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South African teachers work with division actions in Grade 3

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Background: Internationally, the teaching of division has noted that the use of sharing situations with sharing actions (one-by-one distribution) is the predominant division model at the beginning of schooling. In South Africa, research suggests a sharing situation with sharing actions is also preferred in the early grades.

Aim: This paper aims to look at the predominant approaches to the use of division actions that teachers offer in teaching division tasks.

Setting: The study is set in three government schools in Gauteng, South Africa.

Methods: In this qualitative study, the teachers were observed through video recording, and then the video recording was transcribed, and semiotics was used to make sense of their teaching.

Results: The findings of this article suggest that grouping actions and group-based approaches to teaching division tasks are more prevalent than sharing through one-by-one distribution actions, even when sharing situations are used.

Conclusion: This study concludes that grouping actions and group-based approaches are part of how teachers solve sharing situations.

Contribution: This study concludes that in a South African context, identifying the grouping actions and group-based approaches linked to sharing situations is a more efficient way of solving sharing situations and will assist teachers in explaining division tasks more coherently.

Keywords: grouping; sharing; foundation phase; Grade 3; group-based approaches; division model.

Introduction

Literature shows the importance of both division models (sharing and grouping), yet teachers and learners are prone to using a particular division model. Booker et al. (1992) note that learners come to school with informal knowledge about division as sharing. Traditionally, Kinda (2013) says that researchers concluded that learners are more comfortable with partitive models of division as learners are familiar with sharing ideas. Squire and Bryant (2002b) mention that teachers' introduction of division problems usually involves sharing problems. Research evidence shows that teachers are prone to using more sharing actions instead of grouping actions (Carpenter et al. 1993; Fischbein et al. 1985). In South Africa, Roberts (2012) posits in a theoretical paper that when teachers introduce the idea of division, the idea of division is normally linked to a sharing model. The sharing model of division appears to be a predominant entry point to the teaching and learning of division with a sharing action seen in a one-by-one distribution.

In his report, Schollar (2008) does not explicitly mention the importance of sharing or grouping models. However, he notes that counting by ones as an approach has been described as common in solving division tasks in Grade 7 classes in South Africa. Considering the overuse of sharing actions to solve division tasks, looking at the spread of more rudimentary and more efficient actions is also of interest to how the teaching of division should be approached. This article discusses an analysis of division teaching, focusing on division actions drawn from three Grade 3 teachers across three government primary schools within Quintile 4 and 5, with two of the schools identified as underperforming by the district. Through this analysis, the predominant division actions came into focus within these three classrooms in a South African context. I sense that flagging the discussion on the use of division actions that extends beyond our traditional view of division actions develops insights into what needs to happen in pre- and post-teacher development. In other words, the teacher's ability to identify the role of all actions related to solving division tasks in their mathematical instruction helps create clarity for learners when solving division tasks. So, counting in ones and the over-reliance on a sharing model of division produce a limited approach to how division as a concept should be understood. A research question guiding this article is: *What division actions are becoming evident in teaching division at the Grade 3 level in three South African classrooms in Gauteng?*

In this article, the background was laid out to focus on identifying different division actions. Then discussed the theoretical and analytical framing developed for looking at the data. Before presenting the findings and analysis the data sources and the methodology was outlined. The article was concluded with comments on how the solutions to division tasks highlight the predominant use of particular division actions across the categorised episodes. The implication of this article focused on helping teachers identify division actions in the solving of division tasks in their instructional discourse.

Literature

Across the literature, two models are critical in developing the mathematical concept of division. The literature on division distinguishes between quotitive (grouping) and partitive (sharing) models. In quotitive division, the divisor refers to the number in each part or the size of each part, while partitive division refers to the number of parts. Vergnaud (1994) notes that a critical distinction between the two ideas relates to whether we need to consider the relations within one or between two measurement variables. Whereas quotitive division is associated with one measurement variable, a partitive division is associated with two measurement variables. In the quotitive problem: 'How many learners can get 4 apples if there are 20 apples?' – the dividend and the divisor concern apples. Therefore, using 4 apples as a unit of measurement, the 20 apples can be 'measured'.

In contrast to quotitive division, in partitive problems such as: 'If five learners have 20 apples to share equally, how many apples does each child get?' – the dividend concerns apples, but the divisor concerns learners. In other words, there is a relation between two variables, apples and learners. Thus, each learner receives a set of four apples. Using five learners as a unit to 'measure' 20 apples is impossible. In partitive division situations, an object or collection of things is divided into several equal-sized groups by sharing. In quotitive division situations, the focus is on how often the divisor quantity appears in the dividend quantity. Teachers' ability to identify these division models is critical in helping learners solve division tasks.

In the primary grades, division tasks can be presented as sharing situations, grouping situations and bald calculations. In everyday language, partitive and quotitive situations are often expressed in verbal form. Sharing (partitive) situations involve sharing objects one at a time until they are divided equally among a known number of people or groups. The person counting the items in the group helps to establish the answer. Grouping (quotitive) situations involve grouping objects until they are distributed and then counting the number of groups to find the answer. Division bald calculations are calculations that consist only of numbers without the use of any written words. The first number in a division calculation is referred to as the dividend, the second number as the divisor and the answer as the quotient. Division bald calculations can be challenging for learners because one calculation can apply to different situations. For example, the calculation $6 \div 2 = 3$ can be interpreted as either a sharing or a grouping situation, depending on how the divisor is interpreted. Both interpretations are acceptable. And this means that if sharing and grouping situations are linked to a number sentence, they can then also be worked with using either sharing or grouping actions, but the teacher needs to be explicit if they shift from one situation type to the other action type, noting that the action selected links to the bald division calculation rather than to the original situation.

Different division strategies involve varying division actions, with 'sharing action' being the fundamental action for partitive interpretation. 'Share by ones' is the simplest form of sharing action involving distributing one item at a time among people or groups. Another approach to 'sharing by ones' is counting out the total number of objects and distributing objects one by one or mentally keeping track while distributing (Carpenter et al. 1999; Kouba 1989).

Some learners progress beyond one-by-one counting. They use a sharing technique that involves placing multiple objects in a set and adding or removing them as needed. This method of sharing action is referred to as distribution by 'partial composite' units. Another way of solving a sharing situation is through the use of grouping to find the answer. A study found that children can use either sharing or removing equal-sized groups to solve the sharing problems (Brown 1992). This shows that using groups is another method for solving sharing situations.

Another distribution, when sharing, is when the quotient value is distributed to each set. Zweng (1964:553) has noted different ways where learners solve partitive problems and recalls a particular partitive problem solved by learners in which learners needed to assign 18 objects to three groups, the child assigned six elements to each of the groups on the first try using a group-based approach of sharing out six items at a time. Clearly, a progression in division action is seen when solving sharing situations, from one-by-one distribution to sharing making use of the quotient value.

Grouping situations (quotitive division) can also be associated with increasingly efficient calculating actions. In initial direct modelling, learners construct a set containing the number of objects specified by the divisor with a one-by-one construction (grouping through one-by-one distribution) and then replicate this set until the dividend quantity is reached. They then count how many sets or groups there are. In this instance, grouping through a one-by-one distribution is based on the divisor. Squire and Bryant (2003) note that grouping through one-by-one distribution can also occur based on the quotient value, which they refer to as 'Grouping-by-quotient'. There are at least two ways of finding the answer, based on whether the dividend number is counted out at the start or built up as a cumulative total. In relation to 28 counters needing to be packed into bags containing seven counters each, in the first case, a learner can count out 28 counters (making of groups based on the divisor – the dividend value is first drawn, and then groups are made) and then separate (through some form of one-by-one counting) them into groups of seven counters (Carpenter et al. 1999).

In the next case, the learner does not count out 28 objects but instead forms groups of seven until she or he has 28 altogether. Kouba (1989:152) refers to this second strategy of grouping as 'double count'. 'Double count' in the case of grouping problems refers to keeping a running count of the cumulative number of objects in the groups while also counting out the objects to form groups (Kouba 1989). When the cumulative value reaches the desired dividend, the learner stops and reports the number of groups as the answer. Kouba (1989) notes that learners doing grouping division problems employ a repeated building-up strategy; for example, with 15 marbles placed into bags containing 3 marbles each, the double counting would involve words like: '3 marbles, 1 bag; 6 marbles, 2 bags' and so on as evidence of double counting.

Kouba (1989) describes a grouping strategy that is more efficient than 'double count' which she refers to as 'transitional counting'. 'Transitional counting' occurs when learners calculate the answer to the problem by using counting sequences based on multiples of a factor in the problem. Kouba (1989) terms counting by multiples, or skip counting, 'transitional counting' because although it relies on knowledge of a counting sequence, it is related to multiplication more fundamentally than basic grouping strategies. 'Transitional counting' is used as a repeated building-up strategy to solve grouping problems. While in a grouping situation, learners skip-count using the number of objects in each group, not by the number of groups; they keep track of the number of groups on their fingers or in some other way. Repeated 'subtractive' strategies also involve skip counting (Kouba 1989). Repeated subtraction occurs with the dividend until the number in each group is repeatedly taken away, and the number of groups is counted as the answer.

Kouba (1989:153) notes that using recalled facts is an even more efficient strategy related to groups. While solving division problems, some learners obtain the answer by remembering the appropriate multiplication and division combinations. This category includes derived facts that learners could not readily recall but could calculate from a related *recalled fact*. At the most efficient level of the problem-solving strategy, learners use combinations of recalled and derived multiplication and division facts to calculate answers to problems mentally. Like with sharing situations, the actions linked to grouping situations show a progression.

In the study, the following division actions were evident across most of the division tasks, as indicated in Table 1.

Table 1 shows the two basic actions linked to sharing and grouping situations. While the basic sharing actions have a one-by-one distribution (Wright et al. 2010), the basic grouping action has two subcategories. In the first sub-category, groups are formed until the total number of items is used (Kouba 1989). In the second sub-category, all the items are first drawn, and then groups are made until all the items are used (Fosnot & Dolk 2001). Group-based approaches refer to grouping actions that are linked only to sharing situations. In this case, there are three subcategories. In the first subcategory, the answer to sharing situation is already known, and then the answer is used to share items between groups (Zweng 1964). In the second sub-category, the divisor is the basis for the group, and the group of items is shared between the people or the groups (Kouba 1989). In the last sub-category – the answer

TABLE 1: Classification of division actions.

Situations	Variable
Sharing situations	**Sharing through 1 by 1 distribution** - occurs when the number of groups is known, and the items are distributed one at a time until the total number of items is distributed between the groups. The answer to a sharing problem lies in counting the number of items in the group (Wright, Martland, & Stafford, 2010)
Grouping situations	**Grouping through 1 by 1 distribution** - A grouping action depends on knowing the number in a group, so the group is replicated until the total number of items is grouped. The answer to a grouping problem occurs by counting the number of groups (Kouba, 1989).
	Making of groups based on the divisor - in these episodes the dividend value is first drawn, and the groups are made (Fosnot & Dolk, 2001).
Group-Based Approaches Linked to Sharing Situations	**Sharing by CCU** – Within a sharing situation, sharing by complete composite units (CCU) refers to distributing items by the **quotient** value immediately (Zweng, 1964). In other words, if the teacher knows that the answer to 15÷5=3, the teacher will share five groups with three items in each.
	Sharing through Group-Based Distribution' (GBD) - Sharing through Group-Based Distribution (GBD) occurs in a sharing situation in which the divisor is used to form groups up to the dividend value (Kouba, 1989). In other words, a sharing situation is articulated. For example, share 15 sweets between five boys. How many sweets will each boy get? The teacher then wrote 15÷5= on the board, at the moment when the teacher adds in the division number sentence, she then can enact a grouping action. She makes groups of five resulting in three groups which she counts and writes in the answer 3. The grouping action within this sharing situation is referred to as Sharing through GBD
	Sharing through 1 by 1 grouping' – the answer is provided to the division calculation; circles are then drawn based on the answer. The number of dots drawn into each circle is based on the divisor by way of a grouping through 1 by 1 distribution - ([Author/s in press], 2019). For example, share 12 sweets between 4 girls. How many sweets will each girl get? The teacher wrote down the following number sentence with the answer 12÷4=3. She then drew **three circles and placed four dots into each circle**. If this were a grouping interpretation of the division number sentence, she would have drawn four sweets at a time until she reached 12, **forming three circles with four sweets**. In other words, she used a grouping through 1 by 1 distribution to illustrate an answer she already knows within a sharing situation.

to the division calculation is known, circles are drawn based on the answer then groups of items are distributed between each circle based on the divisor (Mathews 2019). While this form of action was visible in the analysis of the episodes, this approach to solving a division task is dependent on a teacher highlighting the shift between division models while solving a division task. These categories of division actions form the basis of the classification of episodes across the three teachers' teaching of division.

Theory and analytical framing

For the last three decades, semiotic approaches have been evident in research focused on mathematics learning and teaching (Presmeg et al. 2017). Semiotics has its origins in Peirce's use of the term 'semiosis' to refer to any sign action or sign process: in general, the activity of a sign. A sign is 'something that stands for something else' (Peirce 1992). Thus, semiotics is the study or doctrine of signs (Presmeg et al. 2017:1). The idea of 'standing for' expresses a process in which some newly introduced entity becomes part of a new sign containing traces of the prior sign (Sáenz-Ludlow 2003). Walkerdine (1988) notes that the chaining of signifiers with linked signifieds that carry and expand mathematical concepts becomes visible in mathematical discourse.

According to Chandler (1994:20), 'contemporary commentators tend to describe the signifier as the form that the sign takes, and the signified as the concept to which it refers'. He further mentions that 'the signifier is commonly interpreted as the material (or physical) form of the sign; it is something which can be seen, heard, touched, smelled or tasted' (Chandler, 1994:21). Chandler (1994:21) further states, 'the relationship between signifier and the signified is referred to as signification'. Researchers have used these elements (signifier or signified) in mathematics education to describe a mathematical sign. A mathematical sign is produced when a mathematical relation links a signifier with a signified. These signifiers link to particular signifieds, which represent the underpinning mathematical concepts. Signifiers found in mathematical discourse are words, symbols, concrete objects and drawings. It is important to note that I employed semiotics as an analytical tool for plotting the presence of signifiers in teachers' explanations. For example, the division task was a sharing word problem. The word problem could contain a number of signifiers. The initial signifier would be the words contained in the word problem; as the explanation develops as the teacher solves the division task, more signifiers such as a drawing and a number sentence become visible. All these signifiers would be linked to a signified, in this case, a sharing situation.

In line with using a teaching episode as the unit of analysis, the summary of each episode includes the following details: the task, the initial signifier and a paraphrased description of how instruction played out in the episode that incorporates the signifiers worked with and accompanying explanations. I then analyse whether a grouping or sharing situation, bald calculation or some other aspect (e.g. the definition of terms) was in focus, any division action that followed, the calculation strategy used and the number range involved. The paraphrased summary with the noting of the initial signifier and associated actions and explanations in terms of subsequent signifiers has enabled me to interpret the analysis of each episode while also noting any other salient points. Further, I illustrate this way of working with an example drawn from my data set for looking at what division actions were evident.

In this episode, the flow of signifiers occurs smoothly, with the initial orally presented sharing situation producing an immediate correct answer from a learner, with the teacher then proceeding to offer counters that allow for a concrete replay, which a learner executes – again smoothly – using the 'complete composite' unit sharing. Thus, this episode division action was described as a sharing situation solved through the use of a group-based approach referred to as 'sharing through CCU'.

While each episode was analysed as indicated in Table 2, I proceeded to analyse each teacher's episodes in the following way. I had three broad categories that indicated the type of action – sharing actions, grouping actions and group-based approaches linked to sharing situations (see Table 3). Each teacher's episodes from their sharing situations, grouping situations and bald calculations were placed in one of the three categories.

Research methods and design

Historically, quantitative and qualitative paradigms have framed methodological approaches to research that were seen as largely incompatible and not used within the same research project. Quantitative researchers typically focus on what factors or variables might cause certain results, and

TABLE 2: Division action (Group-based approach) – School A, Term 2, Tr. Joy L1: E1.

Approach	Variable
Description	Task: Six sweets to be shared between three children
	Summary: Explaining that her mother was a vendor with six sweets to share between three children, the teacher gestured three people, drew "6" in the air and showed six fingers before asking: 'How many sweets does each child get?'
	A learner responded: 'Two'.
	Taking out six counters, the teacher asked a learner to: "share them equally between three learners". One learner came up and distributed two sweets to each of three learners.
Analysis	Situation: Sharing
	Action: Sharing by two counters per learner
	Initial signifier: Oral word problem in the task
	Division calculation strategy: Immediate answer suggested a known fact for the learner, with concrete replay involving sharing by complete composite units (CCU)
	Division number range: Single digit dividend and divisor
Interpretation	The number range is much lower than expected, possibly playing into the correct answer being stated prior to any division action or calculation. The division action that was evident was Sharing by Complete Composite Units (CCU) referred to as the immediate distribution of items by the quotient value (Zweng, 1964). The use of a group-based approach in the solving of a sharing situation.

they carry out tests to either support or reject their hypotheses (Anderson & Arsenault 2005); Leedy and Ormrod (1993:143) note that 'the whole process is cold, calculating, deductive logic – from the positing of a hypothesis to the supporting or not supporting it'. By contrast, qualitative researchers have traditionally emphasised process and meaning, focusing mainly on insight, discovery and interpretation rather than hypothesis testing (Noor 2008). However, there has been increasing use of combinations of qualitative and quantitative methods in contemporary research. Dowling and Brown (eds. 2010:89) note that 'studies that combine or mix qualitative and quantitative research techniques fall into a class of research that is appropriately called mixed methods research or mixed research'. This study is located within a qualitative research paradigm, with some quantification included to understand the prevalence of particular phenomena.

Qualitative research enables the investigator to explore, in detail, a small number of instances or examples that require depth of exploration rather than breadth (Merriam 1998). In such investigations, qualitative researchers aim to record the experiences or practices of people in as much detail as possible. These may include how they think, what they say or do and why they say or do these things. Such approaches permit the researcher to examine the participants' particular practices or experiences and issues related to these and to open possibilities for understanding these issues in different or expanded ways from those already available. Ogegbo, Gaigher and Salagaram (2019) have noted that qualitative research approaches are particularly useful for studying classroom instruction in South Africa because these approaches are able to identify teaching and learning gaps which could be used in further planning and improvement of classroom instruction. The qualitative paradigm usually depends on insights gathered from a small number of cases.

According to Lincoln and Guba (1985), a case study is a detailed examination of a single occurrence within a closed system. The focus of a case study is typically a real-life situation with genuine individuals in a familiar setting, as Silverman (2004) described. Yin (1994) asserts that case study research strengthens the credibility of data-collection processes, while Opie (2004:71) emphasises the importance of transparently presenting data that can be easily re-analysed. It is crucial to explicitly declare the ethical considerations of data collection to everyone involved in the case study. Negative experiences or aspects of practices should be reported, and the researchers should acknowledge their biases during data collection and analysis. Opie also stresses the importance of separating data interpretation from its description.

In this study, we examine the presence of division actions in teaching division through multiple cases of the phenomenon of interest. Multiple case studies involve gathering and analysing data from several cases. Merriam (1998:40) suggests that including more cases in a study and having greater variation across them can lead to a more compelling interpretation. Comparing multiple case studies that focus on a particular research interest can result in more precise, valid and stable findings. Noor (2008:1604) explains that consistent findings across multiple cases are considered very robust. Each of the three teachers' instruction related to division constitutes a 'case' within the overall multiple-case study.

The second level of sampling in this study was related to the selection of the teachers whose division instruction was studied within each of the schools. Having selected the school sample as outlined earlier, I focused on teachers who were familiar with teaching Grade 3 Mathematics. Drawing from the more experienced teacher pool in each grade, a Grade 3 teacher from each school was approached to ask if they would be willing to participate in the study, with its focus on the observation of their teaching of division. All teachers approached gave their informed consent for participation. Thus, three Grade 3 teachers with work located within the South African Foundation Phase became the opportunistic teacher sample for this study. From my side,

TABLE 3: Looking at division actions across all analysed episodes.

Teacher	Division actions	Episodes (N = 74)	Sharing actions (through 1 by 1 - distribution)	Grouping actions		Group-based approaches linked to sharing situations		Total number of division actions
				Grouping through 1 by-1 distribution	Making groups based on the divisor	Sharing by complete composite units	Sharing through group based distribution	
Cathy	Sharing situations	17	4	-	-	7	3	-
	Grouping situations	16	1	4	1	-	-	-
	Bald Calculations	16	-	11	1	-	-	-
	Total	49	5	15	2	7	3	32
Mary	Sharing Situation	1	-	-	-	†	-	-
	Grouping situation	2	-	1	1	-	-	-
	Bald Calculations	8	-	1	1	-	-	-
	Total	11	-	2	2	1	-	5
Hazel	Sharing Situation	1	-	-	-	-	-	-
	Bald calculations	13	4	-	2	-	-	-
	Total	14	4	-	2	-	-	6
Grand total		**-**	**9**	**17**	**6**	**8**	**3**	**43**

Note: the number of dots drawn into each circle is based on the divisor by way of a grouping through 1 by-1 distribution - `sharing through 1 by 1 grouping'.
†, the answer is provided to the division calculation; circles are then drawn based on the quotient.

the selection of the three teachers was based on their familiarity with teaching mathematics at their grade level. The teacher selection was also based on their willingness to participate in the study and their allowing me to observe them teaching division at different times of the year.

Observations are a technique used to observe people in a particular context. Such observation usually entails making detailed notes of behaviour, words and actions. To assist me with observation I decided to video record each lesson. Video recording can capture the richness and complexity of classrooms for later analysis (Andrews 2009). Borko et al. (2008) note that the use of video for educational purposes brings new and imaginative perspectives to almost any subject matter. Santagata & Guarino (2011) note that because a video can be played over and over and accessed digitally, it allows for a depth of reflection and analysis not possible during live observations.

Limitations of lesson observation have been noted in the literature – for example, dangers of bias in 'subjective' viewing, curiosity about the researcher as a 'newcomer' in the classroom and the possibilities of the researcher's presence influencing the activity in the classroom (Merriam 1998). However, lesson observation data were essential given this study's focus on instruction. Through communicating my interests openly and building respectful and non-judgemental relationships with the teachers, and with my presence across a number of lessons with each teacher, these dangers were addressed in the research processes in ways that led to data that I felt were a fair reflection of the participating teachers' normal repertoire of practices related to division instruction – addressing the point made by Sangatata et al. (2007) about typicality.

To protect anonymity, the participating teachers are referred to as Cathy, Mary and Hazel, and the schools are labelled A, B and C. These three teachers were chosen from three government primary schools in Johannesburg with varying socio-economic backgrounds. Although all schools were identified as 'Quintile 5' or relatively advantaged fee-charging government schools during the data collection period, two of the schools (Schools B and C) were labelled as 'underperforming' by the province in terms of language and mathematics learning outcomes.

The presence of division actions occurred in the context of the explanation of division tasks. Each lesson transcript was then broken up into 'chunks' based on division tasks. The start of a 'new' division task is marked by the teacher drawing the learners' attention to a new focus related to division. These 'new' foci occurred in several different forms, which was an indication of a 'chunk' – for example, division problems set either as word problems or as division number sentences, asking or telling learners about the names of parts of a division number sentence (e.g. dividend); working with division diagrams; skip counting as a fluency needed for working with division. Each of these chunks was referred to as an episode. Therefore, ending up with a set of transcripts for each lesson taught by each teacher, divided into episodes based on instruction related to different tasks in the teacher's task sequence. In some instances, the teacher dealt with separate tasks in episodes containing the same example, so they were demarcated as individual episodes. Instructional work occurred in whole-class teaching episodes and in 'mat-work' (teacher worked with a sub-group of learners) episodes.

In some lessons, there were segments of individual seatwork or learner group work with no input from the teacher, such as reading division facts from a division sheet or reciting jumps in multiples on a number line. These types of episodes were not analysed. In addition, when working with small groups, the teacher would repeat the same example and explain it to a new group of learners, and these episodes were also left out. Multiple symbolic calculations were described as 'one episode' in certain instances.

The teacher provided an immediate answer in other episodes without elaborating or explaining how they arrived at the answer. In a different instance, another episode can be described as 'one episode' when a teacher uses another episode to create clarity for the initial episode. Episodes involving counting in multiples were also marked, but they were not focused directly on division; these episodes were also omitted from the analysis. The identification of what constituted an 'episode' for analysis gave me a 'thick description' (Geertz 1973) of the division instruction in each lesson taught by each teacher in episode format. Each episode of division instruction constituted a unit of analysis. In some instances where an episode contains two different codes, the author have coded the episode based on the overarching idea.

Coding is pivotal in some approaches to data analysis. The processed data across all three teachers led to the generation of 109 episodes, of which 35 were uncoded for the reasons outlined earlier. The result was that only 74 episodes were divided between the three teachers. The breakdown of episodes across the three teachers, with detail of task foci, included words, division actions, diagrams and symbols. A grounded theory approach to data analysis was taken; there are three basic types of coding: open, axial and selective. Open coding interprets the data by breaking the data down analytically (Corbin & Strauss 1990). Open coding aims to provide new insights through thinking about the phenomenon reflected in the data. Axial coding focuses on putting together data in new ways by making connections between categories, in other words, looking at the patterns that emerge. Selective coding refers to unifying all categories around the 'core' category –'presence of division actions' in this article. This process reflects Wolcott's (1994) suggestion to work with description, followed by an analysis that remains relatively 'close' to the data and then brings theory and literature into play at the interpretation stage. In other words, all three forms of coding were evident in the data analysis.

Ethical considerations

All ethical criteria were taken into consideration. Ethical clearance was obtained from the Wits School of Education.

The study approval number is 2011ECE010C. All the participants were provided letters in which they needed to indicate that they provided written permission to be part of the study. Participants were also informed in the letters that their participation would be voluntary and that, at any point, they could withdraw from the study without experiencing any consequences. To maintain the confidentiality of data, all the participants, in this case, the three teachers, were given pseudonyms as well as the names of the schools. The location of the schools was also not mentioned in the reporting of the data.

Results

This section of the article will have a summary table (Table 3) that shows division workings across sharing or grouping and bald calculations focusing on sharing actions, grouping actions and grouped-based approaches linked to sharing situations. The author will then analyse how each of the teachers worked with division actions and the identification of the predominant action connected to the three teachers.

Table 3 is made up of five columns. The first column describes each teacher and the number of episodes analysed regarding division tasks as expressed in 'sharing situations', 'grouping situations' and 'bald calculations' (Teacher – 74 episodes). For example, Cathy had 49 division tasks; of the 49, 17/49 were episodes containing 'sharing situations', 16/49 were episodes that pointed to 'grouping situations' and 16/49 were 'bald calculations'. In other words, of the 74 episodes, 49 were episodes taught by Cathy. After the 'bald calculations' is a heading referred to as 'Summary of division actions'. This heading shows the number of division actions linked to column 2 (Sharing actions), column 3 (Grouping actions) and column 4 (Group-based approaches linked to sharing situations). The final column is referred to as the 'Total number of division actions'. The 'Total number of division actions' counts the number of division actions pertaining to each teacher. The last row lists the number of division actions per category (sharing and grouping actions and group-based actions based on sharing situations). The final row is the 'total number of division actions' across the columns and the three teachers; in this instance, it was 43. In other words, of the 74 episodes, only 43 of the episodes contained a division action.

The first teacher was referred to as Cathy (pseudonym). The number of episodes across 'sharing situations', 'grouping situations' and 'bald calculations' was 49/74 episodes. A total of 17/49 were identified as 'sharing situations', with 14/17 of the episodes containing some form of division action (7 CCU, 3 GBD, 4 sharing through one-by-one distribution), 16/49 as 'grouping situations', 6/16 made up of a combination of sharing action and grouping actions and 16/49 episodes labelled 'bald calculations' with 15/16 episodes containing division actions. While there were 49 episodes, only 32 of the episodes contained some form of division actions. The action of 'sharing through one-by-one distribution' was seen in 5/32 episodes. 'Sharing through one-by-one distribution' is associated with a sharing situation which is a distribution of items one at a time between people or groups (Wright et al. 2010). Although 5/32 were identified as 'sharing through one-by-one distribution', 1/5 episodes were associated with a 'sharing action'. In this case, the teacher would begin with 'grouping situations'; however, as the explanation unfolded, the teacher coupled a 'sharing through one-by-one distribution' as opposed to 'grouping through one-by-one distributions'. A total of 15/32 episodes were identified as 'grouping through one-by-one distribution', with 4/15 connected to 'grouping situations' and 11/15 linked to 'bald calculations'. 'Grouping through one-by-one distribution' is making groups until the total number of items is used up (Kouba 1989). A total of 2/32 episodes can be identified as 'making groups based on the divisor'. 'Making groups based on the divisor' is the ability first to count out the total number of items and then make groups until the total number of items is grouped (Fosnot & Dolk 2001). A total of 1/2 episodes were linked with 'grouping situations', and 1/2 episodes were associated with 'bald calculations'. 7/32 episodes were classified as 'sharing through complete composite unit' (CCU). 'Complete composite unit' is the ability to immediately distribute items by the quotient value (Zweng 1964). A total of 3/32 episodes were associated with 'sharing through group-base distributions (GBD)'; this type of distribution is based on the divisor used to form groups up to the dividend value (Kouba 1989). These 10/32 episodes were identified as 'group-based approaches linked to sharing situations'. Overall, 5/32 episodes pointed to a 'sharing through 1 – by – 1 distribution'. A total of 27/32 episodes were linked with a 'grouping action' or 'group-based approach linked to sharing situations'.

Mary (pseudonym) was the second teacher, 11/74 episodes. A total of 1/11 episodes were associated with 'sharing situations', 2/11 were 'grouping situations' and 8/11 were 'bald calculations'. A total of 1/11 episodes were identified as 'sharing through one-by-one grouping'. 'Sharing through one-by-one grouping' happens within a sharing situation when the answer for the division calculation is provided, and based on the answer, circles are drawn, and the number shown by the divisor becomes the number of groups placed in each circle (Mathews 2019). A total of 2/11 episodes were identified as 'making groups based on the divisor'. The 1/2 episode was linked to 'grouping situations', and the 1/2 episode was linked to 'bald calculations'. A total of 2/11 episodes are associated with 'grouping through one-by-one distribution'. A total of 1/2 episodes were linked to 'grouping situations', and 1/2 episodes were linked to 'bald calculations'. Overall, 5/11 episodes were identified as linked to 'grouping actions' or 'group-based approaches linked to sharing situations'. Mary had no episodes which contained sharing through one-by-one distribution.

The third teacher Hasel (pseudonym), produced 14/74 episodes. A total of 1/14 episodes were 'sharing situations' with no actions, and 13/14 episodes were 'bald calculations'. A total of 4/13 episodes identified with bald calculations used ‘sharing through 1 by 1 distribution’. The other 2/13 episodes linked to the ‘bald calculations’ were identified as ‘making groups based on the divisor’. Overall, only 2/13 episodes were grouping actions, while 4/13 episodes were ‘sharing through one-by-one distribution’. Hazel had no division actions linked to ‘sharing situations’ and no episodes allocated to ‘grouping situations’.

Across the 74 episodes of the three teachers, 9/74 episodes were identified as ‘sharing through one-by-one distribution’ (12%). A total of 34/74 episodes were associated with grouping actions or ‘group-based approaches linked to sharing situations’ (46%). A total of 23/34 episodes contained ‘grouping through one-by-one distributions’ and ‘making groups based on the divisor’. A total of 11/34 episodes were linked to ‘group–based approaches linked to sharing situations’, which were ‘CCU’, ‘GBD’ and ‘Sharing through one-by-1 grouping’. From Table 3, most episodes pointed away from only ‘sharing through one-by-one distribution’ to grouping actions and sharing situations linked to grouping actions. In other words, the division actions do not align with the notions in most of the South African and certain international literature that, in most cases, a preferred division model like sharing always has a sharing action (Booker et al. 1992; Carpenter et al. 1999; Fischbein et al. 1985; Roberts 2012; Squire & Bryant 2002a).

Conclusion

International research has shown that teachers are prone to using a sharing model with sharing actions as the predominant division model and action (Kinda 2013). Likewise, in a South African context, Roberts (2012) notes that her research shows that teachers are prone to solving division tasks using sharing models and actions. However, the data from this study show that of the three South African Grade 3 teachers, an overarching division action is a grouping action and a group-based approach to solving sharing division tasks.

This article suggests that the data obtained in these three schools indicate a more widespread occurrence of grouping action and approaches in teaching division in a South African context linked to sharing situations. There needs to be a shift in teachers’ understanding of how solving division tasks happens in South Africa, particularly in sharing situations. The first shift is that grouping actions and group-based approaches to solving division tasks are more prevalent than noted broadly in literature in South Africa. The second shift is that there are more grouping actions and grouped-based actions linked to sharing situations because of the inclusion of a division number sentence while solving the sharing division tasks. The third shift would be to assist teachers in identifying how sharing situations can be solved using a group-based approach. These ways include moving from solving sharing situations from one-by-one distribution of items to the distribution of groups in the solving of sharing situations based on the quotient value. The shifts towards more appropriate group-based approaches must become explicit, particularly when solving sharing situations after teachers have consolidated one-by-one distribution actions.

Further research needs to investigate the extent of the use of grouped-based approaches to teaching division tasks with a greater sample of Grade 3 teachers when solving sharing, grouping situations and bald calculations. The purpose would be to see how division actions are enacted and which division action is the predominant action. At the same time, it is important to teach both the pre-service and post-service teachers about the nuances related mainly to solving sharing situations while using a group-based approach.

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Data availability

The data that support the findings of this study are available on request from the corresponding author, C.D. Mathews. The data are not publicly available because of restrictions; for example, they contain information that could compromise the privacy of research participants.

Disclaimer

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References

Anderson, G. & Arsenault, N., 2005, *Fundamentals of educational research*, Routledge, London.

Andrews, P., 2009, ‘Comparative studies of mathematics teachers’ observable learning objectives: Validating low inference codes’, *Educational Studies in Mathematics* 71, 97–122. https://doi.org/10.1007/s10649-008-9165-x

Booker, G., Briggs, J., Davey, G. & Nisbet, S., 1992, *Teaching primary mathematics*, Longman Chesire Pty Limited, Melborne.

Borko, H., Jacobs, J., Eiteljorg, E. & Pittman, M., 2008, ‘Video as a tool for fostering productive discussions in mathematics professional development’, *Teaching and Teacher Education* 24(2), 417–436. https://doi.org/10.1016/j.tate.2006.11.012

Brown, M., 1992, ‘Second – Grade children’s understanding of the division process’, *School Science and Mathematics* 92(2), 92–95. https://doi.org/10.1111/j.1949-8594.1992.tb12148.x

Carpenter, T., Answell, E., Franke, M., Fennema, E. & Weisbeck, L., 1993, ‘Models of problem solving: A study of kindergarten children’s problem-solving process’, *Journal for Research in Mathematics Education* 24(5), 428–441. https://doi.org/10.2307/749152

Carpenter, T., Fennema, E., Franke, M., Levi, L. & Empson, S., 1999, *Children's mathematics cognitively guided instruction*, Heinemann, Portsmouth, NH.

Chandler, D., 1994, *Semiotics for beginners*, Loyola College, Crawley, viewed n.d., from https://www.semanticscholar.org/paper/Semiotics-For-Beginners-Chandler/d540341246102f1d7f82dcdb62f338548a3ea8b0

Corbin, J., & Strauss, A., 1990, 'Grounded Theory Research: Procedures, Canons, and Evaluative Criteria', *Qualitative Sociology* 13(1), 3–20.

Dowling, P. & Brown, A. (eds.), 2010, *Doind research/ reading research re-interrogating education*, Routledge, London.

Fischbein, E., Deri, M., Nello, M.S. & Marino, M.S., 1985, 'The role of implicit models in solving verbal problems in multiplication and division', *Journal for Research in Mathematics Education* 16(1), 3–17. https://doi.org/10.2307/748969

Fosnot, C.T. & Dolk, M.L.A.M., 2001, *Young mathematicians at work*, Heinemann, Portsmouth, NH.

Geertz, C., 1973, 'Thick description: The interpretation of cultures', *The Interpretation of Cultures*, 3–31.

Kinda, S., 2013, 'Generating scenarios of division as sharing and grouping: A study of Japanese elementary and university students', *Mathematical Thinking and Learning* 15(3), 190–200. https://doi.org/10.1080/10986065.2013.794256

Kouba, V.L., 1989, 'Children's solution strategies for equivalent set multiplication and division word problems', *Journal for Research in Mathematics Education* 20(2), 147–158. https://doi.org/10.2307/749279

Leedy, P. & Ormrod, J., 1993, *Practical research: Planning and design*, Merril Prentice-Hall Inc, Columbus, OH.

Lincoln, Y. & Guba, E., 1985, *Naturalistic enquiry*, Sage, London.

Mathews, C., 2019, 'Analysing coherence in division teaching in South Africa', in Paper presented at the proceedings of the 43rd conference of the International Group for the Psychology of Mathematics Education, Pretoria, July 10, 2019.

Merriam, S.B., 1998, *Qualitative research and case study applications in education*, Josey-Bass, San Francisco, CA.

Noor, K.B.M., 2008, 'Case study: A strategic research methodology', *American Journal of Applied Sciences* 5(11), 1602–1604. https://doi.org/10.3844/ajassp.2008.1602.1604

Ogegbo, A.A., Gaigher, E. & Salagaram, T., 2019, 'Benefits and challenges of lesson study: A case of teaching physical sciences in South Africa', *South African Journal of Education* 39(1), 1680. https://doi.org/10.15700/saje.v39n1a1680

Opie, C., 2004, *Doing educational research*, Sage Publications Ltd, London.

Peirce, C.S., 1992, *The essential Peirce: Selected philosophical writings*, vol. 1, Indiana University Press, Bloomington, IN.

Presmeg, N., Radford, L., Roth, W. & Kadunz, G., 2017, 'Topic study group No 84: Semiotics in Mathematics education', paper presented at the in proceedings of the 18th international Congress on Mathematics, University of Hamburg, Hamburg, 28 July 2016.

Roberts, N., 2012, 'Long division', in search of the primary school mathematics holy grail Paper presented at the AMESA national congress long paper, North West University, Potchefstroom, June 25, 2012.

Sáenz-Ludlow, A., 2003, 'A collective chain of signification in conceptualizing fractions: A case of a fourth-grade class', *The Journal of Mathematical Behavior* 22(2), 181–211.

Santagata, R. & Guarino, J., 2011, 'Using video to teach future teachers to learn from teaching', *Zdm* 43, 133–145.

Santagata, R., Zannoni, C. & Stigler, J, 2007, 'The role of lesson analysis in pre-service teacher education: An empirical investigation of teacher learning from a virtual video-based field experience', *Mathematics Teacher Education* 10, 123–140.

Schollar, E., 2008, 'Final report of the primary Mathematics research project', paper presented at the Presentation to the Conference What's Working in School Development, funded by the Shuttleworth and Zenex Foundations, Eric Schollar and Associates c.c, Johannesburg, 08 Feb. 2023.

Silverman, D., 2004, *Qualitative Research*, Sage Publications, London.

Squire, S. & Bryant, P., 2002a, 'From sharing to dividing: Young children's understanding of division', *Developmental Science* 5(4), 452–466. https://doi.org/10.1111/1467-7687.00240

Squire, S. & Bryant, P., 2002b, 'The influence of sharing on children's initial concept of division', *Journal of Experimental Child Psychology* 81(1), 1–43. https://doi.org/10.1006/jecp.2001.2640

Squire, S. & Bryant, P., 2003, 'Children's models of division', *Cognitive Development* 18(3), 355–376. https://doi.org/10.1016/S0885-2014(03)00039-X

Vergnaud, G., 1994, 'Multiplicative conceptual field: What and why?', in G.H.J. Confrey (ed.), *The development of multiplicative reasoning in the learning of mathematics*, pp. 41–59, SUNY Press, Albany, NY.

Walkerdine, V., 1988, *The Mastery of Reason: Cognitive development and the production of rationality*, Routledge, London.

Wright, R., Martland, J. & Stafford, A. (eds.), 2010, *Early numeracy*, Sage, London.

Zweng, M., 1964, 'Division problems and the concept of rate: A study of the performance of second-grade children on four kinds of division problems', *The Arithmetic Teacher* 11(8), 547–556.