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# Democracy revisited for adults learning mathematics

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## Abstract

This article presents part of a larger study on potential relations between democracy and numeracy and mathematicsi teaching for adults. This study is motivated by the many challenges to democratic institutions and democratic values at local, national and international levels that I have observed in recent years. In this article. I go back to the 1980s to revisit literature on the connections between mathematics education and society, focusing on democracy. The chosen literature is not exclusively, but primarily, of Nordic origin and represents the context within which I was enculturated as a young student. Revisiting this literature and supplementing it with newer literature confirmed that the complex relations between mathematics education and democracy described by prominent scholars back in the 1980s are worth remembering and developing further, even today. Although these Nordic scholars focused on other parts of education rather than adult education, I find it relevant to consider the scholars' descriptions of democratic aspects of numeracy and mathematics education in relation to adult education. The present article suggests a model, a frame, consisting of four boxes that can function as a pragmatic tool for analyses of democratic aspects of present-day adult education. The frame highlights that democracy can be interpreted as more than just the right to vote in elections. The frame shows two interpretations of democracy: representative democracy (RD), i.e. the rules and rights of citizens and representatives, and participatory democracy (PD), i.e. democratic participation and democratic communication among citizens in everyday life. The hope is that this frame may encourage, facilitate reflection on and influence the practice of democracy among teachers and students in adult education. To this end, the frame specifies reflections and practice both in school (IS) and outside school (OS). Examples from numeracy and mathematics instruction practice are presented, and some frame limitations are sketched out. Keywords: mathematics instruction; adult education; representative democracy; participatory democracy; educational environment.

## Examples from 1980s literature

To my knowledge, the review by Aguilar & Zavaleta from 2012 of literature on connections between mathematics education and democracy is the only systematic review that exists. This review includes literature from three sources: journal articles, conference proceedings and books. The authors reported three main conclusions. First, that generally, the literature shows a double view of the relationship between mathematics education and democracy: 'Mathematics education can promote democratic competences and values, but also inhibit them, and create social inequalities' (p.10). Second, the literature documents a widespread interest in the research community in promoting values like equality, democracy and applicable competencies. The review recognises that these values are problematic on theoretical levels as well on practical levels. Third, the review showed that theoretical analyses dominate the picture. Empirical data and investigations are rarely reported.

To supplement this broad review, I revisit a few papers from within a geographically limited area, published within a relatively short period. The literature included in the following short overview is my personal choice. I am revisiting literature from Nordic scholars who inspired me in the 1980s during my earliest research in democracy and mathematics education. The Norwegian scholar Stieg Mellin-Olsen and the Danes Mogens Niss and Ole Skovsmose have made several contributions to mathematics education, and I initiate this paper by paying tribute to a small part of their early works: Mellin-Olsen 1981 and 1987; Niss 1983 and 1994; and Skovsmose 1985, 1990 and 1992. I am investigating the main points in their literature and whether the main points of Aguilar and Zavaleta are also present here. The review by Aguilar & Zavaleta included the works of Skovsmose, but not the works of Mellin-Olsen and Niss.

Mellin-Olsen investigated instrumentalism as a psychological concept; he focused on the learner’s instrumental understanding versus relational understanding. Within the psychological theory, instrumentalism is the tendency to construct bits of knowledge without a structural scheme of internal actions. Mellin-Olsen wanted to supplement psychological studies with social perspectives. He expanded instrumentalism to become an educational concept in which the rationale of learning is to pass exams to uphold the individual's and society's economics. With an instrumental rationale for learning (I-rationale), the student cannot place the learning situation in any context other than school learning. The only meaning related to the situation is that the situation depends on the school, and the school is a place where it pays to learn. Even within a ‘learning by understanding’ perspective, learning may be governed by an instrumental rationale.

Because Mellin-Olsen saw educational instrumentalism as insufficient, he invented a corresponding social, educational concept in which the rationale of learning is to learn significant knowledge for the present social context and a valuable future. Students with a social rationale (S-rationale) for learning aspire for connections between school knowledge and knowledge relevant to themselves or others now or later. Mellin-Olsen regards the educational situation as positive when the S-rationale and the I-rationale overlap. He no longer diagnose I-rationale and S-rationale in learning by inspecting students’ responses to mathematical tasks but in terms of how students interact with them (1981, p.366):

> Do they discuss their results when doing investigations? Do they produce their own ideas for further investigations?
>
> How much do they relate their mathematical knowledge to situations outside the classroom?
>
> Do they have ideas about examining certain situations by means of mathematics?

In 1987, Mellin-Olsen developed these perspectives further in his book *The Politics of Mathematics Education*. Throughout the book, Mellin-Olsen questioned mathematics education with perspectives from psychology, anthropology, sociology, and political theory. His political focus was not on parliamentary elections, nor political parties: ‘The term “politics”, as used here, is related to the position that human beings act, participate and survive in their world as *political human beings.* (…) Men [meaning humans] not only act together with other men in social settings, but they also think differently about important matters in their lives as well, thus being, by nature, political men’ (1987, p. 37-8) (my adding). Also, Mellin-Olsen sees curriculum content as political, for instance it is a political issue, whether vocational examples are included in curriculum.

In my view, the messages for teaching mathematics to adult students today from Mellin-Olsen’s work are that adult students should be allowed to relate classroom activities to situations outside the classroom and that adult students should be recognised as people thinking and doing politics when they engage with mathematics. However, it is outside parliamentary elections and outside political parties. These messages have implications for content and the organisation of teaching mathematics to adults.

In 1983, Niss wrote that mathematics teaching should contribute not only to students' understanding of specific mathematical ideas, concepts, procedures, and knowledge but also to students' understanding of factors outside mathematics that are important for applying mathematics. Focus on potentials as well as limitations should be prioritised. 'The purpose of mathematics education should be to enable students to realise, understand, judge, utilise and also perform the application of mathematics in society' (Niss, 1983, p. 248). He reported that he arranged activities for his students to encourage them to build critical awareness of ideological foundations of mathematics, for example, by examining specific models used by the government. This should include and reflect principles and assumptions that shape the transformation of societal issues into mathematical models. Therefore, Niss recommended that special attention to aspects of mathematical modelling is paid in courses concerned with non-mathematical subjects in which mathematics interests those subjects (1983, p. 249).

In 1994, Niss emphasised the relevance of getting some insight into how professional mathematicians create systems and results that may affect society and the lives of citizens. Students should know about 'the constitutive features of and the essential driving forces behind the development of nature, society, and the lives of human beings. Insight [that] […] serves the acquisition of overview, knowledge and judgment of main patterns, connections and mechanisms in the world, the ultimate end being to create prerequisites for taking positions on and acting towards processes of significance to society and the individual.' (Niss 1994, 376). This statement represents an updated version of former ideas of how mathematical learning can provide more than just mathematical ideas, concepts, and procedures. Previously, the prevailing idea was that mathematics would contribute to general logical thinking and problem-solving, to a sense of order.

The messages for today from Niss' work are that adult students should be allowed to practice the application of mathematics in society, to emphasise mathematical aspects in other subjects, to know the constitutive and other roles mathematics plays and to position themselves as someone that acts in significant societal and individual processes. These messages have implications for content and the organisation of teaching mathematics to adults.

Skovsmose (1985) focused on critical and mathematics education, intending to let students develop a critical distance to content taught in school. Concerning the organisation of learning processes, teachers should ensure the students' involvement in the control of the educational process. (…) Dialogue and the teacher-student relationship are developed from the general point that education must belong to a democratisation process. If a democratic attitude is to be developed through education, education as a social relationship should not contain fundamentally undemocratic features. It is not acceptable that the teacher (alone) has the decisive and prescribing role. Instead, the educational process must be understood as a dialogue (1985, p. 340).

Concerning course content, Skovsmose states that it should be possible for the students to see that the problem they are presented with in class is relevant outside class. Subjective relevance and students' experiences then become decisive. Secondly, the problem presented must be important in a societal context. Thirdly, 'somehow and to some extent the students' engagement in the problem-solving process should prepare for a (later) political and social engagement' (1985, p. 350).

In 1990, Skovsmose emphasised that 'to make it possible to carry out democratic obligations and rights, it is necessary to understand the main principles in the "mechanisms" of the development of society although they may be "hidden" and difficult to identify. Especially, we must be able to understand the functions of applications of mathematics (…) (and) understand how decisions (economical, political) are influenced by mathematical model building processes' (p.111). Skovsmose extends his view when he claims that it is 'an implication of the pedagogical argument of democratisation (…) that we must develop open situations in the educational process, i.e., situations which could take different directions depending on the results of discussions between students and students, and between students and teacher. Opening the situation means creating possibilities for educational decisions to be taken in the classroom. Students must be able to form the educational process if not to adapt to unquestionable rituals of mathematical education. (…) The social argument of democratisation implies that we must develop empowering teaching-learning situations and materials, i.e., situations and materials which, in fact, give information about real mathematical models and their functions' (p.119).

In 1992, Skovsmose used the term democratic competence in a title. He discusses representative democracy, which he characterises as the democratic elections of a government. Procedures and algorithms for elections are to be discussed, whether they are democratic. But education for representative democracy is not essential. Then, Skovsmose adds participatory democracy, where people participate 'in discussions and criticism of the actual ruling.' For participatory democracy, education in critical citizenship is essential. Skovsmose writes: 'I see the development of critical citizenship as one fundamental condition for democratic life'. (p.4)

Skovsmose continues his concept of mathematics' formatting power, which aligns with Niss' constitutive features of mathematics. The thesis that mathematics is formatting our society means that mathematics 'can do something to reality, and I concentrate on this aspect because of my interest in the conditions for democracy'. (p.6)

Performing critical competence may, according to Skovsmose, include the following steps:

- Have we used the algorithm in the right way?
- Have we used the right algorithm?
- Can we rely on the results from this algorithm?
- Could we do without formal calculations?
- How does the actual use of the algorithm (appropriate or not) affect a specific context?
- Could we have performed the evaluation in another way?

( p.9)

The fifth step 'how does the actual use of the algorithm (appropriate or not) affect a specific context?' corresponds to the formatting power of mathematics.

The messages for today's teaching mathematics to adult students from Skovsmose's work are (again) that adult students should be allowed to relate classroom activities to situations outside the classroom, that adult students should be in control when engaging in problem solving situations and processes which prepare for political and social engagement and that adult students should recognize the potential formatting role that mathematics plays. Like for Melling-Olsen and Niss the messages have implications for content as well as for organisation of teaching mathematics for adults.

The three scholars are very much in line. However, in the chosen papers, only Skovsmose underlines the importance of students' control and writes about election procedures and algorithms in representative democracy. Too, issues of equity stand stronger in the work of Mellin-Olsen and Skovsmose than in the work of Niss.

The three scholars align with two of three conclusions from Aguilar & Zavaleta (2012), as the scholars recognise that mathematics learning can potentially support various kinds of participation in democracy and potentially inhibit participation. The three scholars show interest in promoting values like democracy and applicable mathematical competencies for the good of the individual and society and show that these values are in no way simple to extract and put into practice. The three scholars are critical towards much contemporary mathematics education and express their hopes and eagerness towards alternative learning goals, teaching content and organisation. The third conclusion is that theoretical analyses dominate, while empirical data and investigations are scarce. Although the extracts I have chosen to present above are dominated by theoretical analyses, all three scholars did, in these and other parts of their work during the 1980s, report on stories and qualitative data from their own practice as teachers or observers. Mellin-Olsen and Niss describe their teacher experiences from upper secondary and university levels, while Skovsmose describes his observations in lower secondary school.

I find it clarified that educational interest in democracy is directed towards representative democracy (RD) and participatory democracy (PD). However, other terms for PC are in use too, i.e., at ALM8, the term Democracy 2 was used Lindenskov & Valero (2002). Both RD and PD are possible features in public discussions, in social media and in numerous contexts in families, schools, classrooms, unions, workplaces, associations for day-care parents, to national and international organisations. Concerning educational interest in participatory democracy (PD), allow me to point to the historical context of World War II as reflected by some scholars that are still often referred to in Danish educational philosophy: At the end of World War II, scholars reflected upon war crimes against humanity and speculated on possible democratic features which might have had an inhibitory effect. For sure, representative democracy did not prevent war crimes against humanity. The scholars suggested participatory democracy with terms like *direct democracy*, *democracy as dialogue* and *democracy as a way of life* as supplements to representative democracy (Koch, 1945); (Rigsdagens Bureau/The Danish Parliament, 1949) (Jakobsen, 2010).

## Literature from outside the Nordic countries

Many other scholars from around the world, including more recent Nordic scholars, have contributed to the understanding of democracy and mathematics education. It is impossible to pay tribute to them all and to pay tribute to the scholars who from 1990’s on have focused on issues of equity, social justice, power, and identity either to add to or replace issues of democracy.

The Brazilian scholar Ubiratan D’Ambrosio shared the wish of the Nordic scholars for an educational system that not only prepares students for future jobs and consumer markets, but also

> prepares for full citizenship, for the exercise of all the rights and the performance of all the duties associated with citizenship in a critical and conscious way. Mathematics education ought to prepare citizens so that they will not be manipulated and cheated by indices, so that they will be allowed to change and to accept jobs which fulfil and appeal to their personal creativity; that is, so that individuals will be allowed the satisfaction of their own creativity and will be free to pursue personal and social fulfilment thus being able to achieve happiness (1990, p.21).

Even though all citizens, according to democratic constitutions, share the same rights, inequalities exist because power is unequal. D’Ambrosio sheds light on this dilemma as a supplement to the Nordic scholars of the 1980s. D’Ambrosio underlined that such inequalities exist in aristocratic or oligarchic societies as well as in liberal democracies where the education system itself serves as a filtering instrument for who will get which jobs and positions and exercise power. To this end, mathematics education has been a prominent filtering instrument since Greek times.

D’Ambrosio sheds further light on power relations inside education, for instance, between teachers and students, with his clear wish that rather than ‘give instruction’ and ‘impart knowledge’, teachers should be partners with students to build up knowledge commonly.

> Teachers and students should be persistent but humble in the common search so that all bring their previous knowledge, insights, and creativity to the process. Teachers can hardly play their role in building a democratic society if they do not foster democratic and just behaviours. Mathematics is an essential component of this. (p.23)

The American adult researcher and teacher Marilyn Frankenstein (1983) sees knowledge of basic mathematics and statistics as important to understanding and gaining democratic control over societal phenomena and structures. For change to happen, an understanding of how technical language is applicable is necessary: Liberatory social change requires understanding the technical knowledge that is too often used to obscure economic and social realities. When we develop specific strategies for an emancipatory education, it is vital that we include such mathematical literacy. (p. 315)

Frankenstein elaborates on power relations in school between adult teachers and adult students and suggests that teachers listen to students to find which themes can be organized and presented to the students as relevant problems to be mathematically investigated. Frankenstein also suggests that teachers actively find themes they themselves judge as important. Frankenstein cites from the 1981 publication by Paulo Freire:

> The opposite of [teachers'] manipulation is not an illusory neutrality, neither is it an illusory spontaneity. The opposite of being directive is not being non-directive - that is likewise an illusion. The opposite both of manipulation and spontaneity is critical and democratic participation by the learners in the act of knowing, of which they are the subjects. (p.28)

Lately, democracy as a central focus seems to be diluted compared to 1980s. Yasukawa et al. (2018) only includes two contributions that use the term 'democracy', one from South Africa and one from India. Still, the contributions do distil several issues that contribute to the role of numeracy practices in participatory democracy. You may even assume that 'all of our (CME) research is underpinned by a belief – or at least hope – that critical numeracy is a resource for democracy, social justice and emancipation' (Yasukawa, personal communication).

As an example of more contemporary Nordic and South American literature, Valero & Knijnik (2016) confirm and develop D'Ambrosio's focus on inequalities among groups and individuals. They apply the lenses of identity and subjectivities to sharpen the focus on human implications: inequalities cannot be measured only by statistical differences among ethnic, language or socio-economic groups. Implications also concern the fabrication of subjectivities and which are possible, desirable, and excluded. Like Mellin-Olsen, they see curriculum content as political: should people's local mathematics be honored and taught or just serve as the point of departure for the mathematics classes?

As another example of a more contemporary contribution, the American scholar Rochelle Gutierrez (2009) adds issues on identity and power. Gutierrez declares that a focus on achievement gaps between different ethnic and social groups threatens concerns for questions on identity and power and threatens broadened notions of learning from a critical perspective. Gutierrez proposes that one place to start reflections on how to prepare students and future teachers for more participatory democracy (p.10) and more democratic citizenship (p.14) is to embrace three tensions in teaching from an equity stance which addresses issues of power and identity.

One tension is that teachers must do everything to know the students while being aware that the knowledge is delimited. Another tension is that while teachers oversee teaching approaches and examples in the classroom, they still are required to standardise their curriculum according to state or national norms and to other teachers. The third tension is that when teachers wish to use supplementary materials and develop anti-racial and social justice curricula, they must also maintain a high level of mathematical rigour.

I appreciate the questions about notions of democracy raised by works on identity and power, among others by Gutierrez, as these questions add further implications for the content and organisation of teaching mathematics to adults.

## Assumed relevance today

In principle, education professionals in al the present 193 member states of United Nations researchers have an interest in democracy and education as democracy is a core value of the United Nations. United Nations sees 'democratic governance as a set of values and principles that should be followed for greater participation, equality, security, and human development. Democracy provides an environment that respects human rights and fundamental freedoms, and in which the freely expressed will of people is exercised.' (Democracy | United Nations)

The literature sketched above demonstrates the wide interest among scholars in mathematics education to engage in interplay between democratic issues og mathematics and numeracy issues, and the Nordic 1980s literature comprises central parts of the interest. Along democratic states the interest is also seen in official steering documents and guidelines for mathematics teaching for adults. In my local context, Denmark, the relevance is evident in curricula and public discussions hereof (Lindenskov 2018, 2019), which include democracy aspects. The present official national objectives in Denmark for mathematics courses in adult general education, termed preparatory adult education, PAE (in Danish: FVU) and general adult education, GAE (in Danish: AVU) include democratic values:

- PAE § 1: '(…) strengthen their preconditions for active participation in all aspects of society'. (LBK no 1654)
- GAE § 1 Part 3: 'Teaching is based on intellectual freedom, equality, and democracy. The teaching is organized so that the students strengthen their prerequisites for active participation in a democratic society'. (LBK no 603)

The term 'active participation in all aspects of society' points towards representative democracy as well as participatory democracy. But, despite assumed relevance of the ideas from 1980s, the question remains how to place and practice the ideas in adult education today: How to distil and present the ideas in a comprehensible and easy-to-apply manner for today's research and practice, and how to add issues of power and identity?

## Framing complex relations

When I advocate taking up the ideas from the 1980s as a promising perspective, I worry about their complexity. However, I do believe they have the potential to open renewed ways to consider the roles of numeracy and mathematics in society and to open new justifying arguments for why and how mathematics teaching or instruction for adults is relevant in lifelong learning.

My experiences from teacher in-service-training and further mathematics education in Denmark confirm that teachers and adult students alike would find the ideas interesting. It seems that the ideas from the 1980s could be meaningful even today. However, as the reviews and examples show, it is difficult to practice democracy in mathematics education. I hope that a frame can be developed as an effective tool for the communication and discussions of democracy in mathematics teaching and help to practice democracy in mathematics education. Such a frame could be structured in many ways. The frame chosen distils from the 1980s literature two kinds of features. The frame distils the two democratic features, representative democracy and participatory democracy. The frame also honours the 1980s literature concerns for engagement in outside school affairs by distilling two contextual features, outside school and in school.

On the basis hereof, I propose the following frame containing four boxes to distil and present the complex ideas in a comprehensible and easy-to-apply manner for today's research and practice.

The frame allows us to map four kinds of relations between mathematics education and democracy. Each kind may be given specific attention.

| | Representative democracy | Participatory democracy |
|---|---|---|
| Outside school | RDO | PDO |
| In school | RDI | PDI |

*Figure 1. The frame of four boxes*

RDO stands for 'representative democracy outside school'. PDO stands for 'participatory democracy outside school'. RDI stands for 'representative democracy in school'. PDI stands for 'participatory democracy in school'.

In the following sections, questions will be explored to demonstrate the potential of the model to raise awareness of how adults learning mathematics can support adults' engagement in representative and participatory democracy, as well as to describe and develop democratic learning processes in mathematics and numeracy among adults.

## The RDO box – representative democracy outside school

The RDO, the representative democracy outside school, includes the rights and duties of citizens in society. The right to vote, freedom of speech, and the duty of keeping the law and paying taxes are included. In democratic countries, people vote to select the candidates who will propose and decide on political and administrative issues in government and parliament. Still, elections are also happening in unions, housing associations, boards at day-care institutions etc.

All representative democracies include electoral systems or voting systems defined by rules. These rules determine the structure and how to determine the results of elections and referendums. How elections and referendums are conducted is determined by rules, and their results are determined by rules. These rules are defined in constitutions and electoral laws, which often differ between nations, levels, and offices.

All aspects of the political voting process are governed by rules: who or what decides when an election occurs, who can stand as a candidate, who is allowed to vote, how ballots are marked and cast, how ballots are counted, how counted votes are translated into an election result. Contextual factors may also be determined by rules, for example, limits on campaign spending, advertising and communicating polls. Electoral systems exist for governments, non-profit organisations, informal organisations and businesses.

The relations between RDO and mathematics deal with at least two aspects. One aspect concern investigating the mathematical algorithms used in elections to count votes. Another aspect concerns mathematical investigations into voter behaviour and fairness of how specific groups of citizens are represented in parliament and on boards.

My thesis is that information on voting algorithms and data on voter behaviours and selected members is available to all teachers and students in adult education.

Concerning voting algorithms, they differ from country to country and system. Exploring these algorithms can raise awareness of how specific mathematics applications affect society and of the fact that voting rules are human constructs that governments and citizens can decide to uphold or change. Voting algorithms are illustrative examples of the formatting power of mathematics, as voting algorithms are highly influential on voting results. With quantitative reasoning, arithmetic calculations and graphical representations, students and teachers can gain insight into this aspect of the system of democracy.

Looking at today's democratic countries, we see several voting systems with decisive differences in use. Some countries use proportional representation, meaning each party is allocated the number of seats in proportion to the popular vote. In other countries, the winning party in a specific county is allocated all county seats. This is termed 'first-past-the-post' voting.

In some countries, each voter can cast one vote; in others, they can cast two; and in a few countries, voting is mandatory. For some countries, the number of seats in parliament changes when the number of cast vote change.

Common to all systems, however, is that the construction of counties and the procedure for counting the cast votes are governed by mathematical formulas or mathematical algorithms.

That the mathematical algorithms in play for the construction of an election and the counting of votes play a decisive influence on election results is illustrated in the following theoretical example about the first-past-the-post model. Figure 2 shows two parties (red and blue votes) in a region constructed into five counties in three different ways with the count of the votes.

In a 'first-past-the-post' model construction of counties are decisive.

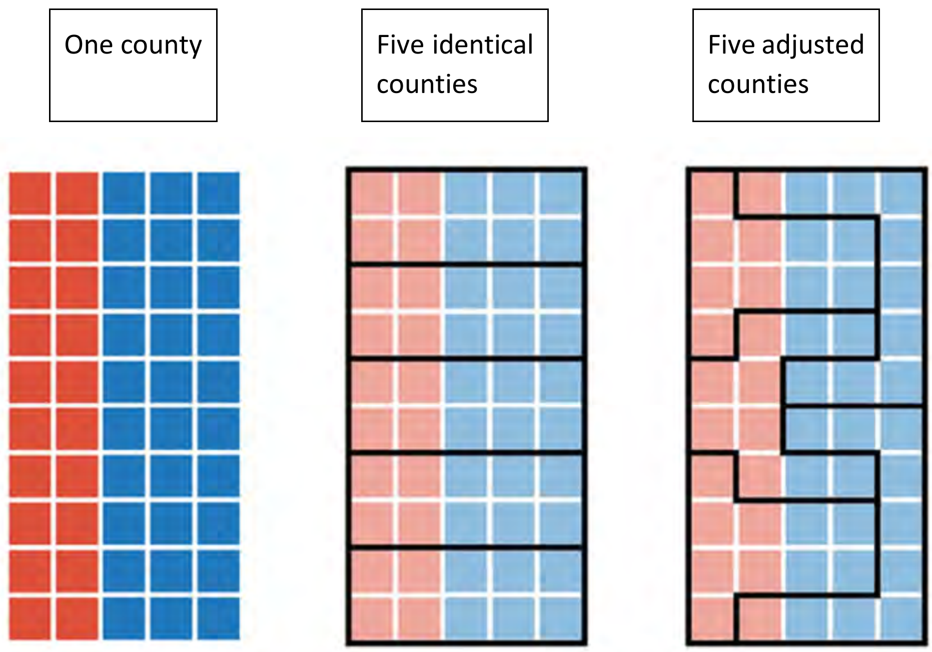



*Figure 2. How counties are constructed influences results in 'first-past-the-post' elections.*

The seats that result from these county constructs differ a lot. To the left, we see the region, for instance, a state in the USA, kept together as one single county. Even though the Red party received 40% of the popular vote, the party will get no seats. As the election winner, the blue party gets all the seats.

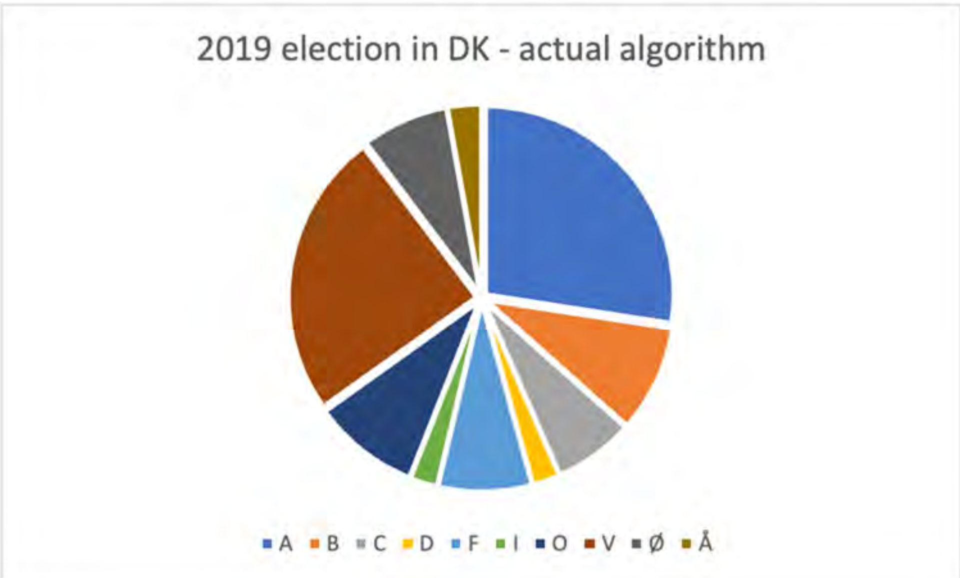
In the middle, the county is split into five identical counties. If the blue party holds the most votes in each county, the party will again get all the seats, while the red party gets no seats.

In the last illustration, we see an alternative division. In this case, the many blue votes are placed in two counties to the right side with only blue votes. The remaining blue votes are placed in the three other counties, where the red party now holds the most votes. With the adjusted counties, a new winner appears. The red party gets 60% of the seats, and the blue party gets 40% of the seats, in contrast to the popular vote. In democratic systems with proportional algorithms, the same vote cast would mean that the blue party would receive 60% of the seats, and the Red party would receive 40%, independently of how counties are constructed (Daley, 2017).

This theoretical example resembles authentic elections in many different countries, and it illustrates how the application of mathematics has a society-shaping function and how mathematics has a formatting power, as termed by Skovsmose. Mathematics not only describes and predicts, but it also prescribes societal matters.

As an activity in adult education, I suggest recalculating actual election results with alternative algorithms to the one used in the election. For example, results from a Danish parliamentary election could be recalculated with a first-past-the-post model, as Figure 3 shows. It is also worth noting that the theme of democracy in the ‘social injustice’ framework of Berry III et al. (2020) includes gerrymandering and that American mathematicians are involved in constructing fairer alternative voting districts. (Arnold, 2017)

In the following Figure 3, the top diagram shows the representation of the various political parties in the Danish Parliament in 2019 in accordance with the algorithm used in Danish parliamentary elections. The bottom diagram illustrates the same election results when applying a first-past-the-post algorithm as in the United Kingdom and the USA.



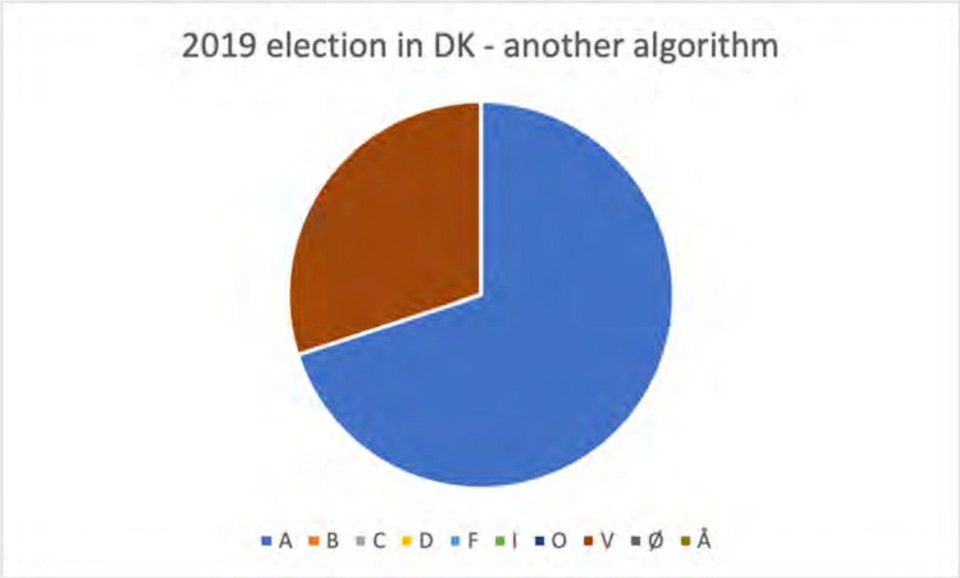



*Figure 3. Actual party representation in the Danish Parliament following the 2019 election (top). Hypothetical party representation in the Danish Parliament following the 2019 election upon application of a first-past-the-post algorithm (bottom).*

Experiences during several years in adult general education in Denmark show that it is possible to engage students in investigations of fairness in relation to different mathematical algorithms in voting, where the used algorithm for municipal elections is in favour of big parties (Danish adult teacher and Ministry consultant Per Bengtson, personal communication). Similar exercises and discussions on election fairness in mathematics classes with adult students could be successfully implemented in other countries.

Another aspect of RDO is voter behaviour. Data on voter behaviour may be collected and owned by private firms, but many are public and communicated in mass media. Further, some international data on adolescents' democratic values and engagement are available. For instance, the International Association for the Evaluation of Educational Achievement (IEA) performs and publishes its International Civic and Citizenship Study (ICCS), which investigates how 13–14-year-olds are prepared to undertake their roles as citizens. ICCS reports on students' civic knowledge levels, their understanding of concepts and issues related to civics and citizenship, and their civic attitudes and engagement. Both declarative knowledge (i.e., facts, concepts and relationships between these), procedural knowledge (i.e., how to carry out actions) and participation are included. (Schulz et.al., 2018)

A final aspect to investigate in adult mathematics classes may be the extent to which elected board and parliament distributions look like the voter populations. Recently, this issue was included in a national exam in general adult education in Denmark. Students were to find and present the percentage of female Parliament members after the 2019 election and investigate development since 1953. (Bengtson, personal communication)

Using such data in adult education could raise awareness of how democratic engagement and values are distributed among different groups and serve as the point of departure for discussions on the fair composition of boards and parliament. It offers students the possibility to learn about citizen values and engagement and, at the same time, to improve their mathematical problem-solving and communication competencies as well as their arithmetic skills. Working mathematically with representative democracy has the potential to support critical insight and engagement in representative democracy and to support relevant mathematical competence. Still, an open question is with Gutierrez when and in which contexts such data would motivate adult students and serve their interests. Will it motivate and serve interest to work with tasks like the following:

> Examine your own parliament or local municipal council – how many representatives from each party do they presently include?
>
> Choose another country. Apply the algorithms used in this country to count the votes cast in your country. What do you think? Are you surprised? Which one do you prefer? Which one is most fair? What do you think about Proportionality, Sainte-Laguës, D'Hondts?
>
> Seek out, process, and disseminate data on the voting behaviour of different population groups. Young people versus elderly people, for instance. How do they differ? Are you surprised?

## The PDO box - participatory democracy outside school

Most critical mathematics education focuses on how citizens can be prepared to engage with applications of mathematics in situations outside school. Critical mathematics education literature often underscores the relevance for society and the students involved. Although most mathematics education research on mathematical modelling, problem-solving and reasoning in context focuses on other parts of education than adult education and vocational education, an unlimited number of sources could provide examples relevant to adult education.

Looking at participatory democracy includes how democracy is 'being done' daily at different societal levels and spheres. It includes public conversations through newspapers, the radio, on TV, in social media (SoMe) or person-to-person conversations, written or orally. Much background information for political decisions locally, nationally, and internationally includes numerical measures, mathematical problem-solving, and modelling results.

So, for participatory democracy outside school, citizenship is more than following society's rules. It includes performative citizenships, i.e., how we perform our citizenship, identify ourselves as active participants and engage and use our critical capacity to influence. Participatory democracy also includes how we collaborate, approach each other, and participate in everyday decisions.

The research documented that many mathematical models on health issues are constructed, used, and communicated to adults (Díez-Palomar & Hoogland, 2022). On a micro-level, this includes, for example, understanding how to calculate medication dosages in medical treatment and reflecting on how it is communicated. On a macro level, it includes surveys on health and healthcare distribution in a population. Many of the media's present-day hot topics would constitute relevant teaching material in the adult classroom. It is relevant to look at how mathematics figures and applications are communicated and to compare how different media communicate – for example, how information was presented by the media during the COVID-19 pandemic.

The COVID-19 pandemic is a useful example of how mathematical modelling and communication play a role in our understanding of a topic and how they influence decision-making on many societal levels: at the personal, workplace, and government levels. During the COVID-19 pandemic, much information, verbally and through figures, was communicated to the public. How citizens understand the information given has a crucial impact on their behaviour, trust in authorities and observance of restrictions in a crisis like the pandemic. Sikunder (2022) sums up that 'participatory citizenship can be conceived of as ethical citizenship as this citizenship will allow us to create a socially inclusive space so that voices of all the citizens are included and negotiated in taking decisions (…)' (p.121). Sikunder suggests investigating mathematics as a tool that formats social systems when involved in vaccine distribution, as not 'securing vaccination to all people around the globe is starting new threat of deaths due to new variant omicron or many other unforeseen variants yet to come into action.' (p.122)

As a Danish COVID-19 example, it was reported on 7 December 2021 that among patients in Danish hospitals, 459 were infected. A national TV station further reported on the vaccination status of the hospitalised individuals:

- 146 not vaccinated.
- 14 vaccinations initiated – first dose received.
- 299 fully vaccinated – the third dose received.

With these absolute figures as background data, you could conclude that being fully vaccinated is not the best protection against the virus. Comparing the numbers 299 by 149 shows that being fully vaccinated doubles your risk of a severe COVID-19 infection compared to people who are not vaccinated. Comparing the numbers 14, 146 and 299 shows that one dose of the vaccine seems to protect you the best against the virus.

Other data were reported, too. Relative figures were provided:

- 26.3 citizens per 100.000 not vaccinated.
- 8 citizens per 100.000 vaccinations initiated – the first dose received.
- 6.7 citizens per 100.000 fully vaccinated – the third dose received.

Comparing the ratios shows that people who are not vaccinated have the highest risk of catching the virus by far; in fact, the risk is more than three times as high as those who are fully vaccinated.

The example shows how mathematical modelling and communication choices play a role in our understanding of a topic. The mathematical choice between absolute numbers versus ratios gives dramatically different conclusions on the efficacy of vaccines. This and other mathematical choices may influence personal behaviour and governmental decisions.

Other examples are found in Elicer's reports from mathematics classes in Chilean upper secondary schools (2020). Examples include using bar graphs of crime rates in a presidential debate and publicly debated use of national test data as measures of school quality (p.116-128). Elicer shows that students' engagement in reading existing graphs and choosing to write alternative graphs can be part of the departure for a reflective discussion on how graphs may shape people's perception of crucial issues.

The above mathematical choices are relevant to the approach in a participatory democracy. In participatory democracy outside school (PDO), citizens read, reflect, and discuss such choices of mathematical data and models. Therefore, it is highly relevant that citizens are aware of choices – for instance – among absolute numbers, ratios, and different graphs and that citizens are motivated to engage in such choices critically. In my view curriculum for mathematics classes for adults may very well consider such examples of how mathematics is used and can be used to understand present circumstances in citizens' life, work, and society. To my knowledge, a typical adult mathematics course primarily takes its examples from the area of consumption. Although the consumption area is of personal relevance, examples from other personal and societal relevance areas should be included.

Again, an open question is with Gutierrez when and in which contexts such data would motivate adult students and serve their interests. Will it motivate and serve interest to work with tasks like the following:

Use video clips from the media or workplaces as a starting point. Participate in formulating and dealing with mathematical questions related to the video clips.

Recalculate the results presented in the video clips.

Compare the communication of mathematics in different media or workplaces.

Illustrate results in alternative ways.

### The RDI box – a representative democracy in school

Representative democracy in school concerns elected fora in the school institution. Are students and teachers allowed, or even obliged, to elect representatives to permanent school boards and temporary committees?

When elected for a board in school institutions, the same investigations on mathematical voting algorithms, voter behaviour and fair representation can be recommended as course content. Recommendations for RDI are like recommendations for representative democracy outside school RDO. Student and teacher organisations may also be interested in investigating these issues in depth.

However, representative democracy also concerns democratic rights to school enrolment and to be given appropriate opportunities to learn in a high-quality mathematical education. A democratic mantra should be that everyone has the right and access to acquire appropriate mathematical expertise and authority. However, throughout history, we find many examples of groups that are denied their democratic rights to high-quality education in mathematics and examples of how the rights are unequally distributed. Historical examples from several countries, for instance, South Africa and the USA, show unequal access to education based on race, formally inscribed in state regulations. (Khuzwayo, 1997) (Moses & Cobb, 2001)

I propose that access to and opportunity for high-quality education – historically and at present – could be part of the curriculum in teacher education and further teacher education. In addition, international organisations and national political parties and movements could put it on their agendas to raise awareness of adults' democratic rights to education. The European Erasmus+ programme has several projects on this matter. For instance, the 'Common European Numeracy Framework' CENF for adults. Democracy is not explicitly mentioned, and the projects do not explicitly investigate whether mathematics education is a democratic right for adults. But the interest is to support a higher level of societal participation and inclusion of adults, thereby to the improvement of the European economy, and the project investigates how up-to-date means and curriculums may look. (Hoogland et. al., 2019).

Once again, an open question is with Gutierrez when and in which contexts such data would motivate adult students and serve their interests. Will it motivate and serve interest to work with tasks like the following:

Has there been and is there presently a legal right for adults to learn mathematics? Why/why not?

Is it a duty to follow adult courses? Is it a prerequisite for receiving social benefits from the state?

Is there funding?

How many adults in your country gain access to adult teaching?

Has this number increased or decreased in the last 20 years? Do you know why/why not?

Is the education offered appropriate? Sufficient?

## The PDI box – participatory democracy in school

The last section concerns participatory democracy in school. Participatory democracy in school is more than legal rights and duties. It includes how students and teachers identify as active participants and engage and use their critical capacity to influence. Participatory democracy includes how students and teachers collaborate, approach each other, and participate in everyday decisions in the classroom.

The bottom right in Figure 1 sheds light on whether and how lessons in mathematics 'are being lived democratically' with participation and communication possibilities for all. Some teaching approaches may socialise into obedience and submissiveness, which is the opposite of encouraging citizenship for democracy. When the mathematics book and the teacher ask questions and decide whether the adult students' questions and answers are relevant and correct, this is a tendency towards authoritarian teaching. Instead, we should aim for democratic learning in which we encourage collaboration between students and collaboration between the teacher and the students.

In the spirit of Freire, the teachers must find a balance between manipulation and spontaneity in their focus on establishing a participatory democratic mathematics classroom, where students and teachers exercise freedom of opinion, discuss, show tolerance, critical attention, trust and justice. You must reflect upon, whether students have the right to expect to be valued and to experience mathematics as meaningful.

Also, for PDI an open question is with Gutierrez when and in which contexts such organisation would motivate adult students and serve their interest? Will it motivate and serve interest to let adult teachers and teacher students discuss what is most important for teachers to consider in their course planning:

How to motivate the students?

How to foster equal dialogue between the teacher and students and among students?

How to provide students with the tools for checking mathematical results for themselves?

How to let the students evaluate meaningfulness, enjoyment, and empowerment?

Finally, will it motivate students and serve their interest to let them discuss:

Do you find the mathematics taught in this course to be relevant – meaningful?

Do you find the collaboration in the course to be relevant – meaningful?

Which parts do you wish to remember in the time to come?

Do you feel that all students in the course can raise their voices?

Do you feel that your voice is heard?

## Classical pedagogical questions

The proposed frame may also inspire formulations of curricula and teaching and assessment activities. Thus, the frame may inspire classical pedagogical questions: what to teach, whom to teach and how to teach?

Such curriculum considerations were the driving force behind the plans of my university department colleagues, formulated at the annual working seminar in 2017, to write an anthology on democracy and 'subject didactics' [German: Fachdidaktik] (Lindenskov, 2020).

The two boxes on top may inspire the question of what kind of mathematics to teach. The box on the bottom right may inspire the question of how to teach mathematics. The box on the bottom left may inspire questions of whom to offer access and opportunities to learn mathematics. This brings us to the following model:

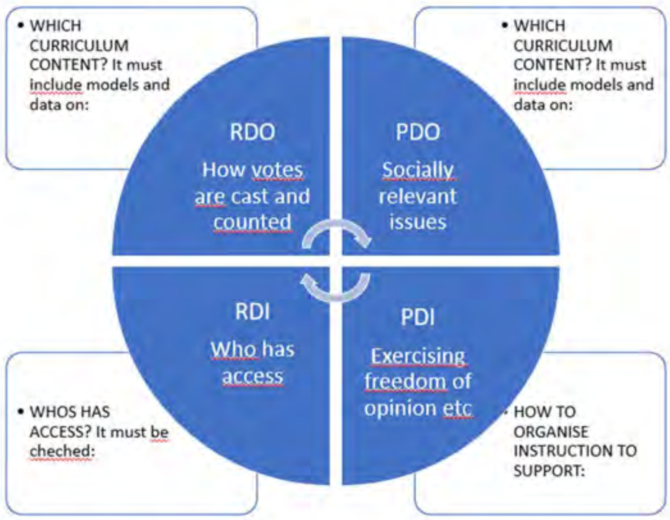



*Figure 4. Curriculum content and organisation*

Hoogland & Díez-Palomar states that there is a growing consensus that a numeracy framework which describes numerate behaviour and numeracy practices should contain much more than only content descriptions. Dispositions, attitudes, higher-order skills, aspects of agency, and self-efficacy are important (2019, p. 131). In line with this statement, a curriculum model like Figure 4 must include issues of course content, but not only issues of content. It must also allow students to redefine dispositions and attitudes and develop higher-order skills, agency, and self-efficacy. This means that questions of what to teach must be supplemented with questions of whom to teach and how to teach.

I hope that this version of the frame can serve as a tool to structure discussions and developments of adults learning mathematics for the sake of democracy.

## Ongoing discussion

This study is motivated by both societal and educational concerns. It is motivated by challenges faced by democratic institutions, threats against democratic values, worries that learning processes in mathematics may be authoritatively affected and worries about educational reforms directed by economic values. In line with Wolfmeyer (2021), you cannot help but question NCTM's suggestion of 'mathematical literacy for all as an "economic necessity" rather than, say, a requirement for a participatory democracy' (p. 1065). We should never let economy and consumerism be the only arguments for learning mathematics. Discussions on the danger of neglecting the broad educational perspective, described as 'Bildung' in German and 'Dannelse' in Danish, is not a new worry; it is as old as education. This worry will always persist.

To me, democracy issues are at least as relevant as in the 1980s to address for practitioners and researchers, and values and theoretical distinctions between representative and participatory democracy and between in-school and outside school from Nordic scholars of the 1980s give insight for us today. Literature of broader origin and more contemporary adds questions on (how) it is in students' interest to motivated engage in classroom activities for democracy and (how) it is possible for the teachers to find and develop appropriate materials and teaching approaches while at the same time upholding conditions for students to develop relevant mathematical rigour. Overall, we are given directions to go.

The assumption that it offers inspiration for rich theoretical and practical development in adult mathematics education can only be tested through ongoing discussions among professionals in research and practice. The model does not encompass all relevant discussions; much more can be said about the relations between mathematics teaching and democracy than what the model can capture. But maybe the frame can help raise awareness of how adults' mathematics and numeracy practices can support engagement in representative and participatory democracy, and maybe some of the mentioned examples can be adjusted to other contexts.

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