A Study of Number Sense and Metacognitive Awareness of Primary School Fourth Grade Students

Alper Yorulmaz[1], Emel Çilingir Altıner[2], Sıtkı Çekirdekci[3]

[1] Muğla Sıtkı Koçman University, Faculty of Education, Muğla, Turkiye, [2] Çukurova University, Faculty of Education, Adana, Turkiye, [3] Sinop University, Faculty of Education, Sinop, Turkiye,

alperyorulmaz@mu.edu.tr , cilingire@gmail.com , cekirdekci-sitki@hotmail.com

Abstract: The purpose of the current study is to determine the number sense performances and metacognitive awareness of primary school fourth grade students and the status of metacognitive awareness in predicting number sense. To this end, the study employed the relational survey model, one of the quantitative research methods. In the study, it was revealed that the number sense performances of the primary school fourth grade students were at a medium level and did not vary depending on gender. In addition, it was determined that the students' metacognitive awareness was at a high level and that metacognitive awareness did not vary significantly depending on gender. It was found that there is a positive correlation between the number sense performances and metacognitive awareness of the primary school fourth grade students. The students' metacognitive awareness was found to predict their number sense performances at a low level.

INTRODUCTION

What is the purpose of mathematics? Is it to reflect on why and how you perform mathematical operations or to remember and apply the right formulas? Mathematics is actually a subject of critical thinking, problem-solving and creativity, contrary to what is known (a subject of the world of formulas and memorization) (Boaler, 2016). People actually organize their own thoughts and decisions while doing math and employing the necessary skills. This requires metacognitive skills.

Metacognition is thinking about thinking. It also includes self-monitoring and correcting one's own learning processes. In mathematics, teachers can use metacognition to teach students how to handle a complex math problem. Just as there is a connection between numbers and operations, there are similar connections between metacognition and problem-solving (McIntosh, Reys & Reys, 1997). When a student first encounters a problem, the student's success in solving that problem depends on his/her ability to be aware of what decision he/she is making and why he/she is making that decision. According to Polya (2014), through the problem-solving stages (understanding, planning, solving and reflecting), students' metacognitive skills and their

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ. MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

awareness of these skills play an important role while applying and arranging their thoughts and decisions. However, metacognition alone is not enough to ensure success. Metacognition serves learning by enabling the use of certain skills (mental calculation, estimation, planning, evaluation, problem-solving…) (Çekirdekçi, 2015). Of these skills, mental calculation and estimation are also important for the development of number sense (Carroll, 1996). When the problem-solving stages of Polya are considered, we see that both number sense and metacognition skills should be used together in order to progress through the stages. Therefore, while number sense is of great importance in developing metacognitive skills (Lee, Ng & Yeo, 2019), metacognitive skills have a significant impact on the performance related to number sense.

Number sense

Mathematics uses number sense, which is the most important component of mathematics, to train the mind in problem-solving and decision making and to foster logical and systematic thinking. Testolin et al. (2020) stated that there is number sense even in animals and they stated that they are numerically encoded in the brain even when we are looking at non-numerical elements we see around us. In addition, Tosto et al. (2014) stated that number sense is not hereditary and that it is associated with general intelligence, but not with gender. From this point of view, it is thought that number sense is a skill that can be developed later.

There are many definitions of number sense. Yang (2003) defined number sense as the ability to produce useful, flexible and efficient strategies for solving numerical problems. Çekirdekçi (2015) defined number sense as a flexible thinking skill based on intuition and senses related to knowing the properties of numbers and operations, working flexibly with numbers in problem situations, making mental calculations, making predictions, developing effective and usable strategies and judging the appropriateness of the results of the problems. Mohamed and Johnny (2010) also stated that individuals with good number sense tend to exhibit characteristics such as logical approach, planning and control, flexibility and conformity and a sense of reasonableness when making mental calculations. They emphasized that these skills are very important for the individual to overcome the numerical problems they encounter in their daily lives.

Enabling students to develop their number sense is considered an important task in national and international mathematics education (National Council of Teachers of Mathematics [NCTM], 2000; Şengül, 2013) because number sense refers to understanding the student's ability to use numbers, operations and their relationships in daily life situations (Yang & Li, 2013). This skill is used to develop useful, flexible and efficient strategies (including mental computation and estimation) for handling numerical situations (Yang & Huang, 2004).

Based on the common ideas about number sense, it is seen that number sense includes knowing the meanings and values of numbers and using this information based on a successful intuition. It can be argued that instead of memorizing rules, the logic of numbers and operations is understood and plans and strategies are developed accordingly with number sense. For example, we can see

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ. MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

that memorization is useless in the learning area of measurement. It is necessary to make a new measurement for each object for which a new measurement value is requested. But a student with developed number sense is adept at approximating without a standard measuring instrument, through his/her experience and self-developed logic and intuitive frameworks. Of course, the student also uses his/her other metacognitive skills in this process. Therefore, it is thought that students' metacognitive awareness will be important in the emergence of this situation.

Metacognitive awareness

As a result of the search for possible solutions to increase students' mathematics achievement, one of the solutions found by educators is to improve students' metacognitive skills (Montero III & Elipane, 2021) because metacognition involves students being aware of their own strengths and weaknesses in their thinking processes, re-checking their thinking processes and managing and arranging their thinking processes while solving a problem (Alindra, Fauzan & Azmar, 2018). Metacognition, which means thinking about thinking, is often related to many skills connected with thinking and learning, such as critical thinking, reflecting on one's own actions and thoughts, solving problems, making decisions, predicting one's own performance on various tasks and monitoring one's own level of understanding (Baş, Özturan-Sağırlı & Bekdemir, 2016). In the field of mathematics, metacognition is explained as a person's knowledge of his/her cognitive processes and his/her perception of a mathematical problem in relation to the process of planning, monitoring and evaluating problem solutions (Flavell, 1979).

Another component of metacognition needed in problem-solving is metacognitive awareness. Metacognitive awareness is defined as the ability of students to reflect on the known and the unknown, to understand how they learn in the context of learning and to control themselves in learning (Herlanti, 2015; as cited in Alindra, Fauzan & Azmar, 2018). According to Boğar (2018), metacognitive awareness is an effort to acquire and use the metacognitive thinking skills that an individual needs throughout his/her life. Metacognitive awareness also refers to the individual's own knowledge, learning processes, knowledge of cognitive and affective states and conscious control and regulation of knowledge. In short, metacognitive awareness can also be expressed as the awareness of what one knows and what one will do in his/her own learning process.

The motto "Knowing when you know is knowing what to do when you don't know" is very suitable for metacognitive awareness. In this way, the student observes himself/herself in the learning processes and tries to compensate for his/her deficiencies. For example, if a student finds that he or she has difficulty measuring an object with a ruler, he/she my try to solve this problem by first checking whether the ruler is broken, whether the point where he/she places the ruler when measuring is the starting point and whether he/she leaves a gap in-between while advancing the ruler or whether he/she uses a ruler of the right size. If he/she realizes that this approach is not working, he/she decides to try a new approach, meaning that he/she has used the metacognitive awareness process.

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ. MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

Studies show that there is a direct relationship between students' metacognition and their success in mathematics (Mevarech & Kramarski, 2003). Rinne and Mazzocco (2014) stated that there is a strong relationship between metacognitive skills and mental arithmetic in children towards the end of primary school and that early metacognitive monitoring ability predicts improvement in mental arithmetic performance. However, it has not yet been determined whether these associations continue to exist and whether the same associations can be observed in young children when different aspects of executive functions and numerical magnitude are included in the process. At the same time, when the studies on metacognitive awareness are examined (Adhytama, 2014), it is seen that students with good metacognitive awareness have better problem-solving strategies and learning outcomes compared to students with poor metacognitive awareness (as cited in Alindra, Fauzan & Azmar, 2018).

The relationship between number sense and metacognition

While solving problems in their classrooms, teachers use helpful and easy strategies specific to the solution of that problem. Students want to use this strategy, but it is important for them to realize that this strategy does not help when faced with different problems. When students realize that not every problem can be solved with the same strategy, that every problem has its own dynamics, then when they encounter different problems, they can develop and use different strategy stages specific to the problem (Waters & Kunnmann, 2010). This process, which holds the student responsible for his/her own learning path, requires teachers to expose their students to various problems until they develop their metacognitive skills. As students acquire a wider range of skills in metacognition, opportunities will arise to develop, evaluate and apply strategies efficiently.

International research on students' problem-solving skills in the context of mathematics education has shown that children do not perform well in multi-stage tasks and that mathematics teachers have difficulty planning and implementing lessons that develop students' problem-solving skills (Kramarski, 2008). Therefore, it has been stated that the concept of metacognition is useful in improving this situation (Schneider & Artel, 2010) because it has been stated that with the inclusion of metacognition in mathematics, executive control, monitoring and self-regulation skills are also included and thus the likelihood of success in mathematics can be increased (Lester, 1982). Verschaffel (1999) pointed out that metacognition has a special importance in the mathematical problem-solving process. In particular, metacognition has a great impact on the processes of using information, distinguishing between necessary and unnecessary information and estimating (Schneider & Artel, 2010). Vo et al. (2014) revealed that students' metacognitive skills specific to the numerical domain predict their mathematical knowledge, and they suggested that children's metacognition is a cognitive ability of mathematics in children. In addition, Bellon, Fias, and De Smedt (2019) stated that the relationship between metacognition and arithmetic performance is a reciprocal relationship where both arithmetic performance and metacognition affect each other. Given that number sense is a necessary process for understanding and learning mathematics in a meaningful way, it can be said that its interaction with metacognition is inevitable.

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ. MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

The development of metacognitive skills and the awareness that a student should not overly rely on a strategy also affect students' number sense skills (Carr, 2010). At the same time, the importance of number sense has been emphasized in terms of providing children with the experience of expressing their thoughts and enabling them to practice about it so that they can develop their metacognitive skills. It has also been emphasized that when metacognition develops, number sense skills will also develop and that there is an active reciprocal interaction between the two (Lee, Ng & Yeo, 2019). Koening (2020) drew attention to the relationship between number sense and metacognition, stating that talks on numbers, which ensure that number sense skills are fluent and automatic, also enable children to practice expressing their thoughts in order to improve their metacognitive skills. For the development of number sense skills, it is important to determine how the student thinks about numbers, operations and results, how he/she reflects this and which strategies he/she prefers. Strategies, along with number sense and metacognition, are no longer limited to what the teacher teaches, but develop further as the student discovers his/her own strategies and adapts them into new complex problems (Ilko, 2021). As a result, metacognition and number sense involve higher-order thinking skills. Number sense is a way of thinking that requires skills such as perception, attention, flexible thinking, fluency, automaticity, strategy development while metacognition is a cognition that controls these skills.

Current study

Even before children enter primary school, they develop strong informal number sense through their interactions with games, older children and adults (Woods, Ketterlin Geller, & Basaraba, 2018). Number sense is very important for primary school students. The first reason for this is that number sense is a way of thinking that encourages meaning-making and directs problem-solving in a flexible and efficient way (Dunphy, 2007). The second reason is that number sense plays a key role in primary school students' achievement in mathematics (Jordan, Glutting, & Ramineni, 2010). The third reason is that number sense triggers a meaningful learning process (Yang & Li, 2008). Although it has been stated that there are positive relationships between mathematics achievement and number sense performance, there are very few studies on this subject in Turkey and this area of research has only just begun to attract attention (Bütüner, 2018).

While trying to develop number sense in students, it is thought that metacognitive skills are also needed to enable students to develop a deeper understanding. When the existing research is reviewed, it is seen that both number sense and metacognition are skills that should be acquired by students (McIntosh, Reys & Reys, 1997; Singh, 2009; Yang & Hsu, 2009). However, it is seen that studies conducted to include metacognition and number sense together are limited in number (Ilko, 2021; Çekirdekçi, Şengül & Doğan, 2016). Moreover, although most of the relational studies on the relationship between metacognition and mathematical performance have been conducted on middle school students and adolescents, the study by Carr, Alexander and Folds-Bennett (1994) investigated the importance of metacognitive knowledge for primary school students' mathematical performance (Schneider & Artel, 2010). The current study was planned due to the

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ. MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

inadequacy of studies conducted on primary school students and the need to address the relationship between number sense and metacognitive awareness, which are considered to be important in terms of increasing mathematics achievement. Thus, in this study, it was aimed to examine the relationship between number sense performances and metacognitive awareness levels of primary school fourth grade students. To this end, the problem statement of the study is worded as follows; "Is the metacognitive awareness of primary school fourth grade students a significant predictor of their number sense performance?". In line with the problem statement of the study, the sub-problems of the study are expressed as follows;

1. What is the number sense performance of primary school fourth grade students?
2. Do primary school fourth grade students' number sense performances vary significantly by gender?
3. What is the metacognitive awareness level of primary school fourth grade students?
4. Do the metacognitive awareness levels of primary school fourth grade students vary significantly by gender?
5. Is primary school fourth grade students' metacognitive awareness a significant predictor of their number sense performance?

METHOD

Research Method

The model of the current study, which examined the relationship between students' number sense performance and metacognitive awareness, was determined as the relational survey model. Relational survey models are research models that aim to determine the existence or degree of change between two or more variables (Karasar, 2011). In the relational survey model, the data obtained by using measurement tools are analyzed with some statistical methods and the possible relationship between the variables is expressed numerically (Creswell, 2014). In the current study, the exploratory correlational research design, one of the relational research methods, was used. In correlational designs, the relationship between the variables is determined without interfering with the variables, while in exploratory correlational studies, the relationships between the variables are tried to be explored (Büyüköztürk et al., 2015).

Participants

The target population is defined by Creswell (2012) as "a sampling framework that includes a group of individuals with some common descriptive characteristics that the researcher can identify and work with". The target population, which is accepted as the universe of the study, consists of fourth grade primary school students attending the schools in a city centre in the north of Turkey in the 2021-2022 school year. The sample of the study consists of 170 fourth grade students attending three primary schools located in a city centre in the north of Turkey in the 2021-2022 school year. The convenience sampling method was used to determine the schools that constituted

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ. MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

the sample group of the study. Convenience sampling is one of the non-random sampling methods in which the data are obtained from a sample easily accessible by the researcher (Büyüköztürk et al., 2015). The reason why the study was conducted on fourth-grade students is that the scales used in the study were aimed at fourth-grade students, as they were thought to be more competent in explaining the solutions of the questions in the number sense test. Of the students participating in the study, 48.2% (f=82) are males and 51.8% (f=88) are females.

Data collection tools

The data of the study were obtained by using two different data collection tools. These two data collection tools used in the study are the "Number Sense Test" developed by Çekirdekci (2015) and the "Metacognitive Awareness Scale Teacher Form" developed by Esmer and Yorulmaz (2017).

The Number Sense Test is a measurement tool with a Cronbach Alpha reliability value of .72, consisting of a total of 11 questions, nine multiple-choice and two open-ended questions, including numbers and operations. A total of 11 items with a discriminant value in the test are gathered under three components with an eigenvalue greater than 1. These components are named as "Knowing the equivalents of numbers and making quantitative reasoning-inferences", "Calculating the effects of operations using a reference point" and "Knowing the meaning of numbers and thinking flexibly". Provided that an explanation is made, the lowest score to be taken from the number sense test is 11 and the highest score is 44. In the current study, the Cronbach Alpha reliability value of the number sense test was calculated to be .87.

Another measurement tool used in the study is the "Metacognitive Awareness Scale Teacher Form" adapted by Esmer and Yorulmaz (2017) from the "Metacognitive Awareness Scale for Children (Form A)" developed by Sperling et al. (2002) and adapted into Turkish by Karakalle and Saraç (2007). The metacognitive awareness scale teacher form was prepared for 3rd and 4th grade students and consists of 12 items. The teacher's form was created by changing the roots and structures of the questions prepared for the students in the original form, and a three-point Likert type (always, sometimes, never) scale was used for respondents to respond the items. The highest score to be taken from the form is 36 and the lowest score is 12. The content and criterion validity of the single-factor form was established and the Cronbach Alpha reliability value was found to be .94. The Cronbach Alpha reliability value of the form in the current study was calculated to be .89.

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ. MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

Data collection

Before the Number Sense Test was started to be answered, one of the researchers informed the students about the purpose and subject of the study and the identities of the researchers. It was emphasized that the explanation part of the questions in the number sense test should not be left blank. Approximately two class hours were given to the students to answer the test. The application of the test was carried out by one of the researchers under the supervision of the classroom teachers. The Metacognitive Awareness Scale Teacher Form was filled in for each student based on the observations of the classroom teachers. The forms were given to the classroom teachers, and after a week, they were taken back from the classroom teachers. The classroom teachers filled the forms by observing the students during this period and taking into account their previous observations of the students.

Data analysis

While analysing the answers given by the students to the number sense test, it was checked whether each item was answered correctly or incorrectly, and then the explanations for the answers to these items were examined. The answers to the items and explanations made were evaluated and scored considering whether they are correct or false and the solution strategy followed and in this way the number sense score was calculated. The strategies used in the solutions of the questions in the test were arranged by taking the coding in the literature as a criterion. The correct answer obtained by using a number sense-based solution (NSBS) in the solutions of the items in the test was given 4 points, and the correct answer obtained by using the rule-operation-based methods (ROBM) was given 2 points. If number sense was used in the solutions of the items, but a wrong answer was found then 3 points were given and 1 point was given if the wrong answer was obtained by using rule-operation-based solutions. If no explanation was given by the students in the solutions of the items or individual generalizations were made while explaining or misconceptions were detected in the explanations (A.C., B.O.A.), then 0 point was given. The items in the metacognitive awareness scale teacher form were prepared in a three-point Likert scale and responses to the items were scored as follows; “always=3”, “sometimes=2”, “never=1”.

A statistical package program was used in the analysis of the data collected with the measurement tools. Before the data were entered into the program, the scores obtained from the metacognitive awareness scale teacher form were entered into the list where the scores taken from the number sense test and the genders of the participants had already been noted.

Missing value analysis was started by checking whether there were missing or incorrect entries for the data group obtained. In the analysis performed to see the rate of missing data, it was observed that there was no missing data in the data group.

Before proceeding to the analysis of the data, the distributions of the data sets were examined. Whether the data sets showed a normal distribution or not was evaluated with reference to the arithmetic mean, mode, median values, and kurtosis and skewness coefficients. In order for the

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ. MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

data to show a normal distribution, the skewness and kurtosis values must be between -1 and +1 (Hair et al., 2013). As a result of the evaluations, it was observed that the mean, mode and median values of the distributions were close to each other, and the kurtosis and skewness values were found to be in the acceptable range. For this reason, it was decided that the data sets had a normal distribution and parametric tests were used in the analysis of the data.

In the first and third sub-problems of the study, descriptive statistics (frequency, percentage calculation, mean, standard deviation) values were calculated, then the general mean scores were calculated and the students were grouped according to their levels. In the second sub-problem, in which whether the students' number sense performances vary significantly by gender was checked, independent samples t-test was used and in the fourth sub-problem, in which whether the students' metacognitive awareness levels vary significantly by gender was checked, Mann-Whitney U, t-test was used. In order to determine whether there is a significant relationship between the students' number sense performances and their metacognitive awareness levels, the Pearson Moment Product Correlation coefficient was calculated in the fifth sub-problem. Regression analysis was used to determine whether the students' metacognitive awareness was a significant predictor of their number sense performance. In the findings section, solutions based on number sense given by the students to the questions in the number sense test and solution examples based on rule-operation-based methods are also included.

FINDINGS

In this section, the analysis findings obtained as a result of the statistical processes carried out for each sub-problem of the study will be given. The findings related to the first sub-problem of the study "What is the number sense performance of primary school fourth grade students?" are presented in tables including the number sense strategies used in the solutions of the questions in the number sense test, operation-rule-based calculations used in solving the questions in the number sense test, and the arithmetic mean values obtained from the test. The distribution and percentages of the solution strategies used by the students in the solutions of the questions in the number sense test are shown in Table 1.

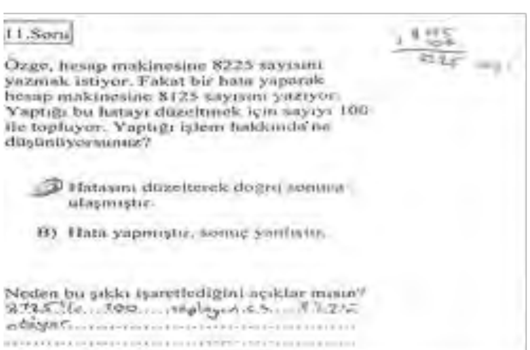
Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ. MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

Question Number	Types and Percentages of the Strategies					
	A.C.	NSBS		ROBM		
	B.O.A.(%)	Wrong answer (%)	Correct answer (%)	Wrong answer (%)	Correct answer (%)	Total (%)
1	8.8	50.6	23.5	2.9	14.1	100.0
2	7.6	25.9	35.3	1.2	30.0	100.0
3	0.6	49.4	25.3	17.1	7.6	100.0
4	2.4	21.8	25.9	22.4	27.6	100.0
5	2.4	15.9	76.5	0	5.3	100.0
6	4.7	20.6	66.5	1.2	7.1	100.0
7	1.8	22.4	71.8	0	4.1	100.0
8	13.5	54.1	20.0	4.1	8.2	100.0
9	2.9	18.8	62.4	0	15.9	100.0
10	4.7	27.6	42.4	11.2	14.1	100.0
11	4.1	14.7	40.6	5.3	35.3	100.0

Table 1: Analysis of the strategies used by the primary school fourth graders in the solutions of the questions in the number sense test

When Table 1 is examined, it is seen that in general the students used rule-operation-based methods more and number sense skills less in the solutions of the questions. In the fourth question, the students used rule-operation-based solutions and number sense skills at approximately the same levels. Number sense skills were used the most in the fourth question (50.0%) and the least in the seventh question (4.1%). Examples of the strategies that students used in solving the questions in the number sense test are given in Figure 1.

11th question: Özge wants to write the number 8225 into her calculator. But she makes a mistake and writes the number 8125 into the calculator. To correct this mistake she made, she adds the number by 100. What do you think about the operation?
A) She corrected her mistake and reached the correct result.
B) She made a mistake, the result is wrong.
Can you explain why you ticked this option?



1st question: Which of the following is a number smaller than 4/8?
Can you explain how you found the answer?
The higher the numerator of the fraction, the smaller it gets

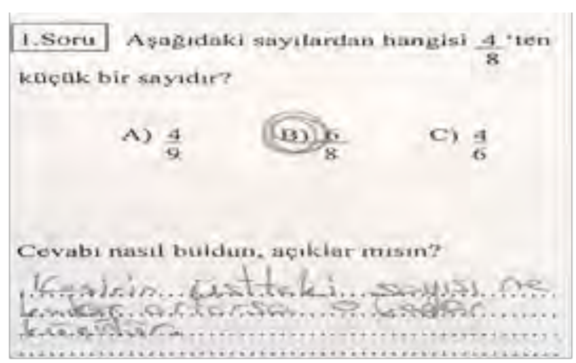
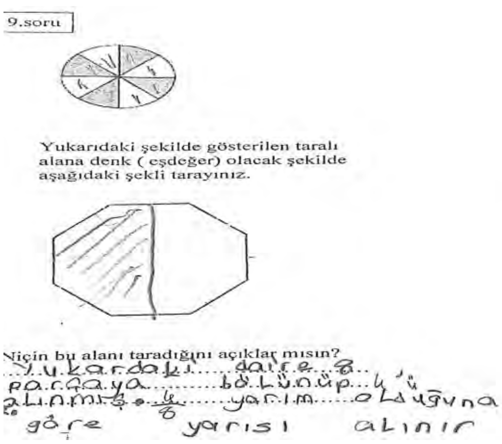


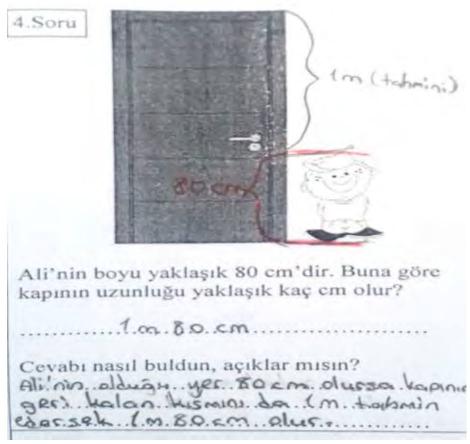
Figure 1: Sample correct and incorrect answers obtained by using the rule-operation-based solution strategies by the students in solving the questions in the number sense test

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ.
MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

In the first image given in Figure 1, the eleventh question in the number sense test is shown. In the rule-operation-based correct answer given to this question, the student answered by performing operations that when 100 is added to the given number, the correct result will be obtained. In the first question of the number sense test, which is another example, the student resorted to the rules used to compare fractions. However, here the student misremembered the rule and gave a wrong answer to the question. The student's misremembering of the rule also exemplifies students' misconceptions. The generalization of a feature of the denominator which is used to compare fractions for the numerator can be expressed as the reason for this situation.



9th question: Scribble the figure below to be equivalent (equivalent) to the shaded area shown in the figure above.
Can you explain why you scribbled this area?
The circle above is divided into eight parts and 4 of them are taken. Since 4/8 is half, half is taken



4th question: Ali's height is about 80 cm. Accordingly, how many centimetres will the length of the door be approximately?
Can you explain how you found the answer?
If the place where Ali stands is 80 cm, if we estimate the rest of the door as 1 meter, it will be 1 m and 80 cm.

Figure 2: Sample correct and incorrect answers obtained by using number sense-based solution strategies by the students in solving the questions in the number sense test

The student who used the number sense-based solution to solve the ninth question in the number sense test and answered the question correctly realized that the shaded area represented in the image in the question represented 4/8 and this value was equal to half. The student divided the blank shape given to him/her into eight equal parts and instead of scribbling the four parts, he/she divided it into two equal parts and took one part. In the fourth question given in Figure 2, another student used the solution based on number sense, but gave an incorrect answer. In order to find the length of the door given in the image, the student made use of the height of Ali standing next to the door and used it as a reference point. As seen in the student's answer, he/she made drawings on the door according to Ali's height, but he/she gave an estimated answer by comparing the remaining part with the reference point. Thus, he/she got the wrong answer.

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ.
MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

The arithmetic mean and standard deviation values of the scores obtained from the number sense test by the students participating in the study are given in Table 2.

	n	$\bar{X}$	ss
Students' number sense performances	170	21.75	5.84

Table 2: The arithmetic mean and standard deviation values related to the students' performances in the number sense test

As seen in Table 2, the arithmetic mean of the scores taken by the primary school fourth grade students from the test is $\bar{X} = 21.75$. The ratio of the mean obtained to the highest score to be taken from the test corresponds to 49.43%. The arithmetic mean of the students participating in the study is approximately the half value of the highest score to be taken from the test. According to this mean, it can be said that the number sense performances of the students are at the "medium" level. In Figure 3, the categorical distributions of the number sense performances of the students participating in the study are given.

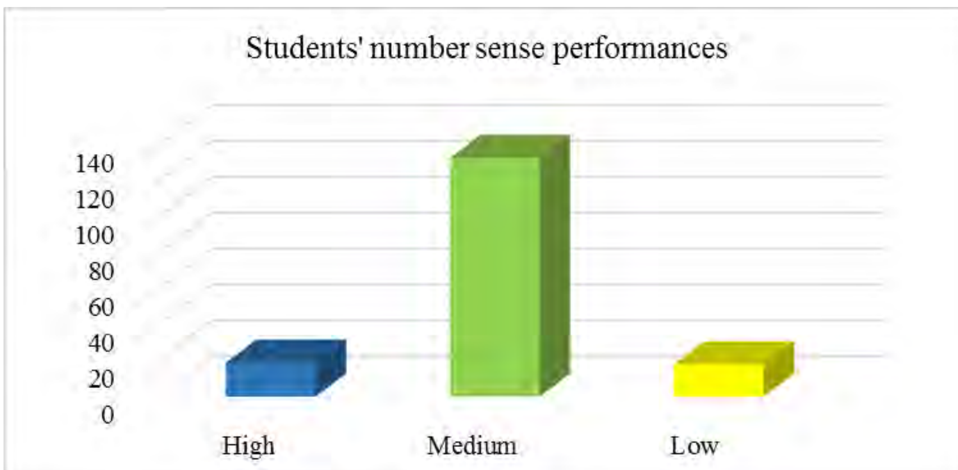


Figure 3: Categorical distributions of the students' number sense performances

When Figure 3 is examined, it is seen that 11.17% of the students (f=19) performed at a "high" level according to the scores they got from the 11-question number sense test, while 78.23% (f=133) performed at a "medium" level and 10.58% (f=18) at a "high" level.

The second sub-problem of the study is "Do primary school fourth grade students' number sense performances vary significantly by gender?" T-test analysis was used to determine whether the number sense performances vary significantly by gender and the analysis results are given in Table 3.

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ. MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

Gender	n	$\bar{X}$	ss	t	sd	p	Cohens's d
Female	88	21.92	5.78	-.386	168	.70	.05
Male	82	21.57	5.93				

Table 3: Results of the t-test conducted to determine whether the students' number sense performances vary significantly by gender

The table shows the results of the t-test to determine the relationship between the number sense performances of the female and male students in the number sense test. When the results of the analysis were examined, no statistically significant difference was found between the number sense performances of the female and male students [$t_{(113)}$ =-.386, p>.05].

In addition, the Cohen's d value calculated to find the effect size of gender on number sense performance was found to be .05. A Cohen's d value of .05 is interpreted as a medium effect size (Büyüköztürk, 2007; p. 48). According to this result, the gender variable has a medium effect on the number sense performance of the primary school 4th grade students. This effect does not create a significant difference between the genders. It can be said that gender explains about 5% of the total variance in students' number sense performance.

Within the context of the third sub-problem of the study, the arithmetic mean and standard deviation values obtained as a result of the analyses conducted to determine the metacognitive awareness levels of the students are given in Table 4.

	n	$\bar{X}$	ss
Students' metacognitive awareness	170	30.81	5.93

Table 4: Arithmetic mean and standard deviation values related to the students' metacognitive awareness

When Table 4 is examined, it is seen that the arithmetic mean of the metacognitive awareness scores of the primary school fourth grade students is $\bar{X} = 30.81$. The mean of the metacognitive awareness scores of the primary school fourth grade students is seen to be high. This finding indicates that the primary school fourth grade students have high metacognitive awareness.

In the fourth sub-problem of the study, the state of variation of the students' metacognitive awareness levels depending on the gender variable was examined. In order to determine the analyses to be made, the normality distribution was examined with the Kolmogorov-Smirnov Normality test and the distribution graph was examined. As a result of the analyses, it was observed that the data group did not have a normal distribution in terms of metacognitive awareness levels by gender and the Mann-Whitney U t-test, one of the non-parametric tests, was used to determine

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ. MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

the status of variation depending on the gender variable. The results of the analysis are given in Table 5.

Gender	n	Mean Rank	Sum of Ranks	U	p	Cohens' d
Female	88	85.97	7565.00	3567.00	.897	.009
Male	82	85.00	6970.00			

Table 5: The state of variation in the students' metacognitive awareness levels by gender

As can be seen in Table 5, there is no statistically significant difference between the scores of the female students and male students according to the Mann Whitney U-test results (U= 3567.00, p<.05).

In the fifth sub-problem of the study, the relationship between the two variables was questioned. Thus, the fifth sub-problem of the study is worded as follows; "Is primary school fourth grade students' metacognitive awareness a significant predictor of their number sense performance?" In order to find an answer to this sub-problem, the normality of the data groups was examined and it was observed that the data showed a normal distribution and the Pearson product-moment correlation coefficient test was performed and the analysis results are presented in Table 6.

		Number sense	Metacognitive awareness
Number sense	r	1	$.34^{**}$
	p		.00
Metacognitive awareness	r	$.34^{**}$	1
	p	.00	

$^{**}p<.01$

Table 6: Correlation values showing the relationship between the students' number sense performance and their metacognitive awareness

The result of the simple linear correlation analysis performed to reveal whether there is a relationship between fourth grade students' number sense performance and their metacognitive awareness is presented in Table 6 and here it is seen that there is a positive and significant correlation between the two variables (r=.34, p<.01). Based on the relationship between the two variables, it can be stated that as students' metacognitive awareness levels increase, their number sense performance will increase, and as their metacognitive awareness levels decrease, their number sense performance will decrease.

Simple regression analysis was used to reveal whether the metacognitive awareness of the primary school fourth grade students was a significant predictor of their number sense performance. The results of the regression analysis performed to determine the common variance between metacognitive awareness and number sense are given in Table 7.

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ. MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

Variable	B	Standard error	β	t	p	R^2	F
Constant	11.41	2.24	-	5.09	.00	.11	22.07
Metacognitive awareness	.33	.07	.34	4.69	.00		

Table 7: Simple linear regression analysis results related to the prediction of the students' number sense performance

As a result of the regression analysis performed, the explained variance value, which is expressed as the coefficient of determination (R^2) and used to interpret how much of the situation that occurs in one variable is explained by the other variable, was found to be .11 (F= 22.07, p<.05). When the coefficient of determination is considered, it can be said that 11% of the total variance in the number sense performance of the primary school fourth grade students is due to metacognitive awareness. Moreover, according to the t-test results related to the significance of the regression coefficients, metacognitive awareness can be expressed as a significant predictor of number sense performance (p<.05).

DISCUSSION

While 11.17% (f=19) of the primary school fourth grade students performed at a high level according to the scores they got from the 11-item number sense test, 78.23% (f=133) performed at a medium level and 10.58% (f=18) at a low level. In addition, the ratio of the mean obtained to the highest score to be taken from the test corresponds to 49.43%. The arithmetic mean of the students participating in the study is approximately the half value of the highest score to be taken from the test. According to this mean, it can be said that the number sense performances of the students are at the "medium" level. Thus, it can be said that very few of the primary school students have a high level of number sense performance, while the majority of them have a medium and low level of number sense performance. This result is similar to the results of the study conducted by Uluçay (2021) to determine the number sense performances of primary school first grade students. However, the results of the current study are different from the results by Yang and Li (2008), Çekirdekci (2015), Can (2019), Gökçe, Güner and Baştuğ (2022) showing that the number sense performances of primary school students are low. They are also different from the studies conducted at different grade levels and in different countries such as Reys and Yang (1998), Yang and Huang (2004), Yang and Li (2008), Singh (2009), Facun and Nool (2012), Purnomo et al., (2014), Cheung and Yang (2020), Yang and Sianturi (2021) also showing that number sense performances are low. The reasons why different results are obtained in the current study may be that the majority of the primary school students in the study group received pre-school education and that the classroom teachers offered a learning environment that would support the number sense performance since their professional experience was ten years or less. In addition, it was revealed that the primary school fourth grade students mostly used rule-operation-based methods in solving the questions in the number sense test and they used solutions related to number sense

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ.
MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

skills less. This result is similar to the results of the studies conducted by Yang (1995), Reys and Yang (1998), Yang (2005), Mohamed and Johnny (2010), Gülbağcı Dede (2015), Çekirdekci, Şengül and Doğan (2020). The reason why the students used more rule-operation-based methods may be that primary school teachers rely more on the activities in the mathematics textbooks during the lesson because, in the study conducted by Cheng and Wang (2012), it was stated that mathematics textbooks have an important place in the development of number sense performance. Another important factor in the emergence of this situation may be the fact that the objectives in the Primary School Mathematics Curriculum include number sense components. In the study conducted by Çekirdekci and Yorulmaz (2021), it was stated that all of the number sense components should be addressed in mathematics curriculums implemented in primary schools in Turkey and that increasing the objectives related to the number sense components in the curriculum would reduce the use of rule-operation-based methods and increase the number sense performance.

It was determined that the number sense performance scores of the primary school fourth grade students who participated in the study did not vary significantly depending on gender. Most of the research results in the literature support this situation (Birgin & Peker, 2022; Can, 2019; Çekirdekci, 2015; Gökçe, Güner & Baştuğ, 2022; Menon, 2004; Reys & Yang, 1998; Yang & Li, 2008). Thus, it can be argued that the gender of primary school fourth grade students does not cause a significant change in their number sense performances, and therefore, gender is not a factor that creates a significant difference. As number sense performance is related to mathematics achievement (Çekirdekci, Şengül & Doğan, 2016; Mohamed & Johnny, 2010; Olkun, Mutlu & Sarı, 2017; Reys & Yang, 1998; Yang, Li & Lin, 2008), it can be argued that male and female students can be equally successful in mathematics (Ministry of National Education [MoNE], 2019).

It was revealed that the mean score of the metacognitive awareness of the primary school fourth grade students is 30.81, and this result indicates that their metacognitive awareness is at a "high" level. This result is similar to the study conducted by Akaydın, Yorulmaz and Çokçalışkan (2020) for primary school third and fourth grade students. In line with this result, it can be said that primary school fourth grade students are adept at controlling and directing their mental processes, determining new strategies and developing self-awareness within the context of metacognitive awareness. Primary school students' having high metacognitive awareness affects their academic achievement in the teaching process to a great extent (Coutinho, 2007; Landine & Steward, 1998; Young & Fry, 2008). When considered in this context, it can be stated that it is an inevitable fact

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ. MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

that students whose metacognitive awareness is developed exhibit higher performance in primary school lessons.

In the study, the metacognitive awareness scores of the primary school fourth grade students were found to be not varying significantly depending on the gender variable. This result is similar to the results obtained from the studies conducted by Özsoy et al. (2010), Hashempour, Ghonsooly and Ghanizadeh (2015), Jaleel (2016), Akaydın, Yorulmaz and Çokçalışkan (2020). Thus, it can be said that the gender variable does not play an important role in the formation of metacognitive awareness. The reasons why the gender factor does not make a difference in the formation of students' metacognitive awareness may be the fact that primary school programs have a structure that provides gender equality, the learning environments are suitable for gender differences, and the behaviors of primary teachers do not differ according to the gender of students.

It was revealed that there is a low level of positive correlation between the number sense performances and metacognitive awareness of the primary school fourth grade students ($r=.34$, $p<.01$). The use of different strategies in the problem-solving process by primary school teachers has an important place in the development of students' metacognitive skills. Metacognition has an important place in using information, distinguishing necessary and unnecessary information and making predictions in the problem-solving process (Schneider & Artel, 2010). Thus, it can be said that metacognition affects the solution of mathematical problems and is also related to arithmetic performance according to Bellon, Fias and Smedt (2019). It was also stated by Lee, Ng and Yeo (2019) that when metacognition develops, the sense of number will also improve, and that they interact with each other. The close relationship of problem-solving and arithmetic in mathematics with metacognition enables us to express that there is a positive relationship between number sense, which is one of the basic components of mathematics, and metacognition. The result of the study indicates that increasing the metacognitive awareness of primary school students will also increase their number sense performance. In addition, it was revealed that metacognitive awareness has a predictive power of 11% in the formation of number sense performances of primary school fourth grade students. In this context, it can be said that metacognitive awareness significantly predicts the development of number sense performance. It was also stated by Vo et al. (2014) that metacognitive skills specific to the numerical domain predict mathematical knowledge. It was also stated by Carr (2010) that the development of metacognitive skills affects students' number sense performance, which is in line with the result obtained in the current study. Therefore, it can be stated that it is important to develop metacognitive awareness in order to increase the number sense performance. As a result, number sense is a way of thinking that requires skills such as perception, attention, flexible thinking, fluency, automaticity and strategy

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ. MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

development. Since metacognition is a cognition that controls these skills, it affects number sense performance.

Number sense is a type of problem-solving competence related to following rule sets and deciding on the appropriate strategy, especially in problem-solving situations, rather than adhering to the algorithm (Fosnot & Dolk, 2001; Putrawangsa, Evendi, & Hasanah, 2020). In addition, problem-solving approaches such as prediction and mental processing, which contribute to controlling the outcome of the problem, involve higher-level thinking skills because they enable students to develop their own strategies (Carroll, 1996; Tsao, 2004). According to the research findings, doing activities such as making predictions, performing mental operations, and developing strategies about numbers and operations in order to improve students' number sense skills will also indirectly affect metacognitive awareness. In this context, conducting activities within the classroom that promote a sense of numbers will contribute to the development of metacognitive awareness. For example, an individual trying to mentally find the answer to the 168+75 operation may use the completion strategy. To do this, the number 25 required to complete 75 to 100 can be reached by separating the number 168 into its parts and obtaining an equivalent representation. He can reconstruct 168 as 125+43 and get the correct answer from his mind using (125+75) + 43. Here, various strategies such as flexible use of numbers, equivalent representation of numbers, reasoning, operational knowledge, and the ability to make evaluations will be included in the process (Çekirdekci, 2023). In order to gain this flexibility about numbers, the relationship between small numbers must be well understood. For example, in order to develop number sense skills, activities related to dot counting, which is the ability to perceive the patterned arrangements of objects that make up the multitude in a very short time and express the number of these objects without counting, can be done in the classroom environment. The number 6 will be perceived in a short time through dot counting, and the number 5 will be seen as (2+2+1, 3+2) during perception. Thus, the student will be able to use the situation appropriate for the context and develop his own strategy. Number talks can also be included in classroom activities to help students develop strategies. Number talks is a teaching strategy that involves classroom conversations around purpose-built calculation problems.

CONCLUSION

In this study, it was determined that the number sense performances of the primary school fourth grade students were medium. It can be said that very few of the primary school students have a high level of number sense performance, and the majority of them perform at a low or medium level. In addition, in this study, it was revealed that number sense performance scores did not vary significantly depending on the gender variable. It was determined that the metacognitive awareness of the primary school fourth grade students participating in the study was at a high level and that the metacognitive awareness did not vary significantly by gender. It was revealed that there is a

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ. MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

positive correlation between the number sense performances and metacognitive awareness of the primary school fourth grade students. In addition, metacognitive awareness was found to predict the students' number sense performances at a low level. In light of the results of the current study, it can be suggested to design activities to develop metacognitive awareness and to include these activities in the classroom in the mathematics teaching process.

Acknowledgements

As the authors, we would like to thank the Turkish primary school fourth grade students who participated in the study and the reviewers who contributed with their valuable comments.

References

[1] Akaydin, B. B., Yorulmaz, A., & Çokcaliskan, H. (2020). Investigation of primary school students' metacognitive awareness and decision-making skill. *International Journal of Progressive Education, 16* (4), 157-171. https://doi.org/10.29329/ijpe.2020.268.10

[2] Alindra, D., Fauzan, A. & Azmar, A. (2018- March) *Metacognitive Awareness and Its Effect on Students' Problem Solving Ability in Implementing RME Approach.* In: Proceedings of the 1st International Conference on Innovation in Education (ICoIE 2018). Retrieved from http://repository.unp.ac.id/id/eprint/26341

[3] Baş, F., Özturan Sağırlı, M. & Bekdemir, M. (2016). The metacognitive awarenesses of preservice secondary school mathematics teachers, beliefs, attitudes on problem solving, and relationship between them. *Eğitimde Kuram ve Uygulama, 12* (2), 464-482.

[4] Bellon, E., Fias, W., & De Smedt, B. (2019). More than number sense: The additional role of executive functions and metacognition in arithmetic. *Journal of Experimental Child Psychology, 182,* 38-60. https://doi.org/10.1016/j.jecp.2019.01.012

[5] Birgin, O., & Peker, E.S. (2022). An investigation of 8th grade Turkish students' performance on number sense, *Educational Studies*, https://doi.org/10.1080/03055698.2022.2049593

[6] Boaler, J. (2016). Mistakes grow your brain. Youcubed at Stanford University Graduate School of Education. Retrieved from https://www.pdst.ie/sites/default/files/Mistakes-Grow-Your-Brain-Revised.pdf

[7] Boğar, Y. (2018). Literature review on metacognition and metacognitive awareness. *Anadolu Öğretmen Dergisi, 2* (2), 136-168.

[8] Büyüköztürk, Ş. (2007). *Sosyal bilimler için veri analizi el kitabı.* Ankara: Pegem Akademi.

[9] Büyüköztürk, Ş., Kılıç Çakmak, E., Akgün, Ö. E., Karadeniz, Ş. & Demirel, F. (2015). *Bilimsel araştırma yöntemleri*. Ankara: Pegem Akademi.

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ. MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

[10] Can, D. (2019). İlkokul dördüncü sınıf öğrencilerinin sayı duyusu performanslarının bazı değişkenler açısından incelenmesi. *Elementary Education Online, 18*(4), 1751–1756. https://doi.org/10.17051/ilkonline.2019.639317

[11] Carr, M. (2010). The importance of metacognition for conceptual change and strategy use in mathematics. In Waters, H. S. & Schneider W. (Edt.) *Metacognition, strategy use, and instruction*, 176-197.

[12] Carr, M., Alexander, J., & Folds-Bennett, T. (1994). Metacognition and mathematics strategy use. *Applied Cognitive Psychology, 8*(6), 583-595. https://doi.org/10.1002/acp.2350080605

[13] Carroll, W. M. (1996). Mental computation of students in a reform-based mathematics curriculum. *School Science and Mathematics, 96*(6), 305-311.

[14] Cheng, Q. & Wang, J. (2012) Curriculum opportunities for number sense development: a comparison of first-grade textbooks in China and The United States. *International Journal for Mathematics Teaching and Learning,* 1-52.

[15] Cheung, K. L., & Yang, D. C. (2020). Performance of sixth graders in Hong Kong on a number sense three-tier test. *Educational Studies, 46* (1), 39–55. https://doi.org/10.1080/03055698.2018.1516631

[16] Coutinho, S.A. (2007). The relationship between goals, metacognition, and academic success. *The Journal of Doctoral Research in Education, 7*(1), 39-47.

[17] Creswell, J. W. (2012). *Educational research: Planning, conducting, and evaluating quantitative*. Upper Saddle River, NJ: Prentice Hall.

[18] Creswell, J. W. (2014). *Research design: Qualitatitive, Quantitative, and mixed methods approaches*, (4th ed.). Sage Publications, London.

[19] Çekirdekçi, S. (2015). *İlkokul 4. sınıf öğrencileri için sayı hissi testinin geliştirilerek öğrencilerin sayı hislerinin incelenmesi*. (Yayımlanmamış Doktora Tezi). Marmara Üniversitesi, İstanbul.

[20] Çekirdekçi, S., Şengül, S., & Doğan, M.C. (2016). Examining the relationship between number sense and mathematics achievement of the 4th grade students. *Qualitative Studies (NWSAQS) 11* (4), 48–66. https://doi.org/10.12739/NWSA.2016.11.4.E0028

[21] Çekirdekci, S., Şengül, S., & Doğan, M. C. (2018). The relationship between number sense and metacognition. *Uluslararası Avrasya Sosyal Bilimler Dergisi, 9*(34), 2465-2481.

[22] Çekirdekçi, S., Şengül, S., & Doğan, M. C. (2020). 4. sınıf öğrencilerinin kullandıkları sayı hissi stratejilerinin belirlenmesi. *Social Mentality and Researcher Thinkers Journal, 6*(31), 680–695. https://doi.org/10.31576/smryj.512

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ. MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

[23] Çekirdekci, S., & Yorulmaz, A. (2021). İlkokul matematik dersi öğretim programlarının sayı hissine göre incelenmesi. *Atatürk Üniversitesi Kazım Karabekir Eğitim Fakültesi Dergisi, 43*, 254-278. https://doi.org/10.33418/ataunikkefd.851683.

[24] Çekirdekci, S. (2023). İlkokulda sayı hissi ve geliştirilmesi. A. Yorulmaz (Ed.), *İlkokulda matematik eğitimi okumaları* (pp. 69-91). Nobel Akademi.

[25] Dunphy, E. (2007). The primary mathematics curriculum: Enhancing its potential for developing young children's number sense in the early years at school. *Irish Educational Studies, 26*(1), 5-25. https://doi.org/10.1080/03323310601125088

[26] Esmer, E., & Yorulmaz, A. (2017). Üst bilişsel farkındalık ölçeği öğretmen formunun geçerlilik ve güvenirlik analizi. *Bartın Üniversitesi Eğitim Fakültesi Dergisi, 6*(3), 955-966.

[27] Facun, R. D., & N. R. Nool. 2012. Assessing the number sense of grade 6 pupils. *International Proceedings of Economics Development & Research, 30*, 297–301.

[28] Flavell, J. H. (1979). Metacognition and cognitive monitoring: A new area of cognitive–developmental inquiry. *American Psychologist, 34*(10), 906 – 911. https://doi.org/10.1037/0003-066X.34.10.906

[29] Fosnot, C. T., & Dolk, M. (2001). *Young mathematician at work: Constructing number sense, addition, and substraction.* Heinemann.

[30] Gökçe, S., Güner, P., & Baştuğ, M. (2022). Monitoring proficiency: Growth of number sense in primary school. *Psychology in the School.* https://doi.org/10.1002/pits.22786

[31] Gülbağcı Dede, H. (2015). *İlköğretim ve ortaöğretim matematik öğretmen adaylarının sayı hissinin incelenmesi,* (Yayınlanmamış Doktora Tezi). Marmara Üniversitesi Eğitim Bilimleri Enstitüsü, İstanbul.

[32] Hair, J. F., Black, W. C., Babin, B. J., Anderson, R. E., & Tatham, R. L. (2013). *Multivariate data analysis.* Pearson Education Limited.

[33] Hashempour, M., Ghonsooly, B., & Ghanizadeh, A. (2015). A study of translation students selfregulation and metacognitive awareness in association with their gender and educational level. *International Journal of Comparative Literature & Translation Studies, 3*(3), 60-69.

[34] Ilko, T. R. (2021). *Metacognition's role in improving the number sense development in students*. Master Thesis, Faculty of Education University of Lethbridge, Canada.

[35] Jaleel, S. (2016). A study on the metacognitive awareness of secondary school students. *Universal Journal of Educational Research, 4*(1), 165-172.

[36] Jordan, N. C., Glutting, J., & Ramineni, C. (2010). The importance of number sense to mathematics achievement in first and third grades. *Learning and Individual Differences, 20*(2), 82-88. https://doi.org/10.1016/j.lindif.2009.07.004

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ. MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

[37] Karakelle, S., & Saraç, S. (2007). Çocuklar için üst bilişsel farkındalık ölçeği (ÜBFÖ-Ç) A ve B formları: Geçerlik ve güvenirlik çalışması. *Türk Psikoloji Yazıları*, *10*(20), 87-103.

[38] Karasar, N. (2011). *Bilimsel araştırma yöntemi.* Ankara: Nobel Yayınevi.

[39] Koenig, J. (2020). *Implementation of number talks to support number sense and fluency, ıncluding automaticity.* Hamline University Saint Paul, Minnesota.

[40] Kramarski, B. (2008). Promoting teachers' algebraic reasoning and self-regulation with metacognitive guidance. *Metacognition and Learning, 3*(2), 83-99. https://doi.org/10.1007/s11409-008-9020-6

[41] Landine, J., & Stewart, J. (1998). Relationship between metacognition, motivation, locus of controli self-efficacy, and academic achievement. *Canadian Journal of Counselling, 32*(3), 200-212.

[42] Lee, N.H., Ng, K.E.D., & Yeo, J.B.W. (2019) Metacognition in the teaching and learning of mathematics. In Toh T., Kaur B., & Tay E. (eds) *Mathematics education in Singapore. Mathematics education – An Asian perspective.* Springer. https://doi.org/10.1007/978-981-13-3573-0_11

[43] Lester, F. K. (1982). Building bridges between psychological and mathematics education research on problem solving. In F. K. Lester & J. Garofalo (Eds.), *Mathematical problem solving* (pp. 55–85). Philadelphia: The Franklin Institute Press.

[44] McIntosh, A., Reys, B., & Reys, R. (1997). *Number sense: Simple effective number sense experiences grades 1-2.* New Jersey: Dale Seymour Publication.

[45] Menon, R. (2004). Elementary school children's number sense. *International Journal for Mathematics Teaching and Learning, 57*, 1–16.

[46] Mevarech, Z.R., & Kramarski, B. (2003). The effects of metacognitive training versus worked-out examples on students' mathematical reasoning. *British Journal of Educational Psychology, 73* (4), 449-471. https://doi.org/10.1348/000709903322591181

[47] Ministry of National Education (MoNE). (2019). *PISA 2018 Türkiye raporu.* [National final report of PISA 2018]. Turkish Ministry of National Education.

[48] Mohamed, M., & Johnny, J. (2010). Investigating number sense among students. *Procedia-Social and Behavioral Sciences, 8,* 317-324.

[49] Montero III, N.A., & Elipane, L.E. (2021). The impact of a metacognitive intervention using I.M.P.R.O.V.E. model on grade 7 students' metacognitive awareness in mathematics. *Turkish Journal of Computer and Mathematics Education, 12* (3), 3881-3894. https://doi.org/10.17762/turcomat.v12i3.1677

[50] National Council of Teachers of Mathematics [NCTM] (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ. MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

[51] Rinne, L. F., & Mazzocco, M. M. (2014). Knowing right from wrong in mental arithmetic judgments: Calibration of confidence predicts the development of accuracy. *PloS One, 9*(7). https://doi.org/10.1371/journal.pone.0098663

[52] Olkun, S., Mutlu, Y., & Sarı, M. H. (2017). The relationships between number sense and mathematics achievement. *International Conference on Education and New Developments*, 24–26, Lisbon, Portugal.

[53] Özsoy, G., Çakıroğlu, A., Kuruyer, H. G., & Özsoy, S. (2010). Sınıf öğretmeni adaylarının üstbilişsel farkındalık düzeylerinin bazı değişkenler bakımından incelenmesi, *9. Ulusal Sınıf Öğretmenliği Sempozyumu,* Fırat Üniversitesi, Elazığ.

[54] Polya, G. (2014). *How to solve it*. Princeton: Princeton University Press.

[55] Purnomo, Y., Kowiyah, K., Alyani, F., & Assiti, S. (2014). Assessing number sense performance of Indonesian elementary school students. *International Education Studies, 7* (8), 74–84. https://doi.org/10.5539/ies.v7n8p74

[56] Putrawangsa, S., Evendi, E., & Hasanah, U. (2018). Assessing students' number sense: What to be considered? In Proceedings of the *International Conference on Mathematics and Islam* (ICMIs 2018), 188-197. https://doi.org/10.5220/0008519201880197

[57] Reys, R. E., & Yang, D. C. (1998). Relationship between computational performance and number sense among sixth-and eighth-grade students in Taiwan. *Journal for Research in Mathematics Education, 29*(2), 225–237. https://doi.org/10.5951/jresematheduc.29.2.0225

[58] Schneider, W., & Artelt, C. (2010). Metacognition and mathematics education. *ZDM, 42*(2), 149-161. https://doi.org/10.1007/s11858-010-0240-2

[59] Singh, P. (2009). An assessment of number sense among secondary school students. *International Journal for Mathematics Teaching and Learning, 155,* 1-29.

[60] Sperling, R. A., Howard, B. C., Miller, L. A., & Murphy, C. (2002). Measures of children's knowledge and regulation of cognition. *Contemporary Educational Psychology*, *27*(1), 51-79. https://doi.org/10.1006/ceps.2001.1091

[61] Şengül, S. (2013). Identification of number sense strategies used by pre-service elementary teachers. *Educational Sciences: Theory & Practice, 13*(3), 1965-1974.

[62] Testolin, A., Dolfi, S., Rochus, M., & Zorzi, M. (2020). Visual sense of number vs. sense of magnitude in humans and machines. *Scientific Reports, 10*(1), 1-13. https://doi.org/10.1038/s41598-020-66838-5

[63] Tosto, M. G., Petrill, S. A., Malykh, S., Malki, K., Haworth, C. M. A., Mazzocco, M. M. M., . . . Kovas, Y. (2017). Number sense and mathematics: Which, when and how? *Developmental Psychology, 53*(10), 1924-1939. http://dx.doi.org/10.1037/dev0000331

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ. MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

[64] Tsao, Y. L. (2004). Effects of a problem-solving-based mathematics course on number sense of preservice teachers. *Journal of College Teaching and Learning, 1*(2), 33-50.

[65] Uluçay, E. (2021). *Sınıf öğretmenleri ile ilkokul birinci sınıf öğrencilerinin sayı duyusu becerilerinin incelenmesi (Uşak ili örneklemi).* Yayımlanmamış yüksek lisans tezi. Afyon Kocatepe Üniversitesi Sosyal Bilimler Enstitüsü, Afyon.

[66] Verschaffel, L. (1999). Realistic mathematical modelling and problem solving in the upper elementary school: Analysis and improvement. In J.H.M. Hamers, J.E.H. Van Luit & B. Csapo (Eds.), *Teaching and learning thinking skills*. Lisse: Swets & Zeitlinger.

[67] Vo, V. A., Li, R., Kornell, N., Pouget, A., & Cantlon, J. F. (2014). Young children bet on their numerical skills: Metacognition in the numerical domain. *Psychological Science, 25,* 1712–1721.

[68] Waters, H. S., & Kunnmann, T. W. (2010). Metacognition and strategy discovery in early childhood. In Waters, H. S. & Schneider W. (Eds.) *Metacognition, strategy use, and instruction* 3-22.

[69] Woods, D. M., Ketterlin Geller, L., & Basaraba, D. (2018). Number sense on the number line. *Intervention in School and Clinic, 53*(4), 229-236. https://doi.org/10.1177/1053451217712971

[70] Yang, D. C. (1995). *Number sense performance and strategies possessed by sixth and eighth grade students in Taiwan*. [Unpublished doctoral dissertation]. University of Missouri, Columbia.

[71] Yang, D. C. (2003). Teaching and learning number sense–an intervention study of fifth grade students in Taiwan. *International Journal of Science and Mathematics Education, 1*(1), 115-134. https://doi.org/10.1023/A:1026164808929

[72] Yang, D. C. (2005). Number sense strategies used by sixth grade students in Taiwan. *Educational Studies,31*(3), 317–334. https://doi.org/10.1080/03055690500236845

[73] Yang, D. C., & Hsu, C. J. (2009). Teaching number sense for 6th graders in Taiwan. International *Electronic Journal of Mathematics Education, 4*(2), 92-109.

[74] Yang, D. C., & Huangh, F. Y. (2004). Relationships among computational performance, pictorial representation, symbolic representation and number sense of sixth grade students in Taiwan. *Educational Studies, 30*, 373-389.

[75] Yang, D. C., & Li, M. N. (2008). An investigation of 3rd-grade Taiwanese students' performance in number sense. *Educational Studies, 34* (5), 443-455.

[76] Yang, D. C., & Li, M. N. (2013). Assessment of animated self-directed learning activities modules for children's number sense development. *Journal of Educational Technology & Society, 16*(3), 44-58.

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ. MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/

[77] Yang, D. C., & Sianturi, I. A. J. (2021). Sixth grade students' performance, misconception, and confidence on a three-tiernumber sense test. *International Journal of Science and Mathematics Education, 19,* 355–375. https://doi.org/10.1007/s10763-020-10051-3

[78] Yang, D. C., Li, M. N., & Lin, C. I. (2008). A Study of the performance of 5th graders in number sense and ıts relationship to achievement in mathematics. *International Journal of Science and Mathematics Education 6* (5), 789–807. doi:10.1007/s10763-007-9100-0

[79] Young, A., & Fry, J.D. (2008). Metacognitive awareness and achievement in college students. *Journal of The Scholarship of Teaching and Learning, 8*(2), 1-10.

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ. MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/