectroscopy (Pritt, 2010). What can be done to help students learn analytic geometry topics better? Considering that knowledge of analytic geometry is necessary for all professions where mathematics is required (Young, 1909), students need to learn analytic geometry in learning environments where they can create products thanks to technology and collaborative learning. Accordingly, the basic structure of this research is to create a useful and applicable dynamic learning environment in which collaborative teams and computer-assisted instruction are integrated into teaching analytic geometry to teachers and students.

The aim of this research is to observe the applicability of the CSCDL environment to the courses and to determine its effect on student success. In this context, this study seeks answers to the following questions:

1. What are the applicability situations of the CSCDL environment in teaching the "Lines" unit?
2. Is there a significant difference between the effect of teaching in the CSCDL environment and teaching using only dynamic materials on the interactive whiteboard on students' academic success and retention of the "Lines" unit?

METHOD

Research Design

The study used a mixed research method. This approach, which supports the research problems both quantitatively and qualitatively by not limiting the collection of data to a single type of method, provides a comprehensive picture by explaining the process of the research, its initial situation, and external influences during the process (McMillan & Schumacher, 2010). Accordingly, the study used embedded design, one of the mixed research approaches. In the embedded design, the researcher collects quantitative and qualitative data simultaneously or sequentially and explains the data they collect in a way that one supports the other. Supporting data can be quantitative or qualitative (Creswell, 2011).

For this study, the researcher preferred the embedded design since it aimed to examine the use of the CSCDL environment, which combines the computer-assisted mathematics teaching method and collaborative learning approach, on the subject of "Lines" through qualitative and quantitative data. This study including the Experiment 1 and Experiment 2 groups, collected quantitative data to determine the effect of CSCDL on students' success. The researcher collected qualitative data throughout the process to illustrate how CSCDL was implemented in the lessons. Therefore, this research revealed the effect of CSCDL on both the observation process and student achievement.

Study Group

The research group consisted of 68 high school students studying in Turkey and their mathematics teacher. The Experiment 1 group consisted of 35 students and the Experiment 2 group consisted of 33 students. The participants were chosen using the convenience sampling method. This sampling method is preferred because of students' convenient accessibility and proximity to the researchers (Yıldırım & Şimşek, 2011). The students in the study had not learned the GeoGebra software from any source before. Previously, the math teacher participated in the CSCDL workshop conducted by the researcher.

Data Collection Tools

Using the data obtained with different methods increases the reliability and validity of the results obtained (Yıldırım & Şimşek, 2011). Therefore, both qualitative and quantitative data collection tools were used in the research study. The study used the Lines Knowledge Test (LKT) and Observation Form as data collection tools. These are presented below.

Lines Knowledge Test

To determine the academic success of the students in the "Lines" unit taught in the research, the researcher prepared the LKT to be used in both experimental groups. The researcher developed this test to be used as a pre-test, post-test, and retention test. The researcher used the high school geometry textbooks (TMoNE (2013); Aksoy & Görçe, 2013) and the geometry course 9-10th grade curriculum (TMoNE (2010) in the development of the test. Care was given to ensure that the items in the knowledge test included the learning objective in the "Lines" unit, that is, to ensure content validity.

The researcher developed the test as 15 open-ended items and gave it to four researchers who are experts in mathematics field education, two doctoral students, one in mathematics education and the other in mathematics, and one mathematics teacher for review. The test, which was revised according to expert opinions, consisted of 14 open-ended items, and the time required for answering the test was determined as 50 minutes.

The pilot study of the prepared knowledge test was applied to 75 11th grade students. The pilot study was conducted in two high schools different from the high school where the research took place According to the data obtained here, Cronbach's α-reliability coefficient, which is the internal consistency coefficient of LKT, was 0.814. This indicates that the scale is highly reliable (Kalaycı, 2010). The study used the final version of the LKT.

Observation Form

In the study, all lessons were observed from the beginning to the end to reveal the applicability conditions of the CSCDL environment. The researcher created the observation form before the implementation process started. First of all, the researcher tried to determine which important situations would shape the characteristics of the CSCDL environment that was going to be observed in general terms by considering the relevant literature. Then they examined the existing literature and the mathematics curriculum (TMoNE (2013).

After the draft form was created, it was presented to two researchers who specialized in field education and one researcher who specialized in education. After reorganizing the form in line with the feedback received, an expert researcher in field education checked it. The form was then used in the pilot study. In the pilot study, the researcher observed 3 objectives in the "Coordinate Systems" unit taught through CSCDL for 4 lesson hours. During the pilot lessons, the researcher observed the lesson by being present among the students. The researcher tried to determine how the students constructed computer-assisted and collaborative learning among themselves and the situations they paid attention to. This way the researcher tried to analyze both the existing situations in the observation form and new situations that may occur. The researcher recorded the data observed for each objective on the form. During the lessons conducted in the pilot study, the observation form was updated and finalized. This study used this final version of the observation form. The observation form consisted of 34 codes under 9 categories, each of which indicated a behavior. The categories included in the form are "teacher's role", "team reward/joint reward", "positive dependency", "individual assessability", "face-to-face (supportive) interaction", "social skills", "assessment", "equal opportunity for success", and "learning environment". While creating the categories, it was taken into consideration that group work should have these characteristics (Açıkgöz, 1992) for it to be collaborative learning. In the creation of the codes, the researcher tried to combine both the features that should be under the relevant category and the dynamic learning features and thus create codes/behaviors that would serve as examples of collaborative dynamic learning.

Teaching the "Lines" Unit with CSCDL

To determine whether the applicability of teaching through CSCDL affects success and retention, the researcher selected the "Lines" unit from among the analytic geometry units. In the study, 4 objectives in the 10th grade geometry course "Lines" unit were taught with CSCDL and only using dynamic materials on the interactive whiteboard in 12 lesson hours. The 6-week implementation of the study took place with the course teacher and 35 Experiment 1 group students and 33 Experiment 2 group students.

In the two weeks before the implementation, the researcher met with the teacher of the course. They analyzed the methodology of the research and the equality between the Experiment 1 and Experiment 2 groups. For this purpose, the researcher analyzed the data obtained through the pre-tests, which were the Geometry Knowledge Test (GKT) and the LKT, which were conducted by the Provincial Directorate of National Education to determine the success of 10th grade students in the geometry course and which contained 15 multiple-choice items. The results of the Kolmogorov-Smirnov test were: ($p_{gkt\text{-}Experimental\text{-}I} < .05$; $p_{gkt\text{-}Experimental\text{-}II} > .05$; $p_{pretest\text{-}Experimental\text{-}I} < .05$; $p_{pretest\text{-}Experimental\text{-}II} < .05$). Measurement results of both groups have to exhibit a normal distribution to choose parametric tests (Büyüköztürk, 2010). Q-Q, histogram, box plot, detrended normality plot, kurtosis, and skewness values had the same trend as the Kolmogorov-Smirnov test. Therefore, the Mann–Whitney test was used for analyzing the quantitative data. According to Mann Whitney U-Test results ($U_{gkt}=473.500$, $p_{gkt} > .05$; $U_{lkt}=531.000$, $p_{lkt} > .05$), there was no significant difference. Based on these data, the class of 35 students was randomly selected as the Experiment 1 group and the class of 33 students was randomly selected as the Experiment 2 group. According to the study plan, in the Experiment 1 group, the lessons were based on CSCDL, while in the Experiment 2 group, the lessons were carried out using only dynamic materials on the interactive board.

Now it was time to form the collaborative teams of the Experiment 1 group. The course teacher determined the starting scores of the students in the Experiment 1 group by taking the average of the students' previous geometry exam grades and their scores on the GKT out of 100 points. Accordingly, they formed heterogeneous collaborative teams according to each student's initial score. They formed teams of 3 people. Accordingly, there was a total of 11 collaborative teams. When forming the teams, the teacher first ranked the students according to how high their initial scores were. The teams were numbered 1, 2,...,11, and the first 11 students with the highest success scores were placed in the 1st, 2nd,..., 11th teams respectively. Then, the second 11 students ranked in the initial score list were placed in the 11th, 10th,...,1st teams respectively, while the third 11 students were placed in the 1st, 2nd,...,11th teams respectively. The last two students remaining in the initial score list were placed in two teams with the appropriate average score. The LKT was administered to both experiment groups as a pre-test the week before the implementation. Both groups had 50 minutes for the knowledge test. Afterward, the students received a briefing about the teams. Students were given the CSCDL Team Study Guide. Students were asked to decide on the team name, team color, and team emblem. Before the start of the course, a poster describing the CSCDL environment prepared by the researcher was placed in the laboratory where the implementation would take place, and the GeoGebra software was installed on the computers and interactive board. Additionally, the researcher uploaded the materials related to the learning objective to the computers before the implementation each week. In the Experiment 2 group, the course teacher taught the objectives using only dynamic materials on the interactive whiteboard. Students did not use computers during the implementation, they learned the unit by listening to their teachers in their own classrooms and occasionally practicing the materials on the interactive board. In both test groups, the same course teacher taught the unit objectives in the same week and for the same duration. Every week the researcher and the course teacher reviewed the dynamic materials and worksheets to be used in teaching the learning objectives during the implementation period. In the Experiment 1 group, each team worked on its own computer and worksheet. Before teaching each learning objective, the teacher went over reminders for the students by using the materials on the interactive board from time to time, taking into account the readiness required for the objective, and initiated the collaborative work in the teams. After each lesson, the teacher collected the worksheets of the teams to check them and provide feedback. These worksheets were photocopied and the original copies were given to the researcher to use in the study, and the worksheets were returned to the teams the following day after examining the photocopies. This way the teams had the opportunity to re-evaluate the feedback they received. During the applications, follow-up tests were conducted after the teaching-learning of both learning objectives. These follow-up tests can be seen as small quizzes. Students participated in the follow-up tests individually. The researcher prepared the follow-up tests by examining the relevant books (TMoNE, 2010; Aksoy & Görçe, 2013). The teacher of the course examined the appropriateness of the follow-up test. Each test was completed in 15 minutes. The purpose of conducting follow-up tests is to determine the scores of the collaborative dynamic groups. Accordingly, two important scores stand out. The starting score and individual progress score. The starting score (SS) was determined as the students' previous geometry written grades. The individual progress score was determined by evaluating the results of the follow-up tests according to the starting scores. The individual progress score was determined as follows: If the result of the follow-up test was 10 points lower than the achievement score, 0 points were given, if 1-10 points were lower, 10 points were given, if 1-10 points were higher, 20 points were given, and if 10 or more points higher, 30 points were given. Accordingly, after the completion of the teaching of the learning objectives with CSCD, each team's score was calculated by averaging the individual progress scores of the students in the team and the successful team was determined. The teams were congratulated in the classroom for their success and the most successful team received a gift and a certificate. The researcher conducted 12 lesson hours of observations using the observation form in the Experiment 1 group. Also, they observed the lessons in the Experiment 2 group without a form. The teacher and students carried out the implementations and the researcher was present in the learning environment only to make observations. The practices were carried out with the following learning objectives:

1. **Parametric and closed equations of a line (Line equation with one known point and straight line vector, line equation with two known points, and line equation with one known point and normal vector).** These were taught through CSCDL in 4 lesson hours. Since one observation form was used to observe each two-hour lesson, 2 observation forms were used for

these subjects. Thus, the "observation status" determined for the evaluated codes was calculated by taking the average of the two forms.

2. **The state of two lines according to one another.** Teaching this subject through CSCDL took 2 class hours.

3. **The slope of a line (The slope of a line and the angle between two lines).** This was taught through CSCDL in 4 lesson hours. Since one observation form was used to observe each two-hour lesson, 2 observation forms were used for these subjects. Thus, the "observation status" determined for the evaluated codes was calculated by taking the average of the two weeks.

4. **The distance of a point to a line (The distance of a point to a line, the distance between two parallel lines, and the geometric location of points equidistant from two parallel lines)**. These objectives were taught through CSCDL in 2 class hours.

The findings present the observation form data according to these objective numbers. The LKT was administered to both experimental groups as a post-test the week after completing the implementations. In addition, 8 weeks after the completion of the study, the LKT was applied to both groups as a retention test. The CSCDL process can be summarized as follows: (see Fig 1)

The course teacher develops dynamic materials and worksheets on the subject.
Determination of students' starting scores.
Formation of heterogeneous collaborative teams based on starting scores.
Informing students about the teams. Presenting the CSCDL Team Work Guide.
Teams agree on the team name, team color, and team emblem.
The course teacher ensures that the dynamic materials are loaded on the teams' computers and provides teams with the worksheets.
The course instructor creates an environment of readiness for the lesson using dynamic materials and initiates the collaborative work of the teams.
Each team works on its own computer and worksheet.
The teacher conducts individual follow-up tests at regular intervals.
Determination of individual progress score based on follow-up tests.
Determination of team score by averaging individual progress scores.
Announcing the successful team and congratulating them with certificates and small gifts.

Figure 1: The CSCDL process

Dynamic Materials and Worksheets

CSCDL was conducted with the 4 objectives mentioned above. Dynamic materials and worksheets that enable students to work in teams were developed for these objectives. Eleven dynamic materials and worksheets was created. An example from the worksheets which were used with the dynamic materials are given the Appendix.

While creating the materials and worksheets, the literature was examined and related books (TMoNE, 2013; Aksoy & Görçe, 2013; Çelik, 2013; Larson, Boswell, Kanold, & Stiff, 2004; Sullivan, 2008; Beecher, Penna, & Bittinger, 2007; Timmons, Johnson, & McCook, 2010; Lial, Hornsby, & McGinnis, 2012; Hungerford & Shaw, 2009; Boyd, Cummins, Malloy, Carter, & Flores, 2008; Burger, et al, 2008), the GeoGebra official website (www.geogebra.org), openly available dynamic materials.

The prepared material and worksheets were presented to a researcher specialized in field education on a computer and corrections were made with the feedback they gave. Additionally, before teaching the units, the researcher and the teacher worked on the materials and worksheets together and made arrangements where necessary.

Data Analysis

The researcher obtained observation data during the teaching of each objective with CSCDL according to the observation form created before the implementation of the study. Accordingly, data analysis occurred under the categories of learning environment, teacher behaviors, and student team behaviors. The study analyzed the observation results obtained from CSCDL environment under the subcategories of "learning environment"; "teacher's role" and "individual assessability"; "team reward/joint reward", "positive dependency", "face-to-face (supportive) interaction", "social skills", "assessment", and "equal opportunity for success". The researcher observed the "Observation Status Coding" for each code under these defined categories during the lesson. Accordingly, a value of "0" was given when the defined behavior was not performed in the classroom environment, "1" when the defined behavior was performed slightly, and "2" when the defined behavior was performed in accordance with the CSCDL environment. The researcher observed the behaviors defined in each code separately for the teams and determined the observation status of each team. The teams were numbered and the same numbered team was observed in each lesson. The observation states obtained were added together for each behavior and divided by the number of teams. This way, for each code, an observation situation was identified in which each team was observed in the classroom and each team was evaluated. Accordingly, the researcher interpreted the data according to the situation defined by the "0", "1", and "2" values in the "Observation Status Coding". An observation form was used to observe each two-hour lesson. For this reason, when more than one observation form was used for a learning objective, the "observation status" for the evaluated codes was determined by taking the weighted average of the observations.

The evaluation of the 14 open-ended items in the LKT, from which the researcher obtained the experimental data of the study, occurred based on the levels of correct, partially correct, and incorrect. A score of "2" was given for a correct answer, "1" for a partially correct answer, and "0" for an incorrect or blank answer. Accordingly, the highest score that one can obtain from the test is 28 and the lowest score is 0. The SPSS 16.0 package program was used for the analysis of the data obtained from the knowledge test. To determine the test to be used in analyzing the data, the researcher first performed a normality analysis of the data. Since the sample numbers were 35 and 33, a Kolmogorov-Smirnov test was used for the normality analysis (Kalaycı, 2010). In addition, Q-Q, histogram, box plot, detrended normality plot, kurtosis, and skewness values were examined to see whether the data distribution was normal (Field, 2009; Kalaycı, 2010). The results of the Kolmogorov-Smirnov test were as follows ($p_{posttest\text{-}Experimental\text{-}I} > .05$; $p_{posttest\text{-}Experimental\text{-}II} < .05$; $p_{retention\text{-}Experimental\text{-}I} > .05$; $p_{retention\text{-}Experimental\text{-}II} < .05$). In addition, the Q-Q, box and whisker plots, detrended normality plot, kurtosis and skewness values were analyzed to determine whether the measurement results exhibited a normal distribution or not (Field, 2009). Measurement results of both groups have to exhibit a normal distribution to choose parametric tests (Büyüköztürk, 2010). Therefore, the Mann–Whitney test was used for analyzing the quantitative data. The study accepted $\alpha=0.05$ as the significance level, which is the most commonly used level in educational studies. The provincial Directorate of National Education gave the results of the GKT used in the study to the high school directorate where the study was conducted. Since the items in the test are multiple-choice, the number of correct and incorrect answers for each student is evident. Accordingly, the researcher determined the geometry scores of each student based on the results obtained to be used in the study. While determining the net score, the basis was that 4 wrong answers eliminated one right answer.

RESULTS

This heading presents the findings obtained from teaching the "Lines" unit in the CSCDL environment.

Table 1 and Figure 2, present the results of the observations regarding the physical environment and layout of the classroom where the "Lines" unit was taught with CSCDL.

The Physical Environment of the Classroom
• The study took place in the information technologies classroom.
• There are 11 teams in the class.
• Two teams consist of 4 people, and the other teams consist of 3 people.
• There is 1 interactive board and 11 computers in the classroom.
• Each team works with 1 computer on a work desk and worksheets.
• The temperature and lighting in the classroom are sufficient.
• The size of the classroom is not sufficient for the number of teams.

Table 1: Observation results regarding the physical environment of the classroom

The figure presents the layout of the teams.

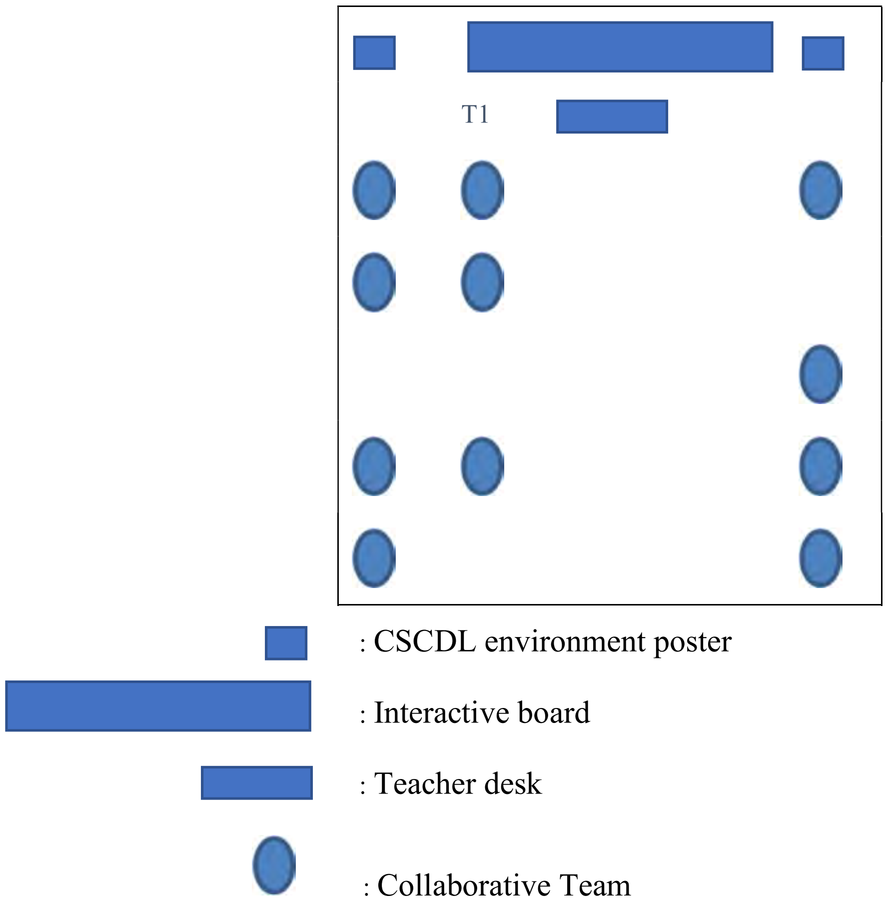


Figure 2: The Layout of the teams

		Behavior	1	2	3	4
Learning Environment	**Learning Environment**	Computers were used effectively in the creation of the CSCDL environment.	1.5	2	2	2
		The interactive whiteboard was used effectively to create the CSCDL environment.	1.5	2	2	2
		Collaborative teams worked effectively in building the CSCDL environment.	1	2	2	2
Teacher Behavior	**Role of the Teacher**	They create a learning environment that enables active participation in accordance with students' levels and interests.	1	2	2	2
		They give students hints instead of giving them information directly.	1.5	2	2	2

		They provide feedback to support learning.	2	2	2	2
		They enable students to share with others.	1.5	2	2	2
		They enable students to make active use of interactive whiteboards and dynamic materials.	1.5	2	2	2
		The teacher facilitates and accelerates the exercises.	0.5	2	2	2
		They question the appropriate level of readiness that involves the objective.	1	2	2	2
		They remind students of the basic concepts and terms related to the subject.	1	2	2	2
		They use dynamic materials effectively on the interactive whiteboard.	1	2	2	2
	Individual Assessability	In the dynamic environment, every student participates in the lesson.	1	2	2	2
		Each student's level of success in generalizations is monitored.	1.5	2	2	2
Student Team Behaviors	Team Award\Joint Award	Team members work to increase team success (a collaborative work structure).	0.95	1.4	1.4	1.5
		Team members can use dynamic materials to reach the generalizations on the worksheets (collaborative reward structure)	1.2	1.5	1.3	1.5
		Team members can to identify algebraic expressions of generalizations using the materials.	0.9	1.3	1.3	1.5
	Positive Dependency	Team members fulfill their responsibilities.	1.35	1.6	1.35	1.5
		Team members act as if they are responsible for each other's learning.	1.35	1.5	1.3	1.5
		Team members use dynamic materials and worksheets effectively.	1.1	1.6	1.35	1.5
		Team members act in cooperation and unity.	1	1.5	1.35	1.5
	Face-to-face (Supportive) Interaction	Team members interact face-to-face (helping).	1.2	1.5	1.35	1.5
		Team members encourage each other (trust).	1.15	1.5	1.4	1.5
		They identify parts of the topic that are not clear (giving feedback).	1.05	1.2	1.15	1.5
		Team members explain the use of dynamic materials to each other.	1.3	1.4	1.25	1.7
		Students correct each other's mistakes.	1.15	1.3	1.15	1.5
		Team members discuss and examine the dynamic materials together and solve the problems they encounter in the generalizations.	0.95	1	1.1	1.5
	Soc ial Ski	Team members stay together for a while before the activities start.	1.35	1.4	1.35	1.4

	Team members can build good relationships with each other.	1.45	1.5	1.35	1.5
	Team members listen to each other.	1.4	1.5	1.35	1.5
Evaluation	Team members evaluate their generalizations at the end of the activity.	1.1	1.4	1.2	1.3
	Team members examine the generalization obtained at the end of the activity together with other teams.	1.2	1.5	1.2	1.5
Equal Opportunity for	Team members improve themselves and thus contribute to generalizations.	1.05	1.5	1.15	1.3
	The contribution of each student is taken into account when making generalizations.	1.1	1.4	1.25	1.5

Observation Status Coding: 0: The described behavior was not performed in the classroom environment, 1: The described behavior was performed partially, 2: The defined behavior was performed in accordance with the CSCDL environment.

Table 2: Observation of applicability conditions of the CSCDL environment

Upon examining Table 2 and the observation data obtained from the teaching of the "Lines" unit in the CSCDL environment, the learning environment and the behaviors of the course teacher were suitable for the CSCDL environment after the first 4 lessons, and the behaviors of the students forming the teams developed in accordance with the CSCDL environment.
This section presents the findings obtained for the statement "Is there a significant difference between the effect of teaching in the CSCDL environment and teaching using only dynamic materials on the interactive whiteboard on students' academic success and retention in the "Lines" unit?"
Table 3 presents the results of the Mann-Whitney U-Test used to determine whether there was a difference between the post-test scores of the Experiment 1 and Experiment 2 groups.

Group	n	Rank Mean	Rank Sum	U	p
Experiment 1	35	45.76	1601.50	183.500	.000
Experiment 2	33	22.56	744.50		

Note: The maximum score for this test is 28.

Table 3: The Mann Whitney U-Test results of post-test scores of the groups

Upon examining Table 3, there was a significant difference between the post-test scores of the Experiment 1 and Experiment 2 group students in favor of the Experiment 1 group ($U=183.500$, $p< .05$).

Table 4 presents the results of the Mann-Whitney U-Test used to determine whether there is a difference between the retention test scores of Experiment 1 and Experiment 2 groups.

Group	n	Rank Mean	Rank Sum	U	p
Experiment 1	35	47.40	1659.00	126.00	.000
Experiment 2	33	20.82	687		

Note: The maximum score for this test is 28.

Table 4: The Mann Whitney U-Test results of retention test scores of the groups

Upon examining Table 4, there was a significant difference between the retention test scores of the Experiment 1 and Experiment 2 group students in favor of the Experiment 1 group ($U=126.00$, $p< .05$).
As a result, it became evident that teaching the subject of "Lines" through CSCDL positively affected students' achievement and ensured the retention of their learning.

DISCUSSION AND CONCLUSION

The study analyzed the observation results obtained from teaching the "Lines" unit in the CSCDL environment under the subcategories of learning environment, teacher behaviors, and student team behaviors.

Accordingly, in the learning environment, in the teaching of the first learning objective of the unit through CSCDL for 4 lesson hours, it was observed that the collaborative teams were working tangentially, and the computer and interactive board were used a little more effectively in the CSCDL environment. Upon examining the results observed in the learning environment in the teaching of the other three objectives, the researcher observed the behaviors of the collaborative teams were carried out in accordance with the CSCDL environment and that the teams and the course teacher were more effective. Accordingly, one can say that the teams and the course teacher behaved in accordance with the CSCDL environment after the first 4 lessons.

According to the analysis of the teacher behaviors, some of the teacher behaviors observed in the 4 lesson hours of the first learning objective of the unit were tangential and some of them were close to being suitable for CSCDL. Additionally, it was observed that in the first 4 lessons, the teacher gave feedback to support learning in accordance with the CSCDL environment, but did not play a role in accelerating and facilitating the work. Upon examining the results observed in teacher behaviors in the teaching of the other three objectives, it was determined that all behaviors were performed in accordance with the CSCDL environment. Accordingly, one can say that the course teacher acted in accordance with the CSCDL environment after the first 4 lessons in teaching this unit.

Upon examining the student team behaviors separately for each objective, the researcher observed that the teams performed most of the behaviors tangentially in teaching parametric and implicit equations of a line. While it can be said that the teams' ability to reach the generalizations in the worksheets was tangential, their ability to define the algebraic expressions of the generalizations using the materials and to solve the problems they encountered in the generalizations was close to tangential. Accordingly, one can say that the teams were able to reach generalizations in teaching parametric and implicit equations of a line, but they had difficulty in defining algebraic expressions of generalizations.

The teams performed some behaviors tangentially during the teaching of the states of two lines with respect to each other, and at the same time, the teams were close to performing most of the behaviors in accordance with CSCDL. Accordingly, one can say that the teams behaved in a manner that was close to being in accordance with CSCDL when learning the states of two lines with respect to each other. Additionally, the level of teams performing all behaviors in accordance with CSCDL increased compared to the previous objective.

It was concluded that the teams performed all of the behaviors partially in teaching the slope of a line. Accordingly, one can state that the teams acted less in accordance with CSCDL in the teaching of this objective compared to the previous objective. However, the teams had the same level of reaching the generalizations in the worksheets and defining the algebraic expressions of the generalizations using the materials. While the level of the teams' ability to reach the generalizations in the worksheets by using dynamic materials decreased compared to the previous learning objective, the level of defining the algebraic expressions of the generalizations did not change.

In the teaching of the distance of a point to a line, the teams performed only three behaviors partially, and most of the behaviors were close to being performed in accordance with CSCDL. Accordingly, one can state that in teaching the distance of a point to a line, the teams were close to acting in accordance with CSCDL. Additionally, the level of teams performing all behaviors in accordance with CSCDL increased compared to the previous objective.

Accordingly, based on the general evaluation of the observation results obtained from teaching the "Lines" unit in the CSCDL environment, the learning environment and the behaviors of the course teacher were suitable for the CSCDL environment after the first 4 lessons and the behaviors of the students forming the teams developed in accordance with the CSCDL environment.

There was a significant difference in favor of the Experiment 1 group between the post-test scores of the Experiment 1 and Experiment 2 group students who were taught using CSCDL and dynamic material only on the interactive board in the "Lines" unit ($U=183.500$, $p< .05$). Accordingly, one can derive that teaching through CSCDL in the "Lines" unit had a more positive effect on the success of high school students. This result is similar to the result of Takači, Stankov, and Milanovic, (2015) who found that the success of the students who applied computer-supported collaborative learning using GeoGebra on the subject of analyzing and graphing functions was higher than the success of the students who only applied collaborative learning. This result is in line with the results of Chiu, Kessel, Moschkovich, and Munoz-Nunez, (2001) and Moschkovich (1999) who found that the use of software is more effective in the subject of lines.

The study determined that there was a significant difference between the retention test scores of the Experiment 1 and Experiment 2 group students in favor of the Experiment 1 group in the teaching of the "Lines" unit ($U=126.00$, $p<.05$). Accordingly, one can derive that the CSCD teaching in the "Lines" unit enabled the retention of high school students' learning. Birgin and Topuz, (2021) found that this environment increased seventh grade students' geometry achievement.

This result is in line with the result of Takači, Stankov, and Milanovic's (2015) study in which they determined that the retention of the learning of the students who used computer-supported collaborative learning using GeoGebra was higher than the learning of the students who only used collaborative learning. Eshuis, Vrugte Anjewierden, Bollen, Sikken, & Jong, (2019) showed that students from the instruction with tool condition out performed the other students as far as their collaborative behavior and their domain knowledge gains. In addition, this result is similar to the result of Ubuz, Üstün, and Erbaş (2009). They found that the retention of seventh-grade students' learning of the concepts of lines, angles, and polygons when they used Geometer's Sketchpad, a dynamic geometry software in the computer laboratory, was higher than traditional learning. This research shows the methods for evaluating CSCDL in the classrooms and the observation form contains CSCDL components in the classroom. Working with scripted activities, students' self- and shared regulation are often limited to understanding the requirements, dividing up the given tasks, and meeting the requirements (Rogat & Linnenbrink-Garcia, 2011); rarely do they have the chance to make transformative changes in inquiry directions and group structures based on emergent interests (Tao & Zhang, 2021). Considering that mathematical expressions contain variables and that students need to learn different situations and features within each concept, the importance of integrating dynamic learning in CSCL environments emerges. According to this, if it is desired to explain the CSCDL revealed in this research; it can be defined as a learning method in which students form collaborative teams and reach generalizations of the definitions and concepts expected from them thanks to the dynamic mathematics software they use on computers, and their teachers guide them with clues. It is hoped that CSCDL will be increased learning success in the classrooms.

Acknowledgements

This study was part of the first author's doctoral doctoral thesis titled "Development, implementation and evaluation of computer supported collaborative dynamic learning environment towards analytic geometry". It was completed under the supervision of the second author. The authors would like to thank the teacher and her students for their invaluable support at diferent stages of this research, as well as the anonymous reviewers for their comments on earlier versions of the manuscript.

Ethical Statement

Ethical guidelines in this study were observed by the researchers. From the initial and implementation phase up to the final writing phase, ethical considerations were strictly followed.

Securing necessary permits to observe the clasrooms and implement the lines knowledge test from respondents underwent proper procedure through the assistance of the Turkish Ministry of National Education. The researcher keeps her documents and permissions and can submit the permission documents at any time. During the conduct of the study in this CSCDL environment, observations made orientation on the nature, purpose, and objectives of the study as well as their roles. The students and teacher were not forced to implement the environment.

References

[1] Adelabu, F. M., Marange, I. Y., & Alex, J. (2022). GeoGebra software to Teach and Learn Circle Geometry: Academic Achievement of Grade 11 Students. *Mathematics Teaching Research Journal,* 14(3), 2-16.

[2] Açıkgöz, K. Ü. (1992). *İşbirlikli öğrenme, kuram, araştırma, uygulama. [Collaborative learning, theory, research, practice]* Malatya: Uğurel Publishing.

[3] Adams, S. (2004). Computers and Cooperative Learning: A Literature Review. [Online] Retrieved February 22, 2015, from http://scottadams.blogs.com/links/2004/11/computers_and_c.html

[4] Aksoy, Ö. & Görçe, Y. (2013). *Ortaöğretim geometri 10. [Secondary education geometry 10]* İstanbul: Fenbil Publishing.

[5] Arnseth, H. C., & Ludvigsen, S. (2006). Approaching institutional contexts: Systemic versus dialogic research in CSCL. *Computer Supported Collaborative Learning, 1*, 167-185. https://doi.org/ 10.1007/s11412-006-8874-3

[6] Beecher, J. A., Penna, J. A., & Bittinger, M. L. (2007). *Algebra and trigonometry.* (4th ed.). Boston: Pearson.

[7] Birgin, O. & Topuz, F. (2021). Effect of the GeoGebra software-supported collaborative learning environment on seventh grade students' geometry achievement, retention and attitudes. The Journal of educaTional research, 114(5), 474–494.

[8] Boyd, C. J., Cummins, J., Malloy C. E., Carter, J. A., & Flores, A. (2008). *California geometry.* Columbus: McGraw-Hill Companies.

[9] Buder, J. (2011). Group awareness tools for learning: Current and future directions. *Computers in Human Behaviour, 27*, 1114–1117.

[10] Burger., E. B., Chard., D. J., Hall., E. J., Kennedy., P. A., Leinwand., S. J., Renfro., F. L., Roby., T. W., Seymour., D. G., & Waits., B. K. (2008). *California geometry.* New York: A Horcoun Education Company.

[11] Büyüköztürk, Ş., Çakmak, E., K., Akgün, Ö., E., Karadeniz, Ş., & Demirel, F. (2010). *Bilimsel araştırma yöntemleri. [Scientific research methods]* Ankara: Pegem Publishing.

[12] Ching Sing, C. Wei Ying, L. Hyo-Jeong S., & Horn Mun, C. (2011). *Advancing collaborative learning with ICT: Conception, cases and design.* Singapore: Ministry of Education.

[13] Chinnappan, M., & Lawson, M. J. (2005). A framework for analysis of teachers' geometric content knowledge and geometric knowledge for teaching. *Journal of Mathematics Teacher Education, 8*, 197–221.

[14] Chiu, M. M., Kessel, C., Moschkovich, J., & Munoz-Nunez, A. (2001). Learning to graph linear functions: A case study of conceptual change. *Cognition and Instruction, 19*(2), 215-252.

[15] Creswell, J. W. (2011). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research.* Baston: Pearson.

[16] Çelik, Ö. (2013). *Ortaöğretim Geometri 11 ders kitabı [Secondary Education Geometry 11 textbook]* Ankara: Bilgetürk Education Publishing.

[17] Eshuis, E. H., Vrugte J., Anjewierden, A., Bollen, L., Sikken, J.., & Jong, T. (2019). Improving the quality of vocational students' collaboration and knowledge acquisition through instruction and joint reflection. *International Journal of Computer-Supported Collaborative Learning*. 14, 53–76. https://doi.org/10.1007/s11412-019-09296-0

[18] Field, A. (2009). *Discovering statistics using SPSS*. London: Sage.

[19] Hungerford, T. W. & Shaw, D. J. (2009). *Contemporary precalculus: A Graphing approach.* Belmont: Thomson Higher Education.

[20] Johnson, D. W., & Johnson, R. T. (1994). Learning together. In S. Sharan (Eds.), *Handbook of cooperative learning methods* (pp. 51-65). London: Westport, Connecticut.

[21] Jones, C., Dirckinck-Holmfeld, L., & Lindström, B. (2006). A relational, indirect, meso-level approach to CSCL design in the next decade. *Computer Supported Collaborative Learning, 1,* 35–56.

[22] Joshi, D. R., & Singh, K. B. (2020). Effect of Using Geogebra on Eight Grade Students' Understanding in Learning Linear Equations, Mathematics Teaching Research Journal, 12(3), 76-83.

[23] Kalaycı, Ş. (2010). *SPSS uygulamalı çok değişkenli istatistik teknikleri. [SPSS applied multivariate statistical techniques]* Ankara: Asil Publishing.

[24] Khalil, M., Khalil, U., & Haq, Z. (2019). Geogebra as a Scaffolding Tool for Exploring Analytic Geometry Structure and Developing Mathematical Thinking of Diverse Achievers. *Internatıonal Electroic Journal of Mathematics Education*, 14(2), 427-434.

[25] Larson, R., Boswell, L., Kanold, T. D., & Stiff, L. (2004). *Algebra 1.* Evanston: McDougal Littell.

[26] Lee, E. Y. C., Chan, C. K. K., & aAlst, J. V. (2006). Students assessing their own collaborative knowledge building. *Computer Supported Collaborative Learning,1*, 57-87. https://doi.org/ 10.1007/s11412-006-6844-4

[27] Lial, M. L., Hornsby, J., & McGinnis, T. (2012). *Beginning and intermediate algebra.* Boston: Pearson Education.

[28] Lipponen L., Rahikainen M., Lallimo J., & Hakkarainen K. (2003). Patterns of participation and discourse in elementary students' computer-supported collaborative learning. *Learning and Instruction 13*, 487–509.

[29] Mapile, R. F. G., & Lapinid, M. R. C. (2023). Online collaborative learning: Applicability in comparison with ındividual learning and face-to-face collaborative learning. *Mathematıcs Teachıng Research Journal, 15*(2), 21-44.

[30] McMillan J. H., and Schumacher S. (2010). *Research in education: Evidence-based inquiry*. Boston MA: Pearson Education.

[31] Moschkovich J. (1999). Students' use of the x-ıntercept as an ınstance of a transitional conception. *Educational Studies in Mathematics, 37*, 169–197.

[32] Pritt, N. (2010). Descartes' Analytic Geometry: Plotting the Future of Mathematics. Junior Division Paper, [Online] Retrieved February 22, 2015, from http://www.mdhc.org/files/579_Paper_Junior_Pritt.pdf

[33] Rogat, T. K., & Linnenbrink-Garcia, L. (2011). Socially shared regulation in collaborative groups: An analysis of the interplay between the quality of social regulation and group processes. *Cognition and Instruction,* 29(4), 375–415.

[34] Schnaubert, L., & Vogel, F. (2022). Integrating collaboration scripts, group awareness, and self-regulation in computer-supported collaborative learning. *International Journal of Computer-Supported Collaborative Learning.* 17, 1–10.

[35] Slavin, R. E. (1990). *Cooperative learning: Theory, research and practice.* Englewood Cliff, NJ: Prentice- Hall.

[36] Slavin, R. E. (1994). Student teams-achievement divisions. In S. Sharan (Eds.), *Handbook of cooperative learning methods* (pp. 3-19). London: Westport, Connecticut.

[37] Stahl, G., Koschmann, T., & Suthers, D. (2006). Computer-supported collaborative learning: An historical perspective. In R. K. Sawyer (Ed.), *Cambridge handbook of the learning sciences* (pp. 409-426). [Online] Retrieved February 22, 2015, from http://GerryStahl.net/cscl/CSCL_English.pdf

[38] Sullivan, M. (2008). *Algebra and trigonometry.* USA: Pearson Education.

[39] Takači D., Stankov G., & Milanovic I. (2015). Efficiency of learning environment using GeoGebra when calculus contents are learned in collaborative groups, *Computers and Education.* https://doi.org/ 10.1016/j.compedu.2014.12.002

[40] Tao, D., Zhang, J. (2021). Agency to transform: how did a grade 5 community co-configure dynamic knowledge building practices in a yearlong science inquiry? *International Journal of Computer Supported Collaborative Learning,* 16, 403–434.

[41] Thapa, R., Dahal, N., & Pant, B. P. (2022). GeoGebra Integration in High School Mathematics: An Experiential Exploration on Concepts of Circle. *Mathematıcs Teaching Research Journal, 14*(5), 16-33.

[42] Timmons, D. L., Johnson, C. W., & McCook, S. M. (2010). *Fundamentals of algebraic modeling*: An *ıntroduction to mathematical modeling with algebra and statistics*. Belmont: Brooks/Cole, Cengage Learning

[43] Turkish Ministry of National Education [TMoNE], (2010). *Ortaöğretim matematik dersi (9, 10, 11 ve 12. sınıflar) öğretim programı* [Secondary mathematics curriculum (Grades 9-12)]. Ankara, Turkey: TMoNE.

[44] Turkish Ministry of National Education [TMoNE], (2013). *Ortaöğretim matematik dersi (9, 10, 11 ve 12. sınıflar) öğretim programı* [Secondary mathematics curriculum (Grades 9-12)]. Ankara, Turkey: TMoNE.

[45] Ubuz, B., Üstün., I., ve Erbaş, A. K. (2009). Effect of Dynamic Geometry Environment on Immediate and Retention Level Achievements of Seventh Grade Students. *Eurasian Journal of Educational Research, 35*, 147-164.

[46] Umay, A. (2004). Primary mathematics teachers' and prospective teachers' perspectives towards ICT use. *Hacettepe University Journal of Education*, 26, pp.176-181.

[47] Uwurukundo, M. S., Maniraho, J. F., & Rwibasira, M. T. (2022). Efect of GeoGebra Software on Secondary School Students' Achievement in 3-D Geometry. *Education and Information Technologies,* 27, 5749–5765.

[48] Yıldırım, A., & Şimşek, H. (2008). *Sosyal bilimlerde nitel araştırma yöntemleri.* [Qualitative research methods in the social sciences] Ankara: Seçkin Publishing.

[49] Young, A. E. (1909). On the teaching of analytic geometry. *The American Mathematical Monthly, 16*(12), 205-212.

Appendix

Worksheet

Objective: The slope of a line. Tools Used: Computer and GeoGebra software

Make sure each of your teammates has learned the subject. You can get help from your teacher.

Group name: ....

Group members:

1)…..................................... 2)... 3)..

Open the slope of a line.ggb file saved on the desktop.

1. Drag the A and B points on the d line on the screen to get the desired values below. (4 different examples are presented consecutively on this tab. But here is one given.)

Example-1

- for the d line passing through the **A = (0, 0) and B= (4, 2)** points
  - ✓ Find the coordinates of the other F, G and D points on the line.
  - ✓ Write **the number** of **horizontal units** (increasing or decreasing according to right or left progression) and **vertical units** (ascending or descending according to progress up or down) from A to B?
  - ✓ Write the number of horizontal and vertical units for the other desired points in the table?

	A=(0, 0) and B=(4, 2)	**B=(4, 2) and F=(..., ...)**	**F=(..., ...) and G=(..., ...)**	**D=(..., ...) and A=(..., ...)**
Horizontal progression				
Vertical progression				
$\frac{\text{Vertical progression}}{\text{Horizontal progression}}$				

What can you say about the ratio of vertical advances to horizontal advances that you found for each pair of points on the line?

....
....

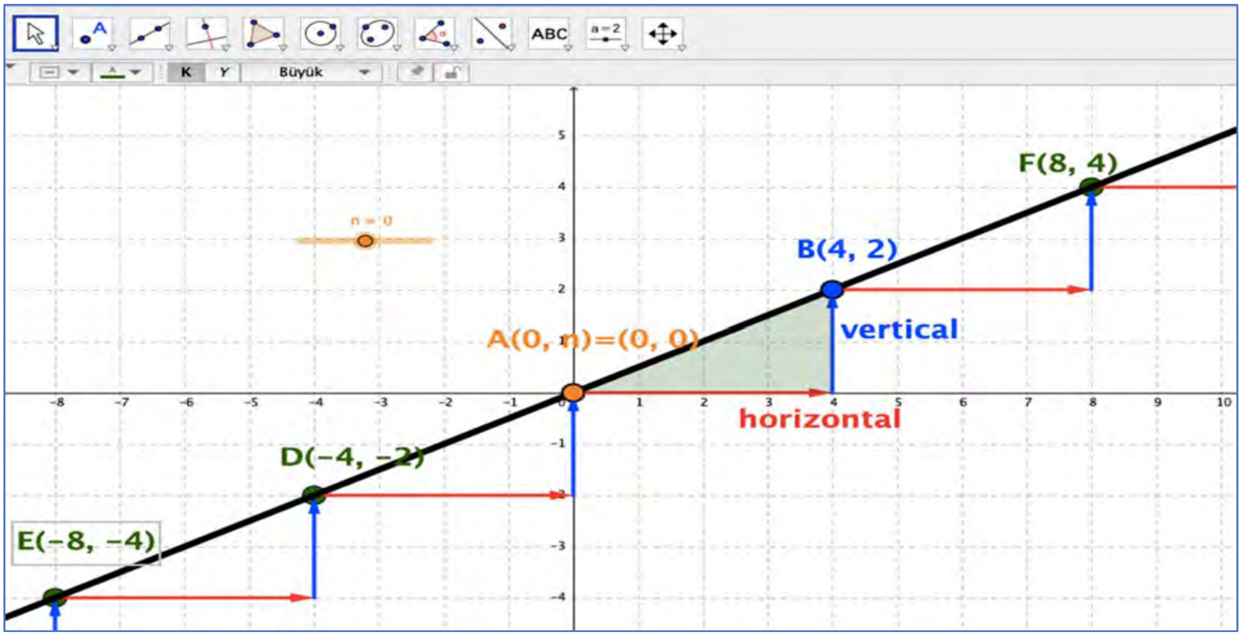
2. According to this, the ratio of vertical advances on the line to horizontal advances is .... to each other. This ratio is called the slope of the line. It is denoted by m.

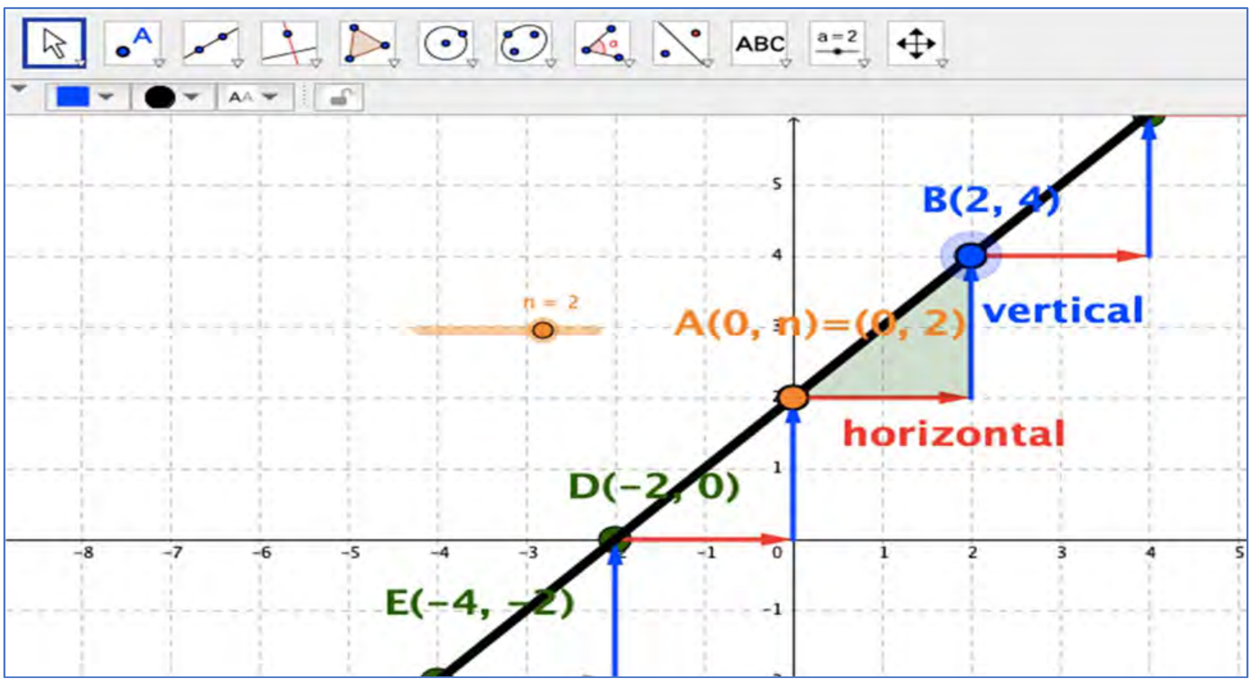
3. Accordingly, let's take the points A= (x_1, y_1) and B= (x_2, y_2) as any two points on a line. Explain how the slope of the line is calculated with the help of these points?

....

Dynamic Material

Screenshots of the dynamic material that are used in worksheet are given below.


ABC
a = 2
K
Y
Büyük
n = 0
F(8, 4)
B(4, 2)
vertical
A(0, n)=(0, 0)
horizontal
D(−4, −2)
E(−8, −4)


ABC
a = 2
n = 2
B(2, 4)
A(0, n)=(0, 2)
vertical
horizontal
D(−2, 0)
E(−4, −2)