

Cognitive Map: Diagnosing and Exploring Students' Misconceptions in Algebra

Analisa Fitria¹, Subanji², Susiswo², Hery Susanto²

¹Antasari Islamic State University, Banjarmasin, Indonesia,

²State University of Malang, Malang, Indonesia

analisafitria@uin-antasari.ac.id, subanji.fmipa@um.ac.id, susiswo.fmipa@um.ac.id,
hery.susanto.fmipa@um.ac.id

Abstract: This study diagnoses and explores year eight students' misconceptions and mistakes in algebra. An in-depth exploration of students' misconceptions were carried out by conducting interviews based on the students' work as outlined in a cognitive map. Based on the cognitive maps, it can be seen that the students emphasize procedural knowledge rather than a comprehensive understanding of the concept. The students had misconceptions about the variable, which is the core of algebra. The students misinterpreted variables, though they sometimes succeeded in determining variable values from algebraic expressions or equations. The students made mistakes in translating literal sentences into algebraic forms, but they succeeded in completing an algebraic expression. In general, the students were able to operate algebraic forms procedurally. The exploration found that these misconceptions or errors were likely caused by their incomplete knowledge of arithmetic. This problem had an impact when they work on the transition from arithmetic to algebra concepts.

Keywords: Misconceptions, Cognitive Maps, Algebra

INTRODUCTION

The Concept image is an individual's overall cognitive structure of a concept. The concept image includes mental image, nature and processes involved in its formation (Tall & Vinner, 1981). Concept images are built over years, originate from individual experiences, and can change when individuals encounter new stimuli. This results in the possibility of incompatibility of students' concept image with the formal definition (Tall & Vinner, 1981). Several studies have shown that individual concept images differ from the formal definition, which is known as a misconception, triggering cognitive conflict (Kang et al., 2010; Ramsburg & Ohlsson, 2016).

Misconceptions are part of the conceptual structure that students have. Holmes et al. (2013) state that misconceptions are part of students' cognitive structures that are not scientifically accurate.

Misconceptions are also described as student conceptions that produce systematic patterns of errors (Smith et al., 1994). When connected with students' prior knowledge, Ojose (2016) argues that misconception is a misunderstanding and misinterpretation caused by students' 'naive theory.' This misunderstanding will cause cognitive conflict in students when faced with the correct concept. Hansen (2020) states that misconceptions indicate a conflict between students' understanding and new mathematical concepts learned (Hansen, 2020). In this study, misconceptions are defined as the incompatibility of students' beliefs with correct scientific concepts.

Misconceptions are different from errors. An error is caused by something that is not patterned and inconsistent. For example, an error happens because of haste and lack of thoroughness (Tooher & Johnson, 2020). However, misconception indicates a person's wrong understanding, and it occurs repeatedly because it is resistant and can produce wrong results (Tooher & Johnson, 2020; Khazanov, 2008). As a result, misconceptions negatively affect learning because they can potentially trigger errors (Zielinski, 2017). Students with misconceptions or insufficient conceptual knowledge perform less optimally in mathematics (Booth & McGinn, 2016). Apart from hindering students' ability to solve mathematical misconceptions, it also hinders students' understanding of new concepts (Stothard, 2021).

According to Irawati et al. (2018), misconceptions can potentially cause errors in forming generalizations about a concept, affecting the learning process in general. Misconceptions among students will be an obstacle to students' success in learning mathematics as a whole, and in the long run, will impact future job opportunities (Ladson-Billings, 1998). Therefore, it is clear that misconceptions negatively influence students' abilities and performance in learning. The results of a literature review conducted by Jamaludin & Maat (2020) on 30 articles that focus on misconceptions show that many students still experience misconceptions in algebra. Students with algebraic misconceptions will likely encounter difficulties using algebra to solve problems (Cline, 2020). Misconceptions about algebra limit students' success in mathematics, thereby limiting their success in school at every level (Zielinski, 2017). Students will also experience difficulties in other branches of mathematics and other related subjects (Efriani et al., 2019; Joanna & Jacquelynn, 2019; Vargová, 2020).

Algebra is one of the main branches of mathematics and has many applications in our daily life. Algebra is closely related to other branches of mathematics, such as probability, geometry, and calculus (Yew et al., 2020). Students are introduced from arithmetic to abstract when learning algebra (Irawati et al., 2018). Algebra uses letters and symbols to represent unknown numbers and describe the mathematical relationship between quantities in formulas and equations (Tooher & Johnson, 2020). Some teachers and students think that algebra is a complicated lesson in mathematics (Das, 2020; Mulungye, 2018; Natalia et al., 2016). One of the factors causing this difficulty is due to the abstract nature of algebra (Rakes & Ronau, 2019). In particular, the abstractness of algebra hinders students from constructing object representations (Kieran, 1992).

Preliminary data shows that students have indications of misconceptions. Students experience confusion in operations involving algebraic forms, for example, $3x + 4 = 7x$ or $3x + 2x = 5x^2$. Students also mistakenly simplify $\frac{a+b}{a}$ dan $\frac{a+x}{b+x}$, into b dan $\frac{a}{b}$. This finding is in accordance with Mulungye (2016), who states that students have misconceptions in operating algebraic forms and mistakenly simplify algebraic expressions. Students also diagnosed misconceptions in the application of the distributive rule. Students work on $(a + b)^5$ as $a^5 + b^5$, and $3(a + b)^2$ as $3a^2 + 3b^2$. When the results of their work regarding the problem were clarified, the students reasoned that they worked based on their intuition (no basis). In Polynomials and Exponents, students also experience misconceptions as they stated that $y^4 + y^4 = y^8$ and $\sqrt{x^2 + y^2} = x + y$. According to Das (2020), most algebraic misconceptions stem from students' misconceptions about arithmetic (Das, 2020).

When and how will someone's misconceptions disappear? This is an important part to be studied. Students with misconceptions tend to continue to experience misconceptions which cause difficulties in learning mathematics (Stothard, 2021). However, misconceptions experienced by students are sometimes not detected by the teacher, so the misconceptions persist for a long time. (Jong et al., 2017; Stothard, 2021). Teachers who are able to diagnose and correct misconceptions can encourage meaningful student learning (Rakes & Ronau, 2019; Vaughn et al., 2020). Therefore, teachers need to detect students' misconceptions to correct and improve them through appropriate learning instructions (Ojose, 2016). Teachers must recognize misconceptions to take preventive and corrective actions for learners (Deringöl, 2019). According to Mulungye et al. (2016), teachers need help identifying misconceptions and knowing the causes of misconceptions in the learning process. If information about student misconceptions is available, it will be easy for teachers to prevent and overcome misconceptions. Ocall (2017) suggests that teachers correct students' misconceptions first before introducing new concepts to students.

This study aims to detect, diagnose and explore students' misconceptions in algebra. Misconceptions in students are often reflected in the results of their work when solving problems (Arnawa, 2019; Russell et al., 2009). Therefore, the thinking students do in solving math problems can be seen from their work in solving problems. The difference between this study and previous research is that the researchers in the current study seek to explore students' algebraic misconceptions by displaying them on a cognitive map.

Cognitive maps are mental images and concepts built to visualize and assimilate information (Sammut-Bonnici & McGee, 2015). Gutiérrez et al. (1991) explained that cognitive maps can be used to examine the uniqueness of one's thinking processes in-depth. Cognitive maps are used to externalize students' thinking, which can help investigate tasks related to students' thinking (Chen et al., 2021). According to Rakes & Ronau (2019), when teachers dig deeper into students' thoughts, they can make interventions that correct misconceptions and strengthen their

understanding of concepts (Rakes & Ronau, 2019). Cognitive maps can be used to explore students' ways of thinking and discover why they experience misconceptions.

This cognitive map can describe causal relationships of various phenomena and concepts and can be modelled (Subanji & Nusantara, 2013). Thus, errors caused by student misconceptions can be corrected with this model. Jacobs (2003) revealed that cognitive maps can show the direction of students' thinking, so that they can be used as a guide for subsequent needs. This study aims to diagnose and explore students' misconceptions and errors in algebra. The exploration was carried out by exploring students' ways of thinking and describing the process of the results of students' work in a cognitive map.

LITERATUR REVIEW

Over the decades, many research papers about misconceptions and errors in algebra have been published. Many students enter high school with algebraic misconceptions that will limit their mathematics success and future educational attainment (Booth & McGinn, 2016; Zielinski, 2017). It is believed that when students build concepts in learning sometimes they develop concepts that are incomplete, immature, still alternative, and transitional (Mathaba & Bayaga, 2021). The concepts built by these students are entirely correct, partially correct, or entirely inconsistent with scientific concepts. Although students use intuition and a process of trial and error while guessing math results and checking them, they build algebraic concepts independently (Kshetree, 2021). When students carry out algebraic operations, it is said that they carry out thinking processes through mental activity in the student's brain. This process is not only for generating numbers and abstract mathematical concepts but also as an essential skill in quantitative analytical and logical thinking (Faizah et al., 2022).

Learning algebra that lacks meaning usually builds procedural skills without considering conceptual understanding, which can cause errors and misconceptions (Mulungye, 2018). Students are expected to know their knowledge, procedures, and concepts to use the knowledge and apply it in solving problems or assignments. The learner is expected to understand the concept first and then follow the procedure. Learners tend to forget concepts and prioritize procedures. Likewise, it is stated that procedural and conceptual errors are caused by a lack of knowledge or misunderstandings from students about the concept itself (Edogawatte, 2011). Students lacking information or understanding of algebra will result in errors. For example, failure to determine the formula used or error in executing operation signs, which are the basis of algebra, will lead to wrong solutions (Mathaba & Bayaga, 2021).

Several studies have identified a number of misconceptions that students tend to have about algebraic content. The misconceptions experienced by most students relate to equations, negativity, variables, fractions, order of operations, and functions (Booth & McGinn, 2016). Edogawatte (2011) further explores misconceptions by dividing them into three major concepts in algebra: variables, algebraic expressions, and equations. A categorization that places more

emphasis on the process of algebraic operations was identified by Zielienski (2018) and Ojose (2018), namely: negative function input, exponential rules, distribution of negative signs, distribution errors, random cancellation (simplification of algebraic fractions), fractional rules and negative exponential rules.

COGNITIVE MAP

A person has cognitive maps since he/she is in the womb, which can be seen from the sensorimotor system (Ahmed et al., 2020). The Cognitive Map describes the interconnection between knowledge, problems, procedures, and concepts from the results of one's thinking (Subanji, 2015). Cognitive maps can reveal students' thoughts when exploring a problem (Jonassen, 2003; Toth et al., 2002). Cognitive maps represent complex thinking from various perspectives on science and its context (Chen et al., 2021; Garoui & Jarboui, 2012). Cognitive maps illustrate the formation of learned knowledge, so that a complete schema is formed in a person (Bottini & Doeller, 2020). Therefore, the cognitive map referred to in this study is an image/scheme used to represent individual cognitive structures.

This study uses the cognitive map framework of Peña, et al. (2007). According to Peña, cognitive maps are the result of a cognitive process which is also called cognitive mapping. Cognitive mapping is an internalization-externalization process that represents the qualitative knowledge of an individual and is believed to be of correct value according to the individual's point of view. Thus, this knowledge is unilateral, uncertain, imprecise, unstable, incomplete, and not universal. Basically, a cognitive map is a graph consisting of concepts (C) and causal relationships (\rightarrow) which can be seen in Figure 1 (Peña et al., 2007).

Figure 1 explains that, according to Peña, et al., the causal relationship between the two concepts can be direct and indirect. The concept of ($Ca \rightarrow Cz$) is said to be directly causal if there is no other concept between Ca and Cz (as shown in Figure 1^a). However, if there is at least one concept, for example Cb appearing between the two ($Ca \rightarrow Cb \rightarrow \dots Cz$), then Ca and Cz have an indirect causal relationship (Figure 1^b). The positive or negative sign on the arc indicates the direction of the causal relationship. Positive means that Ca affects Cz positively. That is, when Ca is positive, Cz will be positive as well. If the sign after the arc is negative ($Ca \rightarrow - Cz$), then the relationship is inverse. This means that if Ca is negative, Cz will be positively affected and vice versa.

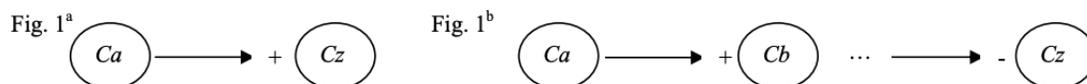


Figure 1: Formal cognitive map model (Peña, et al. (2017))

METHOD

This research is qualitative. According to Cohen et al. (2018), one of the goals of qualitative research is to describe and explain. Cohen suggests types of research questions in qualitative research, namely, questions relating to: (a) describing a state of affairs, their causes and how this state of affairs is maintained; (b) describing the process of change and its consequences (Cohen et al., 2018). This study will diagnose and explore students' misconceptions. The exploration results will be displayed in the form of a cognitive map to see the occurrence of misconceptions through students' thinking. According to Cangelosi (2013), a combination of tests and interviews can be used to diagnose misconceptions. The diagnostic test can be carefully designed using multiple-choice questions that include a misconception response, a correct response, and a detractor (Russell, O'Dwyer, & Miranda; (2009).

In this study, the data collection was done by giving an algebraic diagnostic test to 68 students of year VIII (13-14 years old) in Banjarmasin. The algebra diagnostic test consists of 24 multiple-choice items adapted from Blessing (2004). The misconception score represents the total number of errors in the student's answers out of the 24 questions. From the results of the diagnostic tests, two students with the highest percentage of misconceptions were selected as research subjects. Furthermore, the research subjects were given an Algebraic four-tier test in the form of questions containing three main categories in Algebra: algebraic expressions, algebraic operations, and models to explore misconceptions from the results of diagnostic identification.

The four-tier diagnostic test is the development of four-level multiple choice questions that are used to see students' understanding of the concept. The first level is a multiple-choice question, and the student must choose an answer, while the second level is students' level of confidence in choosing answers. The third level is the student's reason for answering the question, which is a choice of reasons that have been provided. One open reason is also included. The fourth level is students' level of confidence in choosing reasons (Gurel et al., 2015). The profile of the four-tier-test categories can be seen in Table 1.

FINDINGS AND DISCUSSION

The results of the diagnostic test showed that 45 out of 68 students had indications of misconceptions with varying scores, ranging from 20.8% to 58.3%. The biggest misconception lies in students' understanding of variables as 41 students experience this misconception. Most students understand the variable as an object. Another misconception is students' understanding of the concept of the equal sign associated with solving algebraic equations. These two concepts are core concepts in early algebra, which makes it easier for students to understand algebra to the next stage (Xie & Cai, 2022).

This study recruited two students with the highest misconception scores as research subjects, namely S1 and S2. Furthermore, S1 and S2 subjects were given a four-tier test to explore their

misconceptions. Based on the four-tier test, it was decided that S1 was included in the MC (misconception) category, and S2 was included in the LK (lack of knowledge) category. Furthermore, in-depth interviews were conducted with S1 and S2 to explore their responses in the diagnostic and the four tier-tests.

Tier 1	Tier 2	Tier 3	Tier 4	Decision
Correct	Sure	Correct	Sure	SC
Correct	Sure	Correct	Not Sure	LK
Correct	Not Sure	Correct	Sure	LK
Correct	Not Sure	Correct	Not Sure	LK
Correct	Sure	Wrong	Sure	Misconception
Correct	Sure	Wrong	Not Sure	LK
Correct	Not Sure	Wrong	Sure	LK
Correct	Not Sure	Wrong	Not Sure	LK
Wrong	Sure	Correct	Sure	LK
Wrong	Sure	Correct	Not Sure	LK
Wrong	Not Sure	Correct	Sure	LK
Wrong	Not Sure	Correct	Not Sure	LK
Wrong	Sure	Wrong	Sure	Misconception
Wrong	Sure	Wrong	Not Sure	LK
Wrong	Not Sure	Wrong	Sure	LK
Wrong	Not Sure	Wrong	Not Sure	LK

Table 1. Four tier-test categories according to Gurel (2015)

S1 Data Presentation

In the initial identification of the algebraic diagnostic test, S1 achieved a misconception score of 45.8%. Based on the four tier-test, S1 was included in the Misconception (MC) category. S1 is 14 years old and in grade 8 of junior high school. The following is a description of the misconceptions or errors made by S1.

1. Variables representing the number of letters in the expression

S1 was indicated to have a misconception about the concept of variables. S1 assumed that the value of the variable (letter) represents the number of letters in the given algebraic expression. This was obtained from the results of S1 work and interviews.

Q : For $a + 5$, what do you think a denotes?

S1 : 1 (number 1)

Q : Why 1? Not something else, for example, 2 or 3?

S1 : because there is only one a

(The above script was a translation from the Indonesian language. All interview sessions were conducted in the Indonesian language)

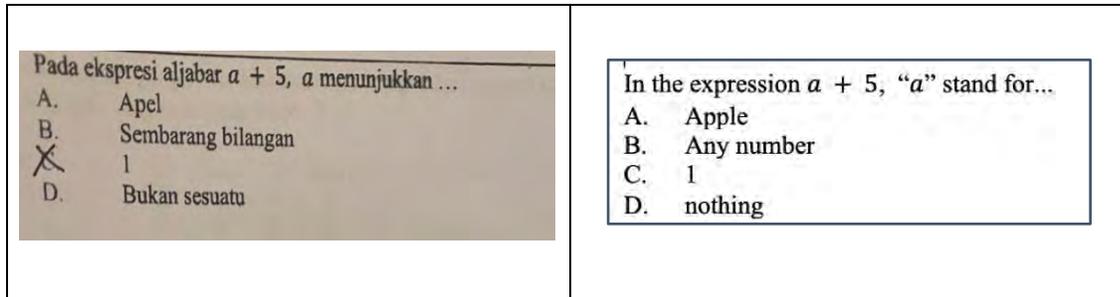


Figure 2: S1's answer sheet

This indication is strengthened by the students' answers on the four-tier test. S1 believes in the wrong concept: that the variable is the number of letters in the algebraic form. Based on this, the researchers wanted to confirm whether S1 can show variables when given any algebraic form.

Q : For $3a + 8$, can you tell the variable in the expression?

S1 : "a" Ma'am

Q : Then what is 3 usually called?

S1 : Hmm.. 3 is the coefficient

Q : What about 8? Is it also a coefficient?

S1 : No, because there are no "letters."

Q : Letters? What do you mean? Can you explain?

S1 : There is no x, ma'am, for $3a$ there is an x, namely a

Q : Well, if 8 is not a coefficient, what is it usually called?

S1 : (long) I don't know ma'am.. just a number.

The results of the interviews show that S1 can identify variables and coefficients, though S1 has a misconception about the definition of a variable.

2. Different letters must represent different values

The misconception about the variable concept that S1 has is believing that different letters (variables) cannot represent the same number.

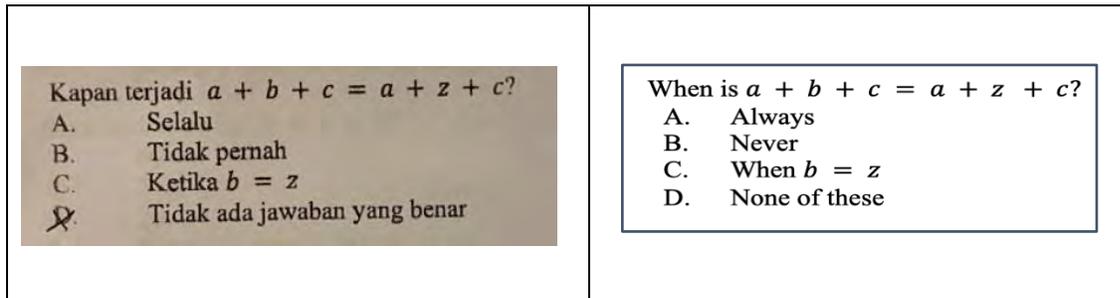


Figure 4. S1's answer sheet

P : in the equation $a+b+c=a+z+c$, when do you think the left side is the same as the right side?

S1 : It can't be the same, ma'am.

Q : So you don't think the two equations will ever be the same?

S1 : No

Q : Why?

S1 : Because a and c are the same, but b and z are different

Q : a and c are the same? Does that mean that both sides have the same a and c ?

S1 : Yes ma'am, and b is not the same as z

The statement ' b is not the same as z ' shows that S1 believes that each different letter must represent a different number. S1 has incomplete knowledge that letters can represent any number, so two different letters can represent the same number. This is because S1 tends to view letters as objects, so when presented with different letters, S1 sees a different object (Yew et al., 2020).

3. *Determining variable values from simple algebraic equations*

S1 was able to manipulate the algebraic form of a simple algebraic equation to determine the value of a variable. In item no 2, namely $a + 5 = 4a$, S1 procedurally found that the value of a is $\frac{5}{3}$. Therefore, even though S1 has a misunderstanding in interpreting variables, S1 can still determine variable values from a simple equation.

Q : For $a + 5 = 4a$, can you find the answer?

S1 : Yes, it's $\frac{5}{3}$

Q : May I know how?

S1 : I moved a to the right, so it was $5 = 3a$, then $a = \frac{5}{3}$ was obtained.

Q : Why must it be moved to the right side?

S1 : Usually like that, ma'am. I also don't know why.

By saying 'I moved a to the right' indicates students' misconceptions about the rules of arithmetic operations (Sarımanoğlu, 2019). As we know, this procedure aims to remove variable a on the left side of the equation by subtracting it with a . When this was clarified, S1 was not aware of the concept. S1 has incomplete knowledge of the concept of arithmetic operations, which can potentially cause errors in algebraic operations. According to Booth (2019), the correct answer can be obtained even though students experience misconceptions. Other studies state that students experience pseudo-true thinking processes in solving problems. Pseudo-true occurs when students correctly answer the questions but cannot give reasons for their answers or are wrong in giving reasons (Subanji, 2015). This reinforces the reason why teachers need to diagnose algebraic misconceptions in students because sometimes the misconceptions experienced by students are invisible without us doing a diagnosis (Das, 2020).

4. *Manipulating the algebraic forms presented in sentences*

S1 made an error when asked to manipulate algebraic forms presented in a sentence. This was

<p>Jika 5 ditambahkan ke $a + 2$, akan menghasilkan...</p> <p>A. $a + 7$</p> <p>B. $5a + 2$</p> <p>C. $7a$</p> <p>D. Tidak ada yang benar</p>	<p>Add 5 onto $a + 2$...</p> <p>A. $a + 7$</p> <p>B. $5a + 2$</p> <p>C. $7a$</p> <p>D. None of these</p>
--	---

because S1 incorrectly interpreted the meaning of the sentence given.

Q : What was your answer to question no 4?

S1 : I answered $5a + 2$.

P : How did you get $5a + 2$? Can you explain?

S1 : 5 plus a plus 2.

Q : Does it mean like this (writing on paper $(5 + a + 2)$).

S1 : No, but $5 + a$ equals $5a$, then plus 2.

The sentence "5 plus a plus 2" indicates that S1 mistakenly interpreted the question presented in the sentence. S1 interpreted the sentence "if 5 is added to $a + 2$ " as adding 5 to a , instead of adding 5 to $(a+2)$. Another finding from the interviews is that S1 believes in the wrong rules for adding integers with variables, namely $5 + a$ becomes $5a$. One study stated that this error occurred due to

the duality of the nature of mathematical notation, namely as a process and an object (Joanna & Jacquelynn, 2019). S1 perceives that the answer cannot contain operator symbols. S1 believes the symbol '+' is a command to do something, and the expression that still contains the operation '+' is not a simple form, so it needs to simplify (Ojose, 2016).

In this regard, in the four tier-test, S1 also experienced a conceptual error: adding up $5x + 3x$ as $8x^2$. By believing that x is the same object, for example, 3 books plus 5 books equal 8 books.

Q : What can you conclude from $5 + a$?

S1 : $5a$

Q : How about $5x + 3x$?

S1 : Hmm.. $5 + 3$ is 8, so $5x + 3x$ is $8x^2$

According to Ojose (2018), S1 applies the wrong rules due to a lack of knowledge. S1 believes in the wrong rule that numbers must be added, and variables must be multiplied to simplify algebraic expressions. This finding is supported by S1's work when asked to solve $\frac{6}{a} + \frac{6}{b}$. S1 simplified $\frac{6}{a} + \frac{6}{b}$ to $\frac{12}{ab}$. S1 believes that the quantifier must be added up because it is a number, while the denominator must be multiplied because it is presented in a variable. According to Ojose (2018), this error is related to students' lack of conceptual understanding.

5. *The order of the letters in the alphabet determines represented numbers*

Algebraic expressions whose variables are written in the form of letters cause misconceptions among students.

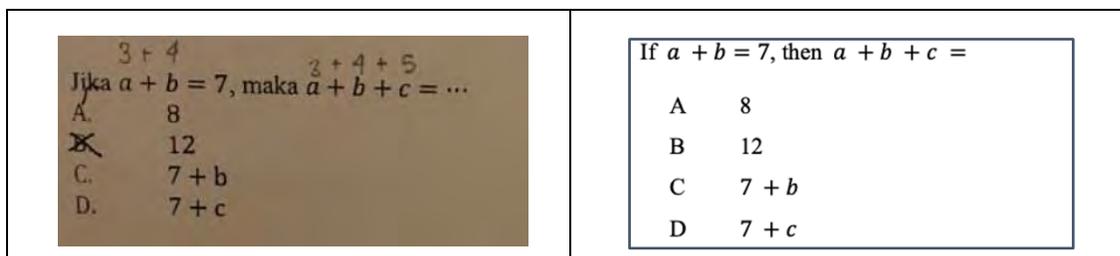


Figure 5: S1's answer

Q : Can you explain why you answered 12 for this question?

S1 : I changed a to 3, b to 4, so that $a + 3$ equals 7

Q : Why not changing a to 2, and b to 5?

S1 : If a is 2, then b is 3, then the sum will not be 7 ($a + b = 7$)

S1 : Because c will be added later, now c is 5, so that $a + b + c = 12$

The sentence "I changed a to 3, b to 4, so that $a + 3$ equals 7" shows that the subject experienced a "trial and error" process in the work. Trial and error were made by selecting numbers based on their intuition. According to Warren (2003), one of the errors that students make is assigning letters a numerical value according to their rank in the alphabet. When students do this, they often assume that variable ' a ' equals 1, variable ' b ' equals 2, and so on. In this case, S1 thought as if there is a correspondence between the order of the linear alphabet and the natural number system, namely $a = 3$ and $b = 4$ because the letters a and b are sequential in the alphabet. S1 believes that 3 and 4 are the appropriate numbers for the expression.

6. Forming algebraic expressions from lateral sentences presented by matching words from "left to right"

In this question, students were asked to form algebraic expressions from the given sentences.

<p>Setengah dari suatu bilangan, dikurangi dari empat hasilnya adalah lima belas. Persamaan yang mewakili pernyataan ini adalah</p> <p>A. $4 - \frac{1}{2}x = 15$</p> <p>B. $x - \frac{1}{2}x = 15$</p> <p><input checked="" type="checkbox"/> C. $\frac{1}{2}x - 4 = 15$</p> <p>D. $\frac{1}{2}x - 4 = 15$</p>	<p>Half a number, subtracted from four is fifteen. The equation best representing this statement is</p> <p>A. $4 - \frac{1}{2}x = 15$</p> <p>B. $x - \frac{1}{2}x = 15$</p> <p>C. $\frac{1}{2}x - 4 = 15$</p> <p>D. $\frac{1}{2}x - 4 = 15$</p>
---	---

Figure 5. S1's answer

When changing literal sentences to form algebraic expressions, S1 arranged them from "left to right." S1 assumed that the word order in the sentence would map directly to the symbol order that appears in the question. S1 also misinterpreted the sentence "subtracted from four." Russell et al. (2009) refer to this error as a direct translation characterized by a phrase-by-phrase translation of the problem into variables and equations.

7. Wrong Interpretation of Sentences

<p>Susan memiliki beberapa kartu gambar. Feby memiliki tiga kali lebih banyak kartu gambar dari Susan. Jika kartu gambar susan dan Feby digabungkan akan berjumlah 36 kartu. Manakah dari persamaan ini yang mewakili koleksi kartu gambar mereka?</p> <p>A. $3x = 36$</p> <p><input checked="" type="checkbox"/> B. $x + 3 = 36$</p> <p>C. $x + 3x = 36$</p> <p>D. $3x + 36 = x$</p>	<p>Susan has some trading cards. Feby has three times as many trading cards as Susan. They have 36 trading cards in all. Which of these equation represent their trading card collection?</p> <p>A. $3x = 36$</p> <p>B. $x + 3 = 36$</p> <p>C. $x + 3x = 36$</p> <p>D. $3x + 36 = x$</p>
---	--

Figure 6: S1 answer sheet

The question above requires students to read the problem presented in the sentence form and change it to an algebraic equation. S1 mistakenly interpreted the sentence given. The sentence "three times as much" was interpreted by S1 as $x + 3$, instead of $3x$

Q : You chose the answer $x + 3 = 36$. May I know what x here indicates?

S1 : x is, for example, the card that belongs to Susan

P : Then why did you add x to 3?

S1 : In this question, it says that there are three times more Feby cards than Susan cards, earlier Susan cards are x , so 3 was added.

The interview revealed the mistakes/misconceptions experienced by S1, namely the wrong interpretation of the given sentence.

8. *Misconceptions in understanding the distributive nature of negative signs*

S1 succeeded in using the distributive rule for algebraic expressions not containing a negative sign. For $2 \times (a + b)$, S1 resolves it into $2a + 2b$. An error occurs when a distribution operation contains a negative sign. S1 had a misconception, believing that $5(p + q) - 3(p + q)$ was $2p + 8q$. This was obtained from $5(p + q) - 3(p + q) = 5p + 5q - 3p + 3q$. According to Zielienski (2017), this misconception happens because students do not interpret brackets as an entity when dealing with addition or subtraction signs outside the brackets. S1 ignored the consequences of sign rules which can involve positive changes to negative or vice versa (Zielinski, 2017).

9. *Invalid generalization of the distribution rule*

S1 interpreted $(5 + x)^2$ as $5^2 + x^2$. This error develops from over-applying the distributive rule. The distributive property states that $a(b + c) = ab + ac$. Subject S1 applied this rule in a new, inappropriate context. According to Chow (2011), this misconception is an invalid distribution. An invalid distribution is also known as an abuse of the distributive nature of algebra (Chow, 2011).

10. *Simplifying superfluous algebraic forms*

For question $\frac{a+b}{b}$, the students were asked to simplify the algebraic expression, and S1 simplified $\frac{a+b}{b}$ as a .

P : For the expression $\frac{a+b}{b}$ do you think this can be simplified??

S1 : Yes, Ma'am, .. $\frac{a+b}{b}$ is the same as a

P : How?

S1 : the letter b above (the numerator) is crossed out along with the letter b below (the denominator) (using the rule of cancellation).

There are two mistakes made in S1's answer for this question. First, S1 believes that the variable b in the numerator and denominator can be removed using the cancellation law. S1 applied the wrong procedure, namely, applying the rules of the cancellation law of multiplication to addition. Second, if cancellation can be made, the variable b in the numerator and denominator change into 1, so that the algebraic form of $\frac{a+b}{b}$ can be changed to $\frac{a+1}{1}$. According to Mulungye (2016), the main cause of this error was that the subject had an incomplete understanding of arithmetic concepts or failed to transfer arithmetic understanding to an algebraic context.

11. Determining the algebraic form of the given geometric pattern

S1 succeeded in determining the number terms of the given geometric pattern. S1 solved the questions that asked to determine the number for the pattern given well. Based on the interview, S1 calculated the number of tiles manually (adding one by one) from pattern 1 to pattern 20. S1 admitted that he did not understand how to generate algebraic expressions to be used as a general formula for a given geometric pattern.

Based on the results of S1's work as a whole, S1's cognitive map is presented in Figure 8 below.

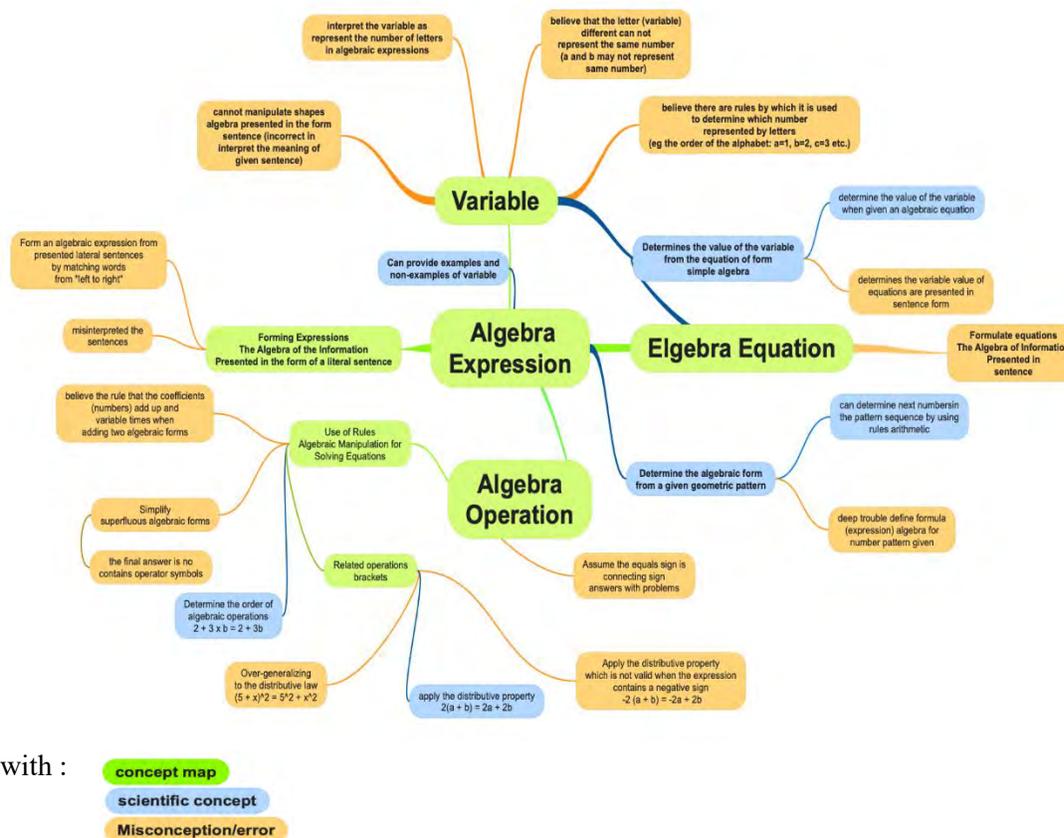


Figure 8: The cognitive map of misconceptions and S1's errors

S2 Data Presentation

Subject S2 is a year eight student at SMP Banjarmasin, and he/she is currently 14 years old. In the initial identification of the algebraic diagnostic test, S2 experienced a misconception of 58.3%. Based on the results of the tier-test, S2 is included in the category of students with a *decision lack of knowledge*. Some of the misconceptions experienced by S2 are also misconceptions experienced by S1. The following is an exploration of the misconceptions and mistakes made by S2 that are different from the misconceptions experienced by S2

1. A variable represents an object

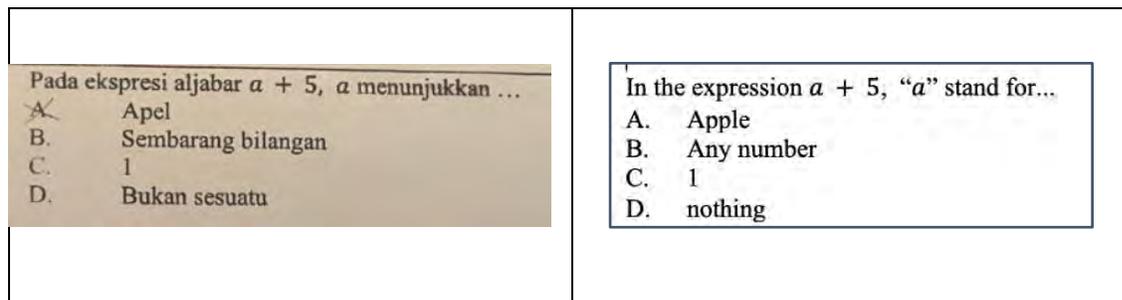


Figure 9: S2's work

S2 interpreted the letter a in the expression $a + 5$ as an object, namely an apple. According to Edogawatte (2011), students interpret that there is a close relationship between the letters used as variables and the real-life context. Sometimes students interpret the expression $8a$ as simple as for 8 apples.

Q : For question $a + 5$, you say that a is an apple?

S2 : Yes, ma'am

Q : Why?

S2 : Because there are no other options that represent letter a .

The sentence " Because there are no other options that represent letter a " indicates that S2 relates the letter a to an object in the real-life context. As in the multiple-choice answers only apple was possible, S2 thought that this option was the answer.

2. The use of rules in manipulating Algebra (operations)

<p>Apa yang dapat kamu simpulkan dari persamaan : $a + 5 = 4a$</p> <p>A. $a = \frac{5}{3}$ B. $4a + 5$ C. $a = \frac{3}{5}$ D. $a = 0$</p>	<p>What can you say about 'a' if $a + 5 = 4a$?</p> <p>A. $a = \frac{5}{3}$ B. $4a + 5$ C. $a = \frac{3}{5}$ D. $a = 0$</p>
<p>Berapakah k jika $k - 12 = 4$?</p> <p>A. -8 B. 3 C. 8 D. 16</p>	<p>What is 'k' if $k - 12 = 4$?</p> <p>A. -8 B. 3 C. 8 D. 16</p>
<p>Jika $5 = 9y$, maka $y = \dots$</p> <p>A. $5 : 9$ B. $9 : 5$ C. $5 - 9$ D. 5×9</p>	<p>If $5 = 9y$, then $y =$</p> <p>A. $5 \div 9$ B. $9 \div 5$ C. $5 - 9$ D. 5×9</p>

Figure 10 : Student S2's work

S2 made several mistakes in manipulating and performing operations to get variable values from the given algebraic expressions. In-depth interviews were conducted with S2.

Q : In the equation $a + 5 = 4a$, can you show which one is the variable?

S2 : Here, Ma'am (pointing to $4a$).

Q : Is a also a variable (referring to $a + 5 = 4a$).

S2 : (pauses) I don't know Ma'am, I don't understand.

Q : Well, for $4a$ above, what's your reason for saying that $4a$ is a variable?

S2 : Because it is different from a and 5

Q : Different? Different shape or what?

S2 : $4a$ is a combination of 4 dan a

From the interview, it was revealed that S2 did not recognize variables when presented in an algebraic expression. S2 could not show which one is a variable and which is not a variable. Regarding his work in writing that $a + 5 = 4a$ was changed to $4a + 5$, S2 admitted that he/she did not know the meaning of the problem. S2 assumed that the chosen answer was the most reasonable one among the other choices.

A similar question was given, $5 = 9y$, and the students were asked to determine the value of y . In this section, S2 could understand the intent of the question, which is to determine the y value of the given equation. S2 confidently determined $y = 5 - 9$. Based on the interview, S2 admitted that he/she found the answer by moving number 9 on the right side to the left, so that the sign for number 9 was changed into -9. The misconception that S2 has was that he/she did not consider 9 and y or $9y$ as an entity (term). He/she believed that 9 and y were separate elements, so when 9 was displaced, y did not follow it. This misconception is consistent with previous research findings that students believe signs, numbers and variables are separate parts (Chow, 2011).

For the next question, in the equation $k - 12 = 4$, S2 used his/her understanding of moving segments to eliminate k . However, the mistake S2 made was not to process the symbols following 12. Therefore, S2 chose the -8 answer with the procedure $k = 4 - 12 = -8$. From the work and interviews conducted, S2 experienced misconceptions regarding the rules for conducting operations. According to previous research, this is caused by students' lack of conceptual understanding of operations on arithmetic applied to algebra. (Bush, 2013; Edogawatte, 2011; Mulungye, 2016; Ojose, 2016).

3. Words and letters matched from "left to right"

Similar to S1, when changing literal sentences to form algebraic expressions, S2 made the same mistake. S2 composed sentences word for word from "left to right" to form algebraic expressions. S1 assumed that the word order in the problem statement would map directly to the symbol order in the question. S1 also misinterpreted the sentence "subtracted from four."

Russell et al. (2009) refers to this error as a direct translation characterized by a phrase-by-phrase translation of the problem into variables and equations.

<p>Jika n adalah jumlah surat kabar yang dikirimkan Lee setiap hari. Manakah dari ekspresi berikut yang mewakili jumlah total surat kabar yang dikirimkan Lee dalam 5 hari jika pada hari kelima ia mengirimkan tiga surat kabar tambahan?</p> <p>A. $5 \times n + 3$ B. $(5 \times n) + 3$ C. $(n + 5) + 3$ D. $3 + (5 - n)$</p>	<p>If n represent the number of newspaper that Lee delivers each day. Which of the following expressions represents the total number of newspaper that Lee delivers in 5 days if on the fifth day he delivers an additional three newspapers?</p> <p>A. $5 \times n + 3$ B. $(5 \times n) + 3$ C. $(n + 5) + 3$ D. $3 + (5 - n)$</p>
<p>Susan memiliki beberapa kartu gambar. Feby memiliki tiga kali lebih banyak kartu gambar dari Susan. Jika kartu gambar susan dan Feby digabungkan akan berjumlah 36 kartu. Manakah dari persamaan ini yang mewakili koleksi kartu gambar mereka?</p> <p>A. $3x = 36$ B. $x + 3 = 36$ C. $x + 3x = 36$ D. $3x + 36 = x$</p>	<p>Susan has some trading cards. Feby has three times as many trading cards as Susan. They have 36 trading cards in all. Which of these equation represent their trading card collection?</p> <p>A. $3x = 36$ B. $x + 3 = 36$ C. $x + 3x = 36$ D. $3x + 36 = x$</p>

Figure 11: S2's work

S2 made a mistake in making algebraic expressions or equations of the problems presented in sentences.

4. Parenthetical Rules

In this section, S2 did not notice the order in performing operations on algebraic expressions. This sequence must be followed even though the expression does not display parentheses (Zielinski, 2017).

Q : For $3 + y \times 2$, did you simplify it to $6y$?

S2 : Hmm yes.

Q : Where does 6 come from?

S2 : I first multiplied 3×2 to get 6, then added y to make $6y$

Q : Did you mean like this? (writing it down on paper: $3 \times 2 = 6$, then $6 + y = 6y$)

S2 : Yes, ma'am.

Q : Do you think $6 + y$ will make $6y$, what about $6 \times y$?

S2 : $6y$ too, ma'am? Well, it should not be the same. It seems I was wrong.

Q : Well, in your opinion, if there is something wrong, what do you think the result of the operation that produces $6y$? Is it $6 + y$ or $6 \times y$?

S2 : Sorry, ma'am, I still don't really understand.

<p>Berapakah nilai dari $3 + y \times 2$?</p> <p><input checked="" type="radio"/> A. $6y$</p> <p>B. $5xy$</p> <p>C. $2y + 3$</p> <p>D. Tidak ada yang benar</p>	<p>Find the value of $3 + y \times 2$</p> <p>A. $6y$</p> <p>B. $5 \times y$</p> <p>C. $2y + 3$</p> <p>D. none of these</p>
<p>Bentuk lain dari $2 \times (a + b)$ adalah...</p> <p>A. $2a + b$</p> <p>B. $2a + 2b$</p> <p><input checked="" type="radio"/> C. $2ab$</p> <p>D. Tidak ada yang benar</p>	<p>Another way of writing $2 \times (a + b)$ would be</p> <p>A. $2a + b$</p> <p>B. $2a + 2b$</p> <p>C. $2ab$</p> <p>D. none of these</p>

Figure 12: S2's work

S2 believes that in algebraic expressions he/she is free to carry out operations without being based on sequences or rules in arithmetic. In this case, according to Ojose (2018), S2's work was based more on his/her intuition rather than his/her formal knowledge. Another finding based on the S2's

interview is that there is a misconception about adding up $6 + y$ to $6y$. As discussed in the findings for S1, this misconception occurs because students believe the operator sign should not be in the final answer. The $+$ sign is an order that students must do so that they work based on their intuition to add $6 + y$ to $6y$. This case also continues when S2 simplified the fraction form, simplifying $\frac{6}{a} + \frac{6}{b}$ to $\frac{12}{ab}$. S2 stated that the results were obtained by adding the same quantifiers and the same denominators. Assuming that $a + b = ab$ was also incorrect.

In the next question, S2 made the same mistake as he/she added arbitrary numbers with variables, $2a + b = 2ab$. Another error found in answer to this question is that S2 did not consider the parentheses contained in algebraic expressions important.

P : For the expression $2 \times (a + b)$, you simplified it to $2ab$?

S2 : Yes, ma'am

Q : Okay, can you explain how you got it?

S2 : I multiply 2 by a then add b

Q : Is it like this? $2 \times a = 2a$ then b was added to the result (while writing $2a + b = 2ab$)

S2 : Yes ma'am.

Q : Okay, what do you think these brackets mean?

S2 : I don't understand, ma'am (S2 admitted that he/she does not understand the parentheses in the expression).

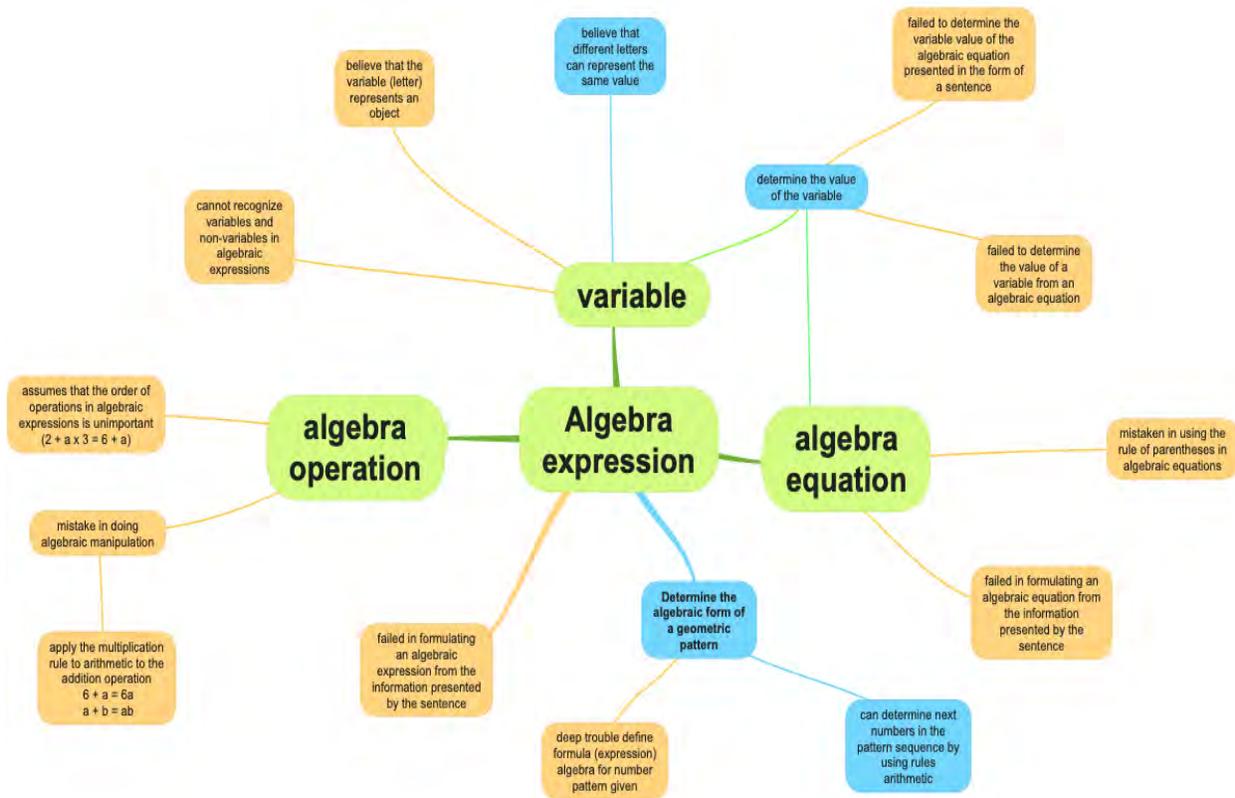
In this regard, the four-tier-test question, S2 also wrote that $5(p + q)$ is $5p + q$. Based on the findings above, it can be stated that S1 has a misconception in or lack of knowledge of using brackets in algebraic expressions. In addition to the above description, S2 also experienced an error in simplifying the algebraic form $\frac{a+b}{b}$ as a , generalizing the distributive law $(5 + x)^2$ as $5^2 + x^2$. The interview results stated that S2 worked for no apparent reason (he/she did not understand the procedure he was using), purely using his intuition.

S2's overall work are presented in S2's cognitive map in Figure 1 below.

S1 and S2's cognitive maps emphasize that the students emphasize procedural knowledge rather than fully understanding the concept. Students with strong conceptual knowledge will be better at solving equations and able to learn new procedures more quickly than students with insufficient conceptual knowledge (Booth & Mc Ginn, 2016). This study shows that S1 and S2 experience misconceptions in the core concept of algebra, namely variables. The basis of algebra is the concept of variables (letters/symbols) and equivalence (Knuth et al., 2005; Weinberg et al., 2016). Referring to Peña et al.'s (2007) cognitive map model, negative direct causal concept refers to

defining variables and determining variable values. S1 and S2 mistakenly interpreted variables, but in some cases, they succeeded in determining variable values from algebraic expressions or equations. The most mistakes made by S1 and S2 are when performing operations in algebra.

The exploration found that this error resulted from their lack of knowledge in the concept of arithmetic operations, so they made mistakes in implementing the concept of algebra. Students who do not understand algebraic concepts, such as expressions, an equal sign, and operation signs, frequently answer math questions using illegal procedures and make mistakes (Edo & Tasik, 2022). This finding is in line with Warren (2003) who asserts that the difficulties experienced by students in algebra stem from inadequate basic knowledge of arithmetic. The potential solution offered is that learning must be meaningful. Students should be encouraged to understand concepts better first so that it is easy to restructure them when accepting new concepts and to master procedural skills (Kshetree, 2020; Mathaba & Bayaga, 2021).



with :

- concept map
- scientific concept
- Misconception/error

Figure 13: S2's Cognitive Map of Misconceptions and Errors

According to Rakes and Ronau (2019), if students are able to connect procedures and concepts, then students' conceptual understanding can produce more robust and consistent procedural skills. Otherwise, students will only use incorrect or incomplete procedures (Skemp 2006). These findings also support the those of several previous studies that misconceptions are often the result of over-generalization or lack of knowledge of previous concepts. Problems arise due to misconceptions or incomplete understanding of concepts or relationships between concepts. In this case, the misconceptions were caused by students' misconceptions of arithmetic and incomplete procedural knowledge of arithmetic. Teachers need to know students' initial knowledge, check it, identify confusion, and then develop new learning ideas (Treagust & Chow, 2013). The negative effects of misconceptions in algebra are emphasized on students' performance and achievement in algebra and other related subjects, which makes detecting these errors an urgent need to overcome them (Yew et al., 2020).

Now, perhaps we can agree that misconceptions in algebra are a problem and can interfere with one's success in mathematics learning. Most likely, instruction in traditional arithmetic and algebra courses is insufficient to address the problem. We attempt to offer instructions that teachers can do in their classrooms to correct students' misconceptions. From the previous discussion, it was found that the instructions carried out were directed at increasing conceptual understanding. The first fundamental thing teachers can do is facilitating students in integrating new information with the knowledge they already know. Teachers must facilitate the transition process from arithmetic to algebra. The results of the restructuring must be made explicit. The teachers can do this by asking students to provide examples and counter-examples of the studied concepts. For example, in the variable concept, when students say that a refers to an apple, which means that $2a$ indicates two apples, the teachers can give a counter-example, such as 'what about a^2 ? What happens if the apple is squared? In this way, students are expected to find their own mistakes from what they understand about the concept of variables.

Second, when teaching, teachers must pay attention to the important concepts in algebra and emphasize them. This way will certainly help students build the correct concept. For example, teachers can explain that the equal sign is not only a pointer to the result of a mathematical work (operation = result), but is also an equivalent relationship between two fields separated by an equal sign. Students should also learn that a result of an operation can contain a variable, and it does not have to be a number. This may seem trivial, but it will significantly affect students' understanding of the concept of equality in algebraic equations.

Third, occasionally, teachers can give scaffolding questions during learning. This question is aimed at confirming students' understanding of a concept. Scaffolding questions can also direct students' thinking to the concept to be achieved. For example, when students say that $\frac{a+b}{b}$ can be simplified to b , the teacher can ask a scaffolding question, such as, does $\frac{a \times b}{b}$ also produce b ?

Students' thinking should be directed to see that it is impossible for $\frac{a+b}{b}$ and $\frac{a \times b}{b}$ to produce the same result. That means there is an error made in one of these works.

The essential thing that needs to be considered by teachers to prevent and overcome misconceptions in students is that teachers should not judge algebraic concepts, especially basic algebra, as easy for students. Teachers need to pay more attention to students' procedural abilities when working on questions, especially in a classroom consisting of students from various backgrounds with different experiences. Of course, these factors affect their initial knowledge and the process of restructuring their knowledge during learning. Finally, the points emphasized above are not only recommendations for teachers in teaching but are expected to be a consideration to help teachers provide meaningful learning to avoid and overcome students' misconceptions in algebra.

CONCLUSION

Students' misconceptions and errors in algebra in the literature review were identified in this study. The cognitive maps drawn from the students' work show that students can recognize algebraic form, and determine variables and non-variables in algebraic expressions/equations. However, they had misconceptions about the definition of the variable itself. The students' cognitive maps also indicate that they had difficulty changing sentences into algebraic expressions/equations but succeeded in determining the variable's value. This finding further confirms that in learning algebra students are likely to master procedural knowledge rather than conceptual one, so they experience misconceptions.

Students' misconceptions in algebra are most likely influenced by their incomplete knowledge of arithmetic, which causes difficulties for students to make the transition from arithmetic to algebra. This study shows that the students' ability to make the transition depends on their ability to understand and use variables to represent unknown entities in mathematical expressions. Their ability to interpret the equal sign as a symbol of mathematical equality between two expressions and solve algebraic problems by representing mathematical ideas expressed in general sentences to mathematical sentences also influence the students' ability to make the transition from arithmetic to algebra.

As mentioned above, teachers can do several things to avoid or overcome misconceptions in students. Teachers should encourage learning to become more meaningful so that students' conceptual understanding is maximized and supports their success in applying procedures in algebra. The teachers also need to facilitate students in integrating new information with prior knowledge they already have by giving examples and non-examples related to certain algebra concepts. Teachers must pay attention to essential algebra concepts and emphasize them when teaching them. Last, the teachers can give scaffolding questions during learning to direct students' thinking.

LIMITATIONS

There are a number of limitations in this study. First, the researcher did not do thorough observations in the classroom. Second, the intervention was carried out by the researcher by giving scaffolding questions to the subjects. However, it was carried out in a limited time so that it could not lead students' thinking completely, considering that many points from basic algebraic concepts needed to be observed. Last, the researchers did not have the opportunity to explore teachers' opinions further regarding their understanding of misconceptions and didactic actions that have been carried out so far to prevent and overcome misconceptions in their mathematics classes.

References

- [1] Arnawa, I. M. (2019). Errors and misconceptions in learning elementary linear algebra. In *Journal of Physics: Conference Series* (Vol. 1321, Issue 2). <https://doi.org/10.1088/1742-6596/1321/2/022095>
- [2] Booth, J., & Mc Ginn, K. (2016). Misconceptions and Learning Algebra. In *And the Rest is Just Algebra* (pp. 1–238). Springer International Publishing. <https://doi.org/10.1007/978-3-319-45053-7>
- [3] Bottini, R., & Doeller, C. F. (2020). Knowledge Across Reference Frames: Cognitive Maps and Image Spaces. *Trends in Cognitive Sciences*, 24(8), 606–619. <https://doi.org/10.1016/j.tics.2020.05.008>
- [4] Bush, S. B. (2013). Analyzing common algebra-related misconceptions and errors of middle school students. In *Dissertation Abstracts International Section A: Humanities and Social Sciences* (Vol. 73, Issues 7-A(E)).
- [5] Chen, J., Wang, M., Dede, C., & Grotzer, T. A. (2021). Analyzing student thinking reflected in self-constructed cognitive maps and its influence on inquiry task performance. *Instructional Science*, 49(3), 287–312. <https://doi.org/10.1007/s11251-021-09543-8>
- [6] Chow, F. T.-C. (2011). Students' Difficulties, Conceptions and Attitudes Towards Learning Algebra: An Intervention Study to Improve Teaching and Learning. In *A Dissertation* (Issue October).
- [7] Cline, K. (2020). Addressing common errors and misconceptions in integral calculus with clickers and classroom voting. *Teaching Mathematics and Its Applications*, 39(2), 71–85. <https://doi.org/10.1093/teamat/hrz002>
- [8] Cohen, L., Manion, L., & Morrison, K. (2018). *Research Methods in Education* (Eight). Routledge Taylor and Franchis.

- [9] Das, K. (2020). A Study on Misconception of Using Brackets in Arithmetic Expression. In *Shanlax International Journal of Education* (Vol. 8, Issue 4, pp. 76–80). Shanlax International Journals. <https://doi.org/10.34293/education.v8i4.3252>
- [10] Deringöl, Y. (2019). Misconceptions of primary school students about the subject of fractions. *International Journal of Evaluation and Research in Education*, 8(1), 29–38. <https://doi.org/10.11591/ijere.v8i1.16290>
- [11] Edo, S. I., & Tasik, W. F. (2022). Investigation of Students' Algebraic Conceptual Understanding and the Ability to Solve PISA-Like Mathematics Problems in a Modeling Task. *Mathematics Teaching-Research Journal*, 14(2), 44–60.
- [12] Edogawatte, G. (2011). *Secondary school students misconceptions in algebra* (Issue 1).
- [13] Efriani, N., Zulyusri, Z., Rahmi, Y. L., & Sumarmin, R. (2019). Identification Of Students Of Misconception Junior High School Class VII Sutera On The Topic Photosintesis Using Two Tier Diagnostic Test. In *Jurnal Atrium Pendidikan Biologi* (Vol. 4, Issue 2, p. 29). Universitas Negeri Padang (UNP). <https://doi.org/10.24036/apb.v4i2.5763>
- [14] Faizah, S., Nusantara, T., Sudirman, & Rahardi, R. (2022). Constructing Students' Thinking Process through Assimilation and Accommodation Framework. *Mathematics Teaching-Research Journal*, 14(1), 253–269.
- [15] Garoui, N., & Jarbou, A. (2012). Cognitive Mapping: Testing Application of Causal Algebra. *Applied Mathematics*, 03(04), 364–372. <https://doi.org/10.4236/am.2012.34057>
- [16] Gurel, D. K., Eryilmaz, A., & McDermott, L. C. (2015). A review and comparison of diagnostic instruments to identify students' misconceptions in science. *Eurasia Journal of Mathematics, Science and Technology Education*, 11(5), 989–1008. <https://doi.org/10.12973/eurasia.2015.1369a>
- [17] Irawati, Zubainur, C. M., & Ali, R. M. (2018). Cognitive conflict strategy to minimize students' misconception on the topic of addition of algebraic expression. *Journal of Physics: Conference Series*, 1088. <https://doi.org/10.1088/1742-6596/1088/1/012084>
- [18] Joanna, S., & Jacquelynn, malloy. (2019). Addressing Misconceptions in Algebra 1. *Mathematics Teacher*, 112(6), 450–454. <https://doi.org/10.5951/mathteacher.112.6.0450>
- [19] Jong, C., Thomas, J. N., Fisher, M. H., Schack, E. O., Davis, M. A., & Bickett, M. E. (2017). Decimal Dilemmas: Interpreting and Addressing Misconceptions. *Ohio Journal of School Mathematics*, 75(1), 13–21. <https://library.osu.edu/ojs/index.php/OJSM/article/view/5731/4590>

- [20] Kang, H., Scharmann, L. C., Kang, S., & Noh, T. (2010). Cognitive conflict and situational interest as factors influencing conceptual change. *International Journal of Environmental and Science Education*, 5(4), 383–405.
- [21] Kshetree, M P. (2021). Eighth grade students' misconceptions and errors in mathematics learning in Nepal. *European Journal of Educational Research*, 10(3), 1101–1121. <https://doi.org/10.12973/EU-JER.10.3.1101>
- [22] Kshetree, Mukunda Prakash. (2020). Exploring and Analyzing Students' Algebraic Misconceptions and Errors. *International Journal of Scientific and Research Publications (IJSRP)*, 10(1), 7–17. <https://doi.org/10.29322/ijsrp.10.01.2020.p9703>
- [23] Mathaba, P. N., & Bayaga, A. (2021). Analysis of Types, Sources of Errors and Misconceptions in South African Algebra Cognition. *Universal Journal of Educational Research*, 9(5), 928–937. <https://doi.org/10.13189/ujer.2021.090505>
- [24] Mulungye, M. (2016). Sources of Student Errors and Misconceptions in Algebra and Effectiveness of Classroom Practice Remediation in Machakos County- Kenya. In *Journal of Education and Practice; Vol 7, No 10 (2016); 31-33*. The International Institute for Science, Technology and Education (IISTE) 2016-04-28. <http://e-resources.perpusnas.go.id:2048/login?url=https://search.ebscohost.com/login.aspx?direct=true&db=edsoai&AN=edsoai.ocn953392885&site=eds-live>
- [25] Mulungye, M. M. (2018). Sources of students' errors and misconceptions in algebra and influence of classroom practice remediation in secondary schools Machakos Sub-County, Kenya. *Angewandte Chemie International Edition*, 6(11), 951–952.
- [26] Natalia, K., Subanji, S., & Sulandra, I. M. (2016). Miskonsepsi Pada Penyelesaian Soal Aljabar Siswa Kelas VIII Berdasarkan Proses Berpikir Mason. *Jurnal Pendidikan: Teori, Penelitian, Dan Pengembangan*, 1(10), 1917—1925-1925.
- [27] Ojose, B. (2016). Common Misconceptions in Mathematics: Strategies to Correct Them. In *Mathematics Teaching in the Middle School* (Vol. 21, Issue 9).
- [28] Peña, A., Sossa, H., & Gutiérrez, A. (2007). Cognitive Maps: an Overview and their Application for Student Modeling. *Computación y Sistemas*, 10(3), 230–250.
- [29] Rakes, C. R., & Ronau, R. N. (2019). Rethinking mathematics misconceptions: Using knowledge structures to explain systematic errors within and across content domains. *International Journal of Research in Education and Science*, 5(1), 1–21.

- [30] Ramsburg, J., & Ohlsson, S. (2016). Category Change in the Absence of Cognitive Conflict. *Journal of Educational Psychology*, 108. <https://doi.org/10.1037/edu0000050>
- [31] Russell, M., O'Dwyer, L. M., & Miranda, H. (2009). Diagnosing students' misconceptions in algebra: Results from an experimental pilot study. *Behavior Research Methods*, 41(2), 414–424. <https://doi.org/10.3758/BRM.41.2.414>
- [32] Sarımanoğlu, N. U. (2019). The Investigation of Middle School Students' Misconceptions about Algebraic Equations *. *Studies in Educational Research and Development*, 3(1), 1–20.
- [33] Stothard, J. (2021). Identifying hidden misconceptions in mathematics. *Mathematics Teaching*, 277, 37–39.
- [34] Subanji, & Nusantara, T. (2013). Karakterisasi Kesalahan Berpikir Siswa Dalam Mengonstruksi Konsep Matematika. *Jurnal Ilmu Pendidikan*, 19(2), 208–2017.
- [35] Subanji, S. (2015). *Teori Kesalahan Konstruksi Konsep dan Pemecahan Masalah Matematika*.
- [36] Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limit and continuity. *Educational Studies in Mathematics*, 3(September), 151–169. <https://doi.org/10.1007/BF00305619>
- [37] Tooher, H., & Johnson, P. (2020). The role of analogies and anchors in addressing students' misconceptions with algebraic equations. *Issues in Educational Research*, 30(2), 756–781.
- [38] Treagust, D. F., & Chow, T.-C. F. (2013). An Intervention Study Using Cognitive Conflict to Foster Conceptual Change. *Journal of Science and Mathematics*, 36(1), 44–64.
- [39] Vargová, M. (2020). Students' misconceptions in definition of a limit of sequence. In *19th Conference on Applied Mathematics, APLIMAT 2020 Proceedings* (pp. 1089–1097). https://api.elsevier.com/content/abstract/scopus_id/85082401092
- [40] Vaughn, A. R., Brown, R. D., & Johnson, M. L. (2020). Understanding Conceptual Change and Science Learning through Educational Neuroscience. *Mind, Brain, and Education*, 14(2), 82–93. <https://doi.org/10.1111/mbe.12237>
- [41] Warren, E. (2003). The role of arithmetic structure in the transition from arithmetic to algebra. *Mathematics Education Research Journal*, 15(2), 122–137. <https://doi.org/10.1007/BF03217374>

- [42] Xie, S., & Cai, J. (2022). Fifth graders' learning to solve equations: the impact of early arithmetic strategies. *ZDM - Mathematics Education*, 54(6), 1169–1179. <https://doi.org/10.1007/s11858-022-01417-8>
- [43] Yew, W. T., AL-Rababaha, Y., & Meng, C. C. (2020). Misconceptions in School Algebra. *International Journal of Academic Research in Business and Social Sciences*, 10(5), 803–812. <https://doi.org/10.6007/ijarbss/v10-i5/7250>
- [44] Zielinski, S. F. (2017). From No to Yes: The Impact of an Intervention on The Persistence of Algebraic Misconceptions among Secondary School Algebra Students [ProQuest LLC]. In *ProQuest LLC*. <http://e-resources.perpusnas.go.id:2048/login?url=https://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=ED582619&site=eds-live>