Comparative Analysis of Students' Argumentation Patterns in the Context of Algebraic Problems

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Abstract: The objective of this study was to evaluate and characterize the argumentation patterns used by seventh-grade students in the context of algebraic addition and subtraction problems. A qualitative case study was conducted using the Claim-Evidence-Reasoning (CER) model (McNeill & Krajcik, 2008) and argument quality framework. To describe the arguments put forth by the participants, a sample of ten students with high mathematics proficiency and ten students with low mathematics proficiency were selected from the target population of junior high schools in a large region of West Java, Indonesia. Data collection was carried out using written argumentation frame for algebraic operations (AFAO). The results revealed that, although none of the students employed the $C_H+E_H+R_H$ *pattern (which represents the highest quality of arguments), students with high mathematics proficiency levels exhibited a greater prevalence of these argumentation patterns compared to those with low mathematics proficiency levels. The findings have implications as a valuable resource for teachers in monitoring the advancement of their students and preventing or alleviating diverse difficulties or inaccuracies that they may face. Based on the findings, a specific design for a classroom teaching activity is proposed.*

Keywords: Achievement Gaps, Argumentation Patterns, Algebraic Expressions, Mathematics Proficiency, Mathematical Argumentation

INTRODUCTION

Argumentation is an essential skill for junior high school students (Knudsen et al., 2014; Ayalon & Even, 2016; Campbell et al., 2019) to develop, construct, and communicate their mathematical knowledge (Stylianides, 2018). In addition, through the process of argumentation, students' reasoning skills, communication, social behavior, and information-gathering abilities also improve, and it can foster students' conceptual understanding and critical thinking (Nussbaum, 2011). The process of argumentation also serves as a foundation for changing individuals' viewpoints, as it enables the development and restructuring of ideas through analytical thinking, leading to the acquisition of knowledge, and at the core of this process is the notion of cognitive transformation, which entails a modification in cognitive frameworks (Chadha & Van Vechten, 2017).

Furthermore, developing students' mathematical argumentation skills has become a focus of attention in curricula in various countries, and cultivating this skill has become a primary goal in education (Schwarz, 2009; Kollar et al., 2014; Fukawa-Connelly & Silverman, 2015). For example, in the United States, the Common Core State Standards for Mathematical Practice (CCSSMP) includes standards for argumentation. The CCSSMP requires students to "construct viable arguments and critique the reasoning of others" (National Governors Association Center, [NGAC], 2010). This approach encompasses the comprehension and application of assumptions, the development of logical arguments, and the formulation and examination of conjectures in a systematic and rational manner (Lesseig et al., 2019). Principles and Standards for School Mathematics states that "instructional programs should enable students to develop and evaluate mathematical arguments and proofs" (NCTM, 2000).

Although the concept of argumentation is not explicitly stated in the current *Indonesian Mathematics Curriculum* (IMC; Kemendikbud, 2018) for junior high school (Years 7-9), some key elements in the standards define the issue. In the current IMC states that the learning process is developed on the principle of active student learning through activities such as observing, questioning, analyzing, and communicating, as well as strengthening critical learning patterns. Therefore, there is a relationship between the demands of the Indonesian Mathematics Curriculum for junior high school and argumentation skills, specifically in the aspect of communication.

Even though argumentation patterns are useful for identifying and evaluating argument structures and for discovering and producing complete and strong arguments (Macagno & Walton, 2015). Argumentation patterns are also useful for assessing the quality of argumentation (Zalska & Tumova, 2015). From this perspective, students' argumentation skills can be enhanced by teaching them how to construct better arguments, which are complete in all their components (including, besides data and claims, other elements that are usually missing, such as backing or warrant). Argumentation patterns are also crucial for junior high school students to evaluate their own or others' arguments. Their arguments must be sufficiently evidence-based to persuade others, and their reasons must be clear to be evaluated by others (Campbell et al., 2019).

In light of many junior high school students in the United States believe that arguments are contrary to mathematical standards when they take math classes (Forman et al., 1998). Students may think that arguments are not necessary because each problem requires a specific strategy of correct solutions (which the teacher can or should provide). This is certainly a problem, contrary to the importance of the mathematics proficiency to argue for students. As a result, students in secondary schools often have difficulty in the process of mathematical argumentation (Schwaighofer et al., 2017). One of them is that secondary school students cannot use justification and reason to support their allegations (Kuhn & Moore, 2015; Mayweg-Paus & Macagno, 2016).

In mathematics education documents, as well as in the mathematical research, arguments are considered vital to mathematics education, but little attention is paid to the pattern of arguments in secondary mathematics classrooms. The objective of this study was to bridge this gap in the literature on mathematical arguments by evaluating and characterizing the written algebraic arguments of students. More specifically, a research question has been raised: How are the patterns structured in students' argumentation during this task?

THEORETICAL FRAMEWORK

In 1958, Stephen Edelston Toulmin introduced a model that represented the "layout of arguments" (van Eemeren et al., 1996) in his book *The Uses of Argument*. This model has been used in numerous textbooks on argumentation for the analysis, evaluation, and construction of arguments. Toulmin's perspective on argumentation has also had a significant impact on a more theoretical level. In a more practical sense, Toulmin's model is frequently used to analyze argumentation (Metaxas et al., 2016; Doğan & Yıldırım Sır, 2022), often used in studies of written or verbal mathematical argumentation (Zambak & Magiera, 2020). Toulmin's model of argumentation has proven to be effective in analyzing argumentation skills and can also be used as a learning approach for students to construct arguments (Jonassen & Kim, 2009; Metaxas, Potari, & Zachariades, 2016).

In mathematics education research, Krummheuer was the first to apply Toulmin's argumentation scheme (Inglis et al., 2007; Moutsios-Rentzos et al., 2019). Specifically, Krummheuer (1995) only selected three core parts of the argumentation in his work, namely data, claim, and warrant (Conner, et al., 2014). Krummheuer believed that the other three components, namely backing, rebuttal, and qualifier, were less relevant to apply in the context of mathematics. Moreover, reducing the complexity of the Toulmin model is highly useful for students at the school level who may have difficulty applying the scheme in full to identify argumentation components (Kollar et al., 2007).

Some other researchers followed Krummheuer's lead by using only the elements of the core argument (Conner et al., 2014). Based on previous research, the study focuses on three main components of arguments, namely data, claims, and warrants. One model adapted to the Toulmin model is the CER model.

The CER (Claim-Evidence-Reasoning) model was derived from the more complex Toulmin argument model, adapted for use in science education (McNeill & Krajcik, 2008). Fielding-Wells (2016) applied the CER model to mathematical argumentation in primary school students in

Australia, while Graham & Lesseig (2018) did so for high school students in America. Zambak & Magiera (2020) used the model to evaluate the mathematical argumentation abilities of prospective primary and secondary school teachers. They simplified the Toulmin model's three main components for analyzing written argumentation abilities into evidence, reason, and claim, as illustrated in Figure 1.

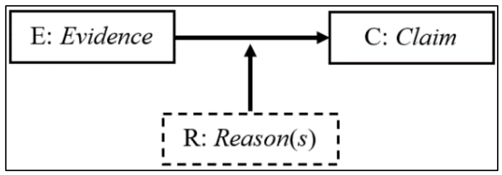

Figure 1: Model for Analyzing Written Argumentation (Zambak & Magiera, 2020)

Claim refers to a student's statement about problem-solving. Evidence is the information collected and used by students to support the truth of the claim. Meanwhile, reason is defined as the justification presented by students to reduce uncertainty and to express a comprehensive solution to all potential issues (Zambak & Magiera, 2020).

Moreover, Krummheuer (1995) asserted that a claim refers to a deduction that can be articulated either prior to or subsequent to the presentation of data (evidence). The relationship between the claim and the evidence can be identified by using terms such as "therefore" or "because." When the evidence is presented first, the reasoning is acknowledged as "evidence, so claim." Conversely, when the claim is presented first, the progression of the argumentation is described as "claim because evidence". Additionally, the term "since" can aid in delineating the correlation between data, claims, and warrants (reasoning). Consequently, the expressions "claim because of data, since reasoning" or "data becomes claim, since reasoning" can portray the progression of an argument (Nordin & Boistrup, 2018).

Within the field of mathematics education, the term "argumentation" can refer to two distinct concepts. Firstly, it may refer to mathematical arguments put forth by both students and teachers within a classroom setting. Secondly, it may refer to arguments made by researchers in mathematics education, pertaining to the nature of mathematical learning and the effectiveness of teaching mathematics in different contexts. According to Sriraman and Umland (2020), mathematical argumentation within the classroom involves presenting a logical sequence of reasoning intended to demonstrate the validity of a mathematical outcome. In the realm of mathematics education research, numerous scholars have emphasized the value of integrating argumentation-based activities in the classroom, as a means of promoting students' comprehension of mathematical concepts and their ability to reason mathematically (Erkek & Bostan, 2019).

Mathematical argumentation is a particular type of conversation characterized by justification, association, and the use of ideas (Ibraim & Justi, 2016; Uygun & Guner, 2019). This discourse is aimed at determining the truth of mathematical statements (Knudsen et al., 2014; Rumsey & Langrall, 2016). It can be defined as a series of statements and reasons aimed at demonstrating the validity of a claim (Cardetti, & LeMay, 2018).

Mathematical argumentation involves a range of activities such as conjecturing, testing examples, thought experiments, representing mathematical ideas, taking other perspectives, analyzing, and revising (Staples & Newton, 2016). It requires students to respond to claims made by others with their own arguments and counterarguments, construct explanations, ask questions, and potentially refute others' arguments. Mathematical argumentation is the process of constructing arguments to demonstrate or explain the truth of mathematical statements or solutions to mathematical problems. Since it is a social activity, it is important for researchers to observe and analyze what is happening so that they can understand the nature of students' argumentation.

METHOD

The present study used a design of qualitative case studies to examine the achievements and skills of seventh grade students in cognitive tasks integrated in the argumentation of selected public schools. To this end, the data collection was carried out using a written argumentation frame for algebraic operations (AFAO). To analyze the data, a specific benchmark is designed to assess content and competence using rating scales and descriptions and existing tools are adapted to measure the learning performance and skill of learners on the task.

Participants and Context

The determination of research participants in this study consists of four stages, namely: 1) selecting a sample from a selected school, 2) administering a mathematics proficiency test, 3) assessing students' mathematics test responses, and 4) determining participants based on differences in mathematics proficiency. The first stage involves selecting a sample. Six classes of seventh-grade students were purposively selected from a public school in a West Java province, Indonesia, out of a total of 12 classes surveyed. The sample was selected through systematic random sampling, using the numerical or alphabetical order of class naming as a basis. For instance, classes A to F were included in the sample.

The second stage is to administer a Mathematics Proficiency Test (MPT) to the selected classes of students. The MPT consists of 23 multiple-choice questions developed by the researcher and validated. This activity was carried out on February 6, 2023, during the first learning session. During the test administration, each class was supervised by one teacher. The participants in the Mathematics Proficiency Test were 225 students, consisting of 90 male students (40%) and 135 female students (60%). The students' ages ranged from 12-14 years (Mean age = 13.08 years, Standard Deviation = 0.55).

The third stage involved scoring students' MPT responses. The scores of the students were then ranked from highest to lowest. Finally, 10 students with the highest MPT scores and 10 students with the lowest MPT scores were selected. The results of the student selection are presented in Table 1.

High Group		N	10	Low Group		N	10
Rank	ID	Class	Score	Rank	ID	Class	Score
1	Student 1	E	22	225	Student 225	B	2
2	Student 2	F	22	224	Student 224	A	3
3	Student 3	F	21	223	Student 223	E	4
4	Student 4	F	21	222	Student 222	A	4
5	Student 5	A	20	221	Student 221	E	6
6	Student 6	A	20	220	Student 220	E	6
7	Student 7	B	20	219	Student 219	C	6
8	Student 8	D	20	218	Student 218	B	6
9	Student 9	F	20	217	Student 217	F	7
10	Student 10	A	19	216	Student 216	F	7

Table 1: Research participants

Data Collection Process

Data were collected by the first author in February 2023. Other teachers were also present to assist with classroom management during data collection. The test was written in Indonesian and was conducted individually in a quiet classroom.

The data in this study were obtained using the Argumentation Frame in Algebraic Operations (AFAO), as presented in Appendix 1. The AFAO was developed based on a question construction framework that was predetermined by the researchers and validated by three expert judges. The CVI scores for the instrument's sufficiency, clarity, coherence, and relevance dimensions ranged from 0.90 to 1.00, indicating excellent content validity (Polit & Beck, 2006). The study utilized the revised Bloom taxonomy cognitive level for evaluating (Anderson & Krathwohl, 2001), which included indicators to verify solutions to algebraic addition and subtraction problems.

The twenty selected participants were provided with AFAO-1, AFAO-2, and AFAO-3 sheets to be completed within a maximum time of 40 minutes. AFAO-2 and AFAO-3 had the same type and content as AFAO-1, with the only difference being the numerical values presented in the questions. AFAO-2 and AFAO-3 were used as triangulation to verify the credibility of the students' answers. The research question was answered using the AFAO instrument (Figure 2). In summary, AFAO is a written survey tool that has been specifically designed to assess and elicit the ideas of learners that are embedded within argumentation.

Ani merupakan siswa kelas VII di salah satu SMP di Kota Bandung. Pada suatu ulangan matematika, Ani diberi soal $(5a - 7b) + (13a + 8b)$. Proses penyelesaian soal yang dikerjakan oleh Ani adalah sebagai berikut: Langkah -1 $(5a - 7b) + (13a + 8b)$ Langkah -2 $5a - 7b + 13a + 8b$ Langkah -3 $-2a + 21b$ Periksa langkah demi langkah proses penyelesaian soal yang dilakukan Ani. Benar atau salahkah pekerjaan Ani? Jelaskan mengapa!	Ani is a student in the seventh grade attending one of the junior high schools in the city of Bandung. While taking a mathematics examination, Ani was presented with a mathematical problem in the form of $(5a - 7b) + (13a + 8b)$. The solution process utilized by Ani is outlined as follows: Step-1: $(5a - 7b) + (13a + 8b)$ Step-2: $5a - 7b + 13a + 8b$ Step-3: $-2a + 21b$ Please verify, in a step-by-step manner, the problem-solving process executed by Ani. Determine whether Ani's work correct or incorrect? Explain why!

Figure 2: The argumentation task frame (AFAO)

The responses in the AFAO instrument were independently coded by the first author and reviewed by the second and third authors. Moreover, efforts were made to establish a coherent link between the task and the inferences drawn from the participants' responses. To ensure the credibility of the data, this study employed the within-method triangulation (Denzin & Lincoln, 2017). The procedure involved comparing the data obtained from AFAO-1 with that of AFAO-2. If there was consistency between the two sets of data, the first data was considered valid, credible, and suitable for analyzing the research student's argumentation process. However, if inconsistency was observed, it was compared with the data obtained from AFAO-3. This process could continue until consistent data was obtained to be used in the data analysis to reveal the pattern of the research student's argumentation.

Based on data obtained from the work of high-mathematical students in AFAO-1, AFAO-2 and AFAO-3, nine ($n = 9$) data were considered reliable. The only data point of student 50 was deemed unreliable (Figure 3). A non-credible data point was considered invalid. Thus, the nine credible data points obtained from the work of AFAO-1 or AFAO-2 are valid and can be used for analysis.

Furthermore, in the data obtained from the low math ability students' work, there was one data point from student 219 that was deemed not credible as it did not follow AFAO-2 and AFAO-3, thus its data credibility could not be tested. Therefore, one non-credible data point was considered invalid. Therefore, the nine credible data points obtained from the work in AFOA-1 or AFOA-2 were valid and could be used for analysis.

1. Salah karena adanya Perpindahan tempat Jadi $5a - 7b + 13a + 8b$ $= -8a + 12b$ AFHA-1	"$\boxed{\text{Wrong}}$ due to the displacement of location. Therefore, $5a - 7b + 13a + 8b = -8a + 12b$."
1. Benar, karena Perpindahan yang sangat tepat AFHA-2	"$\boxed{\text{Correct}}$ due to the very precise displacement."
1. Benar. $-2a + 17b$ itu hitungan yang baik AFHA-3	"Correct $-2a + 17b$ is a good calculation."

Figure 3: Example of non-credible data (Student 50)

Data Analysis Process

We conducted a four-step analysis of the data. In the first step, we identified the arguments that the student understood when examining the algebraic addition and subtraction problem-solving. To accomplish this process, the researchers calculated the percentage of students who answered "*incorrectly*" to the question "*Check step-by-step the problem-solving process done by Ani. Is Ani's work correct or incorrect*?". This step was critical to verify if the question was comprehensible to the students. The second step was to identify the arguments that the student used to justify (evidence and reasoning) when examining the problem-solving. To accomplish this process, the researchers calculated the percentage of students who provided justifications for the prompt, "*Explain why*!".

The third step was to identify patterns in the responses of the students to the algebraic operations problems based on the dimensions of structure, content, and recipient-orientation (Meyer & Schnell, 2020). In terms of the dimension of structure, this research utilized the CER (Claim, Evidence, Reasoning) model. Using the CER model, the researchers focused on identifying the claims made by each student about the process of problem-solving. The researchers also looked for evidence and reasoning provided by the students to support each claim. Furthermore, the researchers created argument maps by summarizing all the arguments formulated by the students. In each map, the researchers explained all the relationships between the evidence, reasoning, and claims included by the student in their solution narrative.

Component of Argument	Quality	Description	Code
Claim	Low	Not making any claims, or making inaccurate or false claims.	C_L
	Moderate	Making claims that are accurate but incomplete	C_M
	High	Making a claim that is both accurate and complete.	C_H
Evidence	Low	Not providing evidence, or only providing inappropriate evidence (evidence that does not support the claim).	E_L
	Moderate	Providing precise but insufficient evidence to support a claim. This may include some incongruous pieces of evidence.	E_M
	High	Providing precise and sufficient evidence to support a claim.	E_H
Reasoning	Low	Failure to provide reasoning, or only providing reasoning that does not connect evidence to claims.	R_L
	Moderate	Providing reasoning that connects claims and evidence, reiterating evidence and/or incorporating some, but insufficient scientific principles.	R_M
	High	Providing reasoning that connects evidence with claims, encompassing appropriate and adequate scientific principles.	R_H

Table 2: Guidelines for assessing argument quality (McNeill & Krajcik, 2008)

In addition, Meyer & Schnell (2020) stated that several questions can be asked to identify the structure of arguments. For instance, does the claim answer the question? Is there evidence to support the claim? Is there reasoning to explain how the evidence supports the claim? and so on. The content dimension is related to the use of mathematical rules included in the argument structure, such as whether the mathematical rules used are correct. Meanwhile, the recipient-orientation dimension is related to the language aspect, such as whether the presented argument can be understood as a whole. The final step was to identify the quality of each student's argument components. The researchers adapted the argument assessment guidelines from McNeill & Krajcik (2008) and as shown in Table 2.

RESULTS AND DISCUSSION

The students' response to the question "*Check step-by-step the problem-solving process carried out by Ani. Is Ani's work correct or incorrect?*" is presented in Table 3.

Group	Percentage of claims	
	"*Incorrect*"	"*Correct*"
High ($n = 9$)	88,9%	11,1%
Low ($n = 9$)	11,1%	88,9%

Table 3: Percentage of responses based on mathematics proficiency levels

As shown in Table 3, from the data of nine high-level mathematics proficiency students that were deemed valid and credible, eight students (88.9%) claimed "*Incorrect*". Only one student (11.1%) claimed "*Correct*". This indicates that the majority of students understood the presented questions and problems. However, these results were in contrast to those of students with low-level mathematics proficiency. From the data of nine students deemed valid and credible, only one student (11.1%) claimed "Incorrect". Eight students (88.9%) claimed "Correct". This suggests that the majority of students with low-level mathematics proficiency struggled to understand the presented questions and problems.

The justification for the "*Explain why!*" question based on the obtained data showed that all students with high-level mathematics proficiency ($n = 9$) provided evidence and reasoning. In contrast, among students with low-level mathematics proficiency ($n = 9$), only three students provided evidence, and six students provided reasoning. Additionally, the distribution of the quality of claims, reasoning, and evidence on the first problem was also identified, as presented in Table 4.

Component of Argument	Quality	High Group Frequency ($n = 9$)	Low Group Frequency ($n = 9$)
	Low	1 (11,1%)	8 (88,9%)
Claim	Moderate	5 (66,7%)	1 (11,1%)
	High	3 (33,3%)	0 (0,0%)
	Low	1 (11,1%)	8 (88,9%)
Evidence	Moderate	7 (77,8%)	1 (11,1%)
	High	1 (11,1%)	0 (0,0%)
	Low	4 (44,4%)	8 (88,9%)
Reasoning	Moderate	0 (0,0%)	1 (11,1%)
	High	5 (55,6%)	0 (0,0%)

Table 4: The distribution of argumentation component levels in the first problem

As presented in Table 4, most high-proficiency mathematics students have moderate levels of claim quality (55.6%) and evidence quality (77.8%), with only few students having low (11.1%) or high (11.1%) quality. Meanwhile, the majority of students have high quality reasoning (55.6%), and 44.4% are categorized as having low quality. No students have moderate quality reasoning. These findings indicate that the majority of high-proficiency mathematics students possess good skills in constructing mathematical arguments.

Furthermore, students with low mathematics proficiency mostly have low quality claims, evidence, and reasoning. Only one student (11.1%) had moderate quality claims, evidence, and reasoning. These results indicate that the majority of students with low mathematics proficiency struggle with constructing mathematical arguments.

Based on the quality of arguments presented in Table 4, this study identifies six patterns of students' mathematical argumentation in solving algebraic addition and subtraction problems as follows: 1) $C_L+E_L+R_L$. Low claim, low evidence, and low reasoning, 2) $C_M+E_M+R_L$. Moderate claim, moderate evidence, and low reasoning, 3) $C_M+E_M+R_H$. Moderate claim, moderate evidence, and high reasoning, 4) $C_H+E_M+R_L$. High claim, moderate evidence, and low reasoning, 5) $C_H+E_M+R_H$. High claim, moderate evidence, and high reasoning, and 6) $C_H+E_H+R_L$. High claim, high evidence, and low reasoning. The distribution of argumentation patterns constructed by students with high mathematics proficiency level (HMP) and low mathematics proficiency level (LMP) is presented in Figure 4.

As shown in Figure 4, the distribution of argumentation patterns is highest among students with high mathematics proficiency levels (6 patterns), while only 2 patterns are found in students with low mathematics proficiency levels. At the HMP level, the majority of students (f=4, 44.5%)

exhibit the argumentation pattern of C_M+E_M+R_H, while only one student each (*f*=1, 11.1%) exhibit the patterns of C_L+E_L+R_L, C_M+E_M+R_L, C_H+E_M+R_L, C_H+E_M+R_H, and C_H+E_H+R_L. At the LMP level, the majority of students (f=8, 88.9%) exhibit the argumentation pattern of C_L+E_L+R_L. Only one student (11.1%) exhibits the pattern of C_M+E_M+R_H. Although there is no high-quality argumentation (C_H+E_H+R_H pattern), the HMP level provides better arguments than the LMP level.

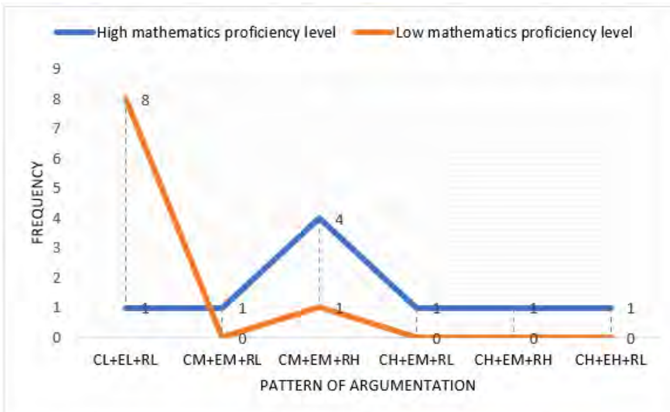

Figure 4: Distribution of argumentation patterns

Argumentation Patterns of Students with High Mathematics Proficiency Level

C_L+E_L+R_L Pattern

Figure 6 below presents the results of the work of student 6 who has high mathematics proficiency using the C_L+E_L+R_L argumentation pattern.

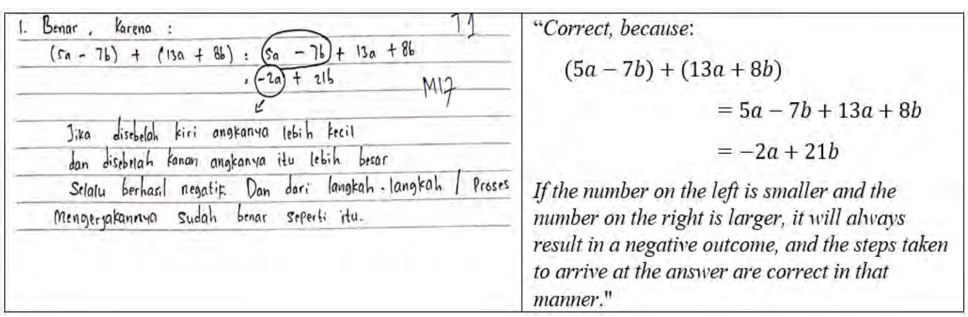

Figure 5: Student 6 Response to problem

As shown in Figure 6, the student made a false claim by stating that Ani's problem-solving process was correct. This resulted in the student providing incorrect or irrelevant evidence and reasoning. The student provided reasoning using the rule of subtracting numbers (incorrect concept) which led to step $5a - 7b = -2a$ being considered correct. The student did not understand the presented problem. The student also did not understand the rules of subtraction or addition in algebraic forms, that subtraction or addition of algebraic forms can only be done if the terms are of the same type.

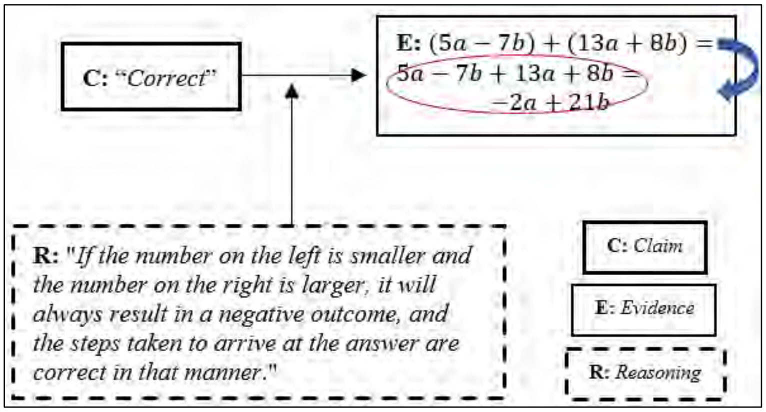

Figure 6: Student 6 Argument map on problem

C_M+E_M+R_L Pattern

The result of the work of student 9, who has high mathematics proficiency with the C_M+E_M+R_L argumentation pattern, is presented in Figure 7.

1. Salah. M198 karena, jawaban yang benar adalah : Langkah -1 : $(5a - 7b) + (13a + 8b)$ Langkah -2 : $(5a \times 13a + 5a \times 8b) + (-7b \times 13a + -7b \times 8b)$ Langkah -3 : $65a^2 + 40ab + -91ab + -56b^2$ (Jumlahkan bilangan yang 1 suku) Langkah -4 : $65a^2 - 51ab - 56b^2$ (hasil)	*"Incorrect.* *Because, the correct answer is:* Step-1: $(5a - 7b) + (13a + 8b)$ Step-2: $(5a \times 13a + 5a \times 8b) + (-7b \times 13a + -7b \times 8b)$ Step-3: $65a^2 + 40ab + -91ab + -56b^2$ (*"Calculate the sum of a single term."*) Step-4: $65a^2 - 51ab - 56b^2$ (*"result"*)

Figure 7: Student 9 Response to problem

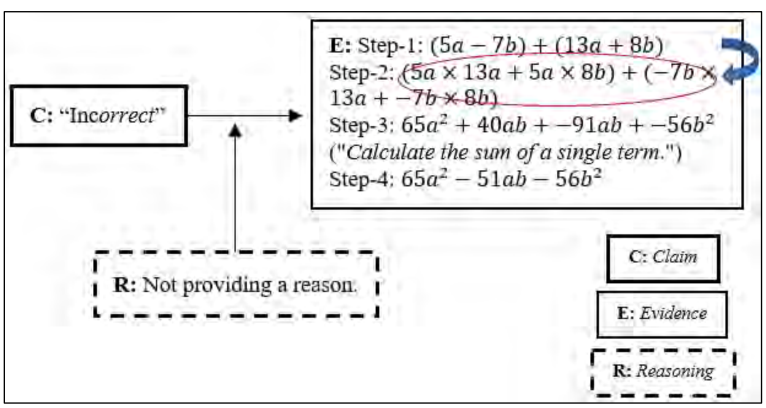

Figure 8: Student 9 Argument map on problem

As shown in Figure 8, the student made a true claim by stating that the solving process performed by Ani was incorrect. However, the student did not provide a reason. Although the student provided evidence, there was evidence that did not match. In the first step, the student provided appropriate evidence by rewriting the problem that needed to be solved. However, in the second step, the student solved the problem using the wrong method ($(5a - 7b) + (13a + 8b) = (5a \times 13a + 5a \times 8b) + (-7b \times 13a + -7b \times 8b)$). The student used the wrong concept by applying the distributive law $a \times (b + c) = a \times b + a \times c$ to solve the problem. The student also did not pay attention to the rule of arithmetic operations in the brackets. These results indicate that the student understands the problem, but does not provide a reason. Although providing evidence, there is an inappropriate process.

C_M+E_M+R_H Pattern

Figure 9 shows an example of the work results of students 1 and 5 who have high mathematics proficiency with the argumentation pattern of C_M+E_M+R_H.

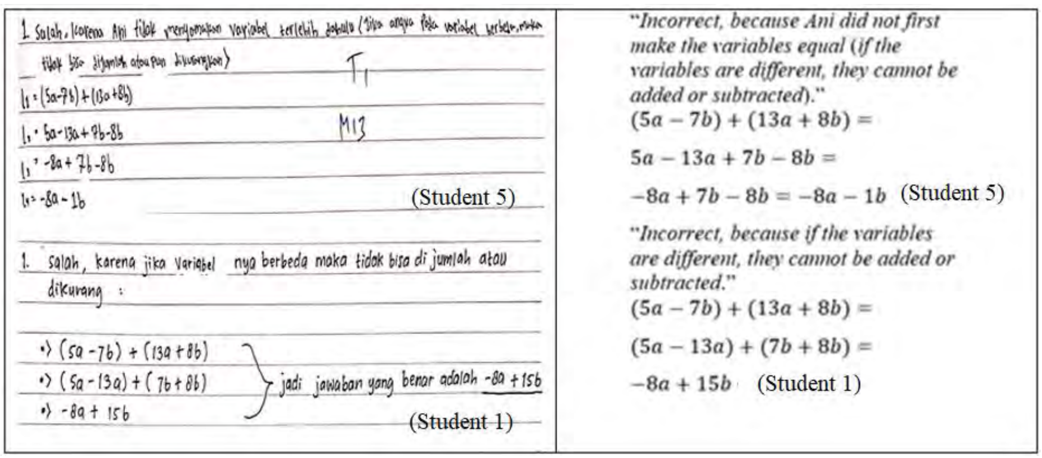

Figure 9: Example responses to problem (Student 1 and 5)

As shown in Figure 10, the student made a true claim by stating that the presented problem-solving process was incorrect. To support this claim, the student provided reasoning that the error in the problem-solving process occurred due to operating (adding or subtracting) different variables. Based on this reasoning, the student presented evidence in the form of problem-solving steps that they believed to be correct. In the process, the initial step taken was correct by grouping like terms. However, in the step $(5a - 7b) + (13a + 8b) = 5a - 13a + 7b - 8b$, there was an incorrect process by changing the operation sign (from + to – or vice versa). This result shows that the student was unable to order the algebraic operation steps involving remove brackets when grouping like terms.

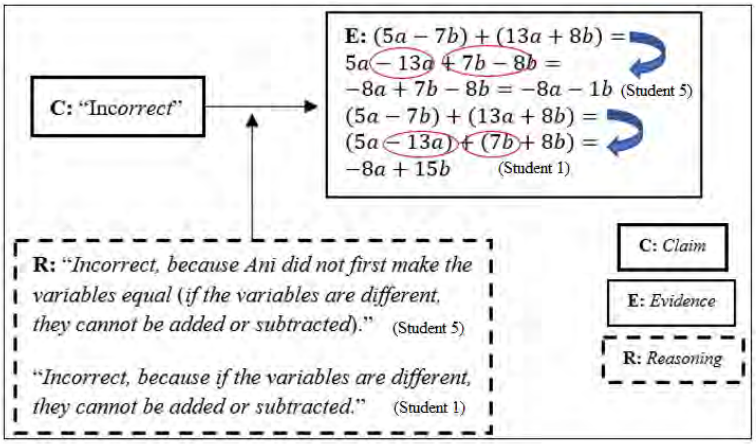

Figure 10: Argument map on problem (Student 1 and 5)

C_H+E_M+R_L Pattern

The results of the work of student 4, who possesses high mathematical abilities and employs the C_H+E_M+R_L argumentation pattern, can be found in Figure 11.

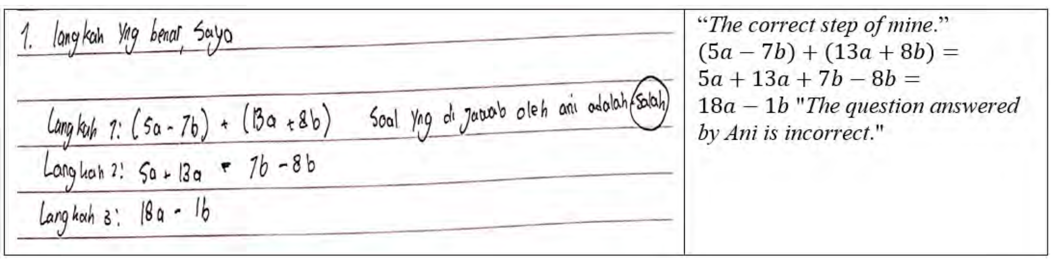

Figure 11: Student 4 response to problem

As shown in Figure 12, the student made a complete claim that "Ani's answer is incorrect". However, the student did not provide reasoning related to the concept used in the presented problem-solving process. Meanwhile, the student presented evidence to support the claim. In the process, the evidence presented was appropriate, but there was an incorrect problem-solving step. In the step $(5a-7b)+(13a+8b)=\ 5a+13a+7b-8b$, there was a change in the operation sign in the term $8b$ (from $+8b$ to $-8b$). This result indicates that the student had difficulty in ordering the operation steps involving remove brackets.

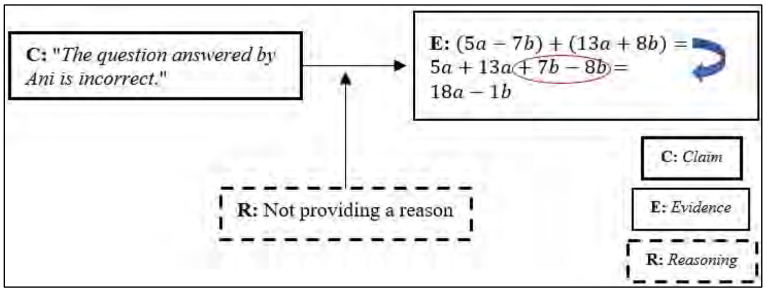

Figure 12: Student 4 argument map on problem

C_H+E_M+R_H Pattern

Figure 13 presents the results of the work of student 8, who possesses high mathematical abilities and employs the C_H+E_M+R_H argumentation pattern.

1.). Jawabannya adalah Salah. Karena : Ani di beri Soal $(5a-7b)+(13a+8b)$. Saat ani menjawab Soal tersebut, Ia tidak menyamakan variabel nya. Seharusnya. Langkah-1 $(5a-7b)+(13a+8b)$ Langkah-2 : $(5a-13a)+(7b+8b)$ Langkah-3 : $-8a+15b$	*"The answer provided by Ani is incorrect."* *"Due to receiving a problem* $(5a - 7b) + (13a + 8b)$*, Ani failed to equate the variables when answering the problem. The correct approach should have been to do so."* Step-1: $(5a - 7b) + (13a + 8b)$ Step-2: $(5a - 13a) + (7b + 8b)$ Step-3: $-8a + 15b$

Figure 13: Student 8 response to the problem

Figure 14 depicts that the student made a claim that the given answer is incorrect due to the use of different variables in the operations. Moreover, the student presented evidence in the form of an alternative problem-solving approach. In step $(5a - 7b) + (13a + 8b) = (5a - 13a) + (7b + 8b)$, the student performed the required steps by grouping and operating with similar terms. However, there were modifications made in the operation signs, specifically in the terms $13a$ (changed from $+13a$ to $-13a$) and 7b (changed from -7b to +7b). Consequently, the final result of the problem-solving process was incorrect.

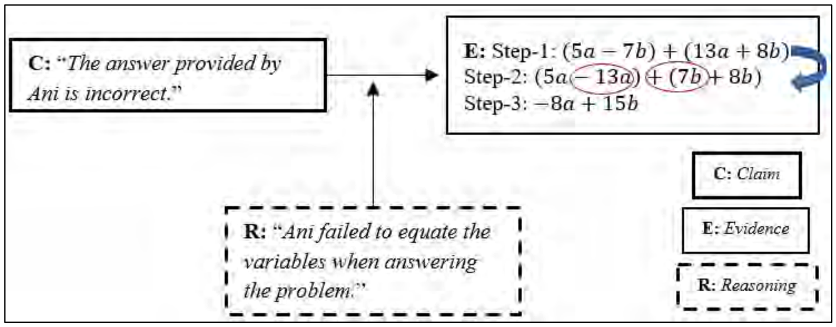

Figure 14: Student 8 argument map on problem

C_H+E_H+R_L Pattern

The findings of the performance of student 3, who exhibits a high level of mathematical proficiency with the argumentation pattern of C_H+E_H+R_L, are displayed in Figure 15.

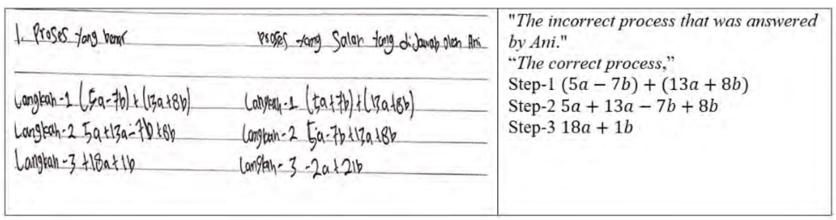

Figure 15: Student 3 response to the problem

Figure 16 illustrates that the subject made a distinct claim by stating that "Ani's answer process is incorrect." However, the subject did not provide reasoning pertaining to the location of the error in the presented problem-solving steps. Meanwhile, the subject put forth alternative evidence of a proper problem-solving approach that is adequate to substantiate the claim.

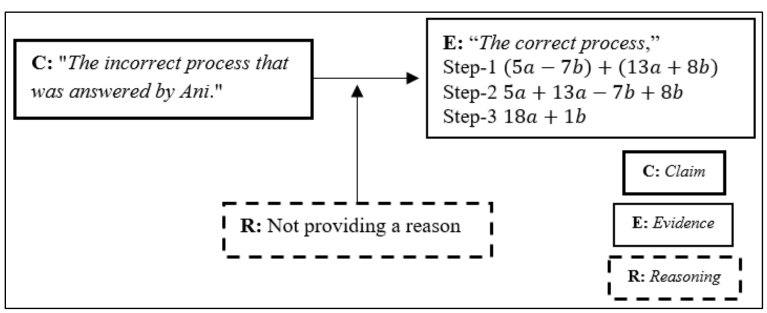

Figure 16: Student 3 argument map on problem

Argumentation Patterns of Students with Low Mathematics Proficiency Level

C_L+E_L+R_L Pattern

The outcomes of the student's responses based on this pattern comprise two types, namely: 1) simply rewriting the problem, and 2) providing arguments, but with inappropriate claim, reasoning, and evidence. Specifically, Figure 17 below presents an example of the student's response.

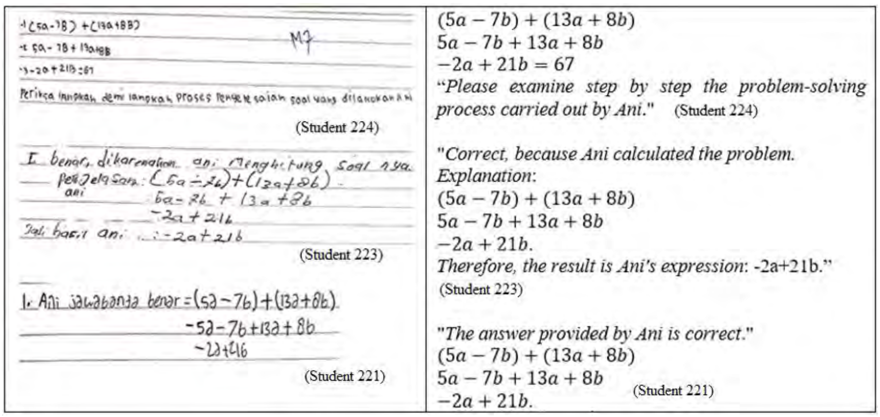

Figure 17: Example of student's response to problem (Student 221, 223 and 224)

As delineated in Figure 18, within the C_L+E_L+R_L argumentation pattern, an LMP student exists who abstains from expressing a viewpoint on the accuracy of the presented problem-solving steps. Additionally, the student refrains from presenting any reasoning or supporting evidence. Rather, the student merely reiterates the problem statement that was initially provided. Moreover, some students make erroneous claims by asserting the veracity of "Ani's answer". Consequently, the student proffers arguments that lack a cogent connection between the evidence and the claim. Furthermore, the student exclusively presents evidence by reciting the problem-solving process that was originally furnished in the query.

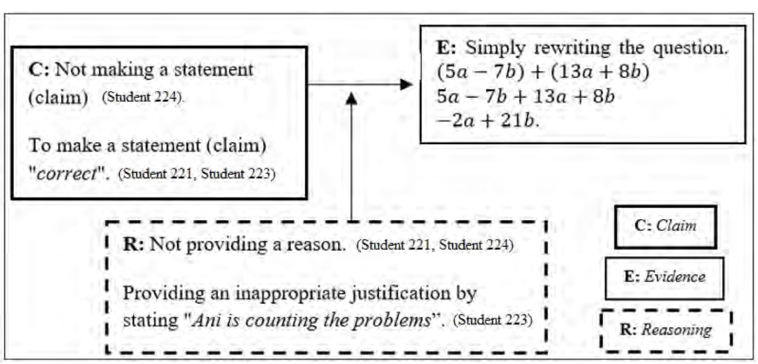

Figure 18: Argumentation map of student's response to the problem (Student 221, 223 and 224)

$C_M+E_M+R_H$ Pattern

The outcomes of the task completed by the student 217 who possesses low mathematical proficiency and employed the $C_M+E_M+R_H$ argumentation pattern, are displayed in Figure 19.

Soal: Ani diberi Soal $(5a-7b)+(13a+8b)$. 1) Jawab: $(5a-7b)+(13a+8b)$. $5a-7b+13a+8b$ $:-8a+15b$ Salah karna ani tdk Menyamakan Variabel.	Ani was given a task $(5a-7b)+(13a+8b)$. Respond: $(5a-7b)+(13a+8b)$ $5a-7b+13a+8b$ $-8a+5b$. *"Incorrect due to Ani's failure to equate the variables".*

Figure 19: Student 217 Response to the problem

As indicated in Figure 20, the student posits that the presented steps for problem-solving are erroneous. Additionally, the student cites the reason for this fallacy as the failure to perform operations using identical variables. Nonetheless, the solution proffered by the student does not correspond with the aforementioned rationale. The student offers an alternative solution; however, it entails performing operations on disparate variables.

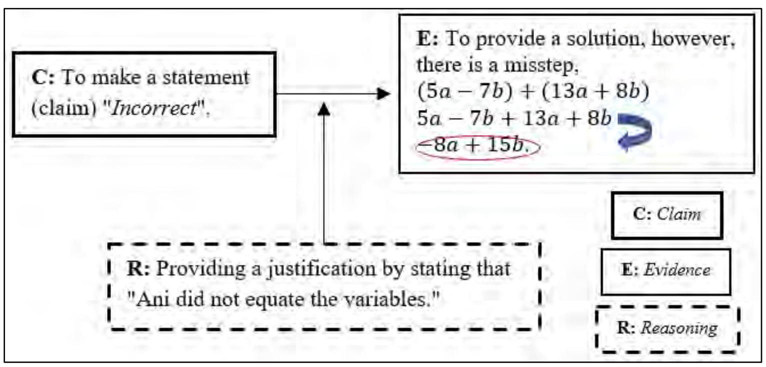

Figure 20: Student 217 Argument map on problem

CONCLUSIONS

The study examined the abilities of high and low proficiency mathematics students in constructing mathematical arguments while solving algebraic addition and subtraction problems. The research concludes that students with advanced mathematics skills are more effective at constructing persuasive arguments than those with lower proficiency. However, even among high proficiency students, limitations in their ability to argue effectively were observed. In accordance with the findings of Farra et al. (2022), students occasionally encountered difficulties in providing justifications for their responses, and a few resorted to reproducing the phrasing used in the questions.

The study identified six distinct patterns of mathematical argumentation that students utilized when solving algebraic addition and subtraction problems. Although none of the students employed the $C_H+E_H+R_H$ pattern (the highest quality of arguments), Students with high mathematics proficiency levels were found to exhibit a greater prevalence of these argumentation patterns than those with low mathematics proficiency levels. Students with high proficiency tended to rely on the $C_M+E_M+R_H$, $C_H+E_M+R_H$, and $C_H+E_M+R_L$ argumentation patterns. Nevertheless, even those students who utilized intricate argumentation patterns were not invariably capable of presenting thorough and precise arguments. In certain instances, they demonstrated inadequate understanding of algebraic concepts, such as the associative and distributive law, or made errors when solving problems, such as incorrect ordering of the algebraic operation steps involving remove brackets. Ojo (2022) stated that the utility and significance of algebra are commonly perceived to derive from its concepts and mode of reasoning.

We argue that the highest quality of arguments does not relate to particular mathematics proficiency levels. This implies that while argumentation patterns can serve as an advantageous instrument for students to enhance their mathematical argumentation and problem-solving proficiencies, they also require a profound understanding of algebraic concepts and procedures. In Addition, through comprehension of students' argumentation patterns, teachers can anticipate the possible procedures that students may undertake while solving a mathematical problem. Consequently, teachers can observe students' progress and preclude or mitigate various challenges or inaccuracies encountered by students.

Teachers should use effective teaching strategies that encourage their students to effectively communicate their mathematical knowledge using various methods, including verbal and non-verbal as well as written argumentation. For low-proficiency mathematics students, interventions that focus on improving their argumentation skills may be particularly beneficial. In this regard, teachers may need individualized support and guidance to help these students develop mathematical argumentation skills. Thus, it is essential to devise an instruction methodology that facilitates the adaptation of students to addressing supplementary algebraic problems. These problems require the use of argumentation patterns and algebraic concepts, while encouraging students to recognize the arguments they use in their explanations and identify any challenges they face during the activity.

RECOMMENDATIONS

Based on the findings of the study, a recommendation is put forth for a specific design of a classroom teaching activity comprising of seven sequential steps. First, introduce the topics of algebraic addition and subtraction problems and explain the importance of constructing persuasive arguments when solving them. Then, give a written argumentation frame for algebraic operations (AFAO) and ask students to solve them individually or in pairs. Subsequently, students should share solutions and arguments with the class and encourage each other to constructively criticize each other's arguments. Explain the six different mathematical argument patterns identified in the study to the students, including the $C_H+E_H+R_H$ pattern, which represents the highest quality of arguments. Provide examples of each argumentation pattern and ask students to identify the pattern(s) they used in their argumentation. Provide feedback on the quality of the students' arguments by focusing on their use of argumentation patterns and their understanding of algebraic concepts and procedures. Lastly, encourage students to reflect on their own learning and identify areas where further practice and support are needed.

Throughout the activity, it is important to emphasize that the pattern of arguments does not relate to particular mathematics proficiency levels. Instead, it requires a profound understanding of algebraic concepts and procedures. By using argumentation patterns as an instrument to enhance their mathematical argumentation and problem-solving proficiencies, students can develop a deeper understanding of algebraic concepts and procedures, and improve their ability to construct persuasive arguments when solving algebraic problems.

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Appendix 1

Argumentation Frame in Algebraic Operations (AFAO)

The questionnaire consists of two sections. The first is intended to simplify analysis. In the second section, you must use your previous knowledge of the algebraic addition and subtraction axioms. This questionnaire is not part of your regular algebra activity, so it does not affect your results. Your name is not linked to your responses.

SECTION A: Demographic information

Personal Particulars	
Name:	**Name of the School:**
Gender:	**Class:**
Date of Bird:	

SECTION B: Algebra Task

Instructions

- *This questionnaire will not affect your grades. Please do not spend a lot of time on a single statement – your first thoughts are usually the best.*
- *Write your responses on the spaces provided after the statement. Please respond to every statement – it's important that you respond to each statement honestly.*
- *All the information will be used for research purposes only. Your responses will be treated confidentially. Your responses will not reveal any information that could identify you.*
- *This survey should take you about 40 minutes to complete.*

Ani is a student in the seventh grade attending one of the junior high schools in the city of Bandung. While taking a mathematics examination, Ani was presented with a mathematical problem in the form of $(5a - 7b) + (13a + 8b)$.

The solution process utilized by Ani is outlined as follows:

Step-1: $(5a - 7b) + (13a + 8b)$

Step-2: $5a - 7b + 13a + 8b$

Step-3: $-2a + 21b$

Please verify, in a step-by-step manner, the problem-solving process executed by Ani. Determine whether Ani's work correct or incorrect? Explain why!