Kindergarten and Primary Teachers' Noticing Within the Context of Vertical Team of Mathematics Lesson Study

Emine Gül Çelebi
TED University

Zerrin Toker
TED University

Çiğdem Alkaş-Ulusoy
TED University

Eçin Emre-Akdoğan
TED University

Ekin Balci
TED University

Gizem Guzeller
TED University

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This study investigated the noticing of teachers who teach mathematics at different grade levels in the context of lesson study. The study specifically focused on teachers' noticing during the planning phase of the lesson. For this, the lesson planning phase of a group of four teachers consisting of kindergarten and primary school 1st grade teachers (vertical teacher team) was investigated. Teachers' planning process for the 1st grade level mathematics lesson was the focus in the context of lesson study. We examined categories and subcategories that emerged in the planning phase of the lesson study and noticing levels dealt with those categories. We examined which categories and subcategories emerged in planning and at which noticing levels they dealt with those categories. The results of the study provide comprehensive data on categories and noticing levels of teachers teaching mathematics at different grade levels in the collaborative planning process of a lesson study focusing on improving problem-solving. The results of the study showed that the teachers' noticing was clustered under three categories: curriculum, teaching methods, and conceptual understanding. In terms of noticing levels, the study's results revealed that noticing is performed at the attending to level in all categories. Although less frequently than the attending to level, teachers noticed making sense of level in all the other categories except the curriculum category. Besides, noticing that the deciding to respond level did not occur at any level might be due to the cultural challenges of adaptation of lesson study and the amount of support, we researchers provided teachers as facilitators.

Keywords • mathematics teacher education research • lesson study • mathematics teacher noticing • •problem-solving • vertical teacher teams

Introduction

To foster students' learning of mathematics and for effective mathematics teaching, teachers should notice and rely on their student thinking and adapt their instruction to foster student learning (National Council of Teachers of Mathematics [NCTM], 2014). Emphasising teachers' practice leads to a better understanding of how teachers' effective mathematics teaching and professional learning take place (Ball et al., 2001). Researchers provided many different views of teacher learning; however, the common criterion for the effectiveness of professional development activities is the quality of resulting change in teachers' professional knowledge, skills, and practices (Cochran-Smith, & Lytle, 1999; Garet et al., 2001; Wei et al., 2009). In this sense, linking teacher learning with teachers' practice is essential.

Mathematics teacher noticing is such a way to improve teaching because what teachers see and how they interpret the instructional events in the classroom is at the heart of mathematics teaching (Mason, 2002; Sherin et al., 2011). Teacher noticing is important because "what teachers attend to as they teach is highly consequential" (Schoenfeld, 2011, p. 223). Efforts to understand the development of teachers' noticing skills enhance teaching expertise and quality (Jacobs et al., 2010; Sherin et al., 2011). However, literature shows that noticing is not necessarily productive and productive noticing occurs when teachers make instructional decisions that support student thinking (Choy et al., 2017), with reasoning and justification (Choy, 2014). Previous studies also indicated teachers do not necessarily notice students' mathematical thinking (Fernandez et al., 2012; Star & Strickland, 2007); lesson study could promote teachers to notice students' thinking (Lewis & Perry, 2017).

As an inquiry-oriented perspective, lesson study provides opportunities to experiment, observe and improve teaching and learning. The inquiry-oriented stand is used here for the teachers "to ask questions and seek answers, tackle problems and seek solutions, explore, investigate, and overall look critically at what we are doing and finding" (Jaworski, 2020, p. 277). A practice-based model of teacher learning is where teachers systematically investigate their teaching while collaborating to plan, teach and discuss their lessons (National Research Council [NRC], 2005). Researchers argue that lesson study, could support teacher learning and focus teachers' attention on student thinking and reform-oriented ideas (Fernandez & Yoshida, 2004; Stiegler & Hiebert, 1999). However, participation in lesson study does not naturally result in effective teaching practice (Lee & Choy, 2017). Besides, to fully realise the potential of lesson study, it is crucial that teachers deliberately focus on three lenses as the researcher, curriculum developer, and student while examining teaching (Fernandez et al., 2003). Here the teachers adopt a researcher role to carry on an inquiry of their practice, a curriculum developer role to understand how their instructional activities connect to student learning and, student role to anticipate students' solutions and use this knowledge to support students' learning of the concept.

There are many studies blending noticing with the context of lesson study aim to explore and support teachers' professional noticing of classroom events and situations (Carter & Amador, 2015; Gonzales & Vargas, 2020; Suh et al., 2021); to support these noticing skills in the teacher education programs (Amador & Weiland, 2015; Güner & Akyüz, 2020; Lee, 2019). Professional noticing is a framework for teaching practice that has of attending, interpreting, and deciding (Thomas et. al. 2020). However, few studies examined teachers' professional noticing that a vertical team of teachers from different grades focuses on covering mathematical topics included in the curriculum (Huang et al., 2019; Suh & Seshaiyer, 2015). Vertical teaming is beneficial for promoting vertical articulation and for improving understanding of the learning progression of the research lessons across grade levels during lesson planning (Suh & Seshaiyer, 2015). Although lesson planning is the phase where teachers make essential decisions for teaching and learning, it is a relatively unexplored process within the processes of noticing (Choy, 2014). Moreover, it is also necessary to understand the planning phase, which is one of the less visible parts (Fujii, 2018).

In Turkish educational context, the national mathematics curriculum has been constructed based on constructivist principles which has a sequential structure of mathematical concepts. Considering the sequence was formed to build mathematical understanding, it is important that teachers should understand their students' learning progression across grades (Suh & Seshaiyer, 2015), of which the first part is developed in kindergarten and primary levels. Unless teachers deliberately focus on the learning progression of their students' understanding, they will restrict themselves only to the grade level they teach, and they may not support their students' understanding sufficiently (Suh & Seshaiyer, 2015). Hence, we aimed to examine a vertical teacher team of kindergarten and primary teachers' professional noticing during lesson planning within a school-based lesson study. The key research question that guided our study is as follows: What and how do a vertical team of kindergarten and primary teachers notice during the planning phase of a mathematics lesson in the context of a Lesson Study?

Theoretical Background

This section presents two theoretical frameworks: Lesson study and mathematics teacher noticing, as these two constructs that are used in Lee and Choy (2017) to illustrate how these two constructs relate to and complement each other in operating to lead teacher learning towards improving teaching practice and student learning.

Lesson Study

Lesson study (LS) targets to improve the quality of teaching by focusing on "improving student learning and understanding" (Yoshida, 2008, p.98), and teachers aim to learn collaboratively from their classroom practice originated in Asia (Japan and China) (Chen & Yang, 2013; Stigler & Hiebert, 1999). Coming to the attention through *Third International Mathematics and Science Study* (TIMMS) video study on eighth-grade mathematics lessons (Stiegler et al., 1999), it has gained wide recognition. Since then, LS has been operationalised in many countries throughout the world. Concerns regarding whether LS can be carried out in other cultures are mainly related to cultural issues such as a collaboration of teachers, self-reflection, and a common curriculum (Lewis & Tsuchida, 1997). Additionally, it requires time for practitioners to understand the main components of this professional development strategy (Gunnarsdóttir & Pálsdóttir, 2019), where outside educational expertise or knowledgeable others is also needed to facilitate the process (Watanabe, 2011). When carried out at a school level, usually a team of teachers associates with the following steps: (1) formulation of long-term goals, (2) planning of a research lesson by building instructional objectives through examining curricular materials, (3) a teacher from the team teaching and the other members observe, to gather evidence for student thinking (4) post-lesson discussion where a reflection on the lesson occurs based on the evidence from observation (Perry & Lewis, 2009). Occasionally, a re-teaching of the research lesson is possible for the sake of improvement in instruction.

Research up to date revealed that LS has promising outcomes in terms of teacher learning (Murata, 2011, Vrikki et al., 2017; Warwick et al., 2016). LS enables teachers to learn about content and student thinking and provides opportunities to reflect on their teaching practice (Murata, 2011). However, the amount of support being given to the teachers would change the results (Takahashi, 2011). Extensive research demonstrated the value of collaborative inquiry that the teachers participate in the LS (Takahashi et al., 2013; Wei et al., 2009). LS enables teachers to concentrate more on students' thinking, learning difficulties, and misconceptions while designing their lessons (González & Vargas, 2020). It is also shown that experienced teachers are more involved in students' thinking through their engagement in the LS (Yang et al., 2021). However, the adaptation attempts of LS in different countries outside of Japan was reported to be challenging due to different norms, belief, and cultural systems (Stiegler & Hiebert, 2016) and the visible impacts of LS may not be seen until the teachers experience multiple LS cycles (Dudley, 2013). Aside from different variations, LS supports the "development of teacher knowledge of content, pedagogy, and children's thinking" (Hart, 2011, p. 290).

Noticing on Mathematics Teaching

According to Mason (2002), noticing is "at the heart of all practice." Although noticing is not unique to teaching, it involves a set of skills unique to experts in any profession (Berliner, 1991). As Miller (2011) argues, the hallmark of expert teachers is dependent on "situation awareness," which involves their ability to perceive and monitor their classroom environment and interpret and act upon this awareness of the elements of the classroom practice concerning student understanding. Hence, teacher noticing consists of active actions and decisions about what to attend to or not to attend, interpreting and responding to events and situations based on these interpretations This ability is viewed as having the potential to develop in time, while it requires rich and in-depth opportunities for bridging teaching and learning (Amador, 2016; Jacobs et al., 2011; van Es & Sherin, 2002).

Efforts in understanding what professional noticing is and how it takes can be categorised based on research on teacher noticing can be categorised into three main perspectives addressing: (a) what teachers attend to, (b) how teachers interpret what they attend to, and (c) the three skills as attending, interpreting, and responding to student thinking (Jacobs et al., 2010; Sherin & van Es, 2009). Furthermore, teachers who can notice how students think mathematically are more likely to make instructional decisions that support and improve students' thinking (van Es, 2011). Engaging with noticing supports teachers in analysing various parts of their teaching and learning in and from their practice (Mason, 2002; Sherin et al., 2011). It is crucial to notice content-specific features to improve mathematics instruction and student learning (Çopur-Gençtürk & Rodrigues, 2021; Sherin & van Es, 2009). Research indicates, however, that teachers could have difficulties in noticing mathematical features of the task (Star et al., 2011), and they may notice general pedagogical features that are not leading to promoting mathematical thinking (Collins et al., 2019). Previous studies indicated that experienced teachers are more likely to analyse student difficulties (Lee & Choy, 2017). Teachers noticing levels can change from attending to making sense of topics from pedagogical events to student thinking (Sherin & Han, 2004; van Es et al., 2017). Additionally, teachers noticing might remain at a more fundamental level for those first-time experiencing LS (Vermunt et al. 2019). If teachers aim to teach effectively in a way to promote student reasoning, then they need to attend aspects of student thinking via students' work and interpret those with a mathematical perspective before, during and after the lesson (Choy et al., 2017). In this study, we used the teachers' noticing framework from Choy's (2016) to understand teachers' noticing during the planning phase; based on the view where teachers attend noteworthy events, reason about them, and make decisions to respond to these events regarding their teaching and student thinking (Jacobs et al., 2010). Choy's framework (2016) describes noticing activities that teachers can perform during lesson planning, breaking down three basic noticing activities into five sequential categories as in the following Figure 1:

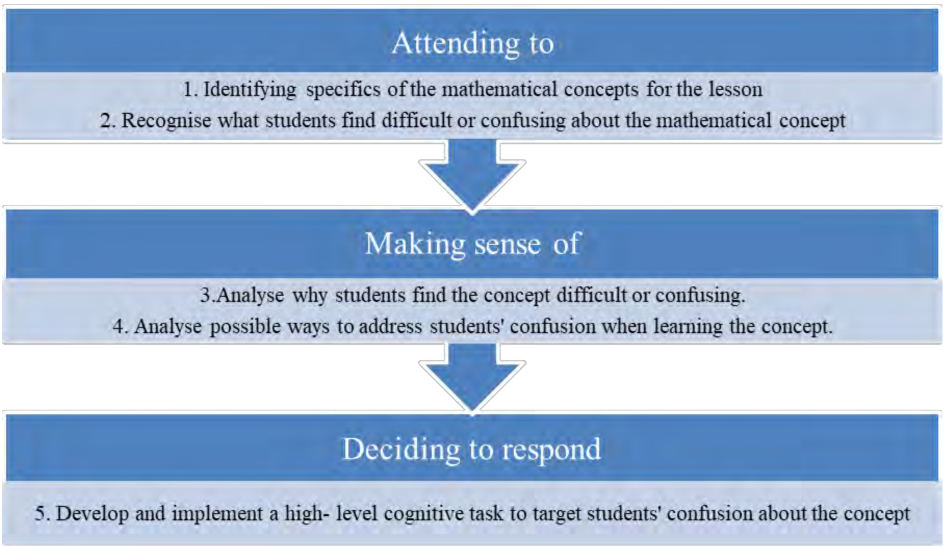

Figure 1. Theoretical framework of noticing during lesson planning (Choy, 2016, p. 429).

Because people only perceive a limited portion of their environment, perception is selective (Goodwin, 1994). This makes it challenging for teachers to notice the critical details of mathematical or instructional events in the classroom setting. For the sake of quality teaching and mathematics learning, teachers will need to focus on critical events in and from their practice. Lesson planning is a phase where 'teachers make decisions that affect instruction dramatically. They decide what to teach, how they will teach, how to organise the classroom, what routines to use, and how to adapt instruction for individuals' (Fennema & Franke, 1992, p. 156). Additionally, LS helps to reveal teachers' thinking and practice and provides opportunities to learn from each other about what is essential to notice and how it should be used to make future decisions to act on for improving student learning (van Es et al., 2017).

The Turkish education system is centralised and administered by the Ministry of Education National Education (MoNE). In Turkey, 12 years of schooling are compulsory, and these years are separated into three categories: 1–4th grades as primary (7–10 years-of-age), 5–8th grades as the middle (11–14 ages), and 9–12th grades as high school (15–18 years-of-age) levels. Early childhood education in Turkey is voluntary and refers to education for children aged 36 to 72 months and is governed by the Ministry of. Pre-school education encompasses the care and instruction of children aged 36 to 60 months, and Kindergarten education is for children aged 60 to 72 months.

The contemporary Turkish mathematics curriculum is influenced by constructivist viewpoints, which have guided many countries' curricular revisions (Zembat, 2010). Kösterelioğlu and Özen (2015) explored Turkish teachers do not recognise the curriculum and its components. For this reason, direct instruction remains the primary modality of training in Turkey (Emre-Akdoğan et al., 2018). The kindergarten curriculum includes cognitive, social, and emotional, psycho-motor, language, and self-care development of children (MoNE, 2013). Relevant to the study are the learning outcomes related to mathematical concepts under the cognitive development area. For example, "*Relating the number of objects and numbers from 1 to 10*", "*performing addition by using objects from 1 to 10*", and "*counting from 1 to 20.*" The primary grade curriculum comprises learning areas, sub-learning areas, learning outcomes, and process skills (like problem-solving) of mathematical concepts (MoNE, 2018). Relevant to our study is the learning area of Numbers and Operations, the sub learning area of Addition of natural numbers, and the learning outcomes are Makes addition with natural numbers whose sum is up to 20 (including 20) and Solves problems that require addition.

Methodology

This research was carried out in a foundation school during the 2020–2021 academic year. The research was a qualitative case study (Yin, 2017) and as an interpretive inquiry (Stake, 2003); to explore and learn about teachers' professional noticing during lesson planning within a school-based LS through information-rich, vertical teams (pre-school and primary teachers). The case of this study was four teachers; two of them were kindergarten teachers, and two of them were 1st grade primary teachers.

Participants

The school administration that the participants of this study worked for planned to implement a teacher professional development activity and hence, contacted the researchers. The researchers had implemented professional development activities about problem-solving skills with K–12 teachers who teach mathematics previously. Participants were selected from the teachers who attended those professional development activities. We conducted this study with the teachers who agreed to participate voluntarily. This study's participants comprised of four teachers, two of which were kindergarten and two were 1st grade primary school teachers, selected purposefully for typical case sampling (Miles et al., 2014). Typically, the teachers had 8–10 years of teaching experience. The teachers had not participated in an intervention related to LS before. In the study, T1, T2, T3, and T4 were used instead of the actual names of the teachers.

The School Curriculum

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Data Collection Process

The LS aims to improve teaching quality by focusing on students' learning and understanding (Yoshida, 2008). LS comprised of three main phases planning, teaching, and assessment (Kriewaldt, 2012). In this study, we have just focused on the planning phase. We conducted a LS with a team of teachers; two are kindergarten teachers, and two are 1st grade teachers. Teams of teachers have not attended any professional development activity on LS. In the first step, we implemented a seminar including the LS's goals and processes with the teachers' teams. Then, researchers shared the lesson plan template they had prepared before with the teachers (Appendix 1). The lesson plan template was designed according to the 5E instructional model based on constructivism. The 5E lesson plan includes five phases of the lesson: engagement, exploration, explanation, elaboration, and evaluation (Bybee et al., 2006). The 5E lesson plan suggests teachers complete the following sequence of activities throughout the five phases:

i) engagement: identify any knowledge gaps and obtain a grasp of the students' previous knowledge and introduce the lesson by engaging students with a new concept,
ii) exploration: have students explore a new concept through learning experiences,
iii) explanation: explain students' thinking about topics based on evidence from their activity,
iv) elaboration: elaborate on each idea or skill through additional practice, and
v) evaluation: evaluate student progress in a new context throughout the lesson.

The team of teachers then determined the learning objectives for their lesson plans from the curriculum. The team of teachers selected the learning outcome: "*Makes addition with natural numbers whose sum is up to 20 (including 20). Solves problems that require addition,*" which is in the numbers and operations learning area. The teachers held three sessions for three weeks to prepare their lesson plan using the lesson plan template. The lesson plan template guided their planning process during the three sessions. Researchers did not participate during the sessions; the team of teachers discussed the planning of their lesson plan amongst themselves.

The data of this study consist of video recordings of the three sessions held once a week for three weeks. Each session lasted about 60 minutes. The team of teachers conducted the sessions on-line through the Zoom platform during the COVID-19 pandemic. The teachers videotaped the sessions and shared them with researchers. The researchers transcribed the videotaped sessions.

Data Analysis

We conducted the interviews in the participants' native language and transcribed and translated them from Turkish to English. We then highlighted the related issues of teachers' noticing during the lesson planning phase in the sessions. Two researchers examined the discussions among the teachers during the planning sessions, and they started coding together to determine how they would code the subcategories. After they reached a consensus, they coded the subcategories independently. Then, they compared these subcategories and wrote brief descriptions of the codes. To illustrate, researchers explored these dialogues as subcategories of symbol usage when the teachers talk about the (+) sign. After subcategories were identified, by reaching an agreement, they gathered related subcategories under the same category. For instance, the subcategories of meaning of addition, symbol usage, context, problem, finding unknown addends, mental computation, number facts and problem posing identified, formed the category of *Conceptual Understanding*. Then, the two other researchers checked the subcategories and categories that emerged, and they compared all the data again. In summary, the coding process was completed with the full consensus of four coders after all the discussion on undecided and different coding was performed. This process was in keeping with qualitative data analysis described by Miles et al. (2014).

Results

The categories that were identified from the data were *Curriculum, Teaching Methods*, and *Conceptual Understanding*. Table 1 shows the list of categories, subcategories, and definitions (see Appendix 2). Categories and subcategories that emerged were organised according to noticing levels. Table 2 displays levels of teachers' noticing during the planning phase of LS processes (Choy, 2016) and includes definitions of the noticing levels and examples from the participants. The categories that emerged during the planning phase were curriculum, teaching methods, and conceptual understanding. The categories and related subcategories that emerged in the analysis of teacher statements during the planning phase are given in Figure 2.

Table 2
Noticing Levels, Their Definitions, and Examples

Level	Definition	Examples
Attending to	Being able to determine the mathematical concepts and possible difficulties caused by these concepts	Problems are always troublesome. We are constantly processing problems. We processed fractions, then problems again.
Making sense of	Being able to identify what caused these difficulties and methods to deal with them.	We already understand what students understand from their problems. They already understand if everyone can figure it out in the next lesson. I write the result in the chat section, if the majority of the results are correct, it 's okay.
Deciding to respond	Being able to generate and apply an extensive and comprehensive construct (lesson plan, task, etc.)	-

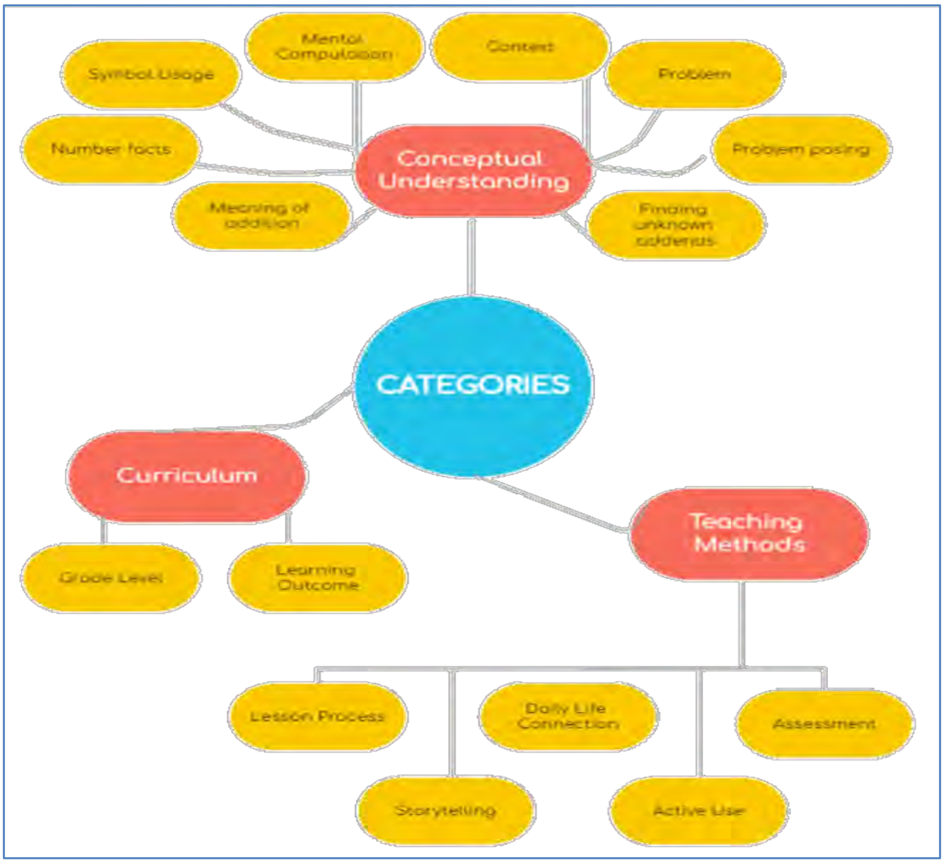

Figure 2. The categories and subcategories that emerged in the analysis of teacher statements during the planning phase.

Although teachers talk about similar subcategories in various dialogues, their level of noticing was different. For example, one of the teachers' statements about student understanding is considered at the *attending to level*, while another statement in the same subcategory was at the level of *making sense*. From here on, the subcategories and noticing levels under each category are explained with examples from teacher statements.

Curriculum

The first category that the teachers noticed is *Curriculum*. This category includes two subcategories that can be interpreted by *attending to the* noticing level. These categories are "grade level" and "learning outcomes." Regarding the grade level, T3 mentioned that the subject could be studied at different grade levels (Kindergarten and 1st grade), saying, "The plan we made fits with 1st grade, or you make it for kindergarten, and I apply it to 1st graders. We would have applied the same plan to both two levels." [T3]

Here we see primary school teacher T3's knowledge of the kindergarten curriculum. She talked about the kindergarten curricula, but her statement doesn't include any relational interpretation between kindergarten and primary school curricula. Therefore, this statement is coded at the *attending to* level. Another example of the grade level subcategory is related to noticing what students should know when they complete certain grade levels. Teachers teaching at different grade levels had a chance to transfer this awareness to each other during the planning process. For example, T1, a kindergarten teacher, explained what students know about addition after completing kindergarten, "They recognise the plus sign as they move from kindergarten to first grade."

This time we see a kindergarten teacher's attending to level of noticing at her grade level. Here, there is no interpretation related to the learning outcome. The planning session revealed an opportunity for teachers to notice not only grade levels but also learning outcomes specified in the curriculum: "Performs the addition of natural numbers whose sum is up to 20. We got this learning outcome. From hereon, it can be a learning outcome specified as 'Solves problems that require addition.'" [T4] In the statement, the primary school teacher (T4) appeared as the first in a conversation episode about a learning outcome. The teacher's noticing about the learning outcome in the episode was evaluated at the attending to level since the statement contains information just about what it is. Overall, the noticing levels of the teachers were limited to the subcategory attending to, and there were no statements related to the making sense of and deciding to respond levels. In the comments the teachers used relation to the grade level (five instances) and learning outcome (three instances) subcategories, knowledge was shared, without including interpretations of the information conveyed.

Teaching Methods

Teachers' noticing regarding teaching methods is classified under the subcategories of lesson process, storytelling, activity use, daily life connection, and assessment. The subcategory of "lesson process" was allocated to larger parts of the transcripts than any other code. Many ideas were expressed in relation to considering student learning. While some of the teachers' expressions were considered at the attending to level, they also made statements that were evaluated as making sense of. To illustrate, T4 (a primary school teacher) initiated a brainstorming activity without explaining its rationale, "We can start with brainstorming. What comes to mind when you think of adding an addition? Or, like what they do when they see the plus sign." Thus, this expression regarding the lesson process subcategory was evaluated under the attending to the level of noticing. [T4]

Storytelling was another subcategory the teachers noticed at the attending to and making sense of levels. Teachers' noticing at attending to level of incorporating storytelling into the process emphasise that it can have both effective and cognitive contributions. For example,

> [T4]: I, for example, use a lot of storytelling to gather the overall focus. I connect it to science; I also connect it to mathematics. It is much better as a warm-up activity.
>
> [T1]: Then we can start the warm-up activity with storytelling. The story is as if this is real life.

In this way, the teachers considered it appropriate to use storytelling at the beginning of the lesson, both to attract students' attention and to use the context of daily life. Here, T1 responds to the suggestion of T4 (initiator of the dialog in the attending to level) about storytelling by commenting on how storytelling can be used, so this dialog is coded at the making sense of level.

The teachers raised the idea that storytelling should be accompanied by activities rather than being used exclusively. However, they expressed different views on what kind of activity it should be and how these activities could be carried out. Also, "activity use" is another subcategory under the "Teaching Methods" category. This subcategory can be examined through an example as follows. T4 suggests an activity for the problem situation that includes visual material:

> [T4]: Something like this can be added: He was going in the forest and had these many fruits in his basket. Then, how many more fruits might he have picked on it ... We can throw something red, for example, around.

> [T3]: What if they rolled the dice and created problems that required addition with the incoming numbers? Then, they compare problems, "This is a problem similar to mine." They can talk about their similarities and differences.

This dialog, initiated by primary school teacher T4, was coded at the level of making sense of, as it included clues about the activities and how they would be.

Another subcategory obtained within the scope of *Teaching Methods* is "daily life." Teachers' noticing regarding association with daily life includes the elements of supporting with materials and adding games. For example, "Well, it can be, you can identify one material, for example, on a tree, with a lot of tangerines on it. So, tangerines can be collected like a game." [T1] The teachers' noticing about associating with daily life were evaluated at the attending to and making sense of levels. Since the following expression of T4, the initiator of the subject, only emphasises the importance of daily life, it was evaluated in attending to level. "We said that we should associate it with daily life, for example, let's send the students to a shopping mall. ... The student will also provide examples from their life, which require performing addition operation." [T4] The second utterance was coded at the level of making sense of, since it provides the rationale for daily life usage examples: "Examples from (students') life already attract a lot of attention." [T4]

The last subcategory under the Teaching Methods category is "assessment." Most the teachers' statements about assessment were evaluated at the level of making sense of, because in the statements, it was seen that the teachers provided evidence about how the students' understanding related to assessment.

> [T3]: We already understand what students understand from their problems. They already understand if everyone can figure it out in the next lesson. I write the result in the chat section; if most of the results are correct, it's okay.

As seen in Table 3, in the *Teaching Methods* category, there was evidence in relation to the for the noticing subcategories at the levels of attending to and making sense of. The highest frequency of noticing level was attending to. The teachers mainly focused on the lesson process. In this subcategory, no statements were evaluated at the deciding to respond level. Noticing levels were limited to attending to and making sense of.

Table 3
Frequency Table for Teaching Methods Category

	Attending to	Making sense of	Deciding to respond
Lesson process	23	16	-
Problem-Solving	7	-	-
Storytelling	12	3	-
Activity use	11	5	-
Daily life connection	6	2	-
Assessment	6	11	-
Total	65	37	-

Conceptual Understanding

When categories are associated with noticing levels, it is noticed that conceptual understanding is observed in attending to and making sense of levels. In any of the "meaning of addition, symbol usage, contexts, problem, and problem-solving" topics reached the attending to level; the teachers in this study, did not go beyond the action of "identifying specifics of the mathematical concepts," which is expected at attending to level. For this reason, these categories were evaluated at the attending to level. Below is a dialog about the "meaning of addition" category:

> [T4]: We are brainstorming. Like what comes to mind when they say addition or what they do when they see the plus sign.

> [T1]: We also give the sign of addition in kindergarten, but I do something so that the sum can be fully coded in their mind: I apply coding-in-mind activities such as picking fruits, collecting items there.

In the dialogue, individual ideas of T4 and T1 were put forward, and information about the practices was given. No analysis was made regarding the meaning of the addition process. Therefore, such statements were coded at the attending to level. A similar situation for the same dialogue applies to the "use of symbols" category. The emphasis on the (+) sign did not include any analysis. The teacher only discussed a brainstorming exercise that had the students' reactions when they saw the (+) sign. Therefore, here, the use of symbols was evaluated at the attending to level.

Another subcategory evaluated at the attending to level is contexts. As seen from the dialogue below, the teachers made appropriate context suggestions for teaching addition, but no deeper interpretation was made regarding the effectiveness of the context in the dialogue. For this reason, these expressions were evaluated at the lower level, attending to.

> [T4]: We can start with brainstorming. What comes to mind when you think of adding and adding? Or like what you do when they see the plus sign. "They know the plus. Coming from kindergarten"

> [T1]: We also teach addition in kindergarten, but for the addition to be fully coded, it is like "picking" and "picking fruit." I think it can be started as an application like let's collect fruit. To add play to fit the pick exactly.

Another subcategory under the *Conceptual Understanding* category is the "problem." There was no opportunity for deep analysis in the dialog for this code, similar to the others. Teachers only recognised that students had difficulties in problems (and problem solving). They did not engage in a deeper dialogue on the reasons for this situation. Since there was no dialogue about why and how the problems create trouble, statements were also coded at the attending to level. The initiator of the following dialog is a primary school teacher. A kindergarten teacher follows the dialog, but we cannot see any inquiry about T3's statement.

> [T3]: Problems are always troublesome. We are constantly processing problems. We processed fractions, then problems again.

> [T2]: We will already focus on problem-solving.

For the *Conceptual Understanding* category, a subcategory obtained jointly at attending to and making sense of levels is the finding "unknown addends." The following dialogue begins with T4's initiation about suggesting adding a "find the missing numbers" activity to the lesson plan. This section is coded in the attending to level as "finding unknown addends," which is identified by T4 to improve students' conceptual understanding of addition. The second teacher (T3), on the other hand, defended the view that it would be more appropriate to include only "basic problems with 2 addends" studies by showing evidence such as "the students have difficulties in finding the missing part activities and there has not been enough work done on that subject yet." This second part of the dialogue, where T3 talked about difficulties that students might have, was coded in the level of *making sense of* since the teacher's comment about the awareness of student knowledge included an analysis of the student's difficulties regarding the learning of addition: "They find it very difficult; we didn't do much work. I have eight marbles; how many marbles are needed to have 16 more marbles." [T3]

For the *Conceptual Understanding* category, the other two subcategories encountered at the making sense of level are "mental computation" and "number facts." These situations were coded at this level because teachers evaluated these situations beyond the teaching addition process as a way to understand addition at a conceptual level. In the dialogue below, T4, the initiator here, proposed to guide the students to make mental additions through a story. At the same time, the other teacher suggested repeating the mental process using paper and pencil as a second step. In this way, teachers try to overcome this difficulty by giving students who may have difficulties in solving addition problems the opportunity to mental addition.

> [T4]: We can ask questions to the children in the story; for example, there were 2 people on the bus, 3 more people got on, and there were 5 people. On top of 5 people, 10 more people got on at the third stop; how many people were there? Perhaps the mental addition process can also come into play in this way.
>
> [T1]: Then we have them mentally figure it out. We can also present a problem where they can do such an operation in the later process. So that's okay, we did it in the head, and we need to move on to the part that is put on paper, right?

Another subcategory of the *Conceptual Understanding* category encountered only at the level of making sense of this study is the "number facts." In the dialogue below, T2 initiates the dialog about number facts by sharing a problem. Then, teachers talked about adding number facts to their lesson plan, and they argued that they used number facts in 10 and then in 20 and that this method was effectively based on their own experiences. Because the evidence statements presented here are based on experience, this part of the dialogue was coded at the making sense of level:

> [T2]: How many spoons and more spoons do you add to have 8 spoons?
>
> [T4]: For example, I had a story when we were talking about numbers that add up to 10 … From now on, it's easy to give numbers that add up to 20. But I think the story is important. How do we choose something?

In the *Conceptual Understanding* category, another subcategory evaluated for the making sense of level was "problem-posing." Here, the teacher proposes to include problem posing in the lesson plan, recognising that it requires a challenging process for first-grade students. The reason for evaluating this expression at the making sense of level is that the teacher thought that students would have a challenge regarding problem posing:

> [T3]: Then if they write a problem themselves ... I don't know if it will be very challenging … They will roll the dice; they will get 3 and 5, they will create the problem with 3 and 5, which requires addition.

Table 4 shows the distribution of noticing levels according to subcategories regarding the conceptual understanding category. As seen in Table 5, there are subcategories from attending to and making sense of noticing levels. However, it is clearly seen that the noticing level with the highest frequency in this category is attending to. In the *Conceptual Understanding* category, the subcategories of the meaning of addition and problems that require addition, which is the lesson plan's objectives, came to the fore. It is suitable for teachers to notice these subcategories to improve students' conceptual understanding, but it is not enough for them to stay at attending to as the noticing level.

Table 4
Frequency Table for Conceptual Understanding Category

	Attending to	Making sense of	Deciding to respond
Symbol usage	3	-	-
Context	1	-	-
Problem	6	-	-
Finding unknown addends	1	2	-
Mental computation	-	1	-
Number facts	1	1	-
Problem posing	4	6	-
Total	22	10	0

When noticing levels in the planning cycle and their distribution according to categories are examined, it is evident that there are differences between categories and levels. Figure 3 shows the content matrix of the three categories and three noticing levels in vertical teams. In this figure, each circle represents a subcategory, and the circle's size is directly proportional to the numbers in the coded data. In addition, Table 5 shows the frequency distributions at three levels of noticing according to the three categories, as a summary of the relevant data.

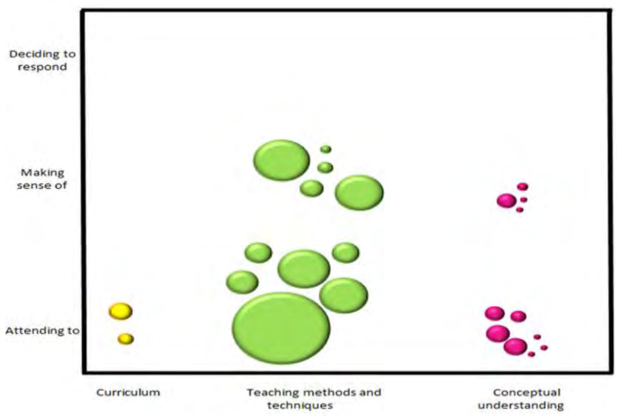

Figure 3. Content matrix of noticing levels and categories in vertical teams.

Table 5
Frequency Distributions at Three Levels of Noticing According to the Three Categories

	Curriculum	Teaching Methods and Techniques	Conceptual Understanding	Total
Attending to	8	65	22	95
Making sense of	-	37	10	47
Deciding to respond	-	-	-	-
Total	8	102	32	142

When Figure 3 and Table 5 are examined, it is seen that noticing is performed at the attending to level for each category. Categories *Teaching Methods* and *Conceptual Understanding* were noticed at the making sense of level. At the deciding to respond level, there was no evidence for any of the categories. The data revealed that noticing was done mainly at the attending to level of the three levels. This was followed by the making sense of level. Noticing was not performed at the deciding to respond level.

Discussion

Our study was motivated by the research showing that teachers' participation in professional development activities such as LS promotes their noticing skills (van Es et al., 2017). The main goal of the current study was to determine a vertical teacher team of kindergarten and primary teachers' professional noticing during lesson planning within a school-based LS. One of the study's main findings is that teachers performed noticing on different themes in the context of LS. It was observed that these categories were *Curriculum*, *Teaching Methods,* and *Conceptual Understanding*, which are fundamental components of teaching and learning. Moreover, it was observed that the team of teachers was basically at the attending to level, their noticing was interpretive about teaching methods and conceptual understanding, yet they did not decide to respond to what they attended to and reasoned about during the planning of the research lesson.

We identified that the *Curriculum* was the least noticed category among the other categories. This category involved grade level and learning outcomes. Regarding the curriculum, the noticing level of the teachers remained at the attending level. One of the reasons might be that teachers do not prefer to use curriculum as the primary resource while planning their lessons. Aligned with the literature, Turkish teachers do not comprehensively notice the curriculum and its components (Kösterelioğlu & Özen, 2015). We found that *Teaching Methods* was the most noticed category compared to curriculum and conceptual understanding categories. The teachers had begun to reason about the pedagogical features of their planning process, which shows that they are making sense of what they attend to regarding teaching methods. The teachers considered the components of teaching methods such as lesson process, problem-solving, storytelling, activity use, daily life connection, and assessment in detail during their lesson plans. Consistent with the previous research (Collins et al., 2019; Çopur-Gençtürk & Rodrigues, 2021; Sherin & van Es, 2009), we established those teachers who mainly noticed pedagogical topics like the *Teaching Methods*, discussed the teaching methods more comprehensively than the other topics they noted while planning. The *Conceptual Understanding* category includes the meaning of addition, symbol usage, context, problem, finding unknown addends, mental computation, number facts, and problem posing. Although we examined some interpretations indicating making sense of mathematical features of tasks that teachers discuss to use in the lesson plan, teachers' noticing mainly remained at the attending to level in the *Conceptual Understanding* category. Aligning with the literature, we determined that some teachers have difficulties noticing components of conceptual understanding (Star et al., 2011), which may restrict their ability to reason and construct instructional decisions that will support students' mathematical understanding.

Engaging with noticing helps teachers to analyse essential parts of their professional practices and provide a shared language to discuss these practices (Sherin et al., 2011). In this study, teachers noticed the main topics of *Curriculum, Teaching Methods,* and *Conceptual Understanding* while planning their lessons. Still, their noticing level was mainly about recognising these categories and making sense of some of them. The teachers, however, did not develop or plan to implement any adjustments to the lesson plan to support students' mathematical learning of the concept. Hence, we did not identify any noticing at the deciding to respond level regarding topics they focus. One of the reasons for not making observations at the deciding to respond level might be cultural issues like teachers' norms, beliefs, and collaborative working habits. Also, the literature stresses that teachers from different countries outside of Japan find it challenging to learn from LS (Stiegler & Hiebert, 2016). Furthermore, since we implemented the study during the COVID-19 pandemic, teachers' sessions were held through online platforms, which might be an additional struggle for collaborative working during the planning phase of LS. Besides, in line with the literature, the amount of support afforded by the researchers and facilitators of this process may have been insufficient (Takahashi, 2011), which may have resulted in productive noticing, like making instructional decisions to support teaching and learning along with reasoning and justification, only being achieved at the deciding to respond level (Choy, 2014; Choy, et al., 2017). Specifically, teachers might focus on different topics covering pedagogical practice to student thinking at different levels (Sherin & Han, 2004; van Es et al., 2017). Besides, research indicates that LS enables teachers to concentrate more on students' thinking, learning difficulties, and misconceptions while designing their lessons (González & Vargas, 2020).

Unlike these arguments, the kindergarten and primary teachers who participated in the LS process did not overtly focus on student thinking but on *Teaching Methods, Curriculum*, and *Conceptual Understanding*. The teachers who participated in our study were not beginning teachers, but their experience level might be considered relatively low (i.e., 8–10 years). According to the literature, more experienced teachers might have focused on student thinking more (Lee & Choy, 2017; Yang et al., 2021). The reasons for student thinking as being a relatively unnoticed category for teachers might relate to: (1) a need for more time to understand the components of professional development (Gunnarsdóttir & Pálsdóttir, 2019), (2) more support from a knowledgeable other as an educational expert to guide the process (Watanabe, 2011), and (3) the expected change parallel to the participation into LS might not come to fruition until participation progresses to multiple LS cycles (Dudley, 2013).

Some limitations of this study include that the teachers who participated in the study performed an LS for the first time, and they completed one full cycle (i.e., planning, teaching, evaluation). Aligned with the previous research, teachers who are involved in a LS for the first time, tend to demonstrate their noticing at the basic levels, like attending to and making sense of levels (Vermunt et al., 2019). In this study, it was both a new experience for them and a new experience working together within a vertical team of teachers. We explored that working vertical teams of teachers with different grade levels not only enabled them to focus on their grade level but also become aware of students' previous learning experiences and prepare more effectively for their future learning progression (Suh & Seshaiyer, 2015). The focus of teacher noticing, and their associated levels provide a body of evidence that illustrates the domains of teacher learning with which teachers engage when involved in LS. It also illustrates how LS can lead to enriched productive noticing and improved mathematics education at kindergarten and early primary grades. Overall, this study presents a list of topics a vertical team of kindergarten and primary school teachers could potentially focus on during the planning phase of LS. In this study, a research question for an analysis based on the teachers was not included in the scope of the study. Future studies could investigate how noticing levels differentiate based on teachers' grade levels, teaching experiences, and LS experiences.

Corresponding author

Emine Gül Çelebi
TED University
Ön Cebeci, Ziya Gökalp Cd. 48/A, 06420
Çankaya, Türkiye

gul.celebi@tedu.edu.tr

Ethics Declarations

Ethical approval

Ethical approval for the research was granted by the TED University and informed consent was given by all participants for their data to be published.

Competing interests

The authors declare there are no competing interests.

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Appendix 1: Lesson Plan Template

Lesson Plan Draft

GRADE		**TIME**	
COURSE DATE			
LEARNING AREA			
SUB-LEARNING AREA			
LEARNING OUTCOME(S)			
TEACHING METHODS & STRATEGIES	*Which teaching methods and techniques do you plan to use? (Problem-solving, collaboration, discussion/debate, role-playing, concept map, brainstorming, etc.)* *How can the teaching method and technique you deem appropriate contribute to the learning objectives of the planned course?*		
MATERIALS	*Write down all the materials you plan to use; you can also add their pictures.*		
RELATED MATH SKILLS	*Which mathematical skills do you aim to develop in students?* ** Among the skills you target, there may be one or more basic mathematical skills such as problem-solving, reasoning, association (with different disciplines, daily life, different subjects of mathematics), communication, using multiple representations, number sense, mental processing, estimation.*		
PRELIMINARY PREPARATION			
Why did you choose this topic? What are the points that attract your attention about the difficulty of students and the way they learn the subject or the way they think?			
What is the general purpose of this course?	Students…. will understand/can/develop…		
What preliminary information should students have about the subject/concept?			
What learning difficulties and misconceptions may students have regarding the subject/concept?			
What is the rationale for your instructional design?	*How would you relate your instructional design to the highlights of the curriculum and other sources you reviewed?* *How did you design your lesson plan? How did you choose the tasks, problems, contexts, demonstrations, etc., prepared for students?* *How would you argue for the relevance of your lesson plan to your research theme?*		

INTRODUCTION	
What warm-up activity did you choose? Write your activity	
Explain the reasons for choosing this activity.	
How do you plan to motivate your students with this warm-up activity?	
What challenges might students have in your warm-up activity?	
What questions can you ask to make sure students fully understand the activity?	
PROCESS	
PRESENTATION OF THE TASK	*Write down the task you chose below to deepen the topic. How will you implement this activity in the classroom? Group work, individual work, etc. How do you plan to ensure that your students understand the task/are aware of your expectations? Please explain.*
PREDICTIONS ON STUDENT LEARNING	*Write down the answers, possible ideas, and ways of thinking that you think students might give about the task.*
DISCUSSION AND COMPARISON	*Have you created a plan or an activity to summarize and compare students' ideas? Please explain.*
ASSESSMENT	*Specify possible tasks and questions that you will use to assess the course process and student learning.*
CLOSING	
SUMMARISING	
HOMEWORK AND/OR FAMILY PARTICIPATION	*Did you design an assignment to reinforce the course process and deepen the results achieved in the course? Have you created a new task requiring family involvement in this process?*
USED RESOURCES	*In addition to printed resources, you can share your materials or images of concrete materials.*

Appendix 2:

Table 1: *Results: Categories, Subcategories, Examples, and Definitions*

Category	Subcategory	Examples	Definitions
Curriculum	Grade Level	*They recognize the plus sign as they move from kindergarten to first grade.*	Dialogues about choosing grade level or reasons why that level was chosen.
	Learning Outcome	*Performs the addition of natural numbers whose sum is up to 20. We got this learning outcome. From here on, it can be a learning outcome specified as "Solves problems that require addition."*	Dialogues about determining the outcome were considered.
Teaching Methods	Lesson Process	*We can start with brainstorming. What comes to mind when you think of adding and addition? Or like what they do when they see the plus sign.*	Dialogues about all plans and expectations regarding the implementation and the flow of the lesson.
	Storytelling	*I, for example, use a lot of storytelling to gather the overall focus. I connect it to science; I also connect it to mathematics. It is much better as a warm-up activity.*	Dialogues about teachers' plan for the story of the problem, choosing the topic, how it would be used, or how they would present it.
	Activity Use	*What if they rolled the dice and created problems that required addition with the incoming numbers? Then, they compare problems. "This is a problem similar to mine." They can talk about their similarities and differences.*	Dialogues about what kind of activity the teacher could develop or how they could use it along with the story.
	Daily Life Connection	*Well, it can be you can identify one material, for example, on a tree, with a lot of tangerines on it. So, tangerines can be collected like a game.*	Dialogues about how the story could be related to daily life, what kind of elements of everyday life could be used, or its advantages for the students.
	Assessment	*We already understand what students understand from their problems. They already understand if everyone can figure it out in the next lesson. I write the result in the chat section; if most of the results are correct, it's okay.*	Dialogues about how and when they would assess the students learning.
Conceptual Understanding	Meaning of Addition	*We are brainstorming. Like what comes to mind when they say addition or what they do when they see the plus sign.*	Dialogues about what addition meant and how they would support it.
	Symbol Usage	*We also give the sign of addition in kindergarten, but I do something so that the sum can be fully coded in their mind: I apply coding-in-mind activities such as picking fruits, collecting items there"*	Dialogues about the symbol of plus sign (+) were coded.
	Contexts	*We also teach addition in kindergarten, but for the addition to be fully coded, it is like "picking" and "picking fruit." I think it can be started as an application like let's collect fruit. To add play to fit the pick exactly.*	Dialogues about suggesting a context for the problem solving.
	Problem	*Problems are always troublesome. We are constantly processing problems. We processed fractions, then problems again.*	Dialogues about the concept of "problem" and its difficulties for the students.
	Finding Unknown Addends	*The students have difficulties in finding the missing part activities, and there has not been enough work done on that subject yet.*	Dialogues about integrating this concept into the lesson plan.
	Mental Computation	*We can ask questions to the children in the story. For example, there were 2 people on the bus, 3 more people got on, and there were 5 people. On top of 5 people, 10 more people got on at the third stop, how many people were there? Perhaps the mental addition process can also come into play in this way.*	Dialogues about how the mental addition process could be integrated.
	Number Facts	*How many spoons and more spoons do you add to have 8 spoons?*	Dialogues about how to use the number facts in the story.
	Problem Posing	*Then if they write a problem themselves, I don't know if it will be very challenging… They will roll the dice and get 3 and 5, creating the problem with 3 and 5, which requires addition.*	Dialogues about how the students would pose their problems or whether the students should pose or not.