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Partitive Fraction Schema: Mental Action Processes Used to Mathematics Construct Concepts in Elementary Students

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Abstract

The concept of fractions given in learning is the concept of part of whole and part of unit. The development of student's concepts of fractions can be built through fraction schemes. A partitive fraction scheme is a scheme that estimates the size of the fraction in the form of non-units to the whole that is not partitioned. The concept of fractions using a partitive fraction scheme uses a strong understanding of the concept of the part unit and part-whole fractions. This study aims to photograph the process of developing students' mental actions in constructing the concept of fractions using a partitive fraction scheme. . This research method uses a qualitative approach with a case study type on students who are able to express partitive fraction schemes. The stages of data analysis consist of three stages, namely; (1) the results of task 1 data are corrected and determined by students who can solve fraction problems with part-whole and partitive unit fraction schemes, (2) students who can complete task 1 are selected as participants (3) students complete task 2 to see their problem-solving abilities fractions using a partitive fraction scheme, (3) students who can use a partitive fraction scheme were interviewed to see the mental action processes used in solving problems. The participants of this study were 35 students of fifth-grade elementary school who already understood the concept of part-whole and part of unit. The results showed that students used two types of mental action processes in the partitive fraction scheme: direct and indirect.

Keywords: Partitive Fraction, Mental Action, Mathematics Construct Concepts.

Introduction

Understanding the concept of fractions is one of the critical components of understanding mathematics (Siegler et al., 2011). The concept of fractions is one of the supporting abilities in solving algebraic problems (Booth et al., 2014; Flores, 2010; Jordan et al., 2011; Rodrigues et al., 2016). The concept of fractions is the basis for understanding other materials such as ratios, proportions, and algebra. One of the concepts of fractions comes from understanding the concept of parts of a whole(Norton et al., 2018).

According to Norton and Wilkins (2010), Students still have difficulty in estimating the size of non-unit fractions, such as 1/2 called half in the whole section until they can classify the overall shape of the unit fraction and the non-unit, as 1/4 is 1 of 4 parts of the whole (Norton & Wilkins, 2010a). Simon et al. (2018) argue that romoting a comprehensive understanding of fraction ideas remains a challenge in mathematics education. Fraction is a difficult material for students (Department of Education, 2008; NCTM, 2010) because students only recognize fractions with part elements of the whole number (denominator and numerator), not as a relationship between parts (Bossé et al., 2018). This resulted in students experiencing misunderstandings about the part-whole relationship in the concept of fractions (Aliustaoğlu et al., 2018). Students also cannot estimate the size of the fraction in the overall form (Norton & Wilkins, 2010). In short, the concept of fractions that students understand is constructed through the concept of equality of all parts consisting of the whole partitioned part (Tzur, 1999). The concept of student fractions can be constructed using the schemas that students have and new schemes for solving fraction problems(Norton & Wilkins, 2013) because the fraction scheme can describe the development of students' concepts (Norton et al., 2018). Student's ability to understand the concept of fractions can be demonstrated through the representation of the fractional scheme used.

Several previous studies have examined the development of students' fraction concepts (Doğan & Tertemiz, 2020; Fuchs et al., 2013a) in using fraction schemes (Norton & Wilkins, 2009b, 2010a, 2013). Norton & Wilkins (2009) analyzed and measured the development of students' fraction concepts through separation operations and fraction scheme construction. The following year, Norton & Wilkins clarified students' schemes in constructing the concept of fractions where the categories of fraction schemes in constructing the concept of fractions were interrelated.

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One linkage is that students can first construct a part-whole scheme (PWS) to construct a partitive unit fraction scheme (PUFS) (Norton & Wilkins, 2010). However, the student's thinking processes have not been seen in constructing a new fraction scheme using the old fraction scheme. In other words, it is necessary to investigate how the thinking process of students in constructing the partitive fraction scheme using the part-whole and partitive of unit schemes.

Partitive fraction schemes can be constructed based on students' abilities in the concept of fractions (Booth et al., 2014; Steffe, 2002; Steffe & Olive, 2010a) using a fraction scheme of parts of a whole and a scheme of fractions of parts of a unit. This investigation is important to see the development of the partitive fraction scheme used by students so that students can construct the concept of fractions using different schemes (new schemes) based on the old schemes they have. This study aims to see the process of developing students' mental actions in constructing the concept of fractions using a partitive fraction scheme.

Theoretical Review

Fraction schemes and mental actions

Development students' fraction concepts can be constructed in a fraction scheme (Norton et al., 2018), where the fraction scheme can construct the concept of fractions to solve problems (Norton & Wilkins, 2013). In mathematics, the schema is defined as the observation of mathematical concepts, referring to students' language (Bossé et al., 2018; Vygotsky, 2012) and mathematical actions (Díez-palomar, 2016; Hinson, 2004; Lewis, 2015; Norton et al., 2018; Steffe & Olive, 2010a) From a cognitive constructivist perspective, Von Glaserfeld (Norton et al., 2018) states that the scheme is the researchers' idea to describe new information obtained by students. Based on the definition of the schema above, in this study, the schema is defined as a structured mental representation (Clapin et al., 2004; Varma & Schwartz, 2011) based on the experience and knowledge of students following the new information obtained. Therefore, the student's fraction scheme can appear as an adjustment in the number calculation scheme (Steffe & Olive, 2010). Fraction schemes and number calculation schemes are two things that are used for different purposes. If students use a number calculation scheme, it is necessary to use new information in the scheme.

There are several schemes used in displaying fractional concepts, such as (1) A portion of the whole, for example 2/5 of the pocket money of 100,000 must be saved; (2) part of the unit, e.g. 1 chocolate is distributed to 4 people where the partitive fraction form is 1/4. The partitive fraction scheme with non-unit fractions seems to require a concept in the description of the measurement, so it is necessary to operate in separating these fractions (Norton & Wilkins, 2010).

The principle of operation on fractions is called mental action (Fuchs et al., 2013b; Norton et al., 2018; Pavlovičová & Vargová, 2020). Operations are some of the students' mental actions in constructing the concept of fractions so that students can understand the concept of fractions correctly. The mental action in the fraction scheme has three operations used, namely; Partitioning is the mental act of breaking the whole into equal parts (e.g., imagining five equal parts as a result of cutting a piece of chocolate), iterating is the mental act of making connected copies of each part (e.g., getting 3/5 of combining 1/5 of 3 parts), and disembedding is the mental act of taking part of a whole without destroying the whole part (e.g., getting 3/5 of the chocolate removed/drawn back from a whole chocolate bar partitioned into five equal parts) (Steffe & Olive, 2010). Table 1 describes the mental actions used in constructing the concept of fractions to produce several fractional schemes.

The five mental actions above show how students develop the concept of fractions from a given new problem. This mental action can occur depending on the ability of students. The fraction scheme that students use to construct the concept of fractions consists of four categories (Norton et al., 2018) as in table 1. This fraction scheme is formed based on mental actions in understanding and solving fraction problems. Table 2 shows the fractional scheme and the actions associated with the operations.

Based on these several categories of schemes, each scheme category has a significant relationship between one scheme and another. One example; is to use the partitive fraction scheme; it is necessary to form a part-whole and a partitive unit fraction scheme for students to construct the concept of fractions (Norton & Wilkins, 2009, 2010). Furthermore, students can

Table 1: Description of mental action (Norton et al., 2018)

Mental action	*Description*
Partitioning	Projection of a composite unit into a continuous whole to create equally sized parts within the whole
Iterating	Repeating a unit of length or area to produce a connected whole
Disembedding	Taking parts out of a whole as separate units while maintaining their relationship with the whole
Splitting	Simultaneous composition of partitioning and iterating as inverse actions
Coordination unit	Maintaining relationships between the various levels of units within a mathematical experience; involves coordinating more uncomplicated actions, such as partitioning, iterating, and disembedding

Table 2: Fraction Scheme (Norton et al., 2018)

Scheme	*Associated action*
Part-whole scheme	Producing m/n by partitioning a whole into n pieces and disembedding m of those pieces
Partitive unit fraction scheme	Determining the size of a unit fraction relative to a given unpartitioned whole by iterating the unit fraction to produce a continuous, partitioned whole
Partial fraction scheme	Determining the size of a proper fraction relative to a given unpartitioned whole by partitioning the proper fraction produces a unit fraction and iterating the unit fraction to reproduce the proper fraction and the whole
Reversible partitive fraction scheme	Producing an implicit whole from a proper fraction of the whole (no referent whole given) by splitting the fraction to produce a unit fraction and iterating that fraction the appropriate number of times

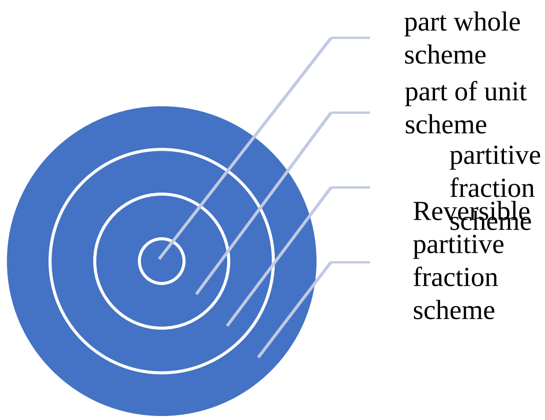


Fig. 1: Student's Fraction Schema Development Model

construct the concept of fractions using the partitive fraction scheme form to estimate the size of the fraction in non-unit form against an unpartitioned whole (Norton & Wilkins, 2010). While the form of the student's partitive fraction scheme in constructing the concept of fractions is a prerequisite or initial support for developing a splitting operation scheme (Norton & Wilkins, 2009). In short, the form of the partitive fraction scheme is one way for students to construct the concept of fractions using separation operations (Steffe, 2002). The following is a picture of the student's fraction scheme development model based on research conducted by Norton and Wilkins;

Each category in each schema has a relationship or relationship, so constructing the concept of fractions is important (Norton et al., 2018). According to the constructivist view, student knowledge is the formation (construction) of the students themselves (Ginsburg & Opper, 1988). The knowledge is built on its own in the minds of students actively (Cobb, 1988; Glasersfeld, 2013) based on previous knowledge (schemata) or cognitive structures (Gagne, 1969).

Method

This type of research is a case study with a qualitative approach. In this study, the phenomenon studied is more about how the development of fraction schemes in constructing the concept of fractions in elementary school students.

Participants

The participants in this study were elementary school students in grade 5. Grade 5 students had the basic knowledge of part-whole and partitive unit fraction schemes. Based on the curriculum analysis, students in grade 4 can identify the form of fractions and solve problems related to different sizes of fractions. Meanwhile, for grades 5 and 6, students can solve fractional problems related to the results of calculations, in addition, subtraction, multiplication, and division operations (Kemendikbud, 2018). The steps for determining participants are as follows; Grade 5 elementary school students, as many as 40 students, were given task 1 to check students' knowledge in constructing part-whole and partitive unit fraction schemes. Thirty-five students who could complete task 1 correctly were taken as research participants.

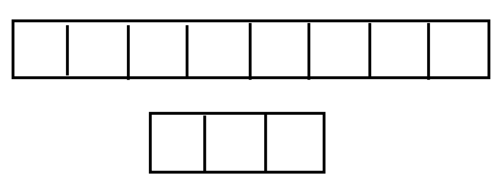
Furthermore, as many as 35 participants in this study were given task 2 to uncover the partitive fraction scheme. Ten participants can use the partitive fraction scheme in completing task 2. Next, interviews were conducted, and there were two ways of mental action flow that students used in constructing the concept of fractions using partitive fraction schemes.

Math task

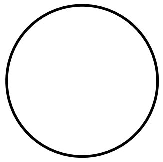
The tasks used in this study consisted of two types: Task 1 is used to determine prospective research participants, and Task 2 is used to investigate students' thinking processes in constructing partitive fraction schemes. Task 1 uses a part-whole and part-of-unit fraction scheme, while task 2 is a task to develop a partitive fraction scheme. Tasks 1 and 2 use questions from Norton et al. (2018).

Task 1

1. Determine the fractional value of the small chocolate pieces compared to the large chocolate in the image below!



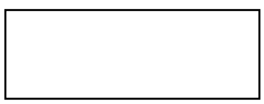
2. Your pizza is worth 1/6 of the pizza below. Draw the shape of your pizza!



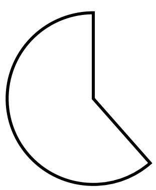
Task 2 is a question that can develop the concept of a student's partitive fraction scheme. This task investigates whether students can use the partitive fraction scheme to complete task 2. Task 2 was taken from Norton et al. (2018), and researchers changed the numbers and shapes of the images used. Norton et al. (2018) used image simulations of candy bars and pizzas, while this study used rectangles and pizzas. In this instrument, the researcher added more explicit instructions for the participants to complete.

Task 2

1. Suppose the rectangle below represents the fraction 3/8,



   Based on the information above, draw the part of the rectangle representing the fraction 1/8!
2. The picture of the pizza below represents the fractional value of 5/8. Based on this information, draw a pizza shape that represents the value 3/8.



Data analysis

The stages of data analysis consist of three stages, namely; (1) the results of task 1 data are corrected and determined by students who can solve fraction problems with part-whole and partitive unit fraction schemes, (2) students who can complete task 1 are selected as participants (3) students complete task 2 to see their problem-solving abilities fractions using a partitive fraction scheme, (3) students who can use a partitive fraction scheme were interviewed to see the mental action processes used in solving problems.

The mental action used in the partitive fraction scheme in constructing the fraction concept consists of five mental actions: partitioning, iterating, disembedding, splitting, and unit coordination. The research conducted by the researcher, namely the transition of representation on the behavior or mental actions of participants in using the partitive fraction scheme, is as follows;

1. The ability of students to partition the parts of the requested fractions
2. The ability of students to partition with the same shape
3. Students know how to partition fractions
4. The ability of students to form fractions part of the whole fraction.
5. Students' ability to reshape whole fractions based on parts of the fraction.
6. Students' ability to take fractions of a whole
7. Students' ability to draw fractional parts outside of the whole partitioned part
8. Students' ability to determine the form of fractions from part pictures outside the whole fraction part

Findings

The participants of this research are students who fall into the part-whole scheme, partitive unit fraction scheme, and partitive fraction scheme. These participants were selected based on the students' accuracy in answering the two problems. The problem given uses two problem models in the form of fractions; namely, the fractional model uses rectangles and circles. These two forms of fractions are developed into problems in the partitive fraction scheme category, which can be solved using students' understanding of concepts in the part-whole scheme and part-unit fraction scheme. The results showed that the mental actions taken by students in solving problems used four stages. The mental actions used in the part-whole scheme are partitioning, iterating, and disembedding.

Furthermore, the mental actions performed on the part of unit scheme consist of non-partitioning, partitioning, and iterating. As for the partitive fraction scheme, students use four stages of mental action: unpartitioning, partitioning, iterating, and disembedding. Participants use mental actions according to the fraction scheme developed by Norton et al. (2018): partitioning, iteration, and disembedding in part-whole, part unit and partitive schemes. This shows that a new mental action is found in the partitive fraction scheme, namely disembedding. Each schema has different operations that are used in each category. The following are the stages of student mental action in terms of the results of answers and participant interviews in solving the problems given correctly.

Mental action on a rectangular instrument

The following table shows students' behaviour in constructing the concept of fractions using the fractional partitive scheme. The operations used in the fractional partitive scheme consist of unpartitioning, partitioning, and iterating (Norton et al., 2018). Table 3 below shows students' mental actions on rectangular tasks based on the results of data analysis using observations, assignment work, and task-based interviews.

Table 3: Mental action on rectangular questions (question no. 1)

Mental action	*Student behaviour*
Unpartitioning	Students understand the non-unit form given in the problem. Students can change or modify the overall form of non-partitioned units.
Partitioning	Students can partition parts of 3/8 using the partitive unit fraction scheme. Students can partition the parts of the non-unit form and the whole that has been modified.
Iterating	Students can make repetitions of parts of units that have been partitioned using a part-whole scheme. Students do not repeat to determine the part of the specified part. Students only show the 1/8th part of the image that has been modified.
Disembedding	Students can determine 3/8 by redrawing the part worth 3/8 of the partitioned image. Students can determine the 1/8 section without destroying the initial partitioned image.

The researcher saw the process of students completing assignments through task-based interviews. The rectangular task-based interview results display the students' mental actions in a transitional manner as follows;

1. students understand fractions as part of a whole (whole number)
2. students determine the overall value first before partitioning the parts of the specified fraction (iterating)
3. Students understand that the picture in the problem/task is a form of an unpartitioning fraction.
4. Students partition the section (partitioning)
5. Students determine the overall value of the fraction (iterating)

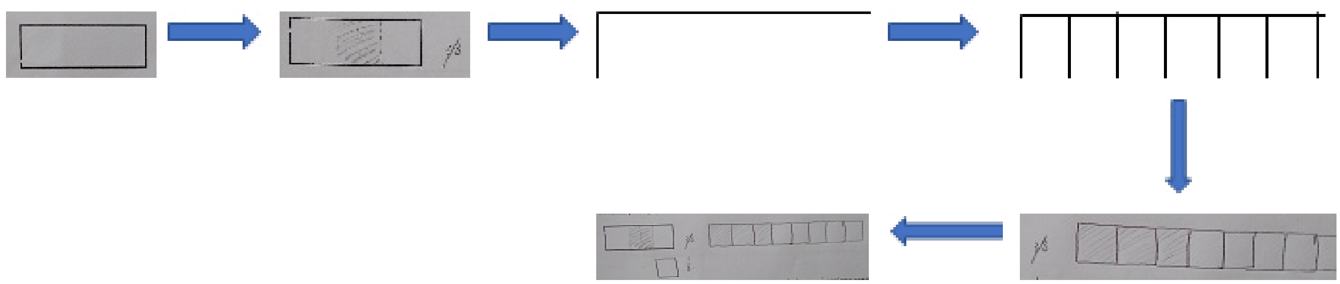


Fig. 2: The students' mental action processes on rectangular questions

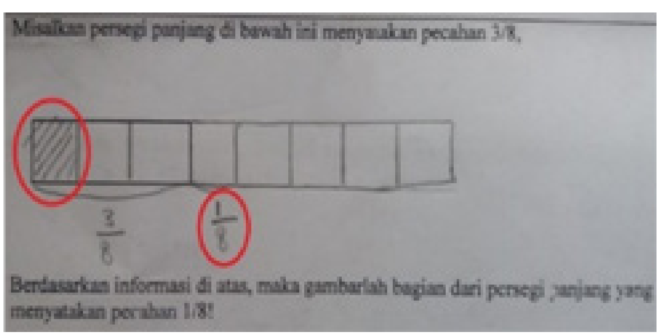
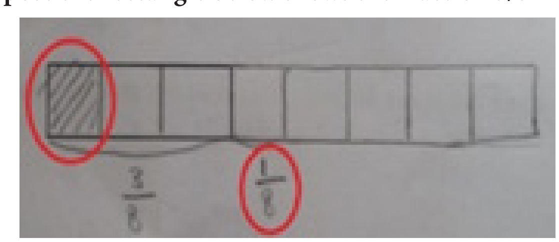


Fig 3: Answers to student assignments in a rectangular shape

The process of switching mental actions above can be shown in the figure below. The mental action process carried out by students in completing the task can be shown in the following figure 2;

The picture 2 shows that the participants understand the problem as part-whole, so students partition the picture given because it is based on the information provided by the question. However, participants did not do shading for the fractional part and immediately determined the answer in the form of 1/8 in the picture. Students think in completing assignments on rectangular fractions by partitioning the part of the given whole, determining the whole number in the form of the fraction, and finally determining the value of 1/8 by shading the partitioned part. The following is a picture of the results of student answers which is used as a reference for researchers to conduct interviews.

Translate;
Suppose the rectangle below shows the fraction 3/8



Based on the information above, then draw a rectangle that represents the fraction 1/8

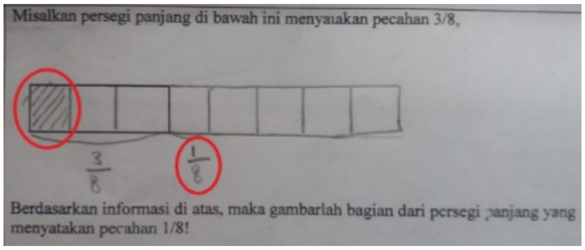


Fig. 3: Answers to student assignments in a rectangular shape

Based on the picture above, the researchers conclude that the interview results were (1) understanding the information and commands given, (2) partitioning the initial image into 3 parts then adding 5 boxes so that the total images became 8 squares, (3) and finally partitioning them into 1/8. In detail, the students' mental action process consists of several ways as follows;

1. Students know that an unpartitioned rectangle is 3 parts of the whole rectangle. (unpartitioning)
2. Students partition the rectangle into 3 parts (partitioning)
3. Students redraw a new rectangle by measuring 3 existing parts (splitting)
4. Students add parts to get 8 parts in the new rectangle (iterating)
5. Students take 1 part of 8 boxes that have been partitioned and shaded (disembedding)

Mental action on a circular instrument

The following table shows the behaviour of students in constructing fractional partitive schemes. The operations used in the fractional partitive scheme consist of unpartitioning, partitioning, and iterating (Norton et al., 2018). Based on the results of data analysis using observations, instrument answers, and task-based interviews, it was found that the flow of mental actions carried out by participants was obtained. Table 4 below shows the process of students' mental action on task number 2 in the form of a circle.

The researcher saw the process of developing mental actions carried out by students through a task-based interview process on circular questions (question number 2). The results of task-based interviews in the form of a circle show the process of transitioning mental actions shown by students as follows;

1. Students understand the problem correctly
1. Students solve by first partitioning part of the whole circle
1. Students create a new image without shading the given command.
1. The logged-in participant uses the disembedding operation to solve the problem.

The results of answers and interviews based on student answers in constructing the concept of fractions in the form of circles are shown in the figure below. The flow of mental actions carried out by students is as follows in Figure 4.;

The picture 4 shows that students understand the information and orders given by the researcher. Students understand that the picture is part of the whole and can be partitioned into 5 parts. Next, students redraw the whole circle to determine the number of partitions of the whole circle and the 3/8 part in question. Concepts constructed by students use the concepts of part of whole, and part of unit. The following are students' answers to the fractional task given in the form of a circle.

Translate;
The picture of the pizza below represents the fraction 5/8. Based on this information, draw a pizza shape that shows the fraction 3/8

Table 4: Mental action on a circular task (Problem no.2)

Mental Action	*Student behaviour*
Unpartitioning	Students understand the non-unit form given to the problem through disembedding on a complete circle. Students can change non-unit forms into units in the form of a complete circle.
Partitioning	Students can partition parts of 5/8 using the partitive unit fraction scheme. Students partition the whole circle into 8 equal parts.
Iterating	Students can make repetitions by removing and making a complete circle to determine the value of 3/8. Students make repetitions of non-unit parts using the partitive unit fraction scheme.
Disembedding	Students can make the overall shape of a circle through the given non-unit repetition. Students make non-unit drawings form a complete circle.

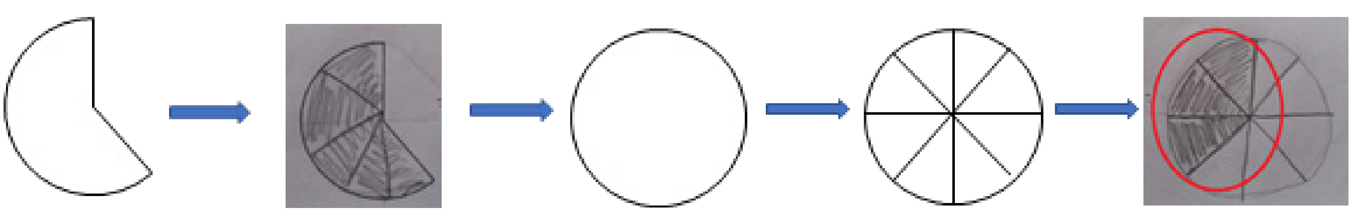


Fig. 4: The student's mental action process on a circular question

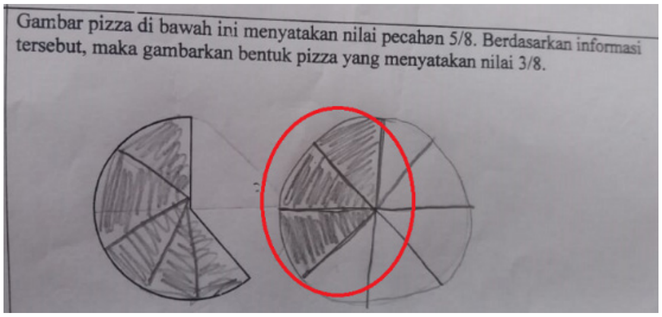


Fig. 5: Student's answer to the question of the shape of a circle

Based on the results of the interview and the picture above, the thinking process of students in completing the task consists of; understanding the problem well, partitioning the part of the picture given, making the whole part of the picture by forming it into a complete circle, understand that the whole part in the circle is 8 parts, and finally draw back outside (disembedding) 3 parts to show the large part of the picture 3/8 fraction value. In detail, the students' mental action process consists of several ways as follows;

1. Students know the picture given is 5 pieces of pizza from one whole pizza (iterating).
2. Students partition 5 pizza parts that have not been divided into 5 parts (partitioning)
3. Students redraw the shape of the pizza as a whole (part unit)
4. Students know that a whole pizza must be divided by 8 to help answer the question (partitioning).
5. Students make a complete repetition of the pizza image (unpartitioning)
6. Students divide the pizza as a whole into 8 equal parts (partitioning).
7. Students shade 3 pizza parts on a pizza picture that has been divided into 8 (iterating)
8. Students take 3 pieces of pizza to represent 3/8 of the pizza (disembedding)

The flow of mental action by students

The students' partitive fraction schemes shown in the rectangular and circular images have different mental actions so that the flow of the mental action process finds the concept of fractions in different tasks 2. Student answers and task-based interviews 2 of the participants showed that participants understood images that were not partitioned as part of a rectangle or a full circle. Before constructing the concept of fractions, participants partitioned the parts in task 2, namely 3/8 parts of a rectangle, by partitioning them into 3 parts and 5/8 parts of pizza that are not whole partitioned into 5 parts. Then, participants repeated the image with the shape of a rectangle and a circle as a whole. The flow of the participants' mental actions in constructing the concept of a rectangular fraction with a partitive fraction scheme can be shown in the figure 6;

The picture 6 shows that the mental action process uses the whole mental action according to the existing scheme in the partitive fraction scheme stage. However, there are additional mental actions performed by the participants, namely, issuing an initial picture with a new picture to show the answers given by participants. In partitioning images, participants use the concepts of part of whole, whole number, and part of unit (Norton & Wilkins, 2009b). Furthermore, the participants repeatedly determined the results ordered on the task.

Furthermore, the process of mental action shows that participants perform mental actions without repetition of other mental actions, so the researcher calls this process a direct process. Participants construct the concept of fractions with a partitive fraction scheme consisting of stages of understanding the shape of a rectangular image that is not partitioned as part of the whole. Next, participants partitioned the rectangle. Participants understand the overall shape of the rectangle. Based on the overall shape of the image, participants understand the parts and issue a new image on the rectangle, then repartition to find parts of the whole image. Furthermore, the flow of participants' mental actions in constructing the concept of a circular fraction with a partitive fraction scheme can be shown in the figure 7.

The picture 7 shows that the participants carried out the mental action process indirectly in completing task 2 on the concept of fractions using a circle shape. The picture shows that the mental action process conforms with the partitive fraction scheme process, where constructing the partitive fraction scheme requires students' understanding of the concepts of

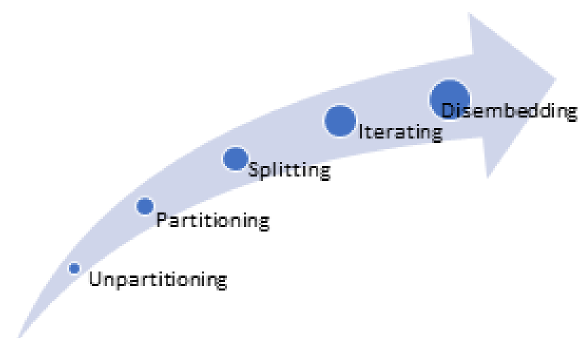


Fig. 6: Process directly on the representation of the rectangular shape

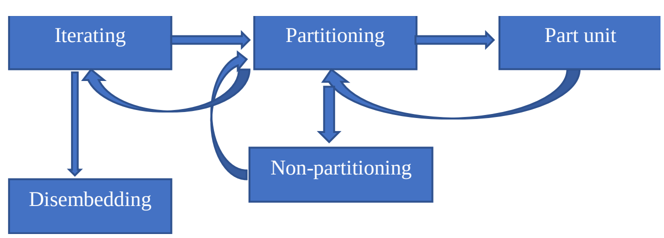


Fig. 7: An indirect process on the representation of the circle shape

part of whole and part of unit. Participants partitioned the whole form and part of whole; then, participants did the disembedding process to determine the part of the unit and repeat the fraction to produce a new fraction.

The mental action used by students when completing task 2 follows the mental action in the scheme developed by Norton et al. (2018). However, the researchers found that there were additional mental actions that were used more by the participants. Participants perform new mental actions (from prior knowledge) that they have. It can be concluded that participants do not forget old knowledge to complete new knowledge received based on interviews conducted by researchers.

The two pictures above show differences in students' mental actions when completing task 2 to construct a partitive fraction scheme. The mental action shown in task 2 is in the form of a rectangle; it can be seen that the mental action process directly reveals the part of each partition formed. While the mental action shown in task 2 is in the form of a circle, it can be seen that students reveal again that the whole circle needs to be redrawn to ensure that the circle can be partitioned into 8 equal parts. The results showed that the mental action process differed in the two forms of the concept of fractions: a rectangle and a circle.

Discussion

The selection of prospective participants by the researcher aims to see the students' fraction scheme. Students' fraction schemes show a variety of frameworks for thinking about the concept of fractions they understand. According to the five fraction schemes found by Norton et al. (2018), some students can use these schemes to construct the concept of fractions. These participants were given assignments and interviewed based on the answers to these assignments. Based on the results of this interview, students' thinking schemes were obtained in constructing partitive fraction schemes. Two participants showed different thought process schemes but contained the same fractional operations and prior knowledge.

Participants understand the form of non-unit and can distinguish between the unit and non-unit on the part of a given whole. Previously, students only understood units and overall values in the form of fractions. Through this stimulus, students can process new schemas using their old schemas. The steps taken at this stage are; to understand the non-unit form given to the problem through disembedding the whole circle, able to partition the unit parts using the partitive unit fraction scheme, able to make repetitions by removing and creating whole circles to determine unit values, and able to make the whole circle shape through repetition non-units given.

Participants could complete the fractional partitive scheme that had been modified using part whole and part unit schemes. Before determining the final score, students modify the image's overall shape to determine the form of units and non-units. The steps taken at this stage are; changing non-unit shapes into units in the form of a complete circle, repeating non-unit parts using the partitive unit fraction scheme, and producing non-unit images into a complete circle.

The results of this study indicate that students use prior knowledge in constructing the concept of fractions. The initial knowledge is used as part of whole, whole number, and part of unit. It can be seen that the mental actions of the participants in constructing the concept of part of the unit are based on prior knowledge in constructing the part of the whole (Norton & Wilkins, 2009a). The concept of part-whole and part of unit is the main part of the type of fraction concept in fractional material (Kennedy et al., 2008). Therefore, these two concepts can be a starting point for building a new fraction concept using the existing five schema categories: partitioning, iterating, disembedding, splitting, and unit coordination (Norton et al., 2018).

Schema is a student's frame of mind in constructing the concept of fractions (Norton et al., 2018). In other words, the schema results from constructing one's mind or showing how one thinks (cognitive construction). In constructing the concept of fractions, a person or student uses a frame of mind, namely their initial knowledge (schemata). The initial knowledge possessed is used to build new knowledge that will be completed(Ausubel, 1964). Therefore, the process carried out by these participants used their prior knowledge to construct the concept of fractions using a partitive fraction scheme. Students construct the concept of fractions using the stages of the partitive fraction scheme (Norton et al., 2018) and fraction operations (Steffe & Olive, 2010a). The participant's partitive fraction scheme shows that in constructing the fraction concept using the partitive fraction scheme, students' initial abilities are needed (Norton & Wilkins, 2010a), namely part of whole and part of unit scheme. The mental action used is according to the partitive fraction scheme. Participants provide additional mental actions to construct the fraction concept, namely disembedding.

Therefore, the process of developing students' mental actions in constructing the idea of fractions using a partitive fraction scheme revealed that students utilized both direct and indirect mental action processes.

Conclusion

The process of students constructing the concept of fractions using partitive fraction schemes is based on their prior knowledge (schemata). The initial knowledge used is; part-whole, part of unit, and whole number. The schematic flow shown by the participants is different when performing fractional operations, namely disembedding. This disembedding is an additional operation performed in constructing the concept of fractions using the partitive fraction scheme, according to Norton et al. (2018). The following is the flow of the scheme shown by the students, namely; (a) Problems are given in the form of rectangles and circles without being partitioned (unpartitioning), (b) Students partition (partitioning) to produce new fractional images. This partitioning is done based on the student's prior knowledge, namely; part-whole, whole number, and part of unit,

This study found differences in how to construct the concept of fractions between rectangular and inner circle images using a fractional partitive scheme. The mental action flow is used in the rectangular form of the mental action process without repeating another mental action. Researchers refer to it as direct flow because participants construct concepts without repeating mental actions. Meanwhile, the flow of mental action used in the circular image contains several repetitions in constructing the concept of fractions, so the researcher calls the flow of mental action indirect.

According to the findings of this study as a whole, it is possible to assert that this partitive fraction scheme needs to be supported by a part-whole scheme as well as a part-unit scheme in order to construct a concept of fractions. In addition, it was discovered through the course of this research that the mental action process of students who were capable of finishing the assignment utilizing a partitive fraction scheme displayed a variety of distinct variations. The distinction that was discovered is that the pattern of the pupils' mental actions has two paths: the direct mental path and the indirect mental path.

References

Aliustaoğlu, F., Tuna, A., & Biber, A. Ç. (2018). Misconceptions of sixth grade secondary school students on fractions. *International Electronic Journal of Elementary Education*, *10*(5), 591–599. https://doi.org/10.26822/iejee.2018541308

Ausubel, D. P. (1964). The Transition from Concrete to Abstract Cognitive Functioning: Theoretical Issues and Implications for Education. *Journal Of Research In Science Teaching*, *2*, 261–266.

Booth, J. L., Newton, K. J., & Twiss-Garrity, L. K. (2014). The impact of fraction magnitude knowledge on algebra performance and learning. *Journal of Experimental Child Psychology*, *118*(1), 110–118. https://doi.org/10.1016/j.jecp.2013.09.001

Bossé, M. J., Bayaga, A., Fountain, C., Lynch-Davis, K., Preston, R., & Adu-Gyamfi, K. (2018). Fraction learners: Assessing understanding through language acquisition. *International Electronic Journal of Elementary Education*, *11*(2), 113–124. https://doi.org/10.26822/iejee.2019248585

Clapin, H., Staines, P., & Slezak, P. (2004). *Representation in Mind New Approaches To Mental Representation*. Elseivier.

Cobb, P. (1988). The Tension Between Theories of Learning and Instruction in Mathematics Education. *Educational Psychologist*, *23*(2), 87–103. https://doi.org/10.1207/s15326985ep2302_2

Department of Education, U. (2008). *The Final Report of the National Mathematics Advisory Panel*. www.ed.gov/

Díez-palomar, E. J. (2016). How can I Help my Students with Learning Disabilities in Mathematics? *Journal of Research in Mathematics Education*, *5*(1). https://doi.org/10.4471/redimat.2016.1972

Doğan, A., & Tertemiz, N. I. (2020). Fraction models used by primary school teachers. *Elementary Education Online*, *19*(4), 1888–1901. https://doi.org/10.17051/ilkonline.2020.762538

Flores, M. M. (2010). Using the concrete-representational-abstract sequence to teach subtraction with regrouping to students at risk for failure. *Remedial and Special Education*, *31*(3), 195–207. https://doi.org/10.1177/0741932508327467

Fuchs, L. S., Schumacher, R. F., Long, J., Namkung, J., Hamlett, C. L., Cirino, P. T., Jordan, N. C., Siegler, R., Gersten, R., & Changas, P. (2013a). Improving At-Risk Learners' Understanding of Fractions. *Joumal of Educational Psychology*, *105*(3), 683–700. https://doi.org/10.1037/a0O32446

Fuchs, L. S., Schumacher, R. F., Long, J., Namkung, J., Hamlett, C. L., Cirino, P. T., Jordan, N. C., Siegler, R., Gersten, R., & Changas, P. (2013b). Improving At-Risk Learners' Understanding of Fractions. *Joumal of Educational Psychology*, *105*(3), 683–700. https://doi.org/10.1037/a0O32446

Gagne, R. M. (1969). Review. Educational Psychology: A Cognitive View (Ausubel, 1968). *JSTOR*, *6*(2), 287–290. http://www.jstor.org/stable/1161899

Ginsburg, H. P., & Opper, S. (1988). *Piaget's Theory of Intellectual Development Third Edition*. www.freepsychotherapybooks.org

Glasersfeld, V. E. (2013). Chapter 2: An Exposition of Constructivism: Why Some Like It Radical. *Journal for Research in Mathematics Education. Monograph*, *4*, 19–29. http://www.jstor.org/stable/749910

Hinson, R. (2004). Elementary and Middle School Mathematics: Teaching Developmentally (Book). *Teaching Children Mathematics*, *10*(5).

Jordan, N. C., Glutting, J., & Ramineni, C. (2011). The Importance of Number Sense to Mathematics Achievement in First and Third Grades. *Learn Individual Differ*, *20*(2), 82–88. https://doi.org/10.1016/j.lindif.2009.07.004.The

Kennedy, L. M., Tipps, Steven., & Johnson, A. (2008). *Guiding children's learning of mathematics*. Thomson/Wadsworth.

Lewis, K. E. (2015). Difference Not Deficit : Reconceptualizing Mathematical Learning Disabilities. *Journal for Research in Mathematics Education*, *45*(3), 351–396.

NCTM. (2010). *Standards for Teaching and Learning Mathematics*. www.nctm.org.

Norton, A., & Wilkins, J. L. M. (2009a). The Journal of Mathematical Behavior A quantitative analysis of children's splitting operations and fraction schemes. *Journal of Mathematical Behavior*, *28*, 150–161. https://doi.org/10.1016/j.jmathb.2009.06.002

Norton, A., & Wilkins, J. L. M. (2009b). A quantitative analysis of children's splitting operations and fraction schemes. *Journal of Mathematical Behavior*, *28*(2–3), 150–161. https://doi.org/10.1016/j.jmathb.2009.06.002

Norton, A., & Wilkins, J. L. M. (2010a). Students' partitive reasoning. *Journal of Mathematical Behavior*, *29*(4), 181–194. https://doi.org/10.1016/j.jmathb.2010.10.001

Norton, A., & Wilkins, J. L. M. (2010b). Students' partitive reasoning. *Journal of Mathematical Behavior*, *29*(4), 181–194. https://doi.org/10.1016/j.jmathb.2010.10.001

Norton, A., & Wilkins, J. L. M. (2013). Supporting Students' Constructions of the Splitting Operation. *Cognition and*

Instruction, *31*(1), 2–28. https://doi.org/10.1080/07370008.2012.742085

Norton, A., Wilkins, J. L. M., & Xu, C. ze. (2018). Brief Report: A Progression of Fraction Schemes Common to Chinese and U.S. Students. *Journal for Research in Mathematics Education*, *49*(2), 210–226. https://doi.org/10.5951/jresematheduc.49.2.0210

Pavlovičová, G., & Vargová, L. (2020). Investigation of selected aspects of fraction understanding. *TEM Journal*, *9*(2), 702–707. https://doi.org/10.18421/TEM92-37

Rodrigues, J., Dyson, N. I., Hansen, N., & Jordan, N. C. (2016). Preparing for Algebra by Building Fraction Sense. *TEACHING Exceptional Children*, *49*(2), 134–141. https://doi.org/10.1177/0040059916674326

Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology*, *62*(4), 273–296. https://doi.org/10.1016/j.cogpsych.2011.03.001

Steffe, L. P. (2002). A new hypothesis concerning children ' s fractional knowledge . *Journal of Mathematical Behavior*, *20*, 267–307.

Steffe, L. P., & Olive, J. (2010a). Children's fractional knowledge. In *Children's Fractional Knowledge*. Springer US. https://doi.org/10.1007/978-1-4419-0591-8

Steffe, L. P., & Olive, J. (2010b). Children's fractional knowledge. In *Children's Fractional Knowledge*. Springer US. https://doi.org/10.1007/978-1-4419-0591-8

Tzur, R. (1999). An Integrated Study of Children ' s Construction of Improper Fractions and the Teacher ' s Role in Promoting That Learning. *Journal for Research in Mathematics Education*, *30*(4).

Varma, S., & Schwartz, D. L. (2011). The mental representation of integers: An abstract-to-concrete shift in the understanding of mathematical concepts. *Cognition*, *121*(3), 363–385. https://doi.org/10.1016/j.cognition.2011.08.005

Vygotsky, L. (2012). *Thought and Language* (A. Kozulin, Ed.; Revised and Expanded). Massachusetts Institute of Technology.