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INTEGRATION OF INDONESIAN CULTURE IN THE DIDACTIC DESIGN OF THE CONCEPT OF FRACTIONS IN ELEMENTARY SCHOOLS

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Abstract

This study aims to develop a didactic design using Engklek games to teach the concept of fractions in elementary schools. This study combines a cultural approach, namely Engklek games with mathematics in the concept of fractions. This study used didactical design research, with 2 research subjects, namely 20 students in grade 5 for learning obstacles as the basis for making didactic designs, and 50 students in grade 4 to determine the impact of implementing didactic designs. Data collection techniques used are tests, interviews, and observation. The data analysis technique uses a qualitative method to see learning obstacles in students and a quantitative method (descriptive and inferential statistics) to determine the impact of didactic design implementation. Based on the results of the Mann-Whitney test that the sig. is .000, meaning that there is a difference in students' mean scores on the concept of fractions before and after the implementation of the didactic design, this indicates that there is a positive influence on students getting the implementation of the didactic design using the Engklek game on the concept of fractions in elementary schools. This research contributes to education in an effort to create effective and meaningful learning by involving real contexts in real life through elements of culture.

Keywords: Ethnomathematics, Engklek games, didactic design, fraction concept.

INTRODUCTION

An important subject to study at all levels is mathematics (Subrahmanyam, 2021), and it relates to other subjects (Lingefjärd & Ahmet, 2022; Tekin, 2022) and will be very useful in work (Escalera-Chávez et al, 2021). Mathematics provides many benefits, assists students in practicing creative and critical thinking, and can change their perspective on things (Baykul, 2009), this is also explained by Schoenfel (1989) in his research on 230 students, that students believe mathematics is not just memorizing but mathematics is a discipline that is creative, useful and teaches how to think. Thus, one of the keys for students to understand the context of mathematics is the teacher's effort to encourage students to be interested in and like aspects of mathematics (Rellensman & Schukajlow, 2020).

Preschool-level comprehension of mathematics contributes to the development of a robust cognitive domain in children (Linder & Simpson, 2018). Additionally, elementary school students' grasp of mathematics serves as a foundational platform for their understanding of the subject in subsequent levels

(Pamudi, 2022). Despite this, it is acknowledged that mathematics is commonly perceived as a challenging subject for students (Harun & Manaf, 2021; Weng & Yang, 2017), including among Indonesian students (Riyanto, 2019).

Basically, mathematics is an abstract science that contains many mathematical symbols, and for elementary school children who are in the phase of concrete operational thinking will be very difficult to understand (Widodo & Kartikasari, 2017).

In mathematics learning, there are several standards that students must have. According to National Council of Teacher Mathematics (2003), mathematical standards are divided into 2, namely material standards and process standards. Material standards are concerned with the content that students must learn, and process standards are concerned with specific skills that students must possess. The material taught in mathematics is arithmetic, geometry, measurement and statistics. One of the materials in learning arithmetic is numbers. Numbers are a very important material in mathematics in elementary school.

Rational numbers are part of several types of numbers learned in elementary school. Rational numbers were first studied in the form of fractional concepts, and this is important to learn in elementary school. In various countries, fractional material has begun to be taught from grade 1 of elementary school (Alajmi, 2012). This means that fractional matter is considered essential in learning mathematics, Students' level of understanding of fractions has a close relationship with their overall success in mathematics (Siegler et al., 2012). So, it is necessary to instill the right fraction concept when students are in elementary school. According to Doğan and Sir (2022), one of the most basic elementary school mathematics subjects is fractions, the concept of fractions plays a key role in the educational background of students. In general, the main idea of fractions can be divided into five parts, part-whole, measure, operation, quotients, and ratios and rate (Eliustaoglu, 2016; Clarke, 2006), This idea is a very important part to be understood by students in the context of mathematics.

Fraction concepts are frequently identified as challenging in elementary school mathematics, as supported by various studies, including those conducted by Heriyani et al. (2022) and Deringol (2019). These challenges persist in the elementary school context, as highlighted in the research of Hariyani et al. (2022), which delves into obstacles encountered by students in learning fractional material.

Based on the results of the report, that in learning mathematics, fractions are the material with the lowest rank (Braithwaite et al., 2017). This certainly reinforces previous studies about students' difficulties in understanding fractional material. This is because students experience many conceptual errors in the material (Hastuti et al., 2020). According to Torbeyns et al. (2015) that conceptual understanding of fractions is one of the biggest challenges in mathematics classrooms that students have faced for several years. The results of research conducted by Hoch (2018) that there are 33 international studies that examine fractions and found 58 empirically typical errors, these errors are often systematic in nature such as implementing the wrong system and showing a lack of conceptual understanding. The difficulty students face in solving problems related to fractions stems from their tendency to memorize formulas and algorithms without grasping the actual meaning of fractions, as indicated by research conducted by Gabriel et al. (2013), and Şiap and Duru (2004). When students have difficulty answering questions especially in fractional material, this indicates that there are some learning obstacles, the causes are the interaction system, the learning process, the nature of the teaching from the teacher, the nature of the subject matter, genetic factors and personal development (Sukirno & Ramdhani, 2016). Obstacles experienced by students can also be caused by gaps between concepts formed by students, teachers, and the presentation of teaching materials, this is also called the zone of concept image different (ZCID). Learning obstacle is divided into 3 of them are, 1) ontogenic obstacle, is a learning obstacle caused by the limitations of the student in self-development or related to the mental readiness of student learning, 2) epistemological obstacle, is a learning obstacle caused by the limited knowledge that students have in certain contexts, and 3) didactical obstacle, is a learning obstacle caused by the method or approach used by the teacher, or it could be from the presentation of textbooks (Suryadi, 2015).

In their professional duties, teachers need to identify learning obstacles and then think about solving problems from these obstacles, when this is not found a solution, it will cause learning difficulties. According to Jamal (2014) student learning difficulties will have an impact on learning achievement, because to obtain good achievement, it can be obtained from the treatment of learning inside and outside of school and on the provisions and efforts in learning. So, what the teacher has to do in overcoming problems related to student learning obstacles is to design a learning plan that is made as a didactic design (Fauzi, 2020). Didactic Design Research (DDR) is to develop didactic design and can overcome student learning obstacles.

DDR is a type of design research method whose goal is to develop learning innovations in the form of didactic designs and teaching materials in mathematics education. According to Gravemeijer and Cobb (2006), philosophy in design research is how researchers try to create various educational innovations.

According to Suryadi (2018), DDR is based on two paradigms, namely interpretive and critical. This paradigm is the main basis for creating an effective design, and can be applied in learning activities. The reality studied in DDR is related to the characteristics of the concept image that is formed in a person as a result of the learning process with a certain didactic design, the process of forming the concept image is not only experienced by students, but also experienced by teachers who have the task of teaching concept knowledge to students. the concept image possessed by the teacher will have an impact on students' understanding and meaning of the concept of knowledge they have. The image of the concept of students and teachers may also not be in accordance with the actual scientific conception. So, this is a DDR reality study to be able to analyze and see the suitability of the Zone of Concept Image Different (ZCID) between students, teachers and scientific conception. Then, another reality that becomes the study of DDR is the impact of didactic design which results in learning obstacles for students.

The two realities described above can bring up various possible responses related to student learning obstacles, which then need to be prepared for pedagogical didactic anticipation in a didactic design, the didactic design developed can form a conceptual image that is in accordance with the scientific conception and can overcome the learning obstacle problems experienced by students. It is on this basis that the critical-interpretation paradigm can be used to encourage the creative thinking of researchers so that they are able to generate positive ideas for the development of better didactic designs.

The didactic design developed in this study is on the concept of fractions, namely fractions as part to whole, compare fractions, equivalent fractions, and add and subtract fractions with the same denominator. Learning in didactic design is made in an Engklek games, this game is often played by children in everyday life. Cultural integration in the context of learning mathematics through Engklek games is very important to be reintroduced in the 21st century so that this is not lost by the times and technological developments. According to Ascher (1984), studies that explain mathematical ideas that contain the context of traditional society are referred to as ethnomathematics. According to Powell and Frankenstein (1997), ethnomathematics invites us to see how mathematical knowledge has been constructed throughout history in different cultural settings. The application of a culture-based learning model is seen as very important to be able to instill positive character formation, and it reflects the nation's cultural values as well as improving the cognitive aspects of students (Arisetyawan et al., 2014).

There have been many studies explaining the integration of culture in learning, especially the Engklek game in mathematics learning, one of which is Supriadi and Arisetyawan (2020) who conducted research entitled the didactic design of Sundanese ethnomathematics learning with the endog-endogan game and the Engklek game in elementary schools, but the learning applied to geometrical concepts. The didactic design in this study was applied to the concept of fractions in elementary schools. This study aims to develop a didactic design on the concept of fractions through cultural integration with Engklek games in elementary schools. This research is something very important to study because it discusses the mathematical and cultural context simultaneously. Engklek games became a culture that was often played by Indonesians, and unconsciously it formed the process of mathematization. This research is different from previous research both in the context of design, material or culture raised.

The several research questions that will be discussed in this research are:

1. What is the concept of image and learning obstacle that occurs to students?
2. How is the Hypothetical Learning Trajectory on the concept of fractions using cultural elements?
3. How is the didactic design of the concept of fractions developed by integrating culture through Engklek games?
4. What is the impact of the didactic design developed on students' understanding of the concept of fractions?

This research can facilitate students to understand the concept of fractions according to the learning trajectory and without experiencing the learning obstacles that were found previously. This research also provides a reference for teachers in planning, implementing and evaluating mathematics learning by integrating the cultural context of the concept of fractions in elementary schools.

METHOD

Research Method

This research uses Didcatical Design Research (DDR). In the DDR stage, In the DDR stage, an analysis of learning barriers is carried out on fractional material, so that this becomes the basis for making didactic designs.

Upon a detailed understanding of the learning obstacles, it serves as the foundation for formulating hypothetical learning trajectories and didactic designs for teaching fractional concepts in elementary schools. The design that has been made is then implemented to see the impact of didactic design on the learning process, the final stage is that the researcher can reflect and revise the didactic design of the concept of fractions in elementary schools.

In developing the didactical design, the researcher uses the DDR stage, namely the process of developing a didactical design consisting of a series of didactical situations, analyzing student responses and the developed didactical situations and decisions taken during the learning process (Suryadi, 2015). DDR consists of three important stages, namely 1) analysis of the didactic situation before the implementation of learning (perspective analysis) in the form of hypothetical didactic design and pedagogical didactic anticipation; 2) didactical-pedagogical analysis of the situation or metapedadidactic analysis; and 3) retrospective analysis linking the results of the hypothetical didactical situation analysis with the results of the metapedia-didactical situation (Suratno, 2016).

Participant

This research was conducted in an elementary school in Serang, Banten Province. The details of the subjects in this study are as follows.

Table 1. Subject of study.

Initial Learning Obstacles	Received a Didactic Design Implementation
20 students	50 students

Data Collection

Data collection was carried out as follows, 1) tests, written tests are given to students in grade 5 who have received previous fractional concept material, the aim is to see learning obstacles, this test is also the basis for seeing the suitability of the concept image formed in students, the tests that have been carried out in grade 5 are the basis for the preparation of hypothetical learning trajectory, and didactic design on fractional material. Tests were also given to grade 4 students to see the impact of the didactic design of the fraction concept before and after it was implemented, 2) interviews, interviews were given to students to see the concept image and mindset or reasons of students in answering the questions that had been given, and interviews were also given to the teacher to see the concept image of the concept of fractions and the teacher's way of teaching the material, this is done to see the cause of didactical obstacles, and 3) observation, observations were made to see the suitability of the hypothetical didactic design developed during implementation. The observed aspects are related to theory of didactical

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situation, didactic contract, pedagogical philosophical theory of mathematics, metapedadidactic, and learning trajectory. These observations form the basis for revising the final didactic design that can be applied in the learning process.

Data Analysis

The instrument that has been made is then tested for validity and reliability, it is used in conducting research and data collection. After the data is collected, the following data analysis is carried out: 1) test, when uncovering learning obstacles in grade 5, the researcher recorded all student responses that appeared, identified and grouped them based on the obstacles. Furthermore, the researcher also conducted a quantitative test stage to see how the impact of the didactic design had on students' understanding of the concept of fractions in grade 4, the analysis used the mean difference test between pretest and posttest, 2) interviews, the researcher conducted interview transcripts, provided coding of the meaning of important sentences, and concluded the results of the interviews on the concept image of students, teachers, and students' mindsets or reasons in answering learning obstacle questions, and 3) observation, researchers take videos and photos of learning activities, then record important things that arise, provide comments and assess the results of observations that have been made.

Research Flow

The didactic design model developed in this study refers to the explanations provided by Mulyana et al (2014), in detail can be seen in the image below.

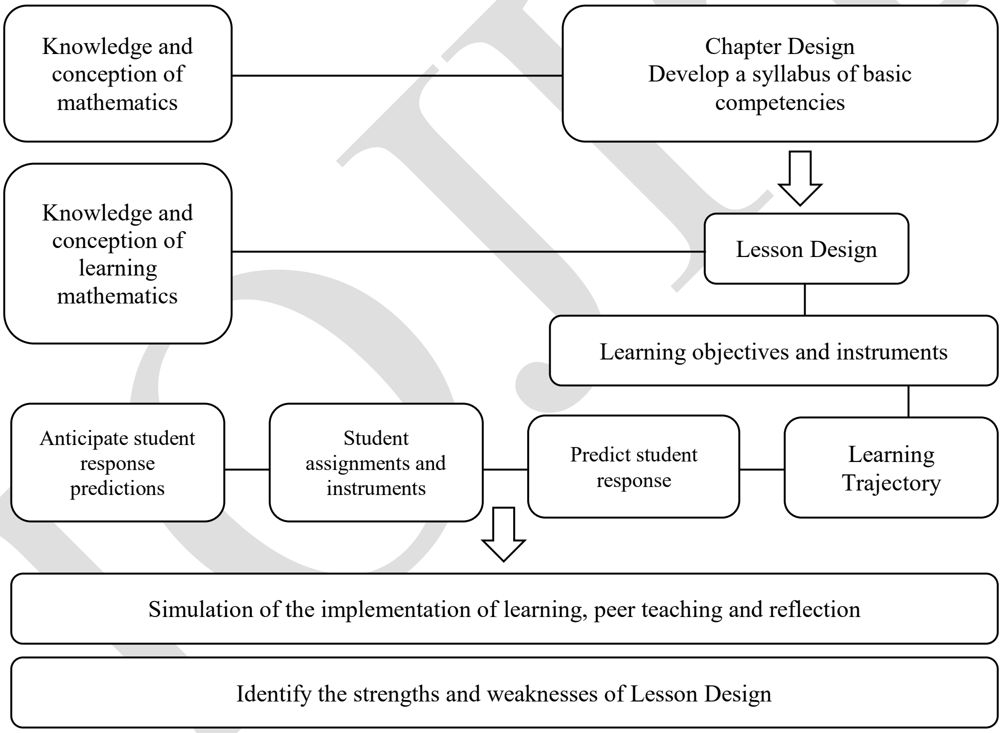

Figure 1. Didactic design development model.

RESULTS

The development of the didactic design of the concept of fractions begins by analyzing in detail the knowledge and mathematical conceptions that occur for both students and teachers, and the analysis of learning obstacles becomes an initial description to reveal these two things, the learning obstacle analysis forms the basis for developing a hypothetical learning trajectory and a hypothetical didactic design, after which it is implemented to see the real picture and the impact of the didactic design on the concept of fractions in elementary schools, this implementation will provide an evaluation and improvement of the didactic design to make it better.

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Learning Obstacles
The learning obstacle test was given to students in grade 5 in a total of 20 students. The types of questions given relate to declaring fractions, comparing fractions, equivalent fractions, addition and subtraction of fractions with the same denominator. Comprehensively the description of students in answering questions is explained in the image below.

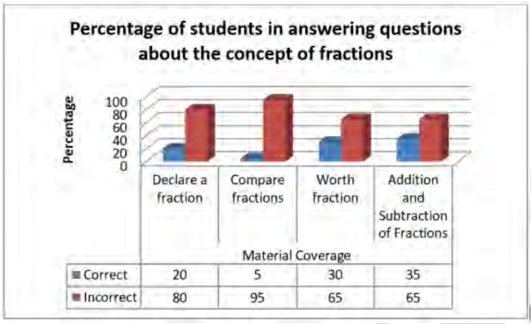

Figure 2. Percentage of students answering questions in the preliminary study.

Based on Figure 2 above, students who have received fractional concept material are still experiencing learning obstacles or their understanding of the material they have mastered is still incomplete, this is evident from the low percentage of students answering questions correctly. Regardless of the low students' ability to answer the questions correctly, it is necessary to analyze in detail the types of learning obstacles experienced by students, so that this provides an overview of the didactic design that will be developed in the material for the concept of fractions in elementary schools.

Ontogenic Obstacle
The ontogenic obstacle found in the concept of fractions is when students have to state the fractions of an image. The obstacle is the inability of students to understand the purpose of the images presented.

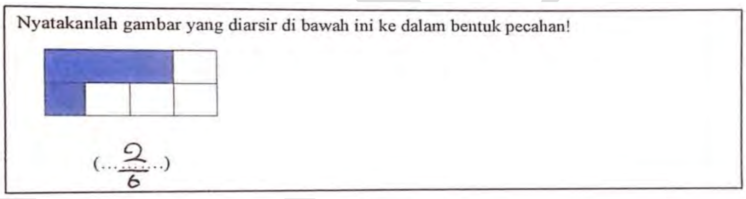

Figure 3. Ontogenic obstacle.

Based on Figure 3 above it shows that students only understand that fractions are part of the whole, without thinking that the parts must be the same, this is also confirmed based on the results of interviews with students which show students only calculate the shaded part (numerator) which amounts to 2, and The total part (the denominator) is 6, so they write down the fraction above is $\frac{2}{6}$, this obstacle is indeed often found from some students, as also explained by Malikha and Amir (2018) that the meaning of fractions which means part of the whole is interpreted differently by students, sometimes students do not pay attention to whether the whole part is fair/equal or not. Supposedly in answering this question, students must involve mental acts in the form of interpretation by making guide lines on parts of the image that are of different sizes, so that later parts of the image with the same size are obtained, only then can they be expressed as fractions whose meaning is part of the same whole.

Epistemological Obstacle
Epistemological obstacles are obstacle to student learning in understanding the concept of knowledge, the obstacles found are very diverse. The obstacle is that when stating fractions from an image, students are not able to understand the concept of fractions as part of the same whole (part to whole).

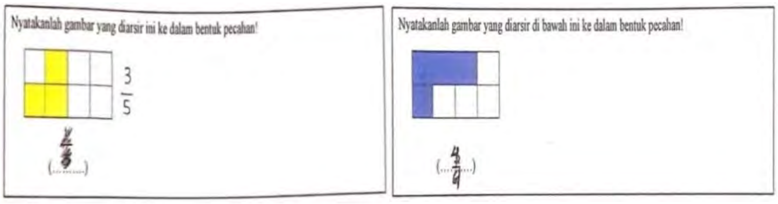

Figure 4. Epistemological obstacle 1.

Based on the results of the interviews, students tend to understand that fractions are the parts that are observed compared to the parts that are not observed. This error is also often found when students state a fraction. According to Şiap & Duru (2004) students assume that the numerator and denominator in fractions are two separate integers, so for that reason they compare the two things. In addition, some students also assume that a fraction is the whole which is expressed as the numerator, and the observed part is the denominator.

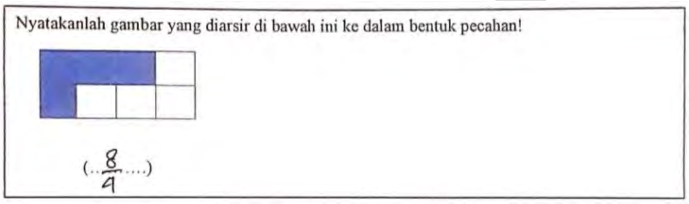

Figure 5. Students consider the whole part (the numerator), and the part that is observed (the denominator).

Based on Figure 5 above, it shows that students are aware that the whole part of the picture is 8, but in stating the fractions, students are mistaken because they think that part 8 is the numerator, and part 4 is the denominator, so they state the fraction is $\frac{8}{4}$

The second epistemological obstacle is related to the inability of students to understand fractions as a measure. This is shown when students are unable to compare fractions with different denominators.

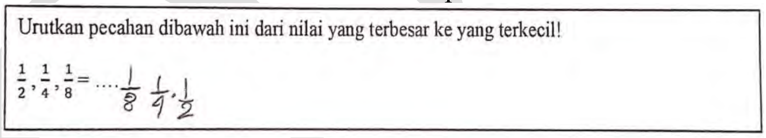

Figure 6. Epistemological obstacle 2.

Based on Figure 6 above, students were asked to sort the values of the fractions from the largest to the smallest, this is to see how students' ability to understand fractions as a measure, but the fact is that out of 20 students, only 1 student answered correctly, and 19 students answered incorrectly as shown in the picture above. Based on the results of interviews that students assume $\frac{1}{8} > \frac{1}{4} > \frac{1}{2}$, whereas to compare fractions with different denominators, students must first equate the parts of the denominator, so that $\frac{1}{8}$, $\frac{1}{4} = \frac{2}{8}$, and $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$ and finally they can conclude that $\frac{1}{2} > \frac{1}{4} > \frac{1}{8}$. According to Obersteiner (2013) that a common mistake is to think of fractions as two separate natural numbers, not as a sum. This becomes a problem when comparing various fractions which assumes that the decision is based on a comparison of the components separately rather than viewed from a holistic comparison of the value of the fractions.

The third epistemological obstacle is related to the inability of students to simplify fractions.

Figure 7. Epistemological obstacle 3.

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Based on the results of the interviews, students tended to be unable to simplify fractions. Another thing to note was prerequisite material that they had not mastered such as multiplication and division of numbers, this caused students difficulties in simplifying fractions. These difficulties are often found, one of them is also described by Hughes (2019) that significantly students are unable to simplify fractions, so that should be one of the targets that must be achieved in learning. Simplifying fractions is closely related to equivalent fractions, and a solid understanding of equivalent fractions is considered the basis for a more complex understanding of the operations of various fractions (Jigyel & Afamasaga-Fuata', 2007).

The fourth epistemological obstacle is related to the inability of students to operate fractions, especially addition and subtraction with the same denominator.

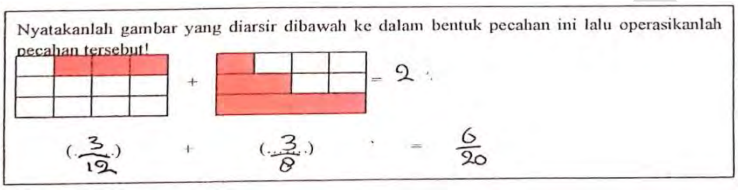

Figure 8. Epistemological obstacle 4.

The first error is seen when the student is unable to express a fraction from an image, thus impacting the addition operation, the second error is when the student is able to state a fraction but is unable to operate it.

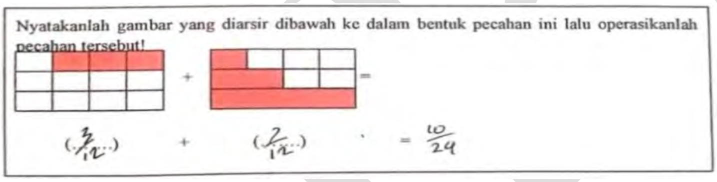

Figure 9. Student mistakes in operating fractions.

Based on the results of the interviews, students completed the addition operation by adding the numerator with the numerator and the denominator with the denominator. This difficulty often occurs because students cannot use modeling correctly in operating fractions (Aksoy & Yazlik, 2017), students only understand that in operating fractions, both (the numerator and denominator) are added up, even though this is a wrong concept.

Didactical Obstacle

The didactical obstacle seen in this study is the teacher's use of methods and strategies in teaching the concept of fractions. Based on the results of interviews with teachers that there are several notes that cause students to experience learning obstacles, one of which is the way the teacher teaches is seen as a conventional way, meaning that the involvement of students in building their knowledge independently is not seen in the learning process, the teacher only presents examples of questions and questions to students based on the handbook. Besides that, in teaching fractions no concrete objects are used, even though according to Piaget that the level of development of elementary school students' thinking is still at concrete operational level. Bruner also explained that learning at the elementary school level must be built from the enactive, iconic, to symbolic stages, so that learning becomes meaningful. Another reason is that the teacher also does not understand the sequence of fractional material based on the actual scientific conception, even though this sequence is the key to students' success in understanding fractional material correctly, this is what causes students' learning obstacles to the concept of fractions.

Hypothetical Learning Trajectory

Based on the analysis of the theory developed by experts about fractions, there are several things that teachers need to pay attention to in relation to learning fractions, this is explained by Nicolaou and Pitta-Pantazzi (2016) in the image below.

The theoretical model formulated for understanding fractions consists of seven abilities including.

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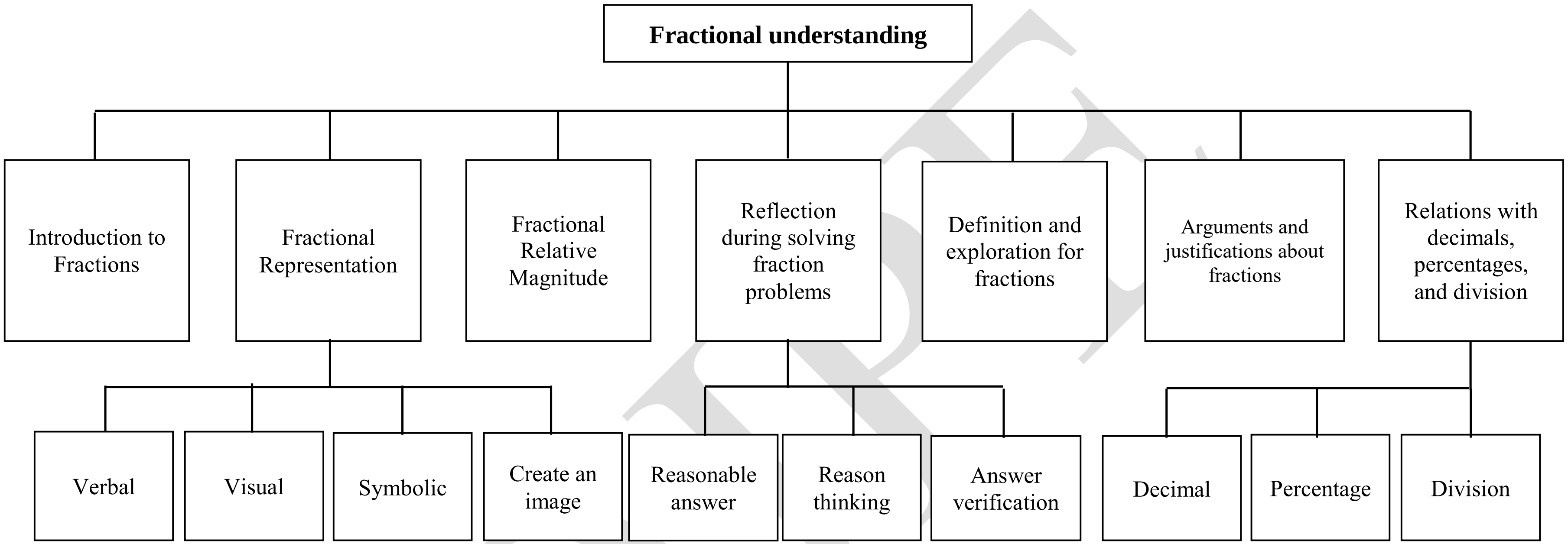

Figure 10. The seven abilities of understanding fractions (Nicalaou & Pitta-Pantazzi, 2016).

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The flow of material above becomes the basis for teachers in teaching fraction material in elementary schools. In addition, according to Nicalaou and Pitta-Pantazzi (2016) the ability to understand mathematical concepts is also very important in understanding fractions, so understanding mathematical concepts in fraction material is as follows: 1) reflection on solving fraction problems, 2) arguments and justifications about fractions, 3) definitions and mathematical explanations for fractions, 4) linking ideas and mathematical concepts, and 5) representation of fractions.

The National Research Council (2001) to develop mathematical abilities in understanding fractional material, mathematics textbooks and teaching materials must assist students in developing an understanding of the concept of fractions, while what must be contained in it is as follows, 1) understanding parts, 2) equivalent fractions by using models and number line, 3) operating fractions, 4) understanding the relative sizes of fractions, 5) comparing fractions and solving problems, and 6) relating fractions to students' daily lives.

Prior to discussing more complex material, namely various fractional operations (addition, subtraction, multiplication, and division), students must first master the prerequisite material as a basis for learning the fractional operations material, The prerequisite material is related to students' understanding of the concept of fractions, namely understanding fractions as part of the same whole, understanding the size of fractions through comparisons of two or more fractions, equivalent fractions, addition and subtraction operations for fractions with the same denominator. Learning the concept of fractions is done by integrating culture in learning mathematics through Engklek games.

In Banten Province, Indonesia, children often play this Engklek game, especially in the early 2000s and it is not uncommon for this game to be played in the present era, the types of plots in the Engklek game that are often used are as follows.

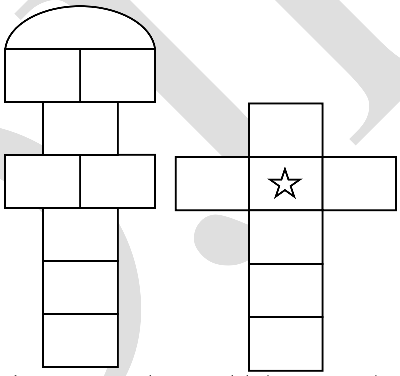


Figure 11. The Engklek game plot.

If you look at the plot above, it is very difficult to introduce the concept of fractions because the pictures are too complex. The fraction plots used in learning mathematics, especially in introducing the concept of fractions, are as follows.

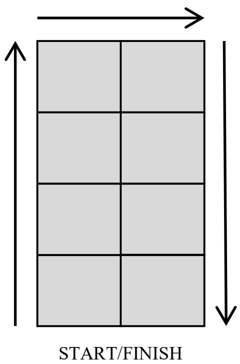


Figure 12. Modification of plots in the Engklek game.

The plot above is much simpler to introduce the concept of fractions, the rules of the game used are the same, including: 1) the children are divided into several groups, 2) each group has 4-5 children to play, 3) each child holds an object (usually a piece of tile or some kind of stone) to be thrown into the plot sequentially, and the child will start jumping using one foot; 4) the thrown object must not stick or go off the line; 4) tiles where objects cannot be stepped on by any player means that the child must jump to the next tile sequentially; 5) when jumping, children also may not step on or go off the line; 6) before reaching the finish, they must take their belongings, 7) a player who has completed one full turn, he or she will throw an object into the square to serve as their home, throwing it facing opposite, 8) the house that belongs to them is marked with an asterisk, the plot may be stepped on using two feet by the owner and other players cannot step on it so they have to jump to the next tile, and 9) the player who has the most number of plots will be the winner.

The lesson begins by introducing the concept of fractions as part of the same whole. According to Van de Wallet et al. (2013) the part to whole interpretation of the concept of fractions is basically about dividing the whole. For example, when the whole is divided into five equal parts, each part is a fifth ($\frac{1}{5}$). Fractions represent the quotient of two quantities $\frac{Part}{Divisor}$, the part (numerator) represents how many parts are taken / paid attention to, and the divisor (denominator) represents the number of parts that are divided (Iulia & Gugoiu, 2006). The introduction of this concept is carried out when students describe parts of the Engklek game plot with size 10 cm × 10 cm this is to show that the total part of the fraction is 8, students then cut out the parts that have been drawn according to size, this is important to show that the fractional parts must be the same size, only then do they count the number of plot parts they have when playing, this is to direct students to introduction to the concept of fractions as part to whole.

After that students calculate and compare the plots they have and also their friends, this is to show the comparison of fractions with the same denominator. What must be considered when comparing fractions is that if the denominators are the same, then the numerator that has the smallest number is clearly smaller than the numerator that has the larger numerator or ($\frac{small}{a} < \frac{large}{a}$), if the denominators are different, you must first equate the denominators, only then to compare them using the rules $\frac{small}{a} < \frac{large}{a}$ (Iulia & Gugoiu, 2006). After that students are given a plot image with 4 parts and the overall size is the same as the plot previously drawn, this is to show that when students get $\frac{4}{8}$ of the plot will be equal to $\frac{2}{4}$ or $\frac{1}{2}$ of the plot, or when students get $\frac{2}{8}$ of the plot will be equal to $\frac{1}{4}$ of the plot, students will also realize that fractions $\frac{1}{2} = \frac{2}{4} = \frac{4}{8} > \frac{1}{4} = \frac{2}{8} > \frac{1}{8}$.

The next step is for students to look back at the pictures that have been made to teach the concept of adding and subtracting fractions with the same denominator. According to National Research Council (2001) when fractions have the same denominator, the addition is the sum of the numerator and numerator, and the denominators are the same number ($\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$) and this is also true in the context of subtracting fractions ($\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$). The hypothetical learning trajectory that has been developed is described in the image below.

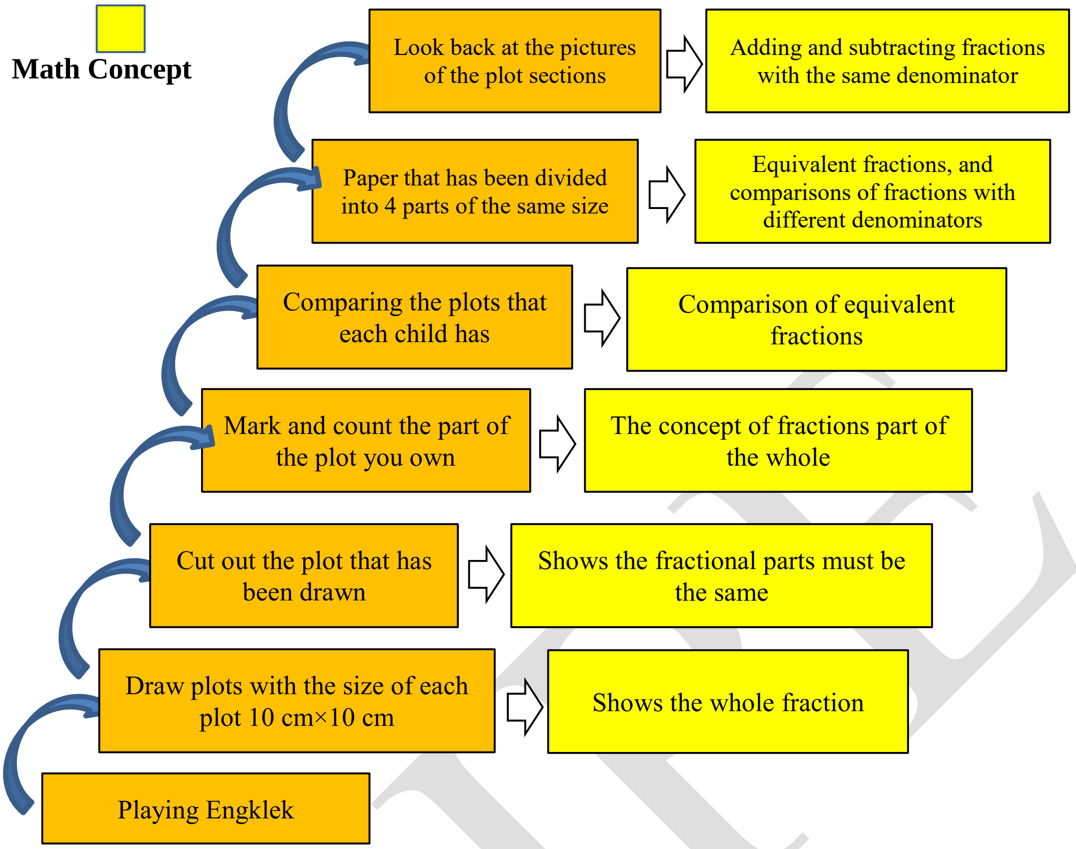


Figure 13. Hypothetical learning trajectory of the concept of fractions.

Implementation of Didactical Design

Before implementing the design, the researcher conducted a pretest to see the students' initial abilities before getting the didactic design, and most of the students got poor results, this was because students' initial understanding of the concept of fractions had not yet been formed. The results can be seen in the image below.

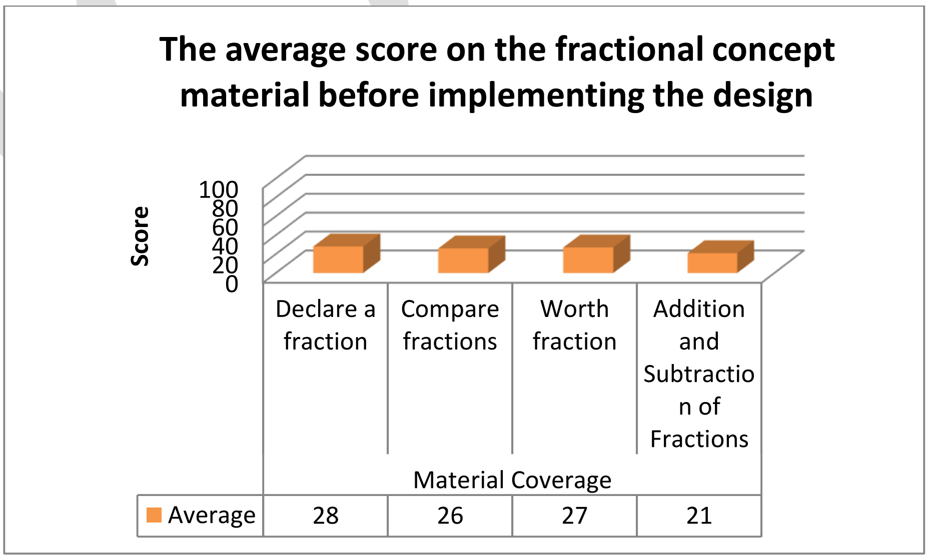


Figure 14. The average score on the fractional concept material before implementing the design.

The didactic design of the concept of fractions that has been developed is implemented in grade 4 of the elementary school, the analysis of the activities is based on the didactic situation that has been planned in the didactic design. Brousseau (2002) dividing the didactic situation into 4 of which are 1) the action stage where students are given the context of the problem for them to solve, 2) the formulation stage where students can find various ways or formulations in solving problems, 3) the validation stage, with the help of the teacher (scaffolding), students can find the correct concept; and 4) the institutionalization stage, namely the process that allows students to change their previous knowledge into new knowledge through other reinforcements provided by the teacher. The results of the implementation analysis are as follows.

The action stage presents a contextual situation that allows students to find the right formulation in finding the concept they are looking for. The contextual activity that is carried out is by playing Engklek games.

Figure 15. Students play Engklek.

The actions carried out above are included in physical action, this action encourages students at the way of thinking stage (WoT) and ways of understanding (WoU) as described by Harel (2008) related to the philosophical-pedagogical theory of mathematics, with this basis they can find the right formulation and the correct concept in understanding the concept of fractions in accordance with the scientific conception. This activity also involves social interaction with the environment, and the interaction process goes well, this game also involves a didactical contract. According to Brousseau (2002) this didactic contract regulates the social responsibility that underlies the devolution process, namely the transfer of problem solving from the teacher to students, in this condition the teacher acts as a supervisor who sees how the didactical contract goes as it should, if the didactic contract does not go well then the teacher can take over to correct and justify it. The various responses formulated at the action stage in the didactic design can be well anticipated, this is because students already understand the rules about playing Engklek, and this facilitates the learning process.

After playing Engklek, students in groups are given cardboard to be directed to the process of drawing part of the plot, the teacher asks students to draw part of the plot from the Engklek game, and each plot size 10 cm × 10 cm, the response that arose when drawing the plot was to divide it into 10 plots, this was because the students made a mistake in making the plot lines.

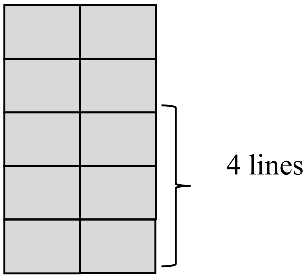


Figure 16. Students' mistakes in making Engklek plots.

There should be 8 plots made, so this requires pedagogical didactic anticipation to correct the concept. From here students are directed to cut out the parts that have been drawn and then mark the parts they get when playing Engklek. Various groups wrote down different parts, by writing down the parts they had compared to the whole part of the Engklek plot, this was where they found an understanding of the concept of fractions being part to whole.

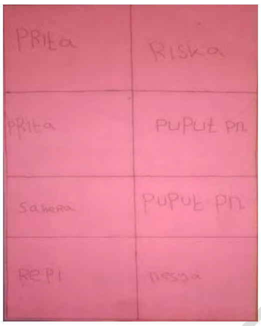


Figure 17. Examples of student drawings and plot sections.

Students begin to understand that the concept they are learning is the concept of fractions, from here students are encouraged to state a fraction. From the picture above students also begin to compare fractions with the same denominator from the largest to the smallest value. Overall students can compare these fractions correctly because they only see and sort according to the numerator value from the largest to the smallest ($\frac{large}{a} > \frac{small}{a}$). After that, the teacher gives the context of another problem by showing a picture of a fraction whose overall parts are the same as the previous picture but the parts are fewer, namely 4 parts.

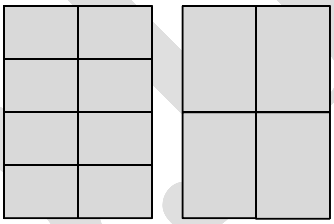


Figure 18. Presenting the context of another problem.

The context of the problem above is given to stimulate students towards equivalent fractions, students realize that the picture given is bigger than the part that was drawn before, by making guide lines so that the parts are the same, students realize that $\frac{1}{4}$ parts if the whole part is changed to 8 parts the fraction will be equivalent to $\frac{2}{8}$, so that the final stage is that students understand that simplifying fractions is done by dividing the numerator and denominator by the same number, until the fraction cannot be divided again. From the picture above, students also understand that the fraction $\frac{1}{4}$ is bigger than $\frac{1}{8}$, this is because $\frac{1}{4}$ has the same value as $\frac{2}{8}$, meaning that in comparing fractions with different denominators what must be done is to first equate the denominator and then compare the fractions with $\frac{large}{a} > \frac{small}{a}$. The process of the activity above also involves mental acts that encourage students to think (WoT) until they find and understand the real concept (WoU).

The last stage, students are asked to look back at their work related to the plots that have been drawn before and ask students to add up their friends' plots with theirs, one example in Figure 17 above.

"If Puput's plots are added up with Rishi, what is the total!"

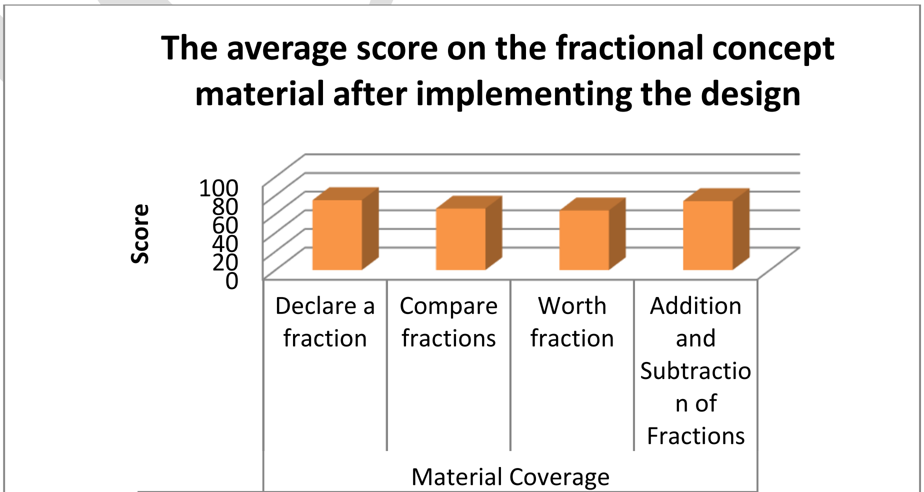
By looking at the part of the picture that has been made, students already realize that the whole part is adding up the numerator and numerator parts, and the denominator numbers are the same ($\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$) or in subtraction ($\frac{a}{b}$ - $\frac{c}{b} = \frac{a-c}{b}$).

In the validation stage, students are directed to find the actual concept. The activities carried out were when they discussed each activity.

Figure 19. Students discussing and working in groups.

Discussion activities in these groups will lead to learning interactions in class, with discussions and collaborating they begin to discover the concepts being studied, including understanding fractions of parts of the same whole, comparing fractions, equivalent fractions, and addition and subtraction operations for fractions with the same denominator. At the discussion stage, students sometimes have difficulty in solving problems, the teacher's job is to provide help so that students understand the problem, and when students begin to be able to solve problems, slowly the teacher releases the help and gives full authority to students to solve it. The help that teachers provide in Vygotsky's theory is called scaffolding.

After students get learning the concept of fractions, the teacher provides the context of other problems to strengthen the new concepts they have understood before, while the context of the problem is by giving questions related to the learning activities that have been carried out, at this stage the teacher notes several important points as material for learning reflection and improvement of the didactic design of the concept of fractions. At the end of the lesson, the teacher also gave a posttest to see students' understanding of the concept of fractions in elementary schools, while the results are as follows.



	Declare a fraction	Compare fractions	Worth fraction	Addition and Subtraction of Fractions
Average	75	66	64	74

Figure 20. The average score on the fractional concept material after implementing the design.

Based on Figure 20, students have begun to understand that fractions are part to whole and the operations of adding and subtracting fractions with the same denominator, but in the section comparing fractions with different denominators and equivalent fractions or simplifying fractions, some errors are still found, and this is also become an important note in the next design improvement.

In addition, based on observations, the didactic design that has been implemented is in accordance with the metapedadidactic theory, in which there are 1) logical integration, namely the presentation of didactic situations that are gradual and in accordance with learning objectives, 2) complete unity, namely the material presented is didactic-pedagogical, the relationship between 3 the elements (student-teacher-material) are well established, and the diversity of responses can be responded to properly, and 3) flexibility, namely the management of learning is flexible and the learning activities are in accordance with the learning flow experienced by students. In addition, if you look at the learning trajectory, the context of the material has been presented systematically and gradually, but the prerequisite material has not been mastered in its entirety, this is the main obstacle in applying the didactic design of the fractional concept.

Retrospective Analysis

At the institutionalization stage, there are several important notes that serve as material for reflection and improvement of didactic design on the concept of fractions, namely in stating fractions with the same size of parts all students understand and can state them correctly, but when given the context of fractional parts with different sizes, the error is still found. In addition, the errors that are often found are when students are asked to simplify fractions and compare fractions with different denominators. This can also be seen from the results of the learning evaluation in Figure 20. The reason is that there are some students whose prerequisite material (multiplication and division of numbers) they have not mastered, or tend not to memorize, even though multiplication and division of numbers are very closely related to comparisons of fractions with different denominators and equivalent fractions. Another important prerequisite material is the greatest common factor and the least common multiple, because this applies to simplifying fractions and equating the denominator of fractions when doing comparisons of fractions with different denominators, so that in the didactic design the revision needs to be linked and discussed the material for the largest common factor and the least common multiple. Another thing to note is that the didactic design that has been implemented needs to be strengthened by revisiting the material that has been taught, so that the hope is that students' understanding will be intact and thorough.

The results of the implementation of the didactical design showed an increase in students' scores on the concept of fractions. This can also be seen from the table below.

Table 2. Statistics on the impact of didactic design implementation.

	Mean	Min	Max	Sig. Normality	Std.Dev.	N-Gain	Sig
Pretest	25.9	5	65	.005	9.87	.59	.000*
Posttest	69.9	33	100	.324	12.3		

*p<.05

From the Table 1 above, the average pretest result is 25.9 with a minimum value of 5 and a maximum of 65, and the posttest average result is 69.9, with a minimum value of 33 and a maximum of 100. The normality test results show that sig. the pretest has a score of .005 meaning that the pretest scores are not normally distributed, and sig. posttest has a score of .324, meaning that the posttest scores are normally distributed. Because one of the data was not normally distributed, the mean difference test was carried out by conducting a non-parametric test using the Mann-Whitney, the results showed that sig. of .000 < .05. This means that there is a significant difference in the mean between students' scores on fractional material before and after implementing the design. This result is also evidenced by the N-Gain score of .59 which indicates an increase in the score in the medium category.

DISCUSSION, CONCLUSION, and SUGGESTIONS

The results showed that there was an increase in students' scores on fractional material caused by didactic designs made according to students' real conditions. Basically, the education and learning system cannot be separated and are an integral part of real life (Annisa, 2019), then, it is very important that the link between real life and learning has a positive influence to attract students' attention in the learning process (Akınoğlu & Tandoğan, 2007). The initial perspective that views mathematics as a difficult subject because it is abstract and formal turns into what they see in everyday life, this indicates that there are mathematical principles as a human activity (Freudenthal, 1971), this means that mathematics cannot be separated from human life (Umbara et al., 2021a). Integration with real life allows students to learn from events encountered in everyday life, this will have an effect on increased educational functioning and individual-centered learning environment (Kaya & Keşan, 2014). The integration of mathematics with real life can improve students' ability to connect mathematics with real life, making it easier for students to understand abstract mathematical concepts (Üredi & Doğanay, 2023). Then, students will feel happy in carrying out learning activities because they relate to their lives. In this context, a pleasant environment will generate motivation and have an impact on student achievement (Gian et al., 2021), besides that, a fun school will have a positive impact, students feel acknowledged, satisfied, safe, and can be themselves (Calp, 2020).

Students feel happy in participating in learning activities, this is because the learning context is integrated with playing. According to Juhász (2021) a game will have a cheerful impact and it will provide a positive element in learning activities. Games provide agreement for children to try new things, explore, and solve problems (Burdette & Whitaker, 2005). In addition, learning through games is a highly recommended activity so as to create a fun classroom environment (Clark, 2019; Sezgin, 2016). Thus, it should be in the context of education, especially in elementary schools, that learning must be integrated with the context of play because it is their nature as children who like games.

The Engklek game is a traditional cultural game that is often played by students in everyday life. This is in accordance with the principles of ethnomathematics, namely integrating learning mathematics into culture. Learning mathematics with a cultural context is not only interesting, but it will be able to involve students in their learning activities, so that students find it easier to understand mathematical concepts and are more likely to bring their experience to the learning process (Snounu, 2019). According to Sunzuma et al. (2021) ethno-matematics is very important in teaching and learning mathematics because it is believed to be able to improve students' understanding and learning achievement in mathematics. Ethnomathematics allows students to communicate the mathematical ideas they find constructed through social phenomena (Umbara et al., 2021b). In addition, students can use mathematical ideas, concepts, and practices in an effort to solve problems in everyday life (Payadnya et al., 2021).

If we see that the Engklek game in learning fractions has a series of mathematization processes, in which horizontal and vertical mathematization processes occur. According to Gravemeijer (1994) the mathematization process that occurs through 2 stages, namely horizontal and vertical mathematization. Horizontal mathematization occurs when students explain contextual problems using their informal strategies to solve them. If the informal strategy leads students to solve problems using mathematical language or to find algorithms, then this movement process shows vertical mathematization. The process of mathematizing in the context of playing Engklek and its relation to the concept of fractions is described below.

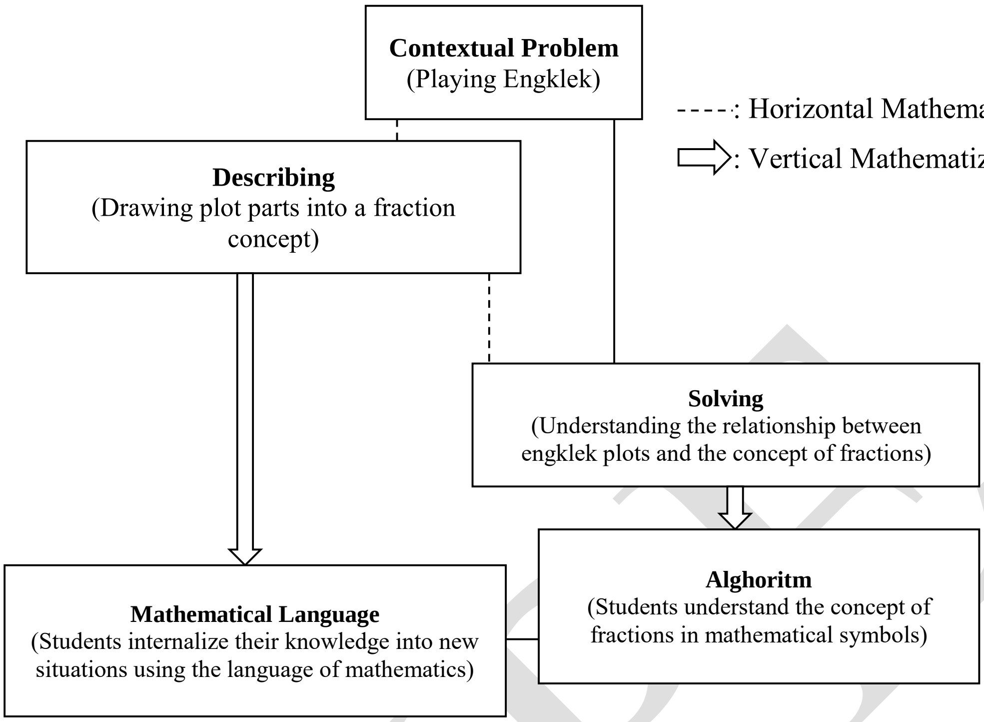


Figure 21. The process of mathematizing in Engklek games and its relation to the concept of fractions.

Based on Figure 21, it can be seen how the transition of the mathematical process is built from the informal stage through the context of play to the formal stage in the language of mathematics. The mathematical process that occurs is in accordance with the stages of thinking development of elementary school children who are at the concrete operational stage (Franzoi, 2011), this makes it easier for students to construct their understanding and experience into new situations to understand the concept of fractions. Students who are classified as having concrete operational development must use proper and direct logic on concrete things, if students are presented on things that are out of a concrete context, students will experience difficulties (Moore, 2012). In the context of learning, especially in elementary schools, there needs to be a learning transition that is built from concrete, semi-concrete to the symbolic/abstract stages.

This principle aligns with Bruner's (1996) assertion that our world can be comprehended and expressed through three stages: enactive (action), iconic (perceptual organization), and symbolic (words and symbols). These stages hold significant relevance, particularly in the context of education and learning, especially in the domain of mathematics education in elementary schools, where cognitive processes are often in the concrete operational stage.

Conclusion

The results of this study indicate that in understanding the concept of fractions, students experience learning obstacles related to ontogenic, epistemological, and didactical obstacles. From this learning obstacle, the researcher created a hypothetical learning trajectory as the basis for making didactic designs on the concept of fractions. The didactic design that was made was related to the cultural context, namely teaching the concept of fractions with Engklek game. After the implementation of the didactic design, there was an increase in the average score of students on the concept of fractions. Before the implementation of the didactic design the average score was 25.9, and after the implementation of the didactic design increased to 69.9, this is also evidenced by the N-Gain score of .59, this indicates that the increase is in the medium category. From the results of the mean difference test using the Mann-Whitney test, it was found that the sig. equal to .000, which means that there is an increase in the average score of students before and after the implementation of the didactic design using the Engklek game. This shows that the didactical design made has a significant impact on students' understanding.

Limitations

The limitation of this study lies in the research subjects who only took 50 students on mathematics learning. In addition, the scope of the material is only on the fractional aspect in grade 4 elementary school. In addition, the cultural context raised in this study is related to Sundanese culture in Banten Province so that this is closely related to the real life of children.

Suggestions

In implementing learning, especially on the concept of fractions, the teacher needs to ensure that the prerequisite material for the concept of fractions must be mastered correctly, such as memorizing multiplication and division, and mastering material related to the largest common factor and the smallest common multiple. This research can provide an overview of the importance of integrating the culture and environment of children's lives into learning, especially in elementary school mathematics. This research can be used as a consideration or reference for conducting similar research on the mathematical context of fractional material by integrating culture. The didactic design in this study is an alternative in mathematics education research, especially in developing the didactic design of the concept of fractions by integrating the cultural context (Engklek game) in elementary schools. However, it does not rule out the possibility that the didactic designs that have been made in this study can be refined or redeveloped for the better.

Acknowledgments

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Ethics and Conflict of Interest

This research has obtained permission from the subject under study and the principal of an elementary school in Banten Province. The study has followed the direction of Research Ethics, and the authors state that they do not have any potential conflict of interest.

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