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Examining Students' Proof Writing and Justification Skills in the Context of Sum of Measures of Polygons' Interior Angles

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Abstract

The present study aims to examine 8th grade students' proof writing and justification skills. The research was conducted using the document analysis method. The participants of the study consisted of 16 voluntaries 8th grade students. The participants were determined according to the convenience sampling method. Data were collected with the "geometric proof writing and justification test" prepared by the researchers. The data collection tool was prepared by referring to the 5th, 6th, and 7th grade curricula. "Proof Writing Rubric" and "Justification Rubric" were used to analyze the data. The results of the study showed that 8th grade students' geometric proof writing and justification skills were at low levels. In addition, it was revealed that their justification skill levels were lower than their proof writing levels.

Keywords:

Proof, Justification Skills, Mathematics Education

Introduction

The mathematical thinking process includes high-level thinking skills such as specialization, generalization, prediction, generating assumptions, and checking the accuracy of assumptions (Mason, Burton & Stacey, 2010). These skills are closely related to proof skills. Proof skill includes understanding the proof of a mathematical statement in addition to being able to recognize and justify the construction of the proof. The importance of mathematical thinking is frequently emphasized in all learning areas of mathematics. Geometry is one of these learning areas. Difficulties and deficiencies in geometry teaching and students' lack of success in geometry learning are among the most emphasized problems by educators (Alex & Mammen, 2012). Geometry teaching is supposed to contribute to the development of students' ability to visualize objects in their minds, reducing the objects they encounter in daily life to two dimensions, solving problems, making assumptions, making logical inferences, and making proofs. Generalization, reasoning, and justification (Ministry of National Education [MoNE], 2020) and the process of constructing geometric ideas in a meaningful way are among the skills that students should acquire in geometry teaching (Driscoll, DiMatteo, Nikula & Egan, 2007). Despite this importance, studies show that students not only have difficulty in understanding rules and operations in the geometry learning domain but also have difficulty explaining their solutions and ways of thinking, understanding proof and writing proof (Almeida, 2000; Jones, 2000; Hadas, Hershkovitz & Schwarz, 2000, Stylianides, Stylianides, & Philippou, 2004; 2007).

Although proof and justification skills have an important place in mathematics in general and geometry in particular, there are limited studies on proof writing and justification (Coşkun, 2019; Dimakos, Nikoloudakis, Ferentinos & Choustulakis, 2007; Senk, 1983;1985;1989; Şen & Güler, 2022, Özmusul, 2018). Senk (1985) stated that, considering the current curriculum and typical teaching practices, students do not master the skills required in a standard geometry course, and even students could not realize the necessity of proof while proving in geometry and were insufficient in proof writing types. Senk (1989) examined the relationships between van Hiele levels, success in writing geometry proofs, and success in geometry. The results of the study showed that students' success in geometry proof writing was positively related to van Hiele's level of geometric thinking and success in standard geometry course. Dimakos, Nikoloudakis, Ferentinos and Choustulakis (2007) revealed that students have difficulties in proof writing in geometry and even they do not know how to start making proofs. Özmusul (2018) examined the levels of justification. The results of the study showed that the justification skills of the participants were low and the students with high achievement levels had high justification skills. Harel and Sowder (1998) made a classification of proof based on the arguments and justifications used by students to explain the decision (correctness or falsity of the statement) about a mathematical statement. In the light of the studies, it is seen that there is a great relationship between proof and justification. Considering these studies, the results of a study in which proof writing and justification skills in geometry are examined together will contribute to the literature. Therefore, the aim of this study is to examine the 8th grade students' proof writing and justification skills in the context of the sum of the measures of the interior angles of polygons and to compare the answers of the students with the lowest and highest skills in both proof writing and justification. In this way, geometric proof writing and justification skills will be evaluated together and contribute to the literature.

Conceptual Framework

Proof writing

Proof has many functions in mathematics education. The most basic role of proof is to show that a claim is correct or incorrect. Students generally perceive the concept of proof in this sense. For many students, the proof is a practice that needs to be memorized by the teacher with standard methods and steps. Knuth (2002) stated that teachers tend to view proof in a pedagogically limited way, that is, as a subject of study rather than a tool for communicating and studying mathematics. According to mathematics educators, proof is a method of thinking as well as an important skill used to explain why a claim is correct. The proof is categorized into two main categories. The first is proofs that show the correctness of a claim superficially and do not involve too much questioning. The second one is explanatory proof that responds to questions of why and why and reveals the correctness of the claim in depth (Bayazıt, 2017). Almeida (2000) emphasizes that understanding proof and writing proof is one of the main distinguishing features of mathematics. Goetting (1995) stated that there are three different understandings of proof in his study in which they examined their understanding of proof - the arguments they found persuasive and the arguments they accepted as valid evidence. These insights are not necessarily imprecise, in the form of a supporting argument, a means to conclusively confirm assumptions, a statement where precise verification is necessary but sometimes not sufficient, or a classroom exercise.

The results of studies on proof writing in geometry showed that students' proof writing skills were weak and they had difficulties (Daguplo, 2014; Güner & Topan, 2016; Şen & Güler, 2022). Güner and Topan (2016) revealed that primary school mathematics teacher candidates have weak geometric proof skills, they have difficulty in proving, they have misconceptions that a single example or numerical representation showing accuracy is sufficient for proof, and they cannot transfer their existing knowledge to the proof process. Şen and Güler (2022) examined the effectiveness of teaching activities based on the Van Hiele model on geometric proof writing skills. The results of the study showed that teaching activities based on the Van Hiele model supported the development of pre-service teachers' proof writing skills. Daguplo (2014) measured students' geometry proof writing performance according to Van Hiele's geometric thinking model levels. At the end of the study, it was seen that the pre-service teachers were not at the highest level in terms of proof writing. In summary, it is seen that they have difficulties in proofs writing in geometry. One of skills that is effective in the development of proof writing skill is justification.

Justification

Justification skill is as important as proof skill in mathematics teaching. There are different definitions of justification in the literature. National Council of Teachers of Mathematics (NCTM) (1989) stated that mathematics is a set of justifications. Likewise, justification is emphasized in the special objectives set by MoNE (2020). In line with these specific objectives, students will be able to express their thoughts and reasoning in the problem-solving process and see the gaps or deficiencies in the mathematical reasoning of their classmates. The realization of these objectives depends on the development of students' justification skills. As Ross (1998) states, if students' reasoning skills are not developed, mathematics will remain a mass calculation without thinking.

Different forms of justification have been defined in the literature by examining students' justification skills (Balacheff, 1988; Bell, 1976; Harel & Sowder, 1998; Marrades & Gutiérrez, 2000). For example, Bell (1976) divided mathematical justification into two categories. The first is "empirical" justification in which the correctness of a mathematical statement is demonstrated with the help of examples. The second is "deductive" justification in which inferences are used in connection with the results. Marrades and Gutiérrez (2000) classified these two categories in more detail. Balacheff (1988) divided the types of justification into two groups. The first group is the "pragmatic justification", which is based on the use of examples or demonstrations. The second is the "conceptual justification", which is based on conceptual facts, abstract formulas, and relationships between properties of mathematical expressions. Harel and Sowder (1998) grouped the justifications used by students to explain the correctness (or incorrectness) of a mathematical statement into three categories as "External Sources-Based, Empirical, and Analytical". On the other hand, justification is discussed in various aspects when the studies in the literature are examined. Justification in the process of generalizing patterns (Akkan, Öztürk & Akkan, 2017; Tanışlı, Yavuzsoy Köse & Camci, 2017; Lannin, 2005); justification in the problem-solving process (Akkuş, 2019), the relationship between justification skills and other variables (Özmusul, 2018) are among these studies. Özmusul (2018) examined whether 7th grade students' justification skills differed according to gender, school, and achievement test score. It was stated that the participants' complete and persuasive justification skills were low, while the justification skills of students with high achievement levels were high. In this study, justification was accepted as correct reasoning to support arguments. According to the arguments put forward in proof writing, it has been discussed as "complete justification" and "partial justification".

The sum of the measures of the polygons' interior angles

The present study's context is "the sum of the measures of the polygons' interior angles". Naturally, students are expected to generalize the reasoning they do in special cases such as triangles and quadrilaterals to polygons while proving and justifying a polygon. Activities related to the topic "Angles in Polygons" are included in the 7th grade textbooks. For example, in the activity "calculating the sum of the interior angles of a polygon" in the MoNE textbook (MoNE, 2014; p.125), while finding the sum of the measures of a polygon's interior angles, the given polygon was divided into triangular parts. The sum of the measures of the polygon's interior angles was found by using the sum of the measures of the polygon's interior angles. In the next activity, the student was asked to find the sum of the measures of the pentagon's interior angles by drawing and explaining the reasons. This way, they are expected to experiment with different polygons. It tried to make students realize that polygons can be divided into triangular regions with diagonals in this activity. Then, a table was created in which the number of these triangular regions was related to the number of sides of the polygons. With the help of this table, students were expected to calculate the sum of the interior angles of polygons with the help of the triangles formed. As can be seen in this activity, the basic skill expected to be developed in students in the context of the sum of measures of the interior of polygons in particular, and geometry teaching, in general, is the ability to reason and justify (NCTM, 1989; MoNE, 2018). In this context, the problem of this study is "What is the relationship between justification and proof writing skills?"

Method

In this study, document analysis was used. Document analysis is the analysis of written materials containing information about the phenomena aimed to be researched (Yıldırım & Şimşek, 2016). Document analysis is used as a solo research method, especially in cases where direct interviews and observations are not possible. This method involves the analysis of written and oral materials containing information about the subjects planned to be researched. Document analysis includes the analysis of written materials containing information about events or phenomena that are intended to be researched. The document review conducted in our research covers the analysis of written materials containing information about the subjects planned to be researched.

Participants

The participants were determined as 8^{th} grade students since it was thought that it would allow the best explanation of the researched topic and provide the best contribution to the solution of the research problem. The participants of the study were 16 volunteer students who continued their education in the 8^{th} grade in the 2020-2021 academic year in a private school in the center of Sivas and participated in the face-to-face teaching process on the day of data collection. Participants were determined according to the convenience sampling method. Seven of these students were female and nine were male. Participants were named S1, S2, S3....

Data Collection Tool

The data collection tool developed to examine 8th grade students' geometric proof writing and justification skills consist of three questions. Data collection tool questions are from specific to general. The students were asked to prove and justify the sum of the measures of the interior angles of a triangle, quadrilateral, and any n-sided polygon. To enable students to do the proof and justification separately, each question consists of two parts option a and option b.

Table 1.
Data collection tool questions

Sum of interior angles measures	1) a) Demonstrate that the sum of the interior angles of a triangle is 180°. b) What can you say to convince others that your result is correct? Explain.
	2) a) Demonstrate that the sum of the interior angles of a quadrilateral is 360°. b) What can you say to convince others that your conclusion is correct? Explain.
	3) a) Demonstrate that the formula for the sum of the interior angles of a polygon with "n" sides is (n-2).180°. b) What can you say to convince others that your result is correct? Explain.

Data Collection

The data were collected by the researchers in a class hour from the 8th grade students who attended a face-to-face mathematics lesson in a private school in Sivas (due to the pandemic conditions). As it was intended to reflect the current situation, the students were not given correction or justification training before the data of the study were collected. The researchers only gave information about how the test should be done during the data collection process. The students' answers were collected in writing. There was no time limit.

Data Analysis

In analyzing the questions in the data collection tool, two rubrics developed by the researchers were used. The first one is the proof writing rubric and the second one is the justification rubric. While creating these rubrics, studies in the related literature (Coşkun, 2009; Senk, 1983; Sowder & Harel, 1998; Özmusul, 2018) were examined. The proof writing rubric is given in Table 2 and the justification rubric is given in Table 4. After the preparation of the rubrics, the opinions of three mathematics educators were obtained. Miles and Huberman's (1994) reliability formula [Reliability=Agreement/ (Agreement + Disagreement)] was used and the compliance rate was determined as 90%.

Table 2.
Proof Writing Rubric

Criteria	Score
Situations where the solution was completely incorrect, the problem is not understood, or nothing is done.	0
Situations where the question is understood. In other words, the question is expressed verbally, the algebraic form of the question is written or short notes are taken about this expression, a graph is drawn, a table is created, and the expression/truth of the given argument/proposition is explained with examples.	1
Situations where the question is comprehended. That is, they understand exactly what needs to be proved, determined the method of proof, and created/realized the logical steps given for this, but can not fully conclude the proof or there are deficiencies/errors in some stages of the proof.	2
Situations where the proof is properly completed in the correct form.	3

As can be seen in Table 2, there is one point for each situation in the proof writing section. If the student does not answer any of the three questions in this test, the student gets a total of zero points and if the student answers all of them correctly, the student gets a total of nine points. Table 3 shows the students' level of proof writing according to their scores.

Table 3.
Proof Writing level

Proof writing level	Proof writing score
Unsuccessful	0, 1,2,3
Moderately successful	4,5,6
Very successful	7,8,9

As can be seen in Table 3, if a student's total proof writing score is three or less than three, the student is considered to be "unsuccessful" in proof writing, if the student scores four, five and six points, the student is considered to be "moderately successful" and if the student scores more than six points, the student is considered to be "very successful". The scoring of the justification rubric consists of four supercodes and seven codes.

Table 4.

Justification Rubric

Codes	Supercodes	Score
Answers proving the question correctly and supporting it with correct mathematical reasoning	Complete justification	3
Proving the question correctly but writing the reasoning incompletely	Partial justification	2
Incomplete proof of the question and incomplete writing of the justification		
Writing the appropriate justification for the answer in the questions where the proof is proved incorrectly, a calculation error is made or concept errors are made	Incorrect justification	1
The proof is correct but no justification	Not writing justification	0
The proof is incorrect and no justification		
The proof is missing and no justification		

As can be seen in Table 4, when evaluating the students' justification skills, if they write a correct justification in a question, they receive three points, and if they fail to write a correct justification, they receive zero points. That is, if the student does not answer any of the three questions in the data collection tool, the student receives a total of zero points, and if the student answers all of them correctly, the student receives a total of nine points. The evaluation criteria in the rubric are the same as the rubric for writing proofs. The justification levels of the students according to their scores are given in Table 5.

Table 5.

Level of justification

Level of justification	Justification score
Unsuccessful	0, 1,2,3
Moderately successful	4,5,6
Very successful	7,8,9

As can be seen in Table 5, if a student's total justification score is three or less than three, the student is considered to be "unsuccessful" in justification, if the student scores four, five and six points, the student is considered to be "moderately successful", and if the student scores more than six points, the student is considered to be "very successful".

Results

The student's ability to write proofs and justification skills related to the sum of the measures of the polygons' interior angles were first given in question-by-question tables with the scores they received. Afterward, for each question, excerpts from the answers written by the students were given. In this way, both the levels of proof writing, justification, and generalization were examined in detail. Table 6 shows the students' proof writing and justification skills scores.

Table 6.

Proof Writing and Justification Scores

Student	Triangle		Quadrilateral		Polygon		Total	
	Justification	Writing proof	Justification	Writing proof	Justification	Writing proof	Justification	Writing proof
S1	2	3	0	2	3	3	5	8
S2	3	3	3	3	3	3	9	9
S3	0	1	0	1	0	0	0	2
S4	0	1	0	1	3	3	3	5
S5	0	2	0	1	0	0	0	3
S6	0	0	0	0	0	0	0	0
S7	3	2	2	2	0	0	5	4
S8	0	1	0	1	0	0	0	2
S9	2	2	0	0	2	2	4	4
S10	0	1	0	1	0	0	0	2
S11	0	0	0	1	0	0	0	1
S12	0	1	0	1	1	2	1	4
S13	2	1	0	0	2	2	4	3
S14	0	1	2	2	0	0	2	3
S15	0	3	2	1	1	3	3	7
S16	0	1	0	1	0	0	0	2

As seen in Table 6, 11 students' justification skills scores are three or below three. Therefore, these students were coded as unsuccessful. One student (S2) received a full score and four students (S1, S7, S9, and S13) received four and five scores, so it was seen that these students were "moderately successful". Nine students had a proof writing score of three or less three and were found to be unsuccessful. One student (S2) received nine full scores. Besides, three students (S1, S2, and S15) scored seven or more points and were "very successful". Four students (S4, S7, S9, and S12) scored between three and six points and were "moderately successful". The findings obtained from both skills are summarized in Table 7.

As seen in Table 7, the justification levels of students who had a very good level of proof (S1, S2, and S15) were very good, moderate, and unsuccessful. The justification levels of the students (S4, S7, S9, and S12) who had a moderate level of proof were moderate or unsuccessful. It can be said from these findings that the proof writing levels of the students participating in the study were higher than their justification levels. Only the S13 justification skill level was higher than the level of writing proof. In the next section, excerpts from the student's answers to each question are given. On the other hand, when proof writing and justification skills are examined in the generalization process, that is, in the context of the sum of the interior of the triangle, quadrilateral, and polygon, the highest total score in proof writing was obtained in the sum of the interior angles of a triangle, as can be seen from Table 8. In the sum of the measures of the interior angles of quadrilateral and polygon, the sum of the scores is equal. In justification skills, the highest total score was obtained in the sum of the internal angles of a polygon. After that, in terms of total points, it was obtained in the sum of the measures of the interior angles of the triangle and then the quadrilateral. In this sense, when analyzed in terms of the total scores obtained, no parallelism was found in terms of proof writing and justification skills. That is, a high score in proof writing skills did not require a high score in justification skills.

Considering the scores obtained in the generalization process from triangle to polygon, it is difficult to say that there is an order.

Table 7.
Participants' levels of writing and justifying proofs

Student	Proof writing level	Justification level
S1	Very good	Moderate
S2	Very good	Very good
S3	Unsuccessful	Unsuccessful
S4	Moderate	Unsuccessful
S5	Unsuccessful	Unsuccessful
S6	Unsuccessful	Unsuccessful
S7	Moderate	Moderate
S8	Unsuccessful	Unsuccessful
S9	Moderate	Moderate
S10	Unsuccessful	Unsuccessful
S11	Unsuccessful	Unsuccessful
S12	Moderate	Unsuccessful
S13	Unsuccessful	Moderate
S14	Unsuccessful	Unsuccessful
S15	Very good	Unsuccessful
S16	Unsuccessful	Unsuccessful

Table 8.
Proof writing and justification skills in the generalization process

	Justification				Proof writing			
Student	Triangle	Quadrilateral	Polygon	Total	Triangle	Quadrilateral	Polygon	Total
S1	2	0	3	5	3	2	3	8
S2	3	3	3	9	3	3	3	9
S3	0	0	0	0	1	1	0	2
S4	0	0	3	3	1	1	3	5
S5	0	0	0	0	2	1	0	3
S6	0	0	0	0	0	0	0	0
S7	3	2	0	5	2	2	0	4
S8	0	0	0	0	1	1	0	2
S9	2	0	2	4	2	0	2	4
S10	0	0	0	0	1	1	0	2
S11	0	0	0	0	0	1	0	1
S12	0	0	1	1	1	1	2	4
S13	2	0	2	4	1	0	2	3
S14	0	2	0	2	1	2	0	3
S15	0	2	1	3	3	1	3	7
S16	0	0	0	0	1	1	0	2
Total	12	9	15		23	18	18	

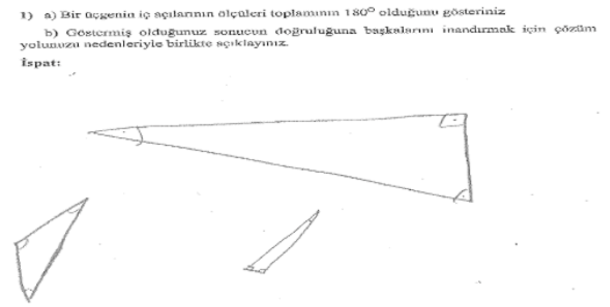
Findings Obtained from the sum of the measures of the triangle's interior angles

Students were asked to prove that the sum of the measures of the triangle's interior angles is 180o and to write their reasons. Here are examples of the answers of the students with the lowest scores and the answers of the students with the highest scores. S6 and S11 got zero points in proof writing and S3, S4, S5, S6, S8, S10, S11, S12, S14, S15, S16 got zero points in justification. The proof of S6, who got zero points in both proof writing and justification, is given in Figure 1a and the justifications are given in Figure 1b.

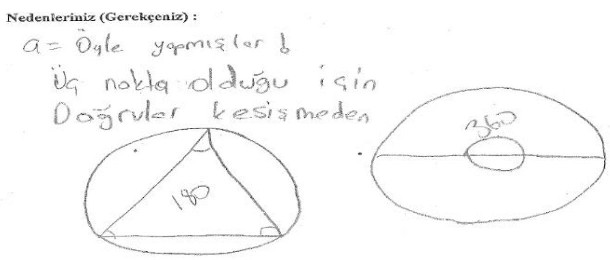
Figure 1.

S6's proof and justifications for the sum of the triangle's interior angles

a)



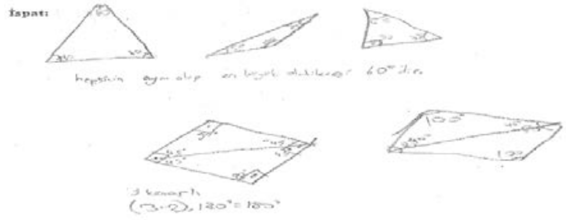
b) Transcription: they did so.. because there are three points...without crossing lines



When Figure 1a is analyzed, S6 drew three different triangles but did not write what the student wanted to explain. According to the proof evaluation rubric, this student's answer was given zero points. S6's justification was neither compatible with the proof nor did the student make a connection with the fact that the sum of the measures of the triangle's interior angles was 180o. Therefore, zero points were given. The students who got one point in this question in writing proof were S3, S4, S8, S10, S12, S13, S14, and S16. According to the justification skills rubric, no student scored one point. That is, no answer fulfills the criterion of not writing a justification appropriate to the answer in the questions where the proof is proved incorrectly, an operation error is made or concept errors are made. The answer of S12 who got one point in this question is given in Figure 2.

Figure 2.

S12's proof for the sum of the measures of the triangle's interior anglesv



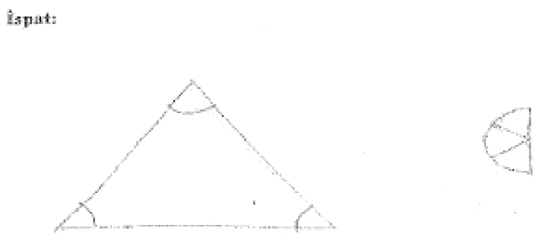
Transcription: They are all the same and the maximum they can be is 60 degrees.

When Figure 2 is analyzed, S12 followed two different ways. In one of them, the student drew different triangles and completed 180o by giving different values to the measures of their interior angles. In the other one, the student drew different quadrilaterals and divided them into two and actually understood the question but could not prove it completely. One point was given according to the proof rubric. S5, S7, and S9 were the students who got two points in proof writing. The proof of S7 is given in Figure 3a. The students who got two points according to the justification rubric were S9 and S13. S9's justifications are given in Figure 3b.

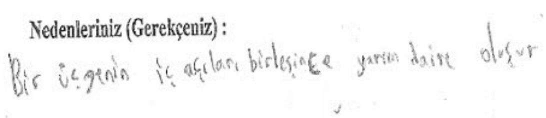
Figure 3.

S7's proof for the sum of the measures of the triangle's interior angles and S9's justifications

a)



b) Transcription: When the interior angles of a triangle converge, a semicircle is formed.



As seen in Figure 3a, S7 drew a triangle and made a right angle by connecting the angles but did not name the angles. Therefore, two points were given because it was not understood which angle was connected where and how in the right angle. As seen in Figure 3b, S9's justification is not clear. The student did not explain what the semicircle formed when the interior angles of the triangle meet. Therefore, two points were given. S1, S2, and S15 in writing proofs, and S2 and

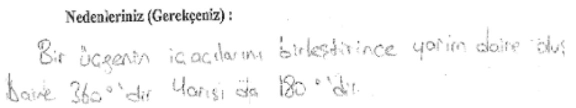
S7 received three points according to the justification skills rubric. The proof of S1 is given in Figure 4a and the justification of S7 is given in Figure 4b.

Figure 4.

S1's proof for the sum of the measures of the triangle's interior angles and S7's justifications

a)

b) Transcription: When the interior angles of a triangle are combined, a semicircle is formed; the circle is 360o, half of which is 180o



As can be seen in Figure 4a, S1 drew the triangle and named the angles, made a right angle by combining them, and got three points for proving the sum of the measures of the triangle's interior angles completely. S7, on the other hand, stated that a semicircle was formed when the interior angles of the triangle were joined and stated that the circle was 360o and half of it was 180o and wrote the justification by the proof.

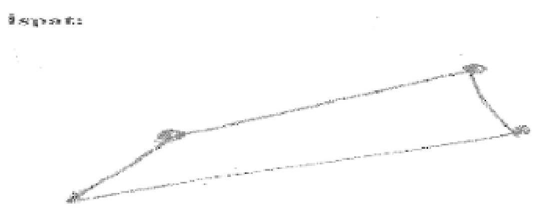
Results obtained from the sum of the measures of the quadrilateral's interior angles

The students were asked to prove that the sum of the measures of the interior angles of a quadrilateral is 360o and to write their reasons. Here are examples of the answers of the students with the lowest scores and the answers of the students with the highest scores. In writing proofs, S6, S9, and S13 received zero points. In justification skills, S1, S3, S4, S5, S6, S8, S9, S10, S11, S12, S13, S16 received zero points. The proof writing answer of S6 is given in Figure 5a and the justifications of S8 are given in Figure 5b.

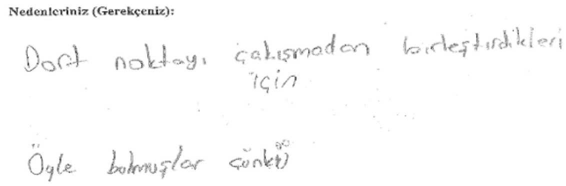
Figure 5.

S6's proof for the sum of the measures of the interior angles of a quadrilateral and S8's justifications

a)



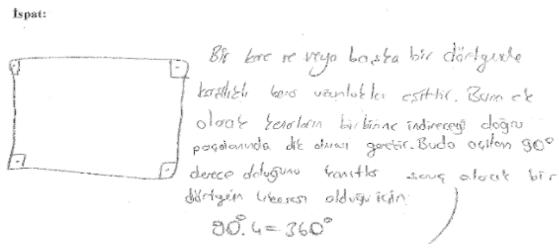
b) Transcription: Since they connect the four points without overlapping. Because that's just as they found it.



As can be seen in Figure 5a, S6 drew only a quadrilateral and was given zero points because the student did not show what s/he was trying to prove or what s/he wanted to explain on the quadrilateral. S8 (Figure 5b) could not present anything as justification. In writing proofs, S3, S4, S5, S8, S10, S11, S12, S15 and S16 received one point. The answer to S3 for writing proof is given in Figure 6 No student got one point in justification skills. That is, in the questions in which the proof was proved incorrectly, operation errors made, or concept errors were made, there was no answer by the criterion of not writing an appropriate justification for the answer.

Figure 6.

S3's proof for the sum of the measures quadrilateral's interior angles

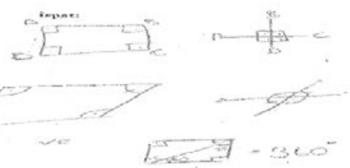


As can be seen in Figure 6, S3 understood the question but did not write an answer to the criteria for writing proofs. The student showed only squares and rectangles and said that one interior angle was 90o and there were four of them, thus s/he showed that the sum of the measures of the interior angles was 360o. However, since what the student wrote was an acceptance and his/her proof would not be accepted for quadrilaterals other than square and rectangle such as rhombus, parallelogram, etc., the student was given one point. S1, S2, S7, and S14 received two points in proof writing and S7, S14, and S15 received two points in justification skills. S1's answer for writing proof is given in Figure 7a. S7's justification answer is given in Figure 7b.

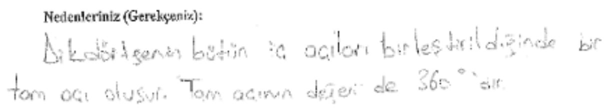
Figure 7.

S1's proof for the sum of the measures of the interior angles of a quadrilateral and S7's justifications

a)

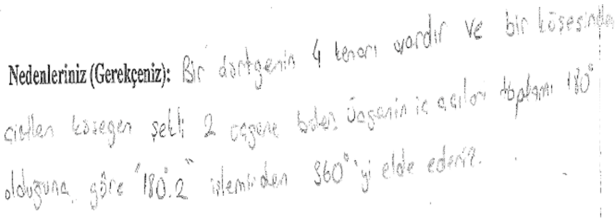


b) Transcription: When all the interior angles of a quadrilateral are combined a complete angle is formed the measure of a full angle is 360o



As can be seen in Figure 7a, S1 drew different quadrilaterals and joined their corners. Then, the student drew a diagonal in a quadrilateral and divided the shape into two. If the student had tried to prove one of the things s/he did and made explanations in that direction, s/he could have got full marks. S7, on the other hand, made his/her justifications only on rectangles, so s/he was given two points. No student got three points in writing proofs. The answer of S2 who got three points in justification skills is given in Figure 8.

Figure 8.
S2's justifications



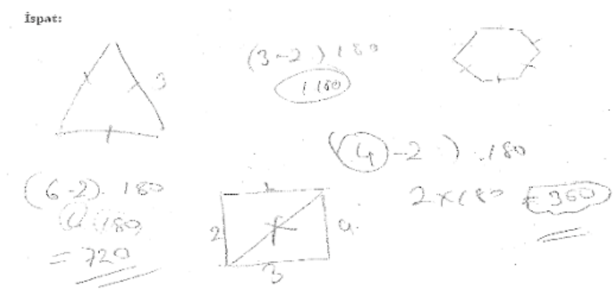
Transcription: A quadrilateral has four sides and the diagonal drawn from one corner divides the shape into two triangles, since the sum of the interior angles of the triangle is 180o, we get 360o from 2.180

As can be seen in Figure 8, S2 made a generalization for all quadrilaterals and wrote that the diagonal drawn from a corner of a quadrilateral divides the quadrilateral into triangles and the sum of the measures of a triangle's interior angles is 180o and the sum of the measures of the interior angles of quadrilaterals is 360o when there are two of them. Therefore, the student received three points.

Results Obtained from the sum of the measures of any polygon's interior angles

Students were asked to show that the sum of the measures polygon's interior angles with n sides is (n-2). 180o and to write their reasons. Here are examples of the answers of the students with the lowest scores and the answers of the students with the highest scores. In writing proofs, S3, S5, S6, S7, S8, S10, S11, S14 and S16 received zero points. In justification skills, S3, S5, S6, S7, S8, S10, S11, S14 and S16 received zero points. S10's proof writing answer is given in Figure 9a and his/her justification is given in Figure 9b.

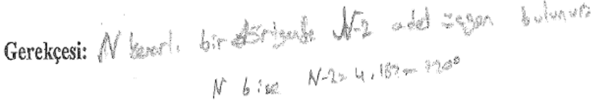
Figure 9.
S10's proof and justifications for the sum of the measures of any polygon's interior angles
a)



b) Transcription: Trial and error

As can be seen in Figure 9a, S10 drew the shapes, and found the sum of the measures of the interior angles, but did not prove what was asked. Therefore, zero point was given. S10 only wrote "trial and error" in the justification. This was not accepted as a justification. No student received a point for writing proof. That is, no answer met the conditions that the question was understood (the question was expressed verbally, the algebraic form of the question was written or short notes were taken about this expression, a graph was drawn, a table was created, the expression/correctness of the given argument/proposition was tested with examples). In justification skills, S12 and S15 got one point. S12's justification is given in Figure 10.

Figure 10.
S12's justification answer



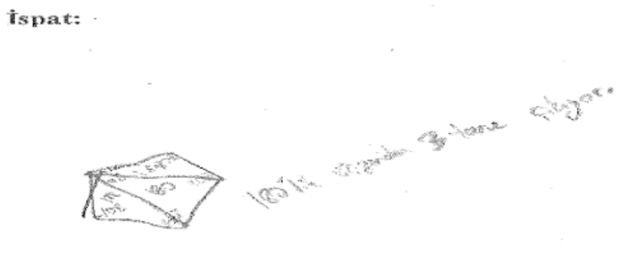
Transcription: A quadrilateral with n sides has n-2 triangles. If n=6, then n-2=4 and 4.180=720.

S12 showed the number of triangles formed by the diagonals drawn from a corner of a pentagon in Figure 11. Here, the student wrote a quadrilateral with n sides. Moreover, he got one point for writing that (n-2) is the number of triangles. S9, S12, and S13 received two points in proof writing and S9 and S13 received two points in justification skills. S12's proof writing answer is given in Figure 11a and S13's justifications are given in Figure 11b.

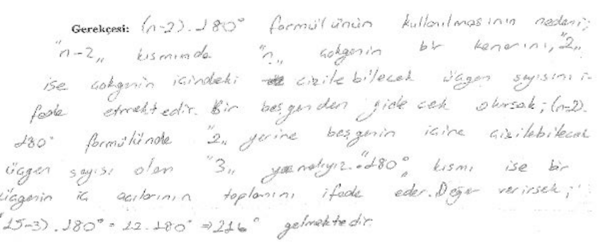
Figure 11.

S12's proof writing and S13's justifications

a)



b)



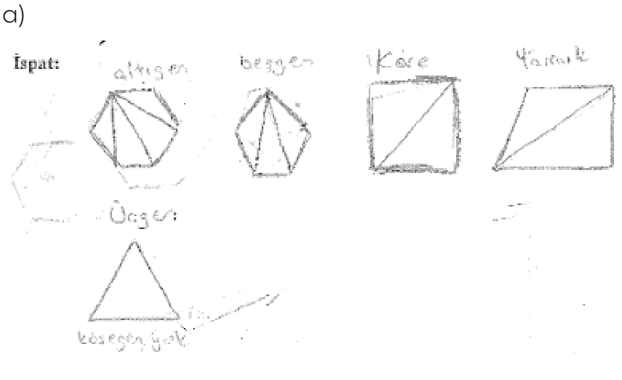
Transcription: The reason for using the formula (n-2)180 is that in the n-2 part, n represents one side of the polygon, and 2 represents the number of triangles that can be drawn inside the polygon. In the (n-2)180 formula, instead of 2, we should write 3, the number of triangles that can be drawn inside the pentagon. The 180 part represents the sum of the interior angles of a triangle. If we give the value (15-3).180=12.180=216.

As can be seen in Figure 11a, S12 drew only one pentagon and started from the number of triangles formed in this pentagon. However, two points were given for drawing only a pentagon. S13 stated the number n as the number of sides of a polygon. But instead of (n-2) being the number of triangles, he said that two is the number of triangles. Therefore, two points were given. In writing proofs, S1, S2, S4, and S15 received three points. The students who got three points in justification skills were S1, S2, and S4. S2's proof is given in Figure 12a and his justifications are given in Figure 12b.

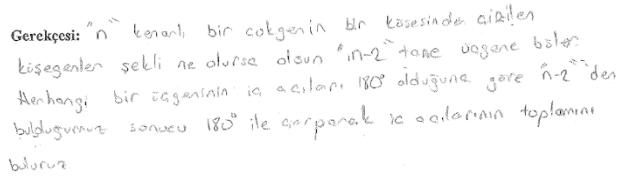
Figure 12.

S2's proof and justifications for the sum of the measures of the interior angles of any polygon

a)



b)



Transcription: The diagonals drawn from one corner of an n-sided polygon divide it into n-2 triangles, regardless of their shape. Since the interior angles of any triangle are 180o, we find the sum of the interior angles by multiplying n-2 by 180o.

As seen in Figure 12a, S2 identified the triangles formed by the diagonals drawn from one corner of the polygons. Besides, the student also showed that the triangle has no diagonal. Therefore, the student got three points. When we analyzed S2's answer, s/he explained exactly what the number n is, what (n-2) constitutes, and why 180o is written. Therefore, three points were given.

Findings obtained from the answers of the students with the lowest and highest scores

In this section, the answers of S6, who scored zero points in total in proof and justification, and S2, who scored nine points, to three questions were analyzed together. In this way, the answers of students at both levels were compared.

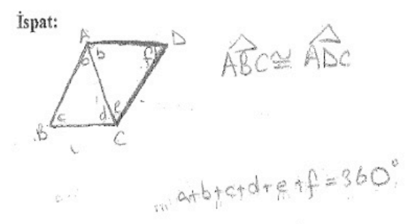
Figure 13.

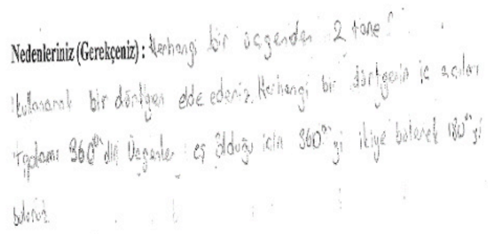
S2 and S6's proofs and justifications about the sum of the measures of the triangle's interior angles

Sum of measures of a triangle's interior angles

S2

Writing proof



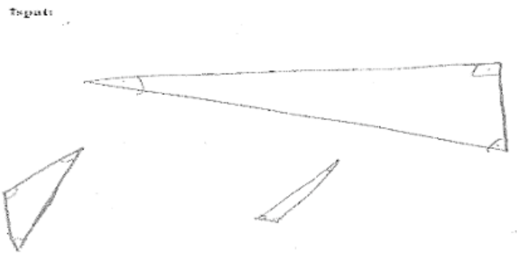


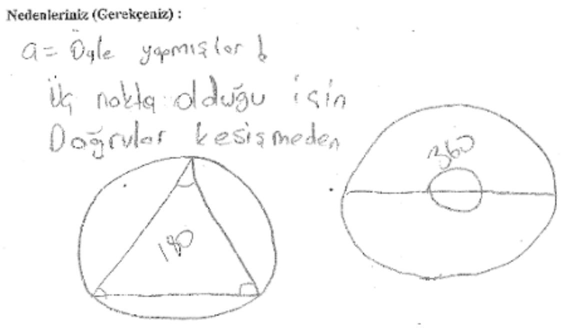
Justification

Transcription: A quadrilateral has 4 and the diagonal drawn from one vertex divides the shape into two triangles. Since the sum of the interior angles of the triangle is 180, we get 360 from 180.2

As can be seen from Figure 13, S2 did the proof and generalized because s/he wrote any quadrilateral in his/her justification and therefore s/he got full points. However, S6 tried to do the proof by taking different triangles. Similarly, s/he tried to justify, but he could neither provide complete proof nor justification. There is a big difference between his/her justification and his/her proof. Accordingly, the student received zero points.

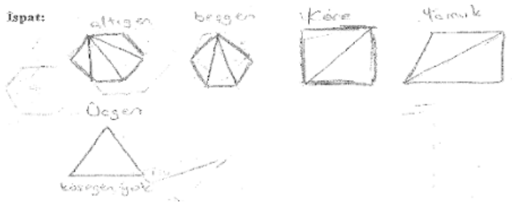
S6
Writing proof





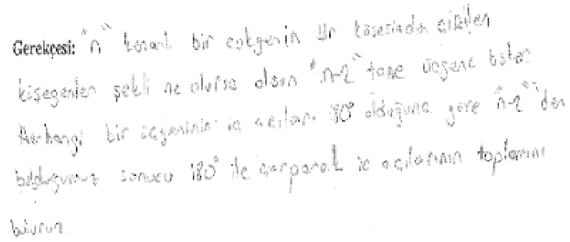
Justification
Transcription: Because they connect the four points without overlapping. because they found it that way

Figure 14.
S2 and S6's proofs and justifications about the sum of the measures of any quadrilateral's interior angles
Sum of the measures of any quadrilateral's interior angles
S2
Writing proof

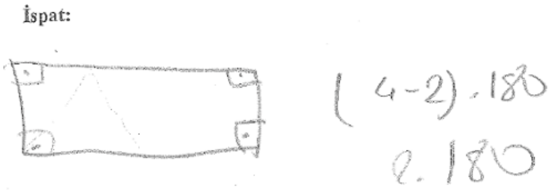


Transcription: Diagonals from one vertex of an n-sided polygon divide the shape into n-2 triangles. Since the interior angles of any triangle are 180, we can find the sum of the interior angles by multiplying n-2 by 180.

Justification

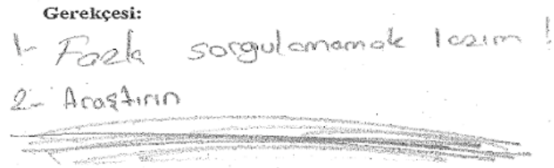


S6
Writing proof



Transcription: No need to questioning so much! Research it.

Justification



As can be seen from Figure 14, S2 showed the number of triangles drawn from a corner of different polygons in his/her proof. In his justification, s/he generalized this and proved the sum of the measures of the interior angles of a polygon with n sides. It was observed that S6 accepted the information without questioning. When the answers of S2 and S6 to three questions were analyzed, it was seen that S2 generalized simple to complex in the light of cause and effect relationship. S6, on the other hand, expressed it as an acceptance without questioning and questioning the cause.

Discussion and Conclusion

In the present study, 8th grade students' proof writing and justification skills in the context of the sum of polygon's angles were analyzed. In light of the findings, it was observed that the levels of justification skills were the same or one level below the levels of proof writing. There is only one student whose both proof writing and justification levels are "very good". This finding reveals the close relationship between proof writing and justification skills. It can be stated that the ability to justify is important for the development of proof writing skills. In addition, proof writing and justification skills are closely related to academic success.

In this study, the relationship between academic achievement and proof writing and justification skills was not examined. Indeed, Senk (1989) stated that the success of writing geometry proofs is positively related to the success of the geometry course. Özmusul (2018) stated that students with high achievement levels have high justification skills. Çalışkan (2012), on the other hand, stated that there is a positive correlation between the mathematics achievement of 8th grade students and their ability to prove. The findings of the Hatisaru (2020) study stated that the students perceive the mathematics classrooms as the teacher being at the center of education and training, explaining the subject and solving routine problems. Really teachers, curriculum, textbooks, etc. play an important role in the development of proof writing and justification skills. Polat (2015) analyzed the tasks requiring explanation and justification in two books determined by the Ministry of National Education for use in 7th grade mathematics teaching. The results of the study showed that tasks requiring explanation and justification are not included in mathematics textbooks.

Nine students were "unsuccessful" in writing proofs, four students were "moderately successful", three students were "very successful" and one student scored full points. The results of the study are consistent with the results of the studies stating that especially 8th grade students have difficulties with proof. Indeed, Albayrak Bahtiyari (2010) found that 8th grade students have deficiencies in both proof and reasoning. Similarly, Zaimoğlu (2012) found that 8th grade students could not fully comprehend the methods and techniques of proof. According to the results obtained from the justification skill levels, 11 students were unsuccessful, four students were "moderately successful", and one student was very successful with full points. This is consistent with the results of Özmusul (2018) that 7th grade students' complete and convincing justification skills are low, and Arslan (2007) that 6th, 7th, and 8th grade students' justification levels are low.

Another important result is that while the level of proof writing and justification level of eleven students were the same, the levels of six students were different. Except for one of these six students, the proof writing levels of the others were higher than the justification levels. If these two results are combined, the proof writing levels of the students participating in the study are either the same or higher than their justification levels. In this study, although it is related to the proof and justification of the sum of the measures of any polygon's interior angles, students are expected to make generalizations for any polygon based on triangle and quadrilateral. In other words, the proof of the sum of the measures of the interior angles of polygons is also related to generalization skills. The results of the study can also be explained by generalization skills. Indeed, there are studies in the literature that emphasize the relationship between generalization and justification skills (Ellis, 2007; Lannin, 2005; Radford, 1996). Ellis (2007) stated that justification affects a student's generalization ability. Similarly, Radford (1996) stated that justification is the process that supports generalization. Akkan, Öztürk, and Akkan (2017) stated in their study that pre-service teachers who generalize patterns correctly provide more justification than pre-service teachers who try to generalize or make no attempt. Yackel (2001) mentioned that providing justification and explanation has a positive effect on the mathematical norm in the classroom. Therefore, studies on justification as well as proof are important. As a result, the findings of this study are in parallel with the studies in the literature on both proof and justification.

Recommendations

The present study was carried out with 16 students who volunteered and participated in the face-to-face teaching process on the day the data would be collected due to the pandemic. This is the limitation of the study. Working with more students can be done.

In this study, proof writing and justification skills are discussed. The literature emphasizes that academic success is also important. Therefore, in future studies, metacognitive levels, academic achievement, geometric thinking levels, justification, and proof writing skills can be examined in detail.

As can be seen from the results of the study, students' proof writing and justification skills are not at the desired level. In this direction, activities aimed at improving students' justification skills can be included in teachers' lessons, textbooks, and curricula. In this way, classroom environments can be created where students can make inquiries, communicate, justify, and share their ideas easily. In addition, this study was conducted with a limited number of participants due to the pandemic. Future studies can be conducted with more participants.

Note: 1. This study was produced from the master thesis prepared by the second author under the supervision of the first author.

2. Within the scope of the research, ethics committee approval was obtained from the ethics committee of Sivas Cumhuriyet University with the decision dated 21.01.2021 and numbered 2021/26.

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