Challenges of Teaching with Challenging Tasks: Teaching Dilemmas Arising From Implementing a Reform-oriented Approach to Primary Mathematics

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Received: June 2022 Accepted: May 2023

The research reported in this paper analyses teaching dilemmas experienced by four in-service teachers in the context of a reform-oriented teaching approach for primary mathematics called Developmental Education in Mathematics (DEM). The findings exemplify three teaching dilemmas: Whether to tell students how to solve a challenging task; what to do when students are bored with an important task; and trying to keep the pace of the lesson while supporting all students in the classroom. Importantly, the origins of these dilemmas were found to lie in internal tensions between the components of the DEM system, which the teachers had implemented. Finally, the implications for development of curricular material, implementation of reform-oriented teaching approaches, and the professional development of teachers are discussed.

Keywords: • teaching dilemmas • challenging tasks • developmental education in mathematics • mathematics teacher development • implementation of reform-oriented teaching

Introduction: Primary Mathematics Teachers as Dilemma Managers

Implementing reform-oriented teaching practices can be difficult. Throughout the past couple of decades, there have been many research studies looking into the various challenges that come with making changes to mathematics teaching. Many of these studies have focused on the difficulties that teachers experience and possible reasons for these difficulties. Sometimes, the causes of the difficulties are sought by focusing on the teachers (e.g., teacher knowledge, beliefs, and competencies), a perspective that often results in a deficit view of the teachers (Skott, 2008). Others have studied reform-oriented practice to understand the kind of problems teachers face when implementing such practices. Lampert (2001) analysed problems related to teaching mathematics as problem-solving. Ball and others identified the tasks of teaching that teachers face, and the knowledge needed to succeed with them (e.g., Ball, 2017: Ball et al., 2008). Many of these teaching challenges or tasks can be described as solvable problems that the teacher can be prepared to meet through pre-service teacher education or professional development. However, some challenges teachers face are, or can seem, impossible to resolve completely. Lampert (1985) called them *teaching dilemmas* and claimed that these are the kind of challenges the teacher must manage and is not always able to resolve. If too many new and demanding dilemmas arise when trying to implement reform-oriented practice, the teacher could give up and return to their previous, more traditional teaching practice (which is often more teacher-centered and easier to manage). This study seeks to identify and analyse dilemmas which arise during the implementation of a specific reform-oriented approach to primary mathematics education, to understand where these dilemmas come from, and to discuss what could be done to leverage these dilemmas for the purpose of professional development.

Context: Using Challenging Tasks within Developmental Education in Mathematics

The context of this study is a reform-oriented approach to primary mathematics education called Developmental Education in Mathematics (DEM). Norwegian schools and teachers have the freedom to choose curricular materials and pedagogical approaches as long as they generally adhere to the national curriculum, and around 100 schools currently use DEM materials in Norway. DEM is based on the system of primary education (originally for all school subjects) developed by Zankov, a student and close associate of Vygotsky (Gjære & Blank, 2019; Zankov, 1977). Based on Vygotsky's (1934/1986) cultural-historical theory and two decades of extensive school research in the Soviet Union, Zankov (1977) formulated five main principles which together form the underpinning theoretical foundation of the system:

1. Teaching at a high level of difficulty
2. A leading role for theoretical knowledge
3. Proceeding at a fast pace
4. Promoting the students' awareness of their own learning processes
5. Systematic development of each individual student

Principles 1, 2 and 3 are of particular importance to this study because they provide most of the framework for designing curricular material. Principle 1 is a direct pedagogical consequence of Vygotsky's concept of the Zone of Proximal Development (ZPD), which means that every lesson should provide the students with some learning challenges appropriate to their level. Principle 2 is based on Vygotsky's (1934/1986) theory of concept formation, which states (broadly explained) that the introduction of scientific concepts in school fundamentally restructures the learner's consciousness and allows for theoretical thinking. Principle 3 reflects the fact that as the students learn and develop over the course of a school year, their ZPDs expand. Consequently, the challenges must be gradually increased to take into account the expansion of their abilities. To avoid misunderstandings, Zankov (1977) stressed that fast pace does not mean record-breaking speeds, but rather a thoughtful increase in challenge as the students grow and develop. This principle primarily reflects his criticism of the traditional system of his days, which relied heavily on rote memorisation and "overlearning", slowing student development (Zankov, 1964).

Textbooks and teacher guides are important resources for teachers to structure their teaching practice (Ball & Cohen, 1996). The implementation of DEM in Norway relies on a series of textbooks (Grades 1–4), originally written in Russian in the 1990s. They were based on Zankov's (1977) principles and translated and adapted to the Norwegian school context by staff at the University of Stavanger, who also wrote teacher guides for the books. Compared to other Norwegian primary mathematics textbooks, the DEM books were found to contain a higher proportion of task types suggesting high cognitive demands (Tokheim, 2015). Following the principle of theoretical knowledge, many of the tasks are designed with clear mathematical goals, which are often stated explicitly in the teacher guide. Consistent with the socio-cultural view of mathematics teaching and learning, the teacher guide generally suggests leading the students to formulate mathematical concepts or relationships through activities and discussions.

Teaching with Challenging Tasks and the Dilemmas that Follow

As can be understood from the pedagogical principles above, providing students with mathematical challenges is a central feature of DEM. In mathematics education research literature, various terms are used to describe tasks aimed at challenging the students: high-level tasks (e.g., Stein et al., 2008), challenging tasks (e.g., Russo & Hopkins, 2017; Sullivan et al., 2015), and cognitively demanding tasks (e.g., Wilhelm, 2014) are used somewhat interchangeably. Definitions of these terms usually refer to the work of Doyle (1988) and Stein and Lane (1996), who classified tasks depending on the type of cognitive activity demanded from the students. Stein and Lane (1996) described two main categories of challenging tasks: *doing mathematics*, referring to solving novel problems to which the students do not have immediate access to a clear solution path or method; and *use of procedures with connections to concepts, meaning, and/or understanding*, meaning that the task "maintains and/or develops deep levels of understanding of mathematical concepts and ideas" (p. 58). Both types of challenging tasks are prevalent in the textbooks used in the context of this study.

The advantages of teaching with challenging tasks are well described. Challenging students to solve complex problems and supporting them to construct solutions and concepts and draw conclusions for themselves can improve general problem-solving traits and abilities, which helps to prepare them for adult life (English, 2011; Lesh & Zawojewski, 2007; Lester & Cai, 2016). Furthermore, challenging tasks are hypothesised to promote deeper learning of mathematical concepts and relationships as well as positive learning experiences, thereby improving students' attitudes toward mathematics (Lesh & Zawojewski, 2007; Lester & Cai, 2016).

Maintaining and succeeding with a mathematics teaching style centered around challenging tasks is not without difficulties. According to Brodie (2010), reform-oriented classrooms may be harder to navigate for both teachers and learners due to the increased complexity of interaction. Many teaching dilemmas are directly linked to tensions that arise when teachers implement various reform-oriented pedagogies. A classic example is the dilemma of telling, which refers to the pedagogical choice between allowing students to find their own solutions and telling them what to do. This dilemma originates in tensions between the need for students to explore, articulate, and make sense of concepts themselves, and the teacher's desire or expectation that the students reach specific learning goals within a limited time (Baxter & Williams, 2010; Philipp, 1995; Roth & Lee, 2007). Even when the teacher can think of a way to help the students without telling directly, deciding how long students should "flounder" on their own with a problem and when to intervene can still pose a dilemma (Ball, 1993). Teachers often remove the challenges for the students by showing how to solve the problem (Lester & Cai, 2016), which could be accounted for by the dilemma of telling. Another set of teaching dilemmas relates to learner contributions; whether the teacher should take them up or ignore them (Brodie, 2010), or how to treat learner contributions respectfully when their reasoning is flawed (Ball, 1993). Such dilemmas can be exacerbated by the complex demands of interpreting and responding to learner contributions "on the fly" (Bass & Mosvold, 2019).

The analysis that follows builds on Lampert's (1985) notion of a teaching dilemma as a situation that cannot be completely resolved, only managed by balancing conflicting responsibilities. Dilemmas are choice situations where there is no obvious best option, and where each option entails both gains and losses. For example, if the teacher never tells, he or she might stay true to a problem-solving epistemology (see Schoenfeld, 1992) but risk that the students become frustrated or miss important mathematics. Osborne (1997) suggested that some teaching dilemmas are inevitable since the tensions they represent are important drivers of learning, such as the tension between the needs of the group and the needs of each individual learner. Moreover, dilemmas should not be thought of as choices between "black and white" but rather as "shades of gray" (Philipp, 1995).

Another important characteristic of teaching dilemmas is that they are personal experiences of each teacher (Battey & Franke, 2008; Brodie, 2010). This has been made especially clear in studies where the author and the teacher are the same person, reflecting on their own teaching experiences and dilemmas (Ball, 1993; Frid, 2000; Lampert, 1985; Osborne, 1997). This entails that different teachers may experience different dilemmas or experience similar dilemmas differently. For instance, Ben-Peretz & Kremer-Hayon (1990) found that novice and senior teachers were concerned with different dilemmas and suggested that as teachers gain experience, they may cease to be confronted by some dilemmas and encounter new ones. Conversely, "the same" dilemma can be experienced differently by novices and experts; compare Philipp's (1995) account of two pre-service teachers who were rather stumped by their black-and-white interpretation of "teaching is not telling" with Ball's (1993) detailed discussion of just how much she should let the students struggle before intervening.

Methods

The participants of this study were four in-service teachers who taught mathematics at different primary schools. Two of the teachers, here given the pseudonyms "Anne" and "Henry", taught Grade 1 (6-year-olds) at the time of data collection. The other two, "Mona" and "Siri", taught Grade 4 (10-year-olds). The teachers volunteered for the study by responding to a general call for research participants distributed to schools and were selected on the basis that they all had four years or more of experience using the DEM textbooks and principles. Data were collected over a period of three months. For each of the teachers, three lessons were observed and video-recorded. The teachers were asked to plan and conduct the lessons as usual. In addition, two focus group interviews were conducted with the four teachers, one before and one after the period of classroom observations.

During instruction, mathematics teachers are inevitably faced with a multitude of choice situations, many of which potentially constitute small or large dilemmas. Therefore, a review of the classroom videos did not easily reveal which of the participants' choice situations constituted significant teaching dilemmas related to the implementation of DEM. Therefore, the focus group interviews were used to clarify the picture. For the analysis of the focus group interviews, reflexive thematic analysis (RTA) was used (Braun & Clarke, 2006; 2019). The purpose of using RTA was not to produce an objective account of the data; instead, it was a sense-making process positioned within an interpretative paradigm and relied on "the researcher's reflective and thoughtful engagement with their data and their reflexive and thoughtful engagement with the analytic process" (Braun & Clarke 2019, p. 594). RTA is suitable for accessing participants' meanings, which fits this study well given the personal nature of teaching dilemmas.

Preliminary codes were developed through several readings of the data material with the research problem as a guide. The codes were revisited and refined them into more complex themes through a six-step process as suggested by Braun and Clarke (2006). This included checking that each theme formed an internally coherent narrative centered around a topic, and that each theme was clearly distinct from the others (Byrne, 2022). Consistent with RTA, no inter-coder reliability measures were sought because another researcher could construct a different set of themes using the same data material. The analysis has both an experiential and a critical orientation (see Byrne, 2022). This means that the aim was to seek both to access the participants' meanings regarding teaching with DEM, and to try to discern the origins of the teaching dilemmas they experienced.

The themes developed do not constitute dilemmas. Rather, they represent various aspects of the participants' experiences or concerns with teaching mathematics using DEM. For example, it was found that *the importance of challenging the students* formed one coherent narrative across both interviews. However, the narratives that exhibit the different themes could conflict with each other. Teaching dilemmas emerge precisely when the teacher must manage conflicting needs or responsibilities. In the analysis below, themes are compared to show how the participants articulated different responsibilities toward their students and toward DEM, and how some of these responsibilities might conflict. These conflicts plausibly give rise to some teaching dilemmas, which are exemplified through teaching excerpts from some of the recordings of the mathematics lessons.

Findings

Using RTA, six distinct and coherent themes were constructed (Figure 1), which together, form a general account of the focus group interviews. The data shed light on important dilemmas experienced by the teachers. As explained above, these themes do not constitute dilemmas themselves. Nevertheless, potentially conflicting responsibilities that could cause the participants to experience teaching dilemmas can be gleaned from the themes. A comprehensive thematic map of the data material is included in the Appendix.

1. The importance of challenging the students.
2. The importance of theoretical knowledge in DEM.
3. Keeping progression and tempo.
4. Teaching using DEM "the right way."
5. A problem-solving approach to teaching mathematics.
6. Adapting lessons to all students.

Figure 1. A list of themes constructed from the focus group interviews.

Comparing these themes, some tensions came out in sharper relief than others, and there were three teaching dilemmas that stood out in the analysis:

- The dilemma of telling.
- When students are bored with important tasks.
- Keeping pace while supporting all students.

Analysing each dilemma separately, an overview of the relevant themes and the tensions among them that led to the dilemma is presented. Examples of how the dilemma emerged during a mathematics lesson are illustrated through excerpts from the classroom videos. The three teaching dilemmas are shown collectively in the thematic map presented In the Appendix. Sections of the thematic map relevant to each teaching dilemma are presented in the text that follows.

Dilemma: The Dilemma of Telling

Conflicting responsibilities that led the participants to experience the dilemma of telling included: keeping progression and tempo, teaching using DEM "the right way", theoretical knowledge, and a problem-solving approach (Figure 2).

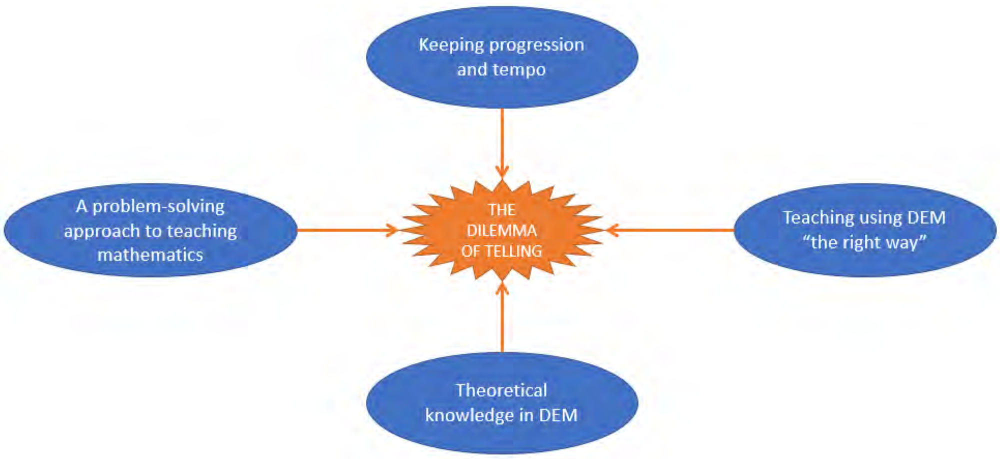

Figure 2. Conflicting responsibilities that led to the dilemma, "Is teaching telling?"

Throughout the interviews, the participants repeatedly voiced several ideas consistent with *a problem-solving approach to mathematics teaching* (Theme 5): to understand mathematics, students should come to their own conclusions through discussing, reasoning about, and solving problems; the teacher should not explain directly but instead build on students' contributions so the class can solve difficult problems together; creativity is an important aspect of mathematics; errors should be treated in a positive light as a step on the way to learning; and emphasis should be on the solution process and not just getting the right answer. Moreover, the participants attributed all these ideas to DEM and used them to contrast DEM with their previous, more traditional approaches. The idea that "teaching is not telling" was clearly expressed by Mona, who said,

Mona	I think I can count on one hand the times I've *explained* something. [laughs] That has become *really* hard for me these days. It hurts my soul if I have to (…) But it could matter for the tempo, the tasks could take some time. Because you depend on the students' answers.

Here, Mona began to put into words one version of the "dilemma of telling", namely, that staying true to a problem-solving approach to mathematics teaching and not telling students directly what to do may conflict with *keeping progression and tempo* of the lessons (Theme 3). Proceeding at a fast pace is one of the main principles of the DEM system, so it is perhaps not surprising that the participants touched on this theme on several occasions:

Henry	[Fast pace means] that you ask about something new each lesson. But it doesn't necessarily mean working through many tasks. You work thoroughly, but fast pace means that you introduce something new all the time. You don't work on the same thing for several weeks. If [the students] do a hundred tasks that are all the same, it's not fast pace, because you are doing the same. You aren't doing anything new. And then you haven't moved forward.
Siri	And moving on even though you know you only got a third of the group with you. It's like, "OK, let's move on, next task!" And you know that there are many [students] who haven't formed a complete understanding, but you still trust that, OK, we'll get back to it.

Siri's quote says something about the way the books are structured; students will meet a given concept many times in different task settings, repeating old content in new contexts. This also touches on another important theme, namely *teaching using DEM "the right way"* (Theme 4). The participants tended to hold the books in high regard and trusted that following the books and the teacher guides would help them maintain a "correct" DEM teaching practice. The books were especially important when they first began using DEM, as all said they experienced a lot of uncertainty in the beginning.

Henry	I didn't have courses [about DEM] the first couple of years, and I always wondered if I was doing it right.
(…)	
Siri	At first, I read the guide like this [pretends to read a book closely], but then I noticed, the more and more secure I got, that I could put the book away. Just read it quickly to get the idea behind a task and then I could teach. (…) And many times, I read a task and I thought, "what's the point of this?" And then when I read ahead, I could see, "Aha, now I get it, that's why." So, I think it's good to have the books…to know the point of a certain task. "Why am I introducing this now?" [laughs] And then I would understand a little later, "Aha, to create more understanding when *that* task comes, that's why I'm doing it."
Mona	After four years we've understood that *every single* task has a point. [laughs] There are no fillers. Everything has its place.

Moreover, all four participants agreed that it had been a major advantage to be able to teach Year 1 to 4 in sequence, since this allowed for an overview of the conceptual structure of the textbooks so they could teach more confidently:

Anne	I'm in my second round [of Year 1–4 of DEM], and I followed the class who started in their first year, I followed them up to the fourth. And now I'm at Year 1 again. And I feel that I'm in control in a different way than I was before.

The participants' narratives about how closely they followed the books and how important the books were to structure their teaching practice, could hint at preoccupations that feed into the dilemma of telling: what do you do when you are using an important task which comes at a specific place in the sequence of the school year, and the students just aren't making the right conclusions? After all, the tasks build on one another to form a coherent mathematical conceptual structure. This led to considering Theme 4, *the importance of theoretical knowledge in DEM.* The participants underlined the importance of developing the students' mathematical vocabulary (the "concept bank", as Henry put it). Furthermore, they valued that the textbooks put a thorough emphasis on students learning *why* rules and algorithms work, instead of just having a lot of repeated practice tasks. In fact, this was something they saw in contrast to their previous practice. They also acknowledged some potential downsides to the rigorous mathematics of DEM, since it could be off-putting to some teachers and parents who were not used to it. Additionally, the teachers felt it necessary to use some time during lessons to read and understand problems properly and came to realise that language could be a barrier for students who moved into a DEM group from the outside. Finally, the introduction of problem-solving strategies was emphasised, such as drawing models and schemas, or students thinking back on their solutions:

Anne — In traditional math, you learn one way to do it, and then you just do that. And if you get the wrong number, well, you just continue. I've seen a lot of students do that. They just race through, and they don't notice their mistakes. While our students now, a lot of them seem more attentive to, "What am I seeing here?" They don't just race through; they stop and think. Some strategies, like, "How many digits must this answer have?" They've learned those, and they discover that, "Oh, this has to be wrong." Well, most do. You can't get to all of them.

The analysis revealed significant tensions present in the teachers' narratives that plausibly exacerbated the dilemma of telling: on the one hand, the teachers wanted their students to *do mathematics*: solve challenging problems themselves, be creative, discuss each other's ideas, and come to conclusions on their own; on the other hand, the textbooks set a rather rigorous mathematical structure where specific tasks are used at specific points in the school year to develop specific mathematical concepts. According to the teachers, it was important to follow the ideas of the books to teach with DEM "the right way." And importantly, there should be a certain pace to both individual lessons and the school year in general. Leading students through explorations and discussions to make the "right" conclusions and formulate the "right" concepts at the "right" time is not easy. Based on this analysis from the interviews, it is not surprising that the dilemma of telling appeared in several of the lessons. As one example, the following teaching episode was from one of the Grade 4 classes.

Teaching Episode 1: The Car Problem

The purpose of presenting this episode is to show how the teacher guide and textbooks, which the participants said were important to scaffold their implementation of DEM, could be an important source of the dilemma of telling during task enactment. The episode takes place in one of the Grade 4 classrooms using the following task from the DEM textbook.

> Two cities are 600 km apart. Two cars started at the same time and drove toward each other from each city. One of the cars drove 12 km/h faster than the other. They met after 4 hours. Find the speed of each car.

In the DEM system, formulating and using equations to solve word problems is a mathematical goal for the students during the 4th year. In the teacher guide, other solutions are suggested for this task, but using an equation is presented as the preferred and most sophisticated solution (in line with the principle of a leading role for theoretical knowledge: Theme 2). During enactment, the teacher began by presenting the task, which the whole class read aloud together. The teacher then opened the whole-class discussion:

Teacher — There was a lot of information in this task. But we're going to solve it using an equation. But before we get started on the equation, it could be smart to make a drawing or a model of this task. So, then I'm wondering if anyone has a suggestion for how to make a drawing for this word problem. (…)

The teacher explicitly stated the intention of using an equation and wanted the students to draw a model (presumably as a support to formulate the equation). The students willingly drew models and offered hypotheses for how to solve the problem but did *not* suggest using a variable or formulating an equation. Many ideas were presented, but they had not yet arrived at a solution after about eight minutes of whole-class discussion. At that moment, the teacher explicitly introduced the idea of using a variable (x) and proceeded to lead the students quite firmly in the formulation and solution of an equation based on the word problem. During the discussion, one student suggested an elegant arithmetic solution to the problem. This contribution, however, was not picked up as such by the teacher but instead used to further the equation:

S4: I just wanted to say that (.) one of them was 12 kilometers faster[1], so I multiplied 12 by 4 and then I got 48. And I think that means that one of the cars went 48 kilometers farther.

Teacher: Yes. And then you are actually thinking that Car 1 drives 12 kilometers extra each hour? (*Proceeds to write "x + 12" below the hourly segments of Car 1, see Figure 3*)

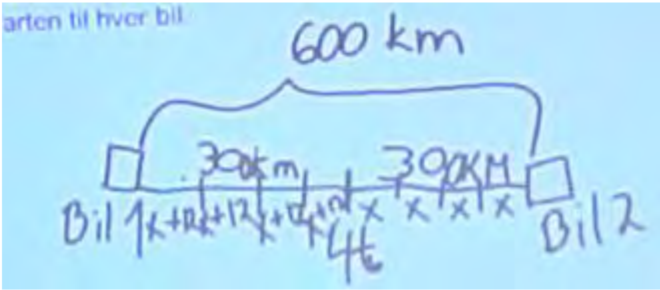

Figure 3. The teacher used and extended a student's model as the task progressed and decided to add variables herself. [Translation: "Bil" means car; "4t" means 4h (i.e., four hours)]

The dilemma of telling can often arise when a teacher gives her students a challenging problem with a specific solution in mind. What is especially striking here, however, is how the origin of the dilemma can be traced to tensions between various principles and tools of the DEM system. One major principle of DEM is that theoretical knowledge should play a leading role in the teaching-learning activities, which follows from Vygotsky's concept formation theory (1934/1986). In Episode 1, this meant using algebra as a problem-solving tool as early as Grade 4. Nevertheless, the participants also underlined the importance of developing students' initiative, creativity, and independent thought. As long as the students provide contributions that can be built on toward the mathematical concept or method the teacher has set as the goal of the lesson, all considerations are aligned. As the episode clearly illustrates, the dilemma of telling arises when the students are *not* coming up with suggestions that leads the discussion toward the planned mathematical goal. In such moments, the teacher faces a choice of pushing through toward the goal (i.e., telling the students what to do) or accepting either "less sophisticated" solutions or not solving the problem at all (at least within that lesson).

It is plausible that the teacher's decision to tell the students directly how to set up an equation was strongly influenced by the teacher guide, which promoted this solution method based on Vygotskian ideas of theoretical concepts. Given the participants' high regard for the textbooks and their dependence on the books for their DEM practice, the consideration to follow the book to teach DEM "the right way" would have weighed strongly in this episode. Additionally, the teacher had other tasks planned for this lesson, and therefore the consideration of getting to a solution within reasonable time to keep the pace of the lesson could also have played a role in the decision to "tell". As a final note, this episode was not unique to this teacher, as more dilemmas of telling, small or large, occurred for all four participating teachers.

[1] Verbatim translation: The student said «kilometers» while referring to speed (it should be km/h).

Dilemma: When Students are Bored with an Important Task

Conflicting responsibilities that led the participants to experience a dilemma what to do when students become bored with a task that was deemed important included: teaching using DEM "the right way", and the importance of challenging students (Figure 4).

Figure 4. Conflicting responsibilities that led to the dilemma, "When students are bored with an important task."

The Car Problem in Teaching Episode 1 was a challenge for the students. Nevertheless, not all DEM tasks are similarly demanding. Moreover, the level of challenge does not necessarily correspond directly to the importance of the task and its place in the sequence of teaching during the school year. As shown in the previous section, the participants praised the logical sequencing of the tasks and were loyal to following this progression.

At the same time, the participants held *challenge* to be an important and integral part of DEM (Theme 1). All agreed that the textbooks placed higher expectations on the students than the teaching materials they had used previously, and that DEM tasks required the students to compare, analyse, reason, and explain (i.e., high cognitive demands). Siri connected such challenges directly to learning, pointing out that learning mathematics necessarily entails some effort and struggle. Henry, in contrast, connected this theme to the students' general development, stating that children need to learn to face challenges later in life, and to develop perseverance in the face of discomfort. Finally, Mona brought up the importance of challenge for the students' motivation on several occasions:

Mona	And high-level tasks that leads them [the students] to experience great successes when they can do it. There are many tasks that seem impossible at first glance, at least for some. But then you get "Yes! We did it!" And then they see, after a few times, that they can do it by themselves.
(...)	
Mona	That's where the motivation comes from. When they have to struggle a bit, and then they can do it. They're like, "oh, it's fun!", right? And that's where the motivation lies to have a go at anything later on.

However, what should the teacher do when a main[2] task containing important mathematics does *not* offer the students any challenge? Should he or she skip ahead, or stick to the task sequence as planned? This dilemma was particularly salient in another Grade 4 lesson.

Classroom Episode 2: Finding the Volume of a Right Rectangular Prism

This episode was comprised of a whole-class discussion about finding the volumes of right rectangular prisms. This task was part of a series of tasks on the same topic and had status as a main task in the textbook, meaning that it could not be skipped. Previously, the class had discussed and concluded that the volume of a right rectangular prism can be calculated by multiplying the length, width, and height of the prism, which the teacher confirmed was the formula. Next, two prisms were shown on the board, together with a table to fill out with length, width, height, and volume of each prism. Prism 1 is shown

[2] In the DEM textbooks, some tasks are marked in red, indicating that they are "main tasks", not to be skipped by the teacher. Other tasks, marked in blue, are optional.

in Figure 5. During the mathematical discussion, some students were moving about on their seats, sighing audibly, or answering oddly.

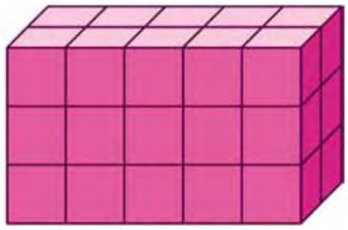

Figure 5. The students were asked to find the volume of this prism using the formula they had just formulated (the prism is called Figure 1 in the transcript.)

Teacher	OK! Let's use the formula to fill out this table and calculate the volume of these two figures. What then, can we say about the length of Figure 1? (*The teacher pauses. The students are unrestful and only a few have their hands up*) The length of Figure 1? Come on, people, the length of Figure 1. S11.
S11	Um, it's five, I think. Yeah.
Teacher	Yes, five (*writes "5" under Length, Figure 1, in the table*)
S11	But I found out what the whole was, too.
Teacher	The length is five. What about the width? (*pause*) S17?
S17	Two.
Teacher	Two. (*writes "2" in the table*) and the height? S6?
S6	THREE! (*answers in an odd voice, perhaps indicating frustration*)
Teacher	And then the volume is?
Students	Twenty-four / thirty (*both numbers are heard*)
Teacher	Five...? Five time two is...? (*speaks very slowly and clearly*)
Students	Ten!
Teacher	Ten times three?
Students	THIRTY! (shouting)

A plausible interpretation of the students' initial reluctance to participate and their odd answers was that they found the discussion uninteresting. This was corroborated later. When given a worksheet on the same topic, some students were heard saying this was not a challenge at all:

Teacher	You are to find the volume of these prisms. But you see, they aren't quite filled up with cubic centimeters. Can you still find the length, the width, and the height of these prisms?
A student	Um, yeah.
Another	That's easy (*heard whispering to his desk mate*)

During this whole-class discussion, the teacher faced a different dilemma from that of telling. The tension here lies between the importance the participants put on mathematical challenges as a driver for the students' learning and motivation, and the importance of the textbook and the sequencing of the tasks. In the teaching episode, it was clear that the students were not challenged, and that their motivation had dropped. They knew the mathematics and were ready to move on, yet the teacher chose to stick to the discussion. One possible explanation is that the teacher could just be following the progression of the textbook without further thought. However, there could also be more to the situation. During the interviews, the participants showed awareness of the way main tasks build on one another. Since there would be more difficult tasks on the topic of volume later, the teacher in this episode might think it important for the students to not miss anything and to build a solid foundation for future problem solving. Moreover, even if many students expressed a lack of challenge does not mean *all* the students felt the same way. Some might have benefited from the extra discussion. These conflicting considerations makes the choice of when to stick to a discussion and when to move on difficult for the teacher.

Dilemma: Keeping Pace While Supporting all Students

Conflicting responsibilities that led the participants to experience the dilemma of how to maintain the pace of learning while supporting all students included: adapting lessons to all students, and keeping progression and tempo (Figure 6).

Figure 6. Conflicting responsibilities leading the participants to experience the dilemma, "Keeping pace while supporting all students."

The participants were generally concerned with *adapting the DEM lessons to all students* (Theme 6) despite the variation in mathematical ability. They all agreed that low-achieving students could benefit from the challenging tasks and claimed that their students now understood more than in their previous teaching practice. They attributed this to the many whole-class mathematical discussions:

Henry	You have to vary, so that everyone can experience some success during the lesson. But I see some who show a certain resistance toward the challenging parts, even with DEM. And yet they participate, and they <u>hear</u> everything. So, I think it trickles … we see a kind of communication that makes it trickle on them too. They catch some of the thinking of the others. (…)
(…)	
Siri	I have this one student, he participates on level with the others when we have whole-class discussions and we talk through problems, but for individual work, he gets adapted tasks.

The other participants agreed that whole-class discussions, as well as peer discussions, was the key to adapting mathematics lessons to the various students. However, Henry pointed out that trying to get all students to participate could also lead to a slower pace:

Henry	I have a goal that everyone should say something aloud in each lesson. Sometimes, the shy ones, maybe they'll just get to speak once per lesson. I think that's too little. I'd like to challenge them more, but it's difficult and time-consuming. (…) If you approach them <u>all</u> the time, it breaks up the effectivity for the others.

The tensions entailed in providing a heterogeneous group of students with appropriate challenges at different levels while moving the group forward as a whole is something most mathematics teachers struggle with. The dilemma, however, is perhaps felt even more keenly in contexts such as DEM, where the group often discusses challenging tasks while the teacher at the same time needs to mind the pace of the lesson and keep a certain progression and effectivity. An example of this dilemma is shown in the third teaching episode.

Classroom Episode 3: The Mystery Figure

This episode took place in one of the Grade 1 classrooms. In this activity, the teacher used a document camera projected on a whiteboard to gradually reveal a geometrical figure, which turned out to be a concave quadrilateral (Figure 7, Picture 3). The students had to guess what the teacher was hiding by looking at the gradually revealed figure.

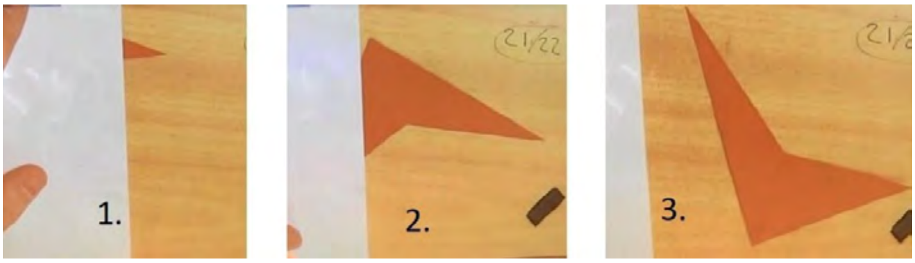

Figure 7. A mystery figure (a quadrilateral) was slowly revealed by the teacher, who called for students to suggest what it could be.

There were 19 students in the room that day, and the teacher asked each of them what they thought the mystery object was. The teacher let the students associate freely and sometimes stopped to ask some of them for their reasons for their guesses.

S1	A star.
Teacher	A star, why do you think it is a star, S1?
S1	Because it has an edge in the beginning.
Teacher	It has an edge in the beginning.
S1	A triangle in the beginning, and stars have that on their sides.
Teacher	Mhm. Yes, we'll have to see if it has that on all the sides. S3, what do you think?
S3	Star!
Teacher	You think it's a star as well. S4?
(...)	

True to the nature of a dilemma, the teacher both gained and lost something by asking each of the 19 individual students in this manner. On the one hand, every student got a chance to contribute to the discussion and to be creative. The students offered varied suggestions such as a star, lightning, a triangle, a rocket, or a boomerang. The teacher also took the opportunity to probe the thinking of some of the students, such as with S1 above. At a general level, participating in mathematical discussions at school is part of what students in this age group must learn, something this activity achieves. On the other hand, the pace of the lesson was slowed considerably, and there was a lot of wait time for each student. The discussion concluded mathematically with a discussion of the concepts of *edge* and *corner* (which was the goal of the activity), but it took a lot of time getting there, and many students remained silent after offering their suggestion for the mystery object. This episode illustrates the trade-off teachers often do between broad adaptation and the pace of the lesson.

Discussion and Conclusion

This study showed how internal tensions between different teaching principles belonging to a reform-oriented teaching system led the participants to experience some dilemmas while teaching mathematics. These dilemmas are not exclusive to the DEM system; any teacher who wishes to challenge their students mathematically by adopting new teaching materials and practices could find themselves experiencing similar dilemmas. Nevertheless, these dilemmas came out in especially sharp relief in this study. A possible reason could be that the theoretical principles and teaching materials for DEM are very explicit. This is a known paradox; the more specific textbook authors try to be when explaining teaching principles and providing suggestions for how to use tasks, the more likely teachers are to interpret the teacher guide as instructions to be followed. This may result in the loss of the spontaneity involved in teaching (Mason & Johnston-Wilder, 2006).

The dilemmas identified in this study provide cause for curriculum developers and researchers studying educational implementation processes to pause and reflect. When developing curricula and teaching materials at various levels, authors should consider whether teaching principles or material components could be in conflict in ways that lead teachers to experience dilemmas, and contemplate how to present their work to interested teachers. Any comprehensive teaching system (such as DEM) will have built-in tensions since the work of teaching is inevitably fraught with competing responsibilities. This again has implications for how to approach implementation of educational innovations. For example, in implementation research, the concept of *fidelity* is often given much attention (Jankvist et al., 2022). However, if the object of implementation itself contains internal contradictions, then the concept of fidelity needs to be reworked as the question arises, "How can the teacher stay completely true to a teaching system if its components conflict with each other?"

Alternatively, focusing on dilemmas could prove fruitful in the context of professional development of mathematics teachers. First, awareness of teaching dilemmas is important. If the dilemmas become too prominent, they can be a source of frustration for teachers, who could feel that they are not succeeding with their change of practice. This could lead to the conclusion that the reform-based practice "does not work" and to the teacher reverting to their previous practice. Knowledge is needed about how to avoid such situations and how to best support teachers trying to change. For instance, both teachers and professional development facilitators should be aware that researchers and curriculum developers sometimes present idealised images of reform-based practices, which do not always correspond to the reality of each classroom. Ideal images of practice are difficult for teachers to live up to, which could lead them to feel insufficient (Brodie, 2010). Making teachers aware that dilemmas are a natural part of mathematics teaching can help in this situation.

Furthermore, dilemmas can be leveraged directly for the purpose of furthering professional development, since they could be growth points for a teacher's practice (Caspari-Gnann & Sevian, 2022). Addressing and struggling with dilemmas of practice can help make teachers' collective work focused and inquiry oriented (Mellroth et al., 2021), and in general, helping teachers notice, articulate, and address dilemmas in their practice could help them move past the *implementation plateau* (Silver et al., 2011). The participants in this study expressed to a certain degree that they had "found their place" within DEM, which is indicative of an implementation plateau. To further their professional development and to gain more knowledge about how DEM could be implemented effectively in schools, it could be productive to form communities of practice with the purpose of addressing the teaching dilemmas experienced.

To support such professional development efforts, there is a need for both better conceptualisation and further empirical research on teaching dilemmas. A common vocabulary would aid in this process. Moreover, appropriate research methods to access and analyse teaching dilemmas should be worked out. This study provides a small contribution to this end. There are, however, limitations to the present study: since teaching dilemmas are personal experiences of the teachers, there is a need to access and analyse the reflections of the teachers themselves. This could, for example, be done through video recall interviews. Another promising method is *teacher time-outs*, which allows teachers and coaches to stop a lesson in the moment and address uncertainties on the spot, combining insider and outsider perspectives (Mosvold et al., 2023).

Finally, the findings of this study could have some implications for pre-service teacher education. Many aspiring teachers come to the university with an expectation that they will learn "how best to teach." Despite the introduction of "high-leverage practices" in mathematics teacher education (e.g., Bailey & Taylor, 2015), the picture is not as clear as that. The findings of this study indicate that some high-leverage practices could potentially conflict with each other. If, as Floden and Clark (1988) suggested, teacher education should be about preparing teachers for the coming uncertainties of the classroom, then teacher students should be offered rich opportunities to experience, analyse, and reflect on important dilemmas of mathematics teaching.

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Appendix: A Complete Thematic Map for the Focus Group Interviews