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Metacognitive Strategies in Mathematical Modelling Activities: Structuring an Identification Instrument

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Abstract

The paper aims to propose an instrument to identify students' metacognitive strategies in mathematical modelling activities. The items were designed from a review regards metacognition and instruments already recognised in literature. This design comprises an instrument that differs from others in which the data were the self-reported students' use of metacognitive strategies. The instrument we propose is aimed at the teacher and it is he/she who, based on an analytical process, will infer whether the students use metacognitive strategies in modelling activities. An empirical research was realized and five modelling problems were solved by students of a mathematics degree. The analysis addressed the groups actions and the individual behaviour within the group when working on modelling processes. The results allow to conclude working successfully and in a goal-oriented manner on modelling problems requires metacognitive strategies and the instrument seems to be adequate for the teacher to identify these students' strategies.

Keywords

Mathematical Modelling, metacognition, metacognitive strategies, students' actions.

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Estrategias Metacognitivas en Actividades de Modelación Matemática: Estructuración de un Instrumento de Identificación

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Resumen

El artículo tiene como objetivo proponer un instrumento para identificar las estrategias metacognitivas de los estudiantes en actividades de modelado matemático. Los ítems fueron diseñados a partir de una revisión sobre metacognición e instrumentos ya reconocidos en la literatura. Este diseño comprende un instrumento que se diferencia de otros en los que los datos fueron el uso autoinformado de estrategias metacognitivas por parte de los estudiantes. El instrumento que proponemos está dirigido al docente y es él quien, a partir de un proceso analítico, inferirá si los estudiantes utilizan estrategias metacognitivas en las actividades. Se realizó una investigación empírica y se resolvieron cinco problemas de modelización por parte de estudiantes de la carrera de matemáticas. El análisis abordó las acciones de los grupos y el comportamiento individual dentro del grupo al trabajar en los procesos de modelado. Los resultados permiten concluir que trabajar con éxito y de manera orientada a objetivos en la modelación de problemas requiere estrategias metacognitivas y el instrumento parece adecuado para que el docente identifique las estrategias de estos estudiantes.

Palabras clave

Modelo matemático, metacognición, estrategias metacognitivas, acciones de los estudiantes.

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The assertion of Levy Vygotsky that "education is just as meaningless outside the real world as is a fire without oxygen, or as is breathing in a vacuum." (Vygotsky, 1997, p. 345) indicates that dialogues between mathematics and reality can become promising and important in the educational field. Henri Pollak, in particular, since the 1960s, has considered that no area of human activity is free from some mathematical interpretation. This interpretation finds support in mathematical modelling, that is the processing of reality-based problems using mathematics in a way that neither mathematics nor the understanding of the real-world situation is sovereign.

Several studies regarding mathematical modelling in educational environments consider the nature of the problems and the referrals required in modelling activities (Cevikbas, Kaiser, & Schukajlow, 2022; Blum, 2015; Almeida 2018). Blum (2015) particularly believes mathematical modelling is a cognitively demanding activity.

This may be a reason why there has been a growing interest in investigations that address psychological processes of students as they engage in mathematical modelling processes. In particular, metacognition is believed to be an aspect that deserves attention in studies about the development of mathematical modelling activities (Vorhölter, 2018; Vorhölter, 2019; Castro & Almeida, 2023; Vorhölter & Krüger, 2021). It is recognised that metacognition, and especially metacognitive strategies, are, among other aspects, important for a successful holistic modelling process (Frenken, 2021).

Nevertheless, just few studies aim to discuss how metacognitive strategies affect students' actions and how teachers can identify this students' strategies, as pointed out by Vorhölter (2018, 2019) and Vorhölter and Krüger (2021), so it is required to know how to characterise, identify, and sometimes, how to measure these strategies.

The research developed by Katrin Vorhölter and her collaborators pointed out that just few instruments make this identification possible and there are still no known instruments that can assist teachers in identifying students' metacognitive strategies in modelling activities.
Taking this lack into account, in this paper we intend to address the interface between metacognition and the identification of metacognitive strategies in mathematical modelling activities. Particularly, the paper presents the structuring of an instrument to identify metacognitive strategies in mathematical modelling.

Mathematical modelling

Mathematical modelling enables the dialogue between mathematics and reality and aims to find a solution to a problem identified in a real situation. So, at the core of modelling problems is a demanding process of translating between the real world and mathematics. This mathematisation leads to the elaboration of a mathematical model so that mathematical procedures can be applied in order to find a mathematical result. Then this result has to be interpreted and validated with regard to the real-world situation (Pollak, 2015; Carreira, Baioa, & Almeida, 2020; Blum, 2015; Frejd & Vos, 2022).

It is often necessary to engage in an iterative refinement of the solution when the intention is to characterise how mathematics and reality are connected in mathematical modelling. To include the refinement, what students do in a mathematical modelling process is recognised in a modelling cycle and include the steps that students can go through in developing a modelling activity. Blum (2015) suggest seven steps: (1) Understanding the real problem; (2) Simplifying the original situation; (3) Mathematising; (4) Working in the mathematical domain; (5) Interpreting the results obtained; (6) Validating; (7) Presenting the results.

The actions in each step require students to have a well-developed repertoire of cognitive and metacognitive strategies [...] In particular, a variety of retrieval, recognition, mental imaging, perceptual, and integration strategies, together with metacognitive strategies for monitoring, regulating, and coordinating the use of these cognitive strategies are necessary (Stillman, 2011, p. 63).

According to Blum (2009, p. 22), "there are many indications that meta-cognitive activities are not only helpful but even necessary for the development of modelling competency". Like Blum (2009), many authors recognise the importance of metacognition in mathematical modelling. Some of the reasons for this recognition lie in the fact that metacognition contributes to math performance, stimulates problem solving skills, improves modelling skills, expands the repertoire of strategies for successfully solving complex problems in a goal-oriented manner (Blum, 2015; Vorhölter & Kaiser, 2016; Vorhölter, 2017, 2018; Vorhölter, et al., 2020; Hidayat, et al., 2020; Vertuan & Almeida, 2016; Castro & Almeida, 2023).

Despite the importance of metacognitive strategies in modelling activities, research on metacognitive modelling strategies is still in its beginning. One reason for this is the complexity of measuring metacognitive strategies, resulting from the complexity of metacognition as a cognitive process as such, resulting in methodological complexity (Vorhölter & Krüger, 2021, p. 179).

Also in this sense, Frenken (2021) points out that the identification of metacognitive strategies requires to use instruments and resources that allow capturing these strategies in students' actions and behaviour.

Metacognitive strategies: knowledge of cognition and regulation of cognition

A definition of metacognition was proposed by Flavell (1976, p. 232), defining it as "one's knowledge concerning one's own cognitive processes and products or anything related to them". Schneider and Artelt (2010) suggested that metacognition also includes executive skills related to monitoring and self-regulating of one's cognitive activity. Brown (1987) understand that metacognition includes two components: knowledge of cognition and regulation of cognition.

Knowledge of cognition, also called metacognitive knowledge, consists of the subject's knowledge and awareness of their own cognition and cognitive resources. Therefore, it is conscious and controllable (Pressley, et al., 1985) but sometimes fallible (Brown, 1987). The metacognitive strategies arising from metacognitive knowledge consist of planning, monitoring, and regulating one's work and can be associated with three kinds of knowledge: declarative knowledge, procedural knowledge, and conditional (or explanatory) knowledge (Paris, et al., 1983; Schraw & Moshman, 1995).

Schraw and Moshman (1995) state that "declarative knowledge includes knowledge about oneself as a learner and about what factors influence one's performance" (p. 352). According to this authors, declarative knowledge, in addition to informing the "know about" and "know what", involves factual knowledge necessary to process or use critical thinking, skills, and intellectual resources and is developed through presentations, demonstrations, and discussions.

In the mathematical modelling context, declarative knowledge leads students to consider their difficulties concerning their way of acting in the activity and recognise relevant aspects of the real situation being studied. For example, when the student signals that he/she knows what is relevant in the situation and identifies mathematical concepts or techniques, discussing it with their colleagues or conducting research about it.

Procedural knowledge refers to "how to apply procedures such as learning strategies or actions to make use of declarative knowledge and achieve goals" (Mahdavi, 2014, p. 530). According to Mahdavi (2014), the more skilled the student is in deciding on techniques and procedures in each situation, the more procedural knowledge will be evidenced. The procedural knowledge, in turn, can emerge through discovery, cooperative learning, and problem-solving.

In mathematical modelling activities, procedural knowledge can be associated with the mathematisation step, in which the students define hypotheses and associate mathematical language with the real situation, and the resolution step, in which they identify reasonable techniques and procedures for the construction of a mathematical model. It is possible that interactions with the teacher or colleagues in the group trigger discussions about the processes used, leading to the appearance of those necessary to solve the problem.

Conditional knowledge refers to "why?" should someone apply specific procedures, externalize specific skills, or use specific strategies. So, it means "knowing when, where, and why to use declarative knowledge as well as particular procedures or strategies (procedural knowledge), and is critical to effective the use of strategies" (Harris, et al., 2010, p. 133). According to Schraw and Dennison (1994), besides indicating when and why to use specific procedures, conditional knowledge favors the determination of particular circumstances, processes, or skills to be used in each situation.

Concerning mathematical modelling the conditional knowledge strategies refer to students being aware of the processes and skills needed to perform a task. These strategies favor students to assess the results during the resolution process and reinforce or change the chosen strategy, according to the evaluation carried out (Vorhölter, 2019; Vorhölter et al. 2020).

The regulation of cognition is the metacognitive skill defined as "a sequence of actions taken by students to control their own thinking or learning" (Mahdavi, 2014, p. 531). It is related to "how students control their learning process, making decisions about how to learn, managing the ongoing process and evaluating their overall performance. It also promotes the ability to evaluate the execution of the task and make corrections, when necessary, provoking the control of the cognitive activity and of the processes that assess and guide the cognitive operations (Desoete & De Craene, 2019).

Although several regulatory strategies have been described in the literature (Schraw & Dennison, 1994), three of them are essential and are included in most studies in the area: planning, monitoring, and evaluating strategies (Schraw & Moshman, 1995; Mahdavi, 2014).

Planning strategies consist of planning how to proceed and involve setting goals or aims, selecting ways to promote learning, making predictions, allocating adequate resources to achieve goals, deciding which steps to take, activating prior knowledge, and organising time (Mahdavi, 2014). According to Price-Mitchell (2020), these strategies are responsible for leading students to examine and develop plans that can be changed. As students learn to plan, they also learn to predict the strengths and weaknesses of their ideas.

In mathematical modelling activities, planning strategies start the understanding of the real problem in which students organise data and seek to understand the situation. In the mathematisation step, planning strategies anticipate necessary procedures based on hypotheses and variables. Those strategies also guide the subsequent choices and decision-making process in the modelling activity (Castro & Almeida, 2023).

Monitoring strategies refer to the awareness of performing a critical analysis of strategies. They involve supervision, control, and self-testing of essential processes to regulate learning (Mahdavi, 2014). Moreover, monitoring includes the ability to control the action and verify that it is adequate to achieve the proposed objective, evaluating the deviation from it, noticing errors, and correcting them, if necessary (Rosa, 2017). This ability develops slowly and allows constant tracking of what has been learned, what is not yet known, and whether study strategies are helping to learn effectively.

Besides strengthening metacognition, the use of monitoring strategies helps students check their progress and review their reasoning in different contexts. Reflective in nature, those strategies enable adjustments while the activity progresses and encourage learning recovery. So, monitoring strategies work as internal signals that serve as a warning and allow to recover or rethink an idea when the student realizes that something is wrong (Price-Mitchell, 2020).

In mathematical modelling activities, students generally can resort to monitoring in contexts that require defining steps to execute and monitor whether the knowledge involved in the activity is coherent regarding the reality, the modelling, and the mathematics involved. Reflections on referrals and procedures adopted demand clarifying doubts, correcting errors, and corroborating successes. Sharing ideas with the group, asking for help to check how they think, and confronting suggestions can configure examples of monitoring strategies used by students (Vorhölter, 2018, 2019; Castro & Almeida, 2023).

The assessment strategy refers to examining the progress made towards goals that can trigger further planning, monitoring, and evaluation. It includes (re)assessing its objectives and conclusions, i.e., whether the results match the intended purpose (Mahdavi, 2014). A strategy of assessment at the end of the task implies the students' awareness of how much they have learnt, in how much time, under what conditions, and what adjustments are still necessary (Frenken, 2021). Inspecting parts of their work, in turn, provides students with learning about the nuances of their thinking processes, and in doing so, they learn to refine their work and apply their learning to new situations (Price-Mitchell, 2020).

Assessment strategies in mathematical modelling activities are associated with verification processes of procedures and partial results during the activity and with the steps (6) Validating and (7) Presenting the results. Those strategies allow students to have a holistic view of the development of the modelling activity (Castro & Almeida, 2023; Frenken, 2021).

In general, as Schraw and Moshman (1995) note, knowledge of cognition and the regulation of cognition can be integrated through peer interaction, which can encourage the construction and refinement of metacognitive strategies. However, what is required is to know means or instruments to identify the students' metacognitive strategies.

Instruments to identify metacognitive strategies: what do we already know?

The instruments to identify and, in some situations, assess metacognitive strategies, come mainly from the field of psychology and are used for educational purposes or research in Education and in Mathematics Education field.

Schraw and Dennison (1994) present an instrument recognised in the research community, the Metacognitive Awareness Inventory (MAI), to be applied to young students. The MAI consists of a questionnaire that students must answer, checking true or false for statements related to strategies of knowledge of cognition and regulation of cognition.

Rosa's (2017) research proposed an observation manner that allows the teacher to identify the use of metacognitive thinking by students during physics classes. The construction observation manner took into account six metacognitive elements that are part of the two components of metacognition: the metacognitive knowledge component (person, task and strategy); and the executive and self-regulatory control component (planning, monitoring, and assessment). Such components and their elements were adjusted according to possible actions developed in the physics laboratory. Its construction is based on the relationship between the students' possible actions, the indicators of metacognitive manifestations, and the item on the observation form. The teacher, or researcher, will be responsible for checking the items with Y (yes), N (no) or D (expressed with difficulty) regarding the students' strategies during the experimental physics classes.

Yildirim (2010) surveyed students of an engineering course and investigated how metacognitive aspects influence students' procedures in a mathematical modelling activity. The author concludes that when students reflect on their thinking as they plan, monitor, and evaluate their learning, they develop modelling competencies. To analyse the impact of metacognitive strategies on modelling, the author uses a self-efficacy scale and a metacognitive inventory in which students themselves assess how well they can perform a modelling activity.

To develop items for measuring students' metacognitive strategies for modelling, Vorhölter (2017) proposed a task-bound questionnaire with 39 five-point Likert items to be answered by the students. Associating the students' answers with videotapes of the working process of several groups the author observed the students' metacognitive strategies regarding competencies related to the different steps of a modelling process, considering: competencies for orienting and planning the solution process; competencies for monitoring and, if necessary, regulating the working process; competencies for evaluating the modelling process, in order to improve it.

The research of Frenken (2021, p.219) begin with the question: "Is it possible to measure metacognitive knowledge of mathematical modelling as a latent construct?" In order to answer this question affirmatively, the author proposes an instrument to assess metacognitive knowledge in mathematical modelling. The instrument consists of 35 items classified into the person, task and strategy categories. The items must be answered by the students on a Likert scale ranging from 1 to 4 (1 being true and 4 being false). The test was administered to five classes, at two schools, of 15-year-old students, whereby 115 students participated in total. The test only provides a quantitative analysis relative to the number of items answered by the students and the data were scaled using a one-parameter Rasch model.

Research, therefore, show that the instruments dealing with metacognition in mathematical modelling are predominantly quantitative and answered by students. What we propose in this article is an instrument of a qualitative nature and to be answered by the teacher, considering his/her observations regarding students' actions and behaviour while developing mathematical modelling activities.

Research design and methodology

Our research aims to propose an instrument to identify students' metacognitive strategies in mathematical modelling activities. Specifically, the instrument includes items in order to signal students' metacognitive strategies in the different steps of a mathematical modelling process.

Two steps were undertaken in the research. First of all the items for the instrument were formulated and designed on the basis of theoretical concepts regards metacognition, metacognitive strategies, and instruments already used in education area to obtain information about students' metacognitive strategies.

Afterwards, an empirical research was realized with students of a mathematics degree. In this research the new instrument was used to capture the peculiarities of metacognitive strategies in modelling processes. We carried out a qualitative research study (Sharma, 2013) with an interpretive nature, focusing on the students' metacognitive strategies while solving modelling activities in the classroom.

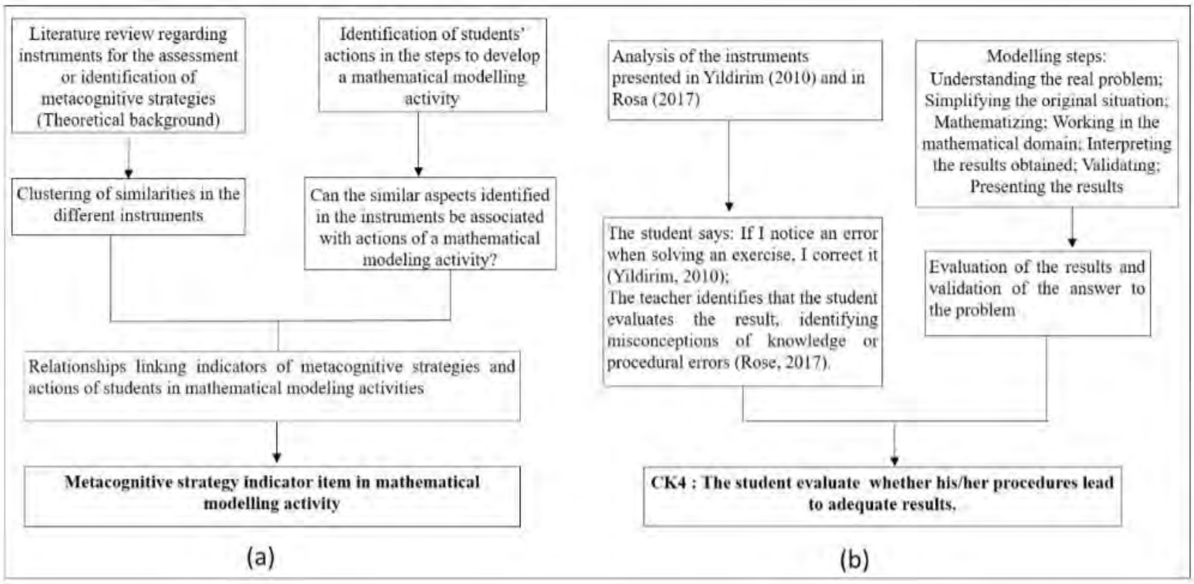
Structuring an instrument to identify metacognitive modelling strategies

In order to structure the instrument to identify metacognitive strategies, we had in mind the regulation of cognition and the associated metacognitive strategies (planning, monitoring, and assessment) and the knowledge of cognition (declarative, procedural, and conditional). The items we designed as possible metacognitive strategies take actions as solution planning, searching for an analogy, a feedback procedure, the verification of the solution, all of them lined up the seven modelling process steps.

In structuring the instrument, we built each item considering a process that follows two phases. First (Figure 1 (a)) we performed the analysis of metacognitive assessment instruments recognized in the literature (e.g. Yildirim, 2010; Rosa, 2017) and proceeded to group similar aspects in these instruments (For example: The student says If I notice an error when solving an exercise, I correct it (Yildirim, 2010); The teacher identifies that the student evaluates the result, identifying misconceptions of knowledge or procedural errors (Rosa, 2017). Beyond that, in a second phase, we took into account the students' actions required in each of the seven modelling steps (Blum, 2015) and associate the students' actions and behaviours necessary for the successful modelling process in each step, a possible metacognitive strategy capable to support this action. For example (Figure 1 (b)), to the items mentioned above by Yildirim (2010) and Rosa (2017), we associate the metacognitive strategy in mathematical modelling CK 4: The student evaluates whether his/her procedures lead to adequate results.

Figure 1

Designing the items of metacognitive strategies



In doing so, we built thirty-three indicators of metacognitive strategies (Table 1), fourteen of them related to the knowledge of cognition component (declarative, procedural and conditional) and nineteen related to the regulation of cognition (planning, monitoring and evaluation).

Table 1

Indicators of metacognitive strategies in modelling activities

Knowledge of cognition	**Declarative knowledge strategies**	1	2
	DK1 The student recognises his/her strengths and weaknesses about of what he/she needs to know to develop the activity.		
	DK2 The student signals to others what he/she knows.		
	DK3 The student considers different ways to solve the problem.		
	DK4 The student recognises the need to collect and organise information about the situation before starting the development of the modelling activity .		
	DK5 The student assesses whether his/her knowledge meets what he/she needs to know to develop the modelling activity.		
	Procedural knowledge strategies		
	PK1 The student mentions using strategies that worked in previous modelling activities.		
	PK 2 The student declares that the construction of the mathematical model is based on the data, on the hypotheses he/she formulated and on the directions defined in the mathematisation step.		
	PK 3 The student shows his/her mathematical knowledge and his/her solution strategies.		

	PK 4 When he/she does not understand some information or concept, he/she reports to colleagues, to the teacher, or conducts research on the subject.		
	Conditional knowledge strategies		
	CK1 The student recognises that he/she uses different strategies to define procedures according to the steps of the modelling activity development.		
	CK 2 The student justifies adequately the use of mathematical concepts and methods.		
	CK 3 The student explains why and how he/she uses the contents, techniques, and strategies to solve the posed problem regarding the messy real world situation.		
	CK 4 The student evaluates whether his/her procedures lead to results that are acceptable.		
	CK 5 The student knows how to enhance his/her knowledge and skills, in face of his/her difficulties.		
	Planning strategies		
Regulation of cognition	**PS1** The student decides what is important to approach the messy real world situation using mathematics.		
	PS 2 The student defines the objectives of the activity before starting its development.		
	PS 3 The student plans the resolution of the problem considering different possibilities		
	PS 4 The student identifies contents or procedures that may be useful to solve the posed problem.		
	PS 5 The student signals to others that he/she identifies in his/her cognitive structure the resources needed to mathematise the situation.		
	PS 6 The student simplifies and organises the data, considering those necessary to solve the problem.		
	PS 7 The student establishes the steps to be followed in conducting the activity development.		
	Monitoring strategies		
	MS1 The student recognises the mathematical model importance and usefulness to study messy real world situation.		
	MS 2 The student recognises that it is necessary to formulate hypotheses and to establish some simplifications to approach de real world situation by mathematics.		
	MS 3 The student realise a change in the planning or request for help when he/she recognises that he/she does not understand something or when he/she is unable to proceed in the modelling process.		
	MS 4 The student mentions spot checks at some moments during the modelling process.		
	MS 5 The student uses analogous examples or colloquial language to explain resolution strategies or to make his/her choices more appropriate.		
	MS 6 The student identifies errors and applies a new strategy to correct them.		
	MS 7 The student exposes strategies to build the model, establishing comparisons with others already studied or suggested by colleagues or teacher.		
	Assessment strategies		
	AS1 The student believes that the model is not suitable and then make an effort to building a new one.		

	AS 2 The student identifies misconceptions or errors about his/her mathematical knowledge.		
	AS 3 The student checks whether his/her final results match the conditions of the real situation.		
	AS 4 The student recognises that there would be other ways to approach the situation using mathematics.		

Although much of what can be considered about metacognitive strategies took into account that they are of a personal (individual) nature, Castro and Almeida (2023)' study pointed out that working in small collaborative groups contributes to individuals developing skills of listening, helping, and sharing, and this can lead to personal metacognitive behaviours associated with learning and questioning. Another important mechanism pointed out by these authors is that interactions between individuals who work together require verbal tools that make it possible to regulate or monitor the behaviour and thinking of the other.

Vorhölter (2018) highlighted that the performance of all group members towards a common consensual goal denotes that metacognitive strategies triggered in the context of a group are a reflection of the collectivity and not just of isolated individual strategies. This author also considered that, although personal, metacognition cannot be explained exclusively by individualistic conceptions, as the interactions with colleagues and teachers are the main sources to encourage and activate triggers that enable students to detect errors and adapt their thinking or solve obstacles and progress in a situation.

In line with this understanding, Castro and Almeida (2023) state that group metacognition seems to be more crucial than individual metacognition in collaborative group work contexts. Vorhölter (2019) asserted that in metacognition of a social nature, i.e., the group's, the individual must make his/her thinking available to others and discuss his/her assumptions, justifications, and conclusions with each other, relating others' thinking to their own.

Therefore, we recognise that, when investigating metacognition in mathematical modelling activities, both individual metacognition and socially shared (collaborative) metacognition must be considered. So, the instrument we propose includes column (1) and column (2) in Figure 2 regarding the indications of individual metacognition or metacognition with a collaborative nature, respectively. This classification criterion depends on the teacher's perspective and provides us with conditions to make the instrument efficient for mathematical modelling contexts, where metacognition is both individual and the result of group actions.

The structuration of this instrument considers that strategies are sometimes used consciously. Still, other times they are used unconsciously or automatically, without students being aware of using them. In this case, the teacher's noticing will be responsible for interpretations of underlying strategies in the activity. On the other hand, the teacher can only evaluate those strategies that are verbalized or shown by the students.

Empirical research

The empirical research refers to the development of mathematical modelling activities in a mathematical modelling discipline taught by one of the authors of this paper at a Brazilian public university for fourth-year students of a Mathematics Degree in the first academic semester of 2021 year. The students had already had contact with mathematical modelling definitions and perspectives and had developed other modelling activities before those in which data were collected.

Considering the limitations posed by the Covid-19 pandemic for face-to-face classes, the activities were developed in remote mode, and the meetings were held through Google Meet. Five modelling activities were developed by 15 students. The students were organized in three groups with five students in each one (G1 (S1…S5); G2 (S6…S10); G3 (S11…S15)). The Poker Game activity and the How to get one million of reais? were developed by all groups. The problem The Covid vaccination coverage in Arapongas city, was realized by group one (G1); Tax collection in the city of Londrina was developed by group two (G2); and Devaluation of a car's price was developed by group three (G3). In the activities The Poker Game and How to get one million of reais? the teacher suggested the problem to be investigated. In the other three, each group of students defined the problem they wanted to study.

The data collection includes videotapes of classes held on Google Meet and reports and slides of activities delivered by students. Furthermore, after working on the modelling activities the students were asked to fill questionnaires and multiple choice questions. In these questions they were asked to judge their motivation and their difficulty of the activity and include a short query on personal data.

Finally, videotapes of the working process of the groups were analysed by identifying metacognitive strategies that could be observed by students' verbal expressions or their behaviour. The reports delivered by the students were analysed in order to identify how they reveal students' metacognitive strategies in their solution plan, their mathematisation procedures, their searching for analogy, their model construction, their verification of the solution and, in general, on the organisation of their actions in the activity development. In the variety of students' behaviours during modelling activities we looked for the metacognitive strategies we characterised as shown in Table 1.

To exemplify the analytical process realised in order to identify the metacognitive strategies designed in the instrument we will keep in mind aspects of the students' performance in the activity How to get one million of reais?

Discussions concerning the looking for answers to this question led to the topic of financial investments and, particularly, to a very popular application in Brazil called savings account. The question How to get a million? was configured like this: What monthly amount (in reais) must be applied to a savings account to obtain one million reais at the end of ten years?

The students built a mathematical model from data obtained for inflation and compound interest rates prevailing in Brazil in the previous ten years. This procedure was associated with simplification and definition of hypotheses in the modelling activity. Using the concept of compound interest and the sum of the terms of a geometric progression, one group presented an answer to the question. Another group, however, went further and used conditions that include monthly gains and losses over time. Table 2 presents some examples of metacognitive strategies identified during the development of this modelling activity.

Table 2

Examples of metacognitive strategies identified by using the instrument

Category of the strategy	**Indicator**	**The students' action or behaviour**	**Nature**
Declarative knowledge	**DK2** The student signals to others what he/she knows	Dialogue between two students at G2 S9: The fact is that inflation in the last year was higher than the saving account in Brazil. See here colleagues (pointing to slide with the data). S10: *Hhh*, therefore, we may only think about long-term yeld, when this situation changes!	2
Procedural knowledge	**PK 3** The student shows his/her mathematical knowledge and the solution strategies	Video transcription and students' report (G2) S6: Now, let's just do the math. Using the compound interest an *x* as monthly kept amount, I obtained: $P_1 = x \cdot 0{,}998$ $P_2 = [(x \cdot 0{,}998) + x]\, 0{,}998$ $= x \cdot 0{,}998^2 + x\, 0{,}998$ $P_3 = [(x \cdot 0{,}998^2 + x 0{,}998) + x]\, 0{,}998$ $= x \cdot 0{,}998^3 + x \cdot 0{,}998^2 + x \cdot 0{,}998$ $\vdots$ $P_{120} = x \cdot 0{,}998^{120} + x \cdot 0{,}998^{119} + \cdots + x \cdot 0{,}998$ $P_{120} = x \cdot \sum_{n=1}^{120} 0{,}998^n$ So, we can obtain: $x\sum_{i=1}^{120}(0{,}998)^n = 1\,000\,000$ $106{,}567x = 1\,000\,000$ $x \approx 9383{,}76$ S8: We found the answer! Is that right?? S6: On the mathematics point ov wiev it is correct! But the *x*-value is too high for the real situation! Students' report (G1) Using a recurrence process we obtained: $S_n = \sum_{i=0}^{n} L \cdot p^n \cdot g^i \Rightarrow S_n = L \cdot p^n \cdot \sum_{i=0}^{n} g^i$	1 2 2
Conditional knowledge	**CK 4** The student evaluates whether his/her procedures lead to results that are acceptable.	Video transcription S10: The resolution we presented is efficient because we used annual contributions and interest and inflation from ten years ago. The Tesouro Direto website, however, uses today's inflation. That's why it makes a difference. Teacher: What can you tell me about the reliability of your answer? S11: Well, we used a fixed rate. This is a weakness of our modelling process. We now that there is no fixed rate […]. Maybe we can try to do another resolution.	2
	PS1 The student decides what is	Áudio trasncription S6: Guys, what do we need to define first? The rates?	2

Category of the strategy	Indicator	The students' action or behaviour	Nature
Planning	important to approach the messy real world situation using mathematics.	S9: First, we need to define the rate we are going to use and try to predict them for ten years. Or are we going to use the same rate for all the years? If so, what will that rate be? S10: Maybe an average over the last few years? G1 report We decided that we want to define the variables in order to use the monthly deposit amount, the gain as well as the loss in each month. Monthly deposit amount: L Gain (g): L=1+interest; Loss (p): L =1-inflation; Time (months): *n;* Total on the end of month n: Sn. […] However, we also organised ourselves to make a resolution with fixed rates and then we will compare the results.	2
Monitoring	**MS 2** The student recognises that it is necessary to formulate hypotheses and to use some simplifications to approach de real situation by mathematics.	Áudio transcriptions (G2) S6: We had some information gaps! S10: We had to overcome them by doing simplifications and formulating hypotheses. Áudio transcriptions (G1) S4: I'm feeling a bit lost! What we may do? S5: We can suppose that the monthly rate is the same! S4: Ok!	1 1
Assessment	**AS 3** The student checks whether his/her final results match the conditions of the real situation.	G2 report The need to save almost nine thousand reais a month to get a million reais is a bad scenario! The salary of most Brazilians makes this monthly savings unfeasible! Therefore, although valid from a mathematical point of view, the answer is unlikely to occur in Brazilian practice. G1 Report It is necessary to take in count that answering "What monthly amount (in reais) must be applied to a savings account to obtain one million reais at the end of ten years?" needs to consider other types of monthly savings!	2 2

Conclusion

The structuring of this instrument proposes an interface between metacognition and mathematical modelling activities that is not very focused in the literature. The proposed items include the two major components of metacognition, the knowledge of cognition (declarative, procedural and conditional) and the regulation of cognition (planning, monitoring and evaluation). In addition, the structure of the instrument includes indicators of metacognitive strategies in each of the seven steps of a mathematical modelling process (Blum, 2015).

The focus on possible metacognitive strategies proposed in this article differs from what the literature already presents on the subject, as is the case of the study realized by Yildirim (2010) and the questionnaires for students proposed by Vörholter (2019) and by Frenken (2021). Our investigation of students' metacognitive strategies in the modelling process proposes some specificities in relation to what is already recognised in the literature.

On the one hand, the look at metacognitive strategies is directed towards the 33 designed items. This design, however, has a theoretical basis that includes metacognition and modelling. On the other hand, unlike previous instruments (Vörholter, 2018, 2019; Frenken, 2021) in which the data were the self-reported use of metacognitive strategies by the students, the instrument proposed here is aimed at the teacher. That is, it is the teacher who, based on a careful and judicious analytical process, will infer whether the students use metacognitive strategies in different steps of a mathematical modelling activity development. Furthermore, in line with previous research (Castro & Almeida, 2023; Vörholter, 2018; Vörholter, 2019), the instrument design agree that group metacognition seems to be more crucial than individual metacognition in collaborative group work contexts. Identification of the nature (individual or collaborative) is therefore included in the instrument.

The inclusion of these two specificities when structuring the instrument considers two aspects. First of all, the instrument is not aimed at identifying students' competencies in modelling activities and does not propose a method of measuring them as proposed by Vörholter's (2018), Vörholter (2019) and Frenken (2021) questionnaires.

A further aspect concerns the fact of attributing to the teacher the role of identifying the student's metacognitive strategies. Proposing the identification of strategies by the teacher is based, on one side, on the assertions of Desoete and De Craene (2019) that providing students with the opportunity to mobilize and develop some metacognitive strategies is relevant when aiming at good mathematical performance and, particularly, the development of problem-solving skills that enable learning by exploring relationships between reality and Mathematics. The teacher, being the one who directly turns his/her attention to the identification of metacognitive strategies, starts to recognise their occurrence in students' actions and behaviour and may organise the task in order to enhance students' metacognition. On the other hand, as Vörholter's studies already indicate, the validity of students' self-reports may be doubted and the teacher observation can offer more information and capture details that the students themselves omit.

The empirical research was conducted in order to value the suitability of the 33 items that make up the instrument. The analysis addressed the groups processes and the individual behaviour within the group and was directed to data obtained with videotapes of the group's working process and of the reports delivered by the students. In the analysis were taken into account their solution plan, their mathematisation procedures, their searching for analogy, their model construction, their verification of the solution and, in general, the organization of their actions in the activity development.

This analytical process is illustrated in the article (Table 1) considering one of the five activities developed by the students. The results allow to conclude that the items offer good indications of metacognitive strategies that may occur in all the steps of the modelling cycle.

Working successfully and in a goal-oriented manner on modelling problems requires metacognition strategies and the instrument seems to be adequate for the teacher to identify these students' strategies. Beyond that the instrument is independent of the specific task and can be used to identify metacognitive strategies in different modelling problems.

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