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A RESEARCH ON MATHEMATICAL EPISTEMOLOGICAL BELIEFS AND MATHEMATICS MOTIVATION OF HIGH SCHOOL STUDENTS

Nur Şebnem BAKIR, İlknur GÜLŞEN TURGUT

Abstract: This research aimed to investigate the relationship between mathematical epistemological beliefs and the mathematics motivation of high school students. The survey model was employed in the research. Four hundred twenty-four high school students constitute the study group from public high school. Data were collected through the "Mathematics-Oriented Epistemological Belief Scale" and the "Mathematics Motivation Scale." The Mann-Whitney U and Kruskal-Wallis H tests were used to analyze the data. The findings showed a significant positive relationship at a moderate level between the belief that learning depends on effort with motivation, a weak and negative significant relationship between the belief that learning depends on ability with motivation and a negative and insignificant relationship between the belief that there is only one truth with motivation. The mathematical epistemological beliefs and mathematical motivations of high school students differ statistically in terms of gender, mother's education level, and daily studying time. However, they do not differ statistically regarding out-of-school support, technology, internet use, class level, father's education level, or perceived income level.

Keywords: Mathematical belief, mathematics motivation, high school student.

1. Introduction

The common competence of different types of knowledge, skills and beliefs provides sensitivity and disposition to use them (Perkins, 1995). Epistemological beliefs express individuals' views on knowledge and knowing (Bråten, 2010). Beliefs serve as an indicator for learning and teaching, as they form a system that organizes the background of thoughts and actions (Pehkonen & Törner, 1996). Thus, it affects learning and the degree to which understanding is advanced (Hofer, 2002). An individual's beliefs about what to consider as mathematical context and what will be essential or exciting are closely related to the situations in which s/he will be engaged and sensitive (Op't Eynde et al., 2002). Beliefs strongly influence pupils' evaluation of their talents, willingness to participate in mathematics tasks, and mathematical tendencies (National Council of Teachers of Mathematics [NCTM] 1989). According to De Corte et al. (2000), an individual's mathematical beliefs are also effective in acquiring mathematical disposition. Therefore, students' mathematical beliefs are closely related to their learning outcomes (Furinghetti & Pehkonen, 2000).

Motivation is described as interrelated beliefs and feelings that affect and direct behavior (Wentzel, 1999). Martin (2005) described motivation as the energy and urge to learn, study effectively, and realize one's potential. Motivation is needed to initiate and maintain the behaviors necessary to achieve a targeted performance. Motivation guides actions by keeping focus on the goal and increasing the effort required to overcome difficulties (Usher & Schunk, 2018). Factors such as belief in the ability to overcome difficulties and achieve results, effort, and allocating time and resources can be determinative in the motivation of the individual (Philippou & Christou, 2002). Students who understand motivation and the factors that affect it will be able to intervene when they feel the need and they are likely to be more successful (Martin, 2005). The value given to a task (Wigfield & Eccles, 2000) and self-belief (Martin, 2007) contribute to student motivation.

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1.1. Mathematical beliefs

Mathematical beliefs include approaches to nature, sources, proof, justifications, and acquisition of mathematical knowledge (Schoenfeld, 1985). Mathematical beliefs help establish general goals that define contexts and meanings for learning mathematics (Cobb, 1986) and are one of the components of mathematical disposition (NCTM, 1989). According to Schoenfeld (1985), mathematical beliefs are a person's mathematical worldview. Mathematical beliefs consist of the individual's subjective, experiential, and implicit knowledge of mathematics, teaching and learning mathematics (Pehkonen, 1998). Op't Eynde et al. (2002) stated that mathematical beliefs are subjective concepts that affect problem-solving and mathematical learning and are either implicitly or explicitly accepted as true. They stated that beliefs about mathematical learning and problem-solving are generally investigated in four dimensions in the literature as "beliefs about the nature of mathematics, problem-solving and mathematical learning; beliefs about self in mathematics learning and problem-solving; beliefs about the social context for learning, teaching, and problem-solving mathematics; epistemological beliefs about the nature of knowledge"(ibid, p.17). Mathematical beliefs are used in various senses emphasizing different points (Pajares, 1992). Elsewhere, Underhill (1988) mentioned four belief states including mathematics as a discipline, math learning, math teaching, and self in a social context where mathematics learning and teaching occur when examining students' mathematical beliefs. McLeod (1992) investigated mathematical beliefs in terms of beliefs about mathematics, mathematics teaching, self, and social context. Smilarly, Pehkonen (1995) discussed mathematical beliefs in four categories as beliefs about mathematics, teaching mathematics, self in mathematics and learning mathematics. Kloosterman (1996) explained mathematical beliefs as beliefs about mathematics and learning mathematics (the learner him/herself, the educator's role, and other beliefs in terms of mathematics learning). Op't Eynde et al. (2002) proposed a more comprehensive framework for students' mathematical beliefs. Beliefs in terms of mathematics education include mathematics as a lesson, learning mathematics and problem-solving and general beliefs about instructing mathematics. Beliefs in terms of the self-include self-efficacy, task value, control, and goal-oriented beliefs. Beliefs in terms of social context include social norms containing the role and task of the teacher and student in the classroom and socio-mathematical norms in the classroom. Although the beliefs mentioned appear to be very similar, they are sometimes classified differently and contain different beliefs. Thompson (1992) stated that beliefs should be evaluated both philosophically and psychologically. According to him, philosophical studies will provide an opportunity to understand the nature of belief, and psychological studies will allow us to understand the relationship between belief and behavior and the function of belief.

Kloosterman (2002) stated that mathematical belief affects the effort to learn mathematics. Beliefs and knowledge are closely related constructs, and their interaction determines students' understanding of mathematical problems (Op’t Eynde et al., 2002). Beliefs have a motivational function in mathematics learning and problem-solving of pupils (McLeod, 1992). Mathematical beliefs are effective in the way students approach a mathematical problem, determine the cognitive strategies and techniques they will use (Garofalo, 1989; Schoenfeld, 1985) and develop emotional reactions towards mathematics (McLeod, 1992). Leedy et al. (2003) revealed that students' beliefs affect their mathematics performance. Beliefs that mathematics is valuable are among the reasons students engage in learning mathematics (Chiu & Xihua, 2008). Ernest (1989) stated that teachers' beliefs about mathematics, teaching and learning mathematics is an important factor affecting their teaching practices. Research show that their teachers' mathematical beliefs affect mathematical beliefs of pupils (Muis, 2004). Therefore, it can be said that mathematical beliefs are a factor that should be considered in terms of both pupils and instructors in the mathematics learning and teaching process.

1.2. Mathematics motivation

Pintrich (2003) stated that motivation is an internal behavior that initiates and maintains action to achieve the goal. Therefore, motivation is closely related to students' beliefs about themselves and the task (Wigfield et al., 2016). While motivated students tend to make an effort to learn, resist difficulties and be successful (Renninger & Hidi, 2019), unmotivated students are less likely to participate in challenging academic tasks, and they avoid learning situations that require effort due to their negative beliefs (Wigfield et al., 2016). Those with high expectations and a high value for the task are more motivated and engaged in its completion (Wigfield & Eccles, 2000). For this reason, motivation is perceived as one of the basic elements that determine the quality of learning (Op't Eynde et al., 2002). It is stated that reasons such as belief that mathematics has no personal value in a person's life (Peterson & Hyde, 2017) and anxiety (Dowker et al., 2016) lead to motivational barriers. Interest and motivation towards learning mathematics and problem-solving are influenced by mathematical beliefs (Kloosterman, 1996, 2002). Focus on studying mathematics content and self-belief are positively related to motivation (Martin, 2010). Motivation has an active role in pupils' interest in studying and school, affecting success (Martin, 2005). It is stated that decreased motivation is associated with low achievement and insufficient engagement problems in mathematics (Martin, 2007). A high level of motivation will mediate higher achievement in mathematics (Köller et al., 2001).

Cognitive skills and motivation are interrelated (Ashcraft & Kirk, 2001; Ferguson et al., 2015) and can help explain math achievement (Atit et al., 2020). Students' emotions and motivations for mathematics affect their effort (Cleary & Chen, 2009; Middleton & Spanias, 1999). Pajares (1996) and Hembree (1990) stated that mathematics motivation affects students' mathematics achievement. Atit et al. (2020) showed that mathematics learning motivation is an important predictor of students' mathematics achievement. Even after mathematics becomes optional, motivation plays an active function in determining how many people are interested in studying it (Hannula et al., 2014).

Motivation is essential when performing challenging mathematical tasks and every student needs to be motivated to participate in mathematical activities (Mueller et al., 2011). In this process, intrinsic and extrinsic motivation can be activated together. Intrinsic motivation in particular enables students to engage in mathematical tasks, recognizing that learning mathematics affects the learner's image and must be successful (Middleton, 1995). Extrinsic motivation, on the other hand, enables engagement in mathematical tasks for reasons such as getting teacher and peer approval, getting good grades, and may not provide a real sense of belonging to mathematics (Middleton & Spanias, 1999). Therefore, it can be stated that students who are intrinsically motivated are more focused on learning. This will be beneficial both during and after the process of performing a mathematical task (Middleton & Spanias, 1999). Lepper and Henderlong (2000) stated that pupils who intrinsically motivated are self assured in choosing different and challenging strategies, are more insistent, and put more effort into solving a mathematical problem. However, Nyman and Sumpter (2019) stated that trying to give students only intrinsic motivation can sometimes fail, and extrinsic motivation should also be taken into account. In her research, Sumpter (2013) showed that extrinsic motivation could compensate for negative cognitive intrinsic motivation. Therefore, it can be said that motivation will contribute positively while performing a mathematical task, especially intrinsic motivation will help focus on learning and being successful, and extrinsic motivation can be activated if needed.

1.3. The present study

Mathematics motivations should be investigated to understand students' positive or negative attitudes towards mathematics and the differences in performance on mathematics tests (Nyman & Sumpter, 2019). However, it is known that there is a relationship between motivation and belief (Kloosterman, 1996, 2000; Martin, 2010; McLeod, 1992). Epistemological beliefs directly or indirectly affect academic achievement through cognitive and motivational factors (Muis, 2004). A limited number of studies have been found in the literature examining high school students' beliefs about mathematics (e.g., Mason, 2003; Mert & Bulut, 2006; Schoenfeld, 1989; Wang et al., 2019) and their motivation for mathematics (e.g., Chen & Stevenson, 1995; Jones et al., 2012, Kesici, 2018; Kim et al., 2015). Based on this, it can be said that there is a need for research that investigates the relationship between mathematical beliefs and mathematics motivation together in the field of mathematics education.

This research aimed to investigate the relationship between high school students' mathematical epistemological beliefs and mathematics motivations. In this context, answers to the following questions were sought.

1. What is the relationship between high school students' mathematical epistemological beliefs and mathematics motivations?

2. Do high school students' mathematical epistemological beliefs and mathematics motivations differ based on gender?
3. Do high school students' mathematical epistemological beliefs and mathematical motivations differ based on whether they receive out-of-school support or not?
4. Do high school students' mathematical epistemological beliefs and motivations differ based on their use of technology and the internet?
5. Do high school students' mathematical epistemological beliefs and mathematics motivations differ based on their grades?
6. Do high school students' mathematical epistemological beliefs and mathematical motivations differ based on their mother's education level?
7. Do high school students' mathematical epistemological beliefs and mathematics motivations differ based on their father's education level?
8. Do high school students' mathematical epistemological beliefs and mathematics motivations differ based on their families' income levels?
9. Do high school students' mathematical epistemological beliefs and mathematics motivations differ based on their daily study time for mathematics?

2. Method

2.1. Research design

The survey model was employed in the research. The survey model aims to understand the characteristics of a group or the current situation through interviews or questionnaires (Fraenkel et al., 2012). It is cross-sectional survey research as the data were collected in one go. In the research, data on the mathematical epistemological beliefs and mathematical motivations of high school students were collected through scales. Through these data, the current situation of high school students' mathematical epistemological beliefs and mathematics motivation was investigated based on the determined variables.

2.2. Study group

The research study group was chosen through convenient sampling, one of the non-random sampling methods. The study group consists of 424 high school students studying in a public high school affiliated to the Ministry of National Education in Ankara in the second semester of the 2021-2022 academic year. 275 (64.9%) of these students were female and 149 (35.1%) were male. 144 (34%) of the students were studying in the ninth grade, 114 (26.9%) were studying in the tenth grade, 109 (25.7%) were studying in the eleventh grade, and 57 (13.4%) were studying in the twelfth grade. The central exam scores of the students' high school transition system (maximum 500 points can be obtained) vary between 400 and 484. The mothers of 74 students (17.5%) are primary school graduate, the mothers of 65 students (15.3%) are secondary school graduate, the mothers of 157 students (37%) are high-school graduate, the mothers of 104 students (24.5%) are college graduate and the mothers of 24 (5.7%) students are postgraduate. The fathers of 41 students (9.7%) are primary school graduate, the fathers of 45 students (10.6%) are secondary school graduate, the fathers of 149 (35.1%) students are high-school graduate, the fathers of 164 (38.7%) students are college graduate, and the fathers of 25 students (5.9%) are postgraduate. The family of 30 students (7.1%) has a low-income level, the family of 375 (88.4%) students has a middle-income level and the family of 19 (4.5%) students has a high-income level. While 339 (80%) of the students do not receive out-of-school education support to learn mathematics, 85 (20%) of the students receive out-of-school education support. One hundred seventy-six of the students (41.5%) spend 0-1 hour daily to learn mathematics, 223 of the students (52.6%) spend 1-3 hours, 21 of the students (5%) spend 3-5 hours, and 4 of the students (.9%) spend 5 hours or more. While 22 (5.2%) of the students do not use technology and the internet to learn mathematics, 402 (94.8%) of the students use technology and the internet.

2.3. Data collection

Research data were collected through the "Mathematics-Oriented Epistemological Belief Scale (MOEBS)" and the "Mathematics Motivation Scale (MMS)".

MOEBS was developed by İlhan and Çetin (2013). The scale consists of three sub-dimensions. The belief that learning is dependent on effort (BLDE) contains ten items, the belief that learning is dependent on talent (BLDT) contains ten items, and the belief that there is only one truth (BTOOT) contains seven items. The scale has a structure that can be answered in the range of “1=strongly disagree” and “5=strongly agree”. The Cronbach's Alpha internal consistency coefficient values calculated for the sub-dimensions of the scale are .84 for BLDE, .81 for BLDT and .71 for BTOOT. The goodness of fit indices were calculated as a result of confirmatory factor analysis as χ^2=772.69, df=315, χ^2/df=2.45, p<.001, RMSEA=.066, SRMR=.089, NNFI=.90, CFI=.90, IFI=.90 and it was concluded that the model was validated. While scoring the scale, the total score obtained from each sub-dimension is considered. The means of a high score obtained is that the higher belief about the relevant sub-dimension. A high score from the BLDE sub-dimension of the scale indicates advanced epistemological beliefs, while a high score from the BLDT and BTOOT sub-dimensions indicates undeveloped epistemological beliefs.

MMS was developed by Kesici (2018). The scale consists of three sub-dimensions. Goal orientation contains four items, expectation-value contains four items and self-efficacy contains four items. The scale has a structure that can be answered in the range of “1=strongly disagree” and “5=strongly agree”. The Cronbach's Alpha internal consistency coefficient value of the scale was calculated as .87. The total variance explained by the scale is 65%. The factor load values of the scale items ranged from .86 to .47. The goodness of fit indices were calculated as a result of confirmatory factor analysis as χ^2=107.83, df=51, χ^2/df=2.11, p<.001, RMSEA=.075, NFI=.90, CFI=.94, IFI=.95, GFI=.92 and it was concluded that the model was validated. The means of a high score obtained is that the higher motivation about mathematics.

The scales were implemented online in a way that did not interfere with the teaching activities of the students. In order to ensure voluntary participation, participant confirmation was obtained at the beginning of the data collection.

2.4. Data analysis

The total scores obtained from each sub-dimension of the MOEBS and the total scores of the whole MMS were used to analyze the data. The normality of the distribution was decided using the Kolmogorov-Smirnova test, measures of central tendency and skewness kurtosis values. The relationship between the sub-dimensions of MOEBS and mathematics motivation was determined by calculating the Spearman Rank Correlation Coefficient. The Mann-Whitney U test was used when comparing the means of the two groups, and the Kruskal-Wallis H test was used when comparing the means of more than two groups. The IBM SPSS Statistics 25 program was used for the analyses.

3. Findings

The distribution was not normal, according to the Kolmogorov-Smirnov[a] test. (p<.05). In addition, the ratios of skewness and kurtosis values to standard error were outside the limits of -1.96 and +1.96, and the measures of central tendency were far from each other, indicating that the distribution was not normal (Can, 2013). As a result, in order to determine whether there is a statistically significant difference between the groups based on the variables, non-parametric tests were used. In this context, the Mann-Whitney U test was used for the variables of gender, out-of-school support, use of technology and the internet, and the Kruskal-Wallis H test was used for the variables of grade, mother's education level, father's education level, family income level, and daily studying time.

The Spearman Rank Correlation Coefficient was calculated to determine the relationship between the sub-dimensions of MOEBS and mathematics motivation, and the findings are shown in Table 1.

Table 1. *Spearman Rank Correlation, mean and standard deviation values*

	1	2	3	4	*M*	*SD*
1. BLDE	-				39.80	7.23
2. BLDT	$-.329^{**}$	-			23.91	8.54
3. BTOOT	.006	$.197^{**}$	-		17.44	6.53
4. Motivation	$.425^{**}$	$-.397^{**}$	-.067	-	3.98	.71

$^{**}p<.001$

Table 1 shows that there is a moderate and positive relationship between BLDE and motivation (r=.425, $p<.001$) and a weak and negative relationship between BLDT and motivation (r=-.397, $p<.001$). It is seen that the relationship between BTOOT and motivation is negative but not significant (r=-.067, $p>.005$).

The test results based on the gender variable of high school students' scores obtained from MMS and the sub-dimensions of MOEBS are shown in Table 2.

Table 2. *Mann-Whitney U test results based on the gender variable*

	Group	N	Mean Rank	Runk Sum	U	*p*
BLDE	Female	275	225.86	62112.00	16813.00	.00
	Male	149	187.84	27988.00		
BLDT	Female	275	194.31	53436.00	15486.00	.00
	Male	149	246.07	36664.00		
BTOOT	Female	275	203.38	55928.50	17978.50	.03
	Male	149	229.34	34171.50		
Motivation	Female	275	223.56	61478.50	17446.50	.01
	Male	149	192.09	28621.50		

Table 2 shows that there is a statistically significant difference between the groups based on the gender variable (BLDE U=16813.00, $p<.05$; BLDT U=15486.00, $p<.05$; BTOOT U=17978.50, $p<.05$; motivation U=17446.50, $p<.05$). According to this, female students' mean scores of BLDE and motivation are higher than the mean scores of male students. On the other hand, the mean scores of the male students in BLDT and BTOOT are higher than the mean scores of the female students.

The test results based on the out-of-school support variable of high school students' scores obtained from MMS and the sub-dimensions of MOEBS are shown in Table 3.

Table 3. *Mann-Whitney U test results based on the out-of-school support variable*

	Group	N	Mean Rank	Runk Sum	U	*p*
BLDE	Getting support	339	214.58	72743.50	13701.50	.48
	Not getting support	85	204.19	17356.50		
BLDT	Getting support	339	209.81	71127.00	13497.00	.36
	Not getting support	85	223.21	18973.00		
BTOOT	Getting support	339	210.64	71407.50	13777.50	.53
	Not getting support	85	219.91	18692.50		
Motivation	Getting support	339	215.65	73104.00	13341.00	.29
	Not getting support	85	199.95	16996.00		

Table 3 shows that there is no statistically significant difference between the groups based on the variable of out-of-school support (BLDE U=13701.50, $p>.05$; BLDT U=13497.00, $p>.05$; BTOOT U=13777.50, $p>.05$; motivation U=13341.00, $p>.05$).

The test results based on the use of technology and the internet variable of high school students' scores obtained from MMS and the sub-dimensions of MOEBS are shown in Table 4.

Table 4. *Mann-Whitney U test results based on the use of technology and the internet variable*

	Grup	N	Mean Rank	Runk Sum	U	*p*
BLDE	Use	402	213.44	85802.00	4045.00	.50
	Do not use	22	195.36	4298.00		
BLDT	Use	402	211.02	84830.50	3827.50	.28
	Do not use	22	239.52	5269.50		
BTOOT	Use	402	209.67	84288.50	3285.50	.05
	Do not use	22	264.16	5811.50		
Motivation	Use	402	213.43	85798.50	4048.50	.50
	Do not use	22	195.52	4301.50		

Table 4 shows that there is no statistically significant difference between the groups based on the use of technology and the internet variable (ÖÇBOİ U=4045.00, p>.05; ÖYBOİ U=3827.50, p>.05; TBDVOİ U=3285.50, p>.05; motivation U=4048.50, p>.05).

The test results based on the grade variable of high school students' scores obtained from MMS and the sub-dimensions of MOEBS are shown in Table 5.

Table 5. *Kruskal-Wallis H test results based on the grade variable*

	Group	N	Mean Rank	df	χ^2	*p*	Difference
BLDE	(1) 9th grade	144	229.42	3	5.531	.13	–
	(2) 10th grade	114	195.72				
	(3) 11th grade	109	204.36				
	(4) 12th grade	57	218.89				
BLDT	(1) 9th grade	144	197.14	3	7.650	.05	–
	(2) 10th grade	114	235.21				
	(3) 11th grade	109	218.38				
	(4) 12th grade	57	194.67				
BTOOT	(1) 9th grade	144	210.50	3	2.965	.39	–
	(2) 10th grade	114	226.39				
	(3) 11th grade	109	210.76				
	(4) 12th grade	57	193.09				
Motivation	(1) 9th grade	144	222.04	3	7.824	.05	–
	(2) 10th grade	114	188.87				
	(3) 11th grade	109	230.17				
	(4) 12th grade	57	201.85				

Table 5 shows that there is no statistically significant difference between the groups based on the grade variable (BLDE $\chi^2_{(3)}$=5.531, p>.05; BLDT $\chi^2_{(3)}$=7.650, p>.05; BTOOT $\chi^2_{(3)}$=2.965, p>.05; motivation $\chi^2_{(3)}$=7.824, p>.05).

The test results based on the mother education level variable of high school students' scores obtained from MMS and the sub-dimensions of MOEBS are shown in Table 6.

Table 6. *Kruskal-Wallis H test results based on the mother education level variable*

	Group	N	Mean Rank	df	χ^2	*p*	Difference
BLDE	(1) Primary school	74	202.61	4	10.360	.03	1-5 2-5 3-5
	(2) Middle school	65	235.18				
	(3) High school	157	215.89				
	(4) University	104	215.98				
	(5) Postgraduate	24	144.35				
BLDT	(1) Primary school	74	204.16	4	4.988	.28	–
	(2) Middle school	65	193.05				
	(3) High school	157	209.45				
	(4) University	104	230.64				
	(5) Postgraduate	24	232.21				
BTOOT	(1) Primary school	74	192.35	4	7.245	.12	–

	(2) Middle school	65	240.65				
	(3) High school	157	219.20				
	(4) University	104	198.70				
	(5) Postgraduate	24	214.35				
Motivation	(1) Primary school	74	224.16	4	9.639	.04	1-5 2-5
	(2) Middle school	65	242.64				
	(3) High school	157	207.59				
	(4) University	104	204.84				
	(5) Postgraduate	24	160.27				

Table 6 shows that there is no statistically significant difference between the groups in terms of BLDT and BTOOT based on the mother's education level variable (BLDT $\chi^2_{(4)}$=4.988, p>.05; BTOOT $\chi^2_{(4)}$=7.245, p>.05), in terms of BLDE and motivation, there is a statistically significant difference between the groups (BLDE $\chi^2_{(4)}$=10.360, p<.05; motivation $\chi^2_{(4)}$=9.639, p<.05). As a result of the multiple comparisons made with the Mann-Whitney U test, it was determined that the difference in BLDE was between the first group and the fifth group, and between the second and third groups and the fifth group. It was determined that the difference in motivation was between the first and second groups and the fifth group.

The test results based on the father education level variable of high school students' scores obtained from MMS and the sub-dimensions of MOEBS are shown in Table 7.

Table 7. *Kruskal-Wallis H test results based on the father education level variable*

	Group	N	Mean Rank	df	χ^2	p	Difference
BLDE	(1) Primary school	41	229.74	4	2.093	.71	–
	(2) Middle school	45	216.10				
	(3) High school	149	209.10				
	(4) University	164	214.19				
	(5) Postgraduate	25	186.90				
BLDT	(1) Primary school	41	182.41	4	3.263	.51	–
	(2) Middle school	45	203.86				
	(3) High school	149	217.01				
	(4) University	164	218.24				
	(5) Postgraduate	25	212.84				
BTOOT	(1) Primary school	41	190.07	4	3.615	.46	–
	(2) Middle school	45	230.53				
	(3) High school	149	221.18				
	(4) University	164	206.66				
	(5) Postgraduate	25	203.42				
Motivation	(1) Primary school	41	258.84	4	9.501	.05	–
	(2) Middle school	45	215.91				
	(3) High school	149	216.64				
	(4) University	164	201.68				
	(5) Postgraduate	25	176.68				

Table 7 shows that there is no statistically significant difference between the groups based on the father education level variable (BLDE $\chi^2_{(4)}$=2.093, p>.05; BLDT $\chi^2_{(4)}$=3.263, p>.05; BTOOT $\chi^2_{(4)}$=3.615, p>.05; motivation $\chi^2_{(4)}$=9.501, p>.05).

The test results based on the family income level variable of high school students' scores obtained from MMS and the sub-dimensions of MOEBS are shown in Table 8.

Table 8. *Kruskal-Wallis H test results based on the family income level variable*

	Group	N	Mean Rank	df	χ^2	p	Difference
BLDE	(1) Low-income	30	206.03	2	.662	.71	–
	(2) Middle-income	375	211.94				
	(3) High-income	19	233.71				

BLDT	(1) Low-income	30	242.45	2	2.507	.28	–
	(2) Middle-income	375	211.27				
	(3) High-income	19	189.45				
BTOOT	(1) Low-income	30	230.22	2	3.237	.19	–
	(2) Middle-income	375	208.93				
	(3) High-income	19	254.97				
Motivation	(1) Low-income	30	222.93	2	.240	.88	–
	(2) Middle-income	375	211.81				
	(3) High-income	19	209.63				

Table 8 shows that there is no statistically significant difference between the groups based on the family income level variable (BLDE $\chi^2_{(2)}$=.662, *p*>.05; BLDT $\chi^2_{(2)}$=2.507, *p*>.05; BTOOT $\chi^2_{(2)}$= 3.237, *p*>.05; motivation $\chi^2_{(2)}$=.240, *p*>.05).

The test results based on the daily study time for mathematics variable of high school students' scores obtained from MMS and the sub-dimensions of MOEBS are shown in Table 9.

Table 9. *Kruskal-Wallis H test results based on the daily study time for mathematics variable*

	Group	N	Mean Rank	df	χ^2	*p*	Difference
BLDE	(1) 0-1 hour	176	187.86	3	19.579	.00	1-2 1-3 2-3
	(2) 1-3 hours	223	224.19				
	(3) 3-5 hours	21	298.31				
	(4) 5 hours or more	4	194.63				
BLDT	(1) 0-1 hour	176	219.24	3	6.145	.10	–
	(2) 1-3 hours	223	206.39				
	(3) 3-5 hours	21	195.67				
	(4) 5 hours or more	4	344.63				
BTOOT	(1) 0-1 hour	176	207.78	3	1.472	.68	–
	(2) 1-3 hours	223	218.43				
	(3) 3-5 hours	21	195.12				
	(4) 5 hours or more	4	181.13				
Motivation	(1) 0-1 hour	176	181.60	3	23.847	.00	1-2 1-3 2-3
	(2) 1-3 hours	223	230.65				
	(3) 3-5 hours	21	285.26				
	(4) 5 hours or more	4	178.25				

Table 9 shows that there is no statistically significant difference between the groups in terms of BLDT and BTOOT based on the daily study time for mathematics variable (BLDT $\chi^2_{(3)}$=6.145, *p*>.05; BTOOT $\chi^2_{(3)}$=1.472, *p*>.05), in terms of BLDE and motivation, there is a statistically significant difference between the groups (BLDE $\chi^2_{(3)}$=19.579, *p*<.05; motivation $\chi^2_{(3)}$=23.847, *p*<.05). As a result of the multiple comparisons made with the Mann-Whitney U test, it was determined that the difference in BLDE was between the first group and the second and third groups, and between the second group and the third group. It was determined that the difference in motivation was between the first group and the second and third groups, and between the second group and the third group.

4. Discussion and Conclusion

As a result of the research, it was found out that there is a moderate and positive relationship between BLDE and motivation. Believing that learning mathematics depends on effort can positively affect mathematics motivation. Kloosterman and Cougan (1994) determined in their research that most of the students believe that mathematics can be learned by everyone, and it requires effort. It was determined that there is a weak and negative significant relationship between BLDT and motivation. Beliefs that the ability is constant and that s/he or she is not capable negatively affect motivation and performance (Alderman, 2007; Graham & Weiner, 1996). Students who believe they are low-skilled are less willing to try (Graham & Weiner, 1996). Garofalo (1989) determined in his research that students believe that only those with extraordinary abilities can do mathematics. Schoenfeld (1988) concluded in his research that students believe that mathematical problems should be solved in a short time and quickly, and that only talented people can do it. Spangler (1992) found that most primary school students believe that only talented students can solve mathematical problems quickly. Therefore, believing that learning mathematics depends on talent may negatively affect mathematics motivation. It was determined that there was a negative and insignificant relationship between BTOOT and motivation. Similarly, Garofalo (1989) in his research found that students believe that memorizing formulas and applying procedures is sufficient for learning mathematics. Diaz-Obando et al. (2003) concluded that students believe that mathematical knowledge is uniform, learned in the same way, and taught in the classroom by the teacher. Elsewhere, Frank (1988), in her research, determined that students believe that the purpose of doing mathematics is to reach a single truth quickly. Spangler (1992) found in her study that students at different grade levels have similar beliefs that doing math is looking for one correct answer. Students generally believe that mathematical knowledge is immutable (Muis, 2004). Therefore, believing that there is only one truth in mathematics can negatively affect mathematics motivation. As a result, in the process of teaching mathematics, students should be made to discover that learning mathematics is dependent on effort, not talent, and mathematical knowledge is not certain and immutable.

As a result of the research, it was found that female students had more advanced epistemological beliefs towards mathematics than male students. In addition, female students' motivation towards mathematics is higher than male students. Fennema et al. (1990) stated that boys performed better in mathematical tasks, girls had more negative beliefs towards math, and career occupations related to math were less preferred by women. Franke and Carey (1997) determined in their research that male and female students have similar beliefs about learning mathematics. Rodríguez et al. (2020) found that female students' motivation towards mathematics is lower than male students. Hyde et al. (2008) found that girls are now as good at mathematics as boys, and Hyde and Mertz (2009) found that women and men perform similarly in mathematics. The results of the current research also indicate that the gender gap may be disappearing.

A great majority (80%) of high school students participating in the research get support (private lessons, courses) out-of-school to learn mathematics. However, getting support out-of-school did not lead to a statistically significant difference between the groups in terms of students' epistemological beliefs and motivations towards mathematics. On the other hand, it was determined that the students who got support out-of-school to learn mathematics had higher scores in terms of developed epistemological beliefs and motivation towards mathematics than those who did not. Private lessons are a phenomenon that has become widespread in many countries of the world (Guill & Bos, 2014; Lee, 2007; Zhang et al., 2020). Although private lessons are seen as a compensatory approach to improve school performance (Mischo & Haag, 2002), it has been stated that the reason for taking private lesson is to increase success in exams in general (Bray, 2009). Guill and Bos (2014) stated that the evidence for the effectiveness of private lessons is rare and contradictory. Positive (e.g., Hamid et al., 2009; Mischo & Haag, 2002), mixed (e.g., Ireson & Rushforth, 2005; Kenny & Faunce, 2004) and partially negative (e.g., Cheo & Quah, 2005; Kenny & Faunce, 2004; Smyth, 2008) results were reported in the research that conducted in different countries on the effectiveness of private lessons. Mischo and Haag (2002) determined that taking private lessons positively affects motivational factors. Both the instructors' and the students' cognitive and motivational abilities have an active role in the effectiveness of the private lesson. In addition, parents' insistence on their children's participation in private lessons can negatively affect students' motivation and beliefs (Guill & Bos, 2014). Therefore, it can be said that the result of the current research is in line with the literature.

A great majority (98.4%) of high school students participating in this research use technology and the internet to learn mathematics. However, using technology and the internet to learn mathematics did not lead to a statistically significant difference between the groups in terms of students' epistemological beliefs and motivations towards mathematics. On the other hand, it was determined that the students who use technology and the internet to learn mathematics had higher scores in terms of developed epistemological beliefs and motivation towards mathematics than those who did not. Moos and Marroquin (2010) stated that the evidence for the motivational effectiveness of technology is mixed. Chen (2019) and Higgins et al. (2017) determined that the use of technology in mathematics teaching has a positive effect on students' motivation and accordingly, there are improvements in their mathematical outcomes. Star et al. (2014) stated that the effect of technology on student motivation is modest. Factors such as purpose of use, intensity, environment, curriculum, mathematical content, teacher, etc. have possible effects on the effectiveness of using technology and the internet on epistemological beliefs and motivation towards mathematics. So, it can be stated that more research is needed to investigate these relationships.

The grade of the high school students participating in the research did not cause a statistically significant difference between the groups in terms of their epistemological beliefs and motivations towards mathematics. Eccles et al. (1993) concluded that younger children's efficacy beliefs were more positive than those of older children. Fredricks et al. (2002) determined that the difference between students' efficacy beliefs decreased from the 1st to the 12th grade. Lepper et al. (2005) and Corpus et al. (2009) found a explicit decrease in students' intrinsic motivation as their grade level progressed (3 to 8). Mason (2003) found that from the first to the last year of high school, students' beliefs about the usefulness of mathematics and the ability to solve difficult mathematical problems decrease. As a result of the current research, it has been determined that the grade of high school students (9 to 12) have no apparent effect on their epistemological beliefs and motivations towards mathematics.

As a result of the research, it was determined that the high school students whose mothers graduated from primary school, middle school and high school had more advanced epistemological beliefs towards mathematics than those whose mothers graduated from postgraduate. In addition, high school students whose mothers graduated from primary school and middle school are more motivated towards mathematics than those whose mothers graduated from postgraduate. Although the result is surprising, it brings to mind the possibility that mothers with postgraduate education have taken advanced mathematics courses and had difficulties in these courses. Tomasetto et al. (2011) stated that the children whose mother have negative beliefs about mathematics tend to show low mathematics performance. Children whose mothers have high mathematics anxiety exhibit lower motivation, poor arithmetic skills, and negative beliefs about mathematics (Cohen & Rubinsten, 2017).

As a result of the research, it was determined that the education level of the fathers did not cause a statistically significant difference in the epistemological beliefs and motivations of high school students towards mathematics. Campbell and Uto (1994) stated that the educational level of parents affects the mathematics achievement of boys and girls in different ways, and cultural factors are effective. Korup et al. (2002) and Crook (1995) stated that the education level of the mother is more effective than the education level of the father in the success of the children. Marks (2008) found that there is a great deal of variation between countries, but in most countries, the education level of mothers is important for children's success. Children are more exposed to their mothers' values, beliefs, attitudes, and wishes because they spend more time with them (Marks, 2008). Considering the relationship of affective factors such as belief and motivation with success, as mentioned before, the results of the current research are in line with these findings.

A great majority (88.4%) of high school students participating in the research stated that their families' income level was middle. The families' income levels of the high school students participating in the research did not cause a statistically significant difference between the groups in terms of their epistemological beliefs and motivations towards mathematics. In the literature, there is no study that directly investigates the relationship between family income level and students' epistemological beliefs and motivations towards mathematics. However, there are different findings in terms of the relationship between family income level and student achievement. Sirin (2005) determined that family income level is associated with student achievement. Dahl and Lochner (2012) and Duncan et al. (2011) found that an increase in family income level has a positive effect on the success of their children. Marks (2016) stated that family income level has a weak effect on student achievement. Marks and Pokropek (2019) found that family income level has a non-negligible effect on student achievement in some countries. Humlum (2011) determined that the effect of family income level on student achievement is small and statistically insignificant. Aughinbaugh and Gittleman (2003) determined that the effect of family income level on children's test scores is quite small. Brown et al. (2011) concluded that family income is not related to children's test scores. Orr (2003) stated that family income level has no effect on mathematics achievement. Findings of different research indicate that the relationship between family income level and student achievement differs according to countries. It can be said that there is a need for research investigating the relationship between family income level and students' epistemological beliefs and motivations towards mathematics.

Only four (.9%) of the high school students who participated in the research stated that they spent five hours or more for studying mathematics in a day. In general, it has been determined that students with longer studying time have advanced epistemological beliefs towards mathematics and their motivation towards mathematics is higher than those with less studying time. Studying time is generally associated with learning and success, and it is accepted that performance will increase with increasing studying time (Rosário et al., 2013; Plant et al., 2005). Cheema and Sheridan (2015) determined that the study time allocated for mathematics homework positively affects mathematics achievement. However, it cannot be said that studying time directly increases success (Lahmers & Zulauf, 2000; Plant et al., 2005; Rosário et al., 2013). Plant et al. (2005) stated that the quality of study time is effective in the relationship between studying time and performance. Rosario et al. (2013) determined that motivational factors mediated the effect of studying time on mathematics achievement. Spitzer (2022) concluded that the study time positively affects the mathematics achievement of students, especially those with low performance, and the quality of study time is effective. He also stated that motivational factors are effective in the quality of studying time. The reciprocal relationship of motivation with learning and performance (Pintrich, 2003) and the effect of mathematical beliefs on motivation towards learning mathematics and problem-solving (Kloosterman, 1996, 2000) are known. Based on this, it can be stated that the study time is indirectly related to the epistemological beliefs and motivations of high school students towards mathematics. Considering that factors such as characteristics of the learner, the learning environment, and the mathematics content etc. may have a common effect on study time and the quality of study time, it can be said that more evidence and research is needed on this subject.

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Authors

Nur Şebnem BAKIR, Dr., Mathematics Teacher, Aydınlıkevler Anatolian High School, Ankara, Turkey. E-mail: bakirnursebnem@gmail.com ORCID: https://orcid.org/0000-0002-0610-8851

İlknur GÜLŞEN TURGUT, Asst. Prof. Dr., Department of Mathematics Education, Faculty of Education, Kütahya Dumlupınar University, Kütahya, Turkey. ilknurgulsen@gmail.com ORCID: https://orcid.org/0000-0002-1721-7498