Journal on Mathematics Education
Volume 14, No. 3, 2023, pp. 503-524

Textbook praxeological-didactical analysis: Lessons learned from the Indonesian mathematics textbook

Tri Nova Hasti Yunianta[1,2,*], Didi Suryadi[1], Dadan Dasari[1], Tatang Herman[1]

[1]Mathematics Education Department, Universitas Pendidikan Indonesia, Bandung, Indonesia
[2]Mathematics Education Study Program, Universitas Kristen Satya Wacana, Salatiga, Indonesia
*Correspondence: trinova.yunianta@uksw.edu

Received: 16 March 2023 | Revised: 6 June 2023 | Accepted: 15 June 2023 | Published Online: 19 June 2023

Abstract

The objective of textbook study is to create high-quality textbooks. Analysis was done using a praxeological-didactical analysis (PDA) method. PDA offers space to analyze curriculum materials, such as math textbooks, which are the outcome of human action in the anthropology of a specific nation's society. There are 10 types of tasks given in the mathematics textbook on measurement of spatial figures and seven techniques are identified as possible ways to complete the tasks. The justification of praxis is that there are three emerging technologies and two theories that are used as the final direction of the given task type. There is a sequence of task composition at the start with loads at level 2 and level 3 that can affect student readiness. Psychologically, it is regarded as difficult at first, which can lead to students becoming disinterested as well as bored, thus experiencing difficulties, creating ontogenetic obstacles. The other predicted learning obstacles identified in this textbook are epistemological and didactic obstacles. All the findings from this analysis can be applied to continuously raise the standard of the currently available mathematics textbooks.

Keywords: Analysis, Didactical, Mathematics, Praxeological, Textbook

How to Cite: Yunianta, T. N. H., Suryadi, D., Dasari, D., & Herman, T. (2023). Textbook praxeological-didactical analysis: Lessons learned from the Indonesian mathematics textbook. *Journal on Mathematics Education, 14*(3), 503-524. http://doi.org/10.22342/jme.v14i3.pp503-524

For quite an extended period of time, school textbooks have been a topic of international research (Fan, 2013; Fan et al., 2018). Textbooks have the potential to be a powerful instrument for assisting students in deepening their understanding of mathematics (Purnomo et al., 2019; Weinberg & Wiesner, 2011). According to Sievert et al. (2019), textbooks play a significant role in the teaching of mathematics because they have an impact on how teachers explain concepts and apply them in the classroom. According to Kang & Kilpatrick (1992), the didactic transposition sequence heavily relies on the use of mathematics textbooks as a tool for maintaining mathematical knowledge. Valverde et al. (2002) explains that textbooks have a significant impact on how teachers explain mathematical topics and use their knowledge of learning trajectories in the classroom. This is one of the key relationships between textbooks and classroom instruction. Therefore, since they are the product of human thoughts, actions, and behaviors recorded in a document for learning mathematics, mathematics textbooks are regarded as important in this context.

Given the importance of textbooks in influencing mathematics teaching in the classroom, mathematics textbooks must be continuously developed. The ultimate goal of textbook research is not textbook development, but rather the production of high-quality textbooks (Yang & Sianturi, 2017). Continuous textbook research is required to make sure that the standard of textbooks keeps rising. According to several studies that examined mathematics textbooks (Wang et al., 2017; Ramelan & Wijaya, 2019; Löwenhielm et al., 2017), each country has a unique approach or emphasis when it comes to teaching mathematics. Many studies in several countries have found different portions in the content of mathematics textbooks such as the findings of textbooks in Indonesia focusing on procedural or algorithms (Wijaya et al., 2015), the findings of mathematics textbooks in Sweden still need to increase opportunities related to problems related to proportional reasoning (Ahl, 2016) and other findings in Turkey by (Bayazit, 2013) revealed that mathematics textbooks in elementary schools support student reasoning. This is in line with the statement that textbooks are part of a curriculum document in a country (Pepin et al., 2001; Bittar, 2022).

In recent years, research on textbooks has been conducted, and various findings have been obtained to improve the quality of mathematics textbooks. By analyzing learning documents, Miyakawa (2017) discovered differences in the properties of proofs taught in textbooks in French and Japanese junior high schools. Different approaches to teaching geometry proofs result in various methods of teaching geometry. Only 10% of the tasks in school textbooks, according to Wijaya et al. (2015), are context-based tasks, and only 2% of contextual tasks are also reflection tasks, which are regarded as having the highest cognitive level task demands. Takeuchi and Shinno (2020) found that an important difference stemmed from the constraint that symmetry and transformations are heavily influenced by the teaching of geometric proofs in Japan, which is not the case in the UK, where transformations have many connections to other contexts or content across domains. Recommendation results from textbook research are important because they can find weaknesses, advantages and produce recommendations for improving and enhancing the quality of mathematics textbooks for classroom learning.

The learning material in the textbook that is of concern in this case is one of the materials that continues to be considered challenging for students to master. In Indonesia, student ability in Geometry content is still regarded as poor, and it has received significant attention in the last ten years. AL-salahat (2022) asserts that one of the foundational subjects of school mathematics instruction is geometry. Geometry also offers a foundation for reasoning and justification (NCTM, 2000) and opportunities to develop cognitive skills, communication, and language comprehension (Cawley et al., 2009). The issue with geometry content in Indonesian junior high schools is with the material of Spatial Geometry Measurement. As an illustration, the average score that can be obtained from the range of 0-100 for the material in the last five years was 44.42 (Kementerian Pendidikan dan Kebudayaan, 2019), as shown in Figure 1. The results of the national examination that were reported by the Center for Educational Assessment, under the Indonesian Ministry of Education and Culture serve as a justification for the analysis of Indonesian junior high school mathematics textbooks, with a specific emphasis on the topic of Measurement of spatial geometry.

Every mathematics textbook undoubtedly contains author-generated ideas that are related to the didactic situation in the classroom. Textbooks are used by teachers in the classroom as knowledge to be taught (Putra, 2020). It depicts ideas in explicit or implicit form. A complex method relating to content knowledge must be justified to understand how ideas are logically connected (Staples & Truxaw, 2009). One of the most important purposes of justification, and perhaps the most widely recognized by teachers, is to contribute to and improve student learning (Dündar & Gündüz, 2017). As a result, the findings of this study require strong justification to be learned from Indonesian mathematics textbooks, which can have an impact on improving the quality of book and impacting the curriculum and learning of mathematics.

Therefore, this study aimed to investigate how praxeological-didactical analysis can enhance mathematics textbook, particularly when analyzing an Indonesian textbook on spatial geometry measurement, task composition, and the identification of task type, technique, technology, learning obstacle, and underlying theories used in the textbook.

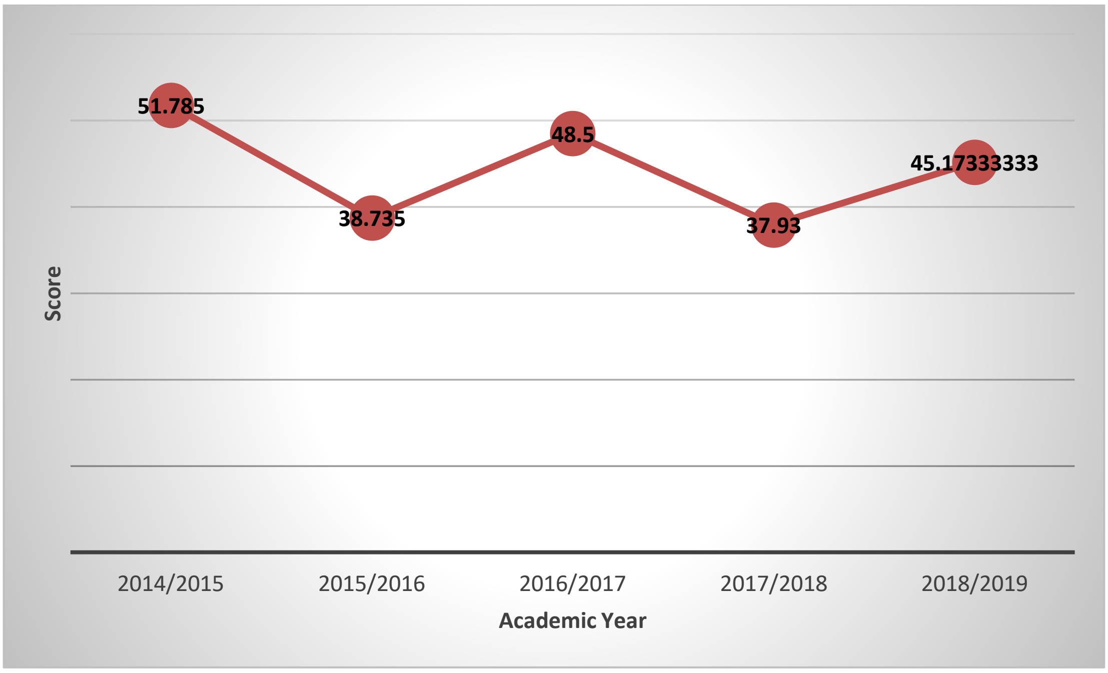


Figure 1. Graph of National Exam scores on spatial geometry measurement materials in Junior High Schools in Indonesia

Theoretical Framework of Praxeological-Didactical Analysis

Praxeological-Didactical Analysis (PDA) is an analytical method for understanding learning and teaching activities in a particular context, which combines two main theoretical approaches, namely Praxeology and Didactics. The idea of the emergence of this PDA stems from the inspiration provided by the theory of mathematical praxeology (Bosch et al., 2005) and didactic praxeology (Chevallard, 1998), as well as the Didactic Situation Theory (Brousseau, 2002). Praxeology can be used to express implicitly written knowledge to be transformed into open-ended knowledge and contexts of reflective action using common sense (Formosinho & Formosinho, 2012). Didactics is the study of the process of teaching and learning. The context of analysis here is related to the use of Praxeology Analysis in Anthropological Theory of the Didactics (Chevallard, 2006; Chevallard, 2007; Chevallard, 2019) and Learning Obstacle Analysis in Didactic Situation Theory (Brousseau, 2002). This analysis included two stages of analysis, first using praxeology to explain the being conveyed by Chevallard (1998). Then, it was followed by analyzing the didactic organization (see Chevallard, 1998), considering both results of praxeological analysis that are related to the content and context in the textbook, to observe the potential the learning obstacles that occurred when a textbook would be transposed didactically in the classroom according to the concept of Brousseau's Theory. This framework was divided into two consecutive parts.

Before looking further into the PDA framework, a different perspective on this method of analysis is required. (Robutti, 2018) introduced meta-didactical praxeology is defined as a theoretical framework utilized to explicate the professional development process of mathematics teachers, involving multiple variables and potential changes that may occur over the course of time in her study. The tasks, techniques, and justification discourses used by researchers and teachers are included in these meta-didactical praxeologies. In simple terms, teachers are introduced to a specific task within their institutional framework, then required to solve the task using a variety of professional development techniques, and the teachers must understand why they chose that solution and be able to justify it. PDA differs from this view in that it justifies epistemological artifacts related to learning and teaching activities in a specific context, looking at the reality and predicting possible learning barriers that students and teachers may experience.

According to Chevallard (2006), the two basic principles of the Anthropological Theory of Didactics (ATD) are that no human action can exist without being and no human doing is unquestionable. This demonstrates how every action we take as humans can take the shape of something that is both explicitly visible to our perceptions and implicitly invisible to them. At the very least, we can question it and offer explanations or justifications for it if it is explainable, understandable, and justifiable. The word being can be interpreted as something material and something invisible immaterial, something that is caught directly perceptually and something that cannot be caught directly perceptually, resulting in two blocks, praxis, and logos. In this study, mathematics textbooks can also be viewed as the result of human action, which can call into question the knowledge that is built because of the ideas of the community about human ways of doing and thinking mathematically.

Mathematics textbooks that are the result of human action can be explained and justified by praxeology. This is conveyed by Chevallard (2006) who states that praxeology is a study conducted in relation to human action and behavior. That implies that praxeology is one of the studies that is not only related to what people do, and how they do it, but also related to what they think, and how they do it. Kleeberg (2019) states that this praxeological approach can be related to the problem of truth that can be proven either historically, discursively or epistemologically where this approach must fulfill two functions, which are related to the norm of truth and assert its validity while being able to see various deviations that exist.

Etymologically, one can analyze every human action with the components or blocks of praxis and logos. Praxis is the practical part and is usually explicit, while logos has been used with stable reference to human thought and reason or can be said to be a theory or as something implicit. Chevallard (2007) explains that praxis is the combination of a type of task with a technique. The praxis part of praxeology contains a type of task T together with a corresponding technique τ (used to perform task $t \in T$ in scope τ). The logos part includes two levels of description and justification: the technology θ, which is a justification of the technique used, and the theory Θ that often unifies several technologies. In short, praxeology, as the basic unit of human activity, is represented by the four parts $[T/\tau/\theta/\Theta]$. The relationship between Praxis and Logos and their development can be seen in Table 1.

Unlike praxeology, which focuses on the study of human actions and behaviors, didactics is a form and tradition of European education that contributes to the way we think about how to plan and develop learning sequences (Duschl et al., 2011). These learning sequences become important in terms of new knowledge production. According to (Chevallard, 1989), academic knowledge is nothing more than knowledge that is applied to the creation of new knowledge as well as the organization of that new knowledge into a coherent theoretical structure. Similarly, ineffective information is often not retained by students (Kang & Kilpatrick, 1992). The didactic process, in this sense, pays attention to the planning and sequence of learning that is continuously developed in order to learn new information, comprehend it, identify particular contexts in mathematics teaching and learning, and anticipate learning obstacles.

Table 1. Praxeological Analysis (Adapted from Chevallard (2006) and Takeuchi & Shinno (2020))

Block Praxis		**Block Logos**	
Type of Tasks (T)	**Technique (τ)**	**Technology (θ)**	**Theory (Θ)**
Type of task (a problem that has a specific goal to be solved).	A way for someone to act in completing a given type of task.	A reason for performing the technique.	The basis or reference of the existence of Technology.
Development of Praxeology Analysis for Textbook			
Textbook Praxeological Analysis (TPA)			
Block Praxis		**Block Logos**	
Type of Tasks (T)	**Technique (τ)**	**Technology (θ)**	**Theory (Θ)**
Types of student tasks given in the textbook (problems that students need to solve)	Possible ways for students to solve the type of task given (solution of the problem given in the textbook)	Justification of the ways in which students complete the tasks in the textbook.	Reliable and reasonable basis/reference for justification of the ways in which students complete the tasks in the textbooks

Learning obstacles in Brousseau's Didactic Situation Theory are divided into three obstacles that include epistemological, ontogenetic, and didactic obstacles. Bachelard became an initiator of ideas related to "epistemological obstacles" that appear as a basis for considering the problem of scientific knowledge (Brousseau, 2002). Brousseau (2002) suggests that epistemological obstacles are proven by errors in the answers given by students in response to selected tasks and questions. The errors in question are not accidental errors. These errors can be caused by the limited concepts introduced so that students tend to follow what has been previously exemplified. This error is called a context error. Limited context in didactic design can lead to epistemological obstacles (Suryadi, 2019a). Therefore, in mathematics textbooks, epistemological obstacles may occur for students where there is a context presented and it will be shown whether the mathematics textbook is epistemic or not through praxeology analysis.

Obstacles caused by factors of sequence and or stages of presentation are referred to as didactical obstacles (Suryadi, 2019b). Didactical obstacles can emerge because of instructional choices (Brown, 2008). These obstacles by Brousseau (2002) have three possible origins, namely ontogenetic origin, which lies in the cognitive abilities of students associated with the limitations of the cognitive development of students; didactical origin, which originates from the teaching strategies used in supporting the learning of certain mathematical ideas (in this case the sequence in the mathematics textbook); and epistemological origin, which occurs when the understanding of certain mathematical concepts of students interferes with the understanding of more complex mathematical concepts. The sequence and stages of the curriculum, as well as how it is presented in the classroom, can result in didactic obstacles (Suryadi, 2019b). Therefore, the concepts written in the textbook can be viewed with a praxeology-didactic analysis of the sequence of presentation so that a learning trajectory is formed that allows students to succeed in learning mathematics, or instead, there is a possibility of a learning trajectory that hinders students in learning.

Ontogenetic obstacles arise because of students' developmental limitations Brousseau (2002). Students inherit parts of their physiology with an innate nervous system from birth. This barrier is related to a person's learning stages and modes of development (accommodation and assimilation). Suryadi (2019a) emphasizes that this ontogenetic obstacle is related to the learning difficulties of student related to their readiness to learn. Suryadi (2016) further explains that ontogenetic obstacles are classified into three types: psychological ontogenetic obstacles, instrumental ontogenetic obstacles, and conceptual ontogenetic obstacles. Student unpreparedness resulting from a conceptual level mismatch between the learning design and the child's psychological state as observed from prior learning experiences is one example of an onto genic obstacle that can cause frustration and a loss of learning orientation.

METHODS

This study used Chevallard's Anthropological Theory of Didactics (ATD) approach and Brousseau's Learning Obstacle in Didactic Situation Theory. Every human action, according to ATD, can be broken down into a series of different task types (Chevallard & Bosch, 2020). The task types listed here are also found in the textbook. As a result, the textbook in question can be examined using the praxeology method because it is a product of human action or activity.

The praxeology analysis here is based on Chevallard's ideas. Regarding the results of human action, the praxeology analysis in ATD enables us to reasonably draw connections between institutional and individual praxeologies (Bosch & Gascón, 2014). The institution here is the government, and the person is the author of the mathematics textbook. The existence of written mathematics books is primarily characterized by the interrelationships between individuals and institutions (Chevallard & Bosch, 2020). A book is the product of a person's writing that was inspired by the interaction between a writing community and an educational institution for didactic purposes.

Furthermore, didactical analysis here refers to the analysis conducted by examining three key elements in the learning context: student activities, knowledge required by students, and teacher instructions according to Brousseau's Didactical Situation Theory. As scholarly knowledge, the book provides task instructions that include the three key elements that can be uncovered by praxeological analysis and followed by didactical analysis, here abbreviated as praxeological-didactical analysis (PDA). PDA is used to identify difficulties or obstacles in learning and teaching, and to formulate recommendations or improvement strategies. The PDA framework related to the textbook analyzed can be seen in Figure 2.

The textbook here, which is the result of human action, can be observed and understood through direct realism. Direct realism draws experience from the textbook being studied by consciously using our perceptual faculties (Pritchard, 2013). The analyzed textbook was created by an institution, in this case the Ministry of Education, Culture, Research, and Technology (Kemendikbudristek), and it was written by Tosho (2021) as a Mathematics for Junior High School 1st Level book for teachers' guidance. In 2022, the Ministry of Education, Culture, Research, and Technology launched the Independent Curriculum in Indonesia (Kemendikbudristek, 2022). The selected Mathematics book is one written for the 2022 Independent Curriculum at the Junior High School Level. The Indonesian Ministry of Education, Culture, Research, and Technology released the first edition of this Mathematics textbook in 2021. The chosen material was the topic of Spatial Geometry Measurement, which has four themes as shown in Table 2. Each Theme was examined for the type of task contained within it and was followed by an examination of the technique used to complete the task. The reason for selecting the technique, in this case technology, can be used to explain the technique. The technology was drawn into a conclusion that is referred to as Theory and was used as a basis for the reasons made.

Table 2. Topic focus on measurement of spatial figures

Topic Chapter	Sub Chapter	Objective
Measurement of Spatial Figures	Surface Area of Spatial Figures (7th grade, 7 pages)	1. Can find the surface area of pyramids and cones based on the nets. 2. Understand how to calculate the lateral surface area of a cone based on the sector property. 3. Can find the surface area of pyramids and cones.
	Volume of Spatial Figures (7th grade, 3 pages)	1. Understand how to determine the volume of prisms and cones based on observations and experiments. 2. Able to find the volume of cylinders and cones.
	Surface Area of a Sphere (7th grade, 6 pages)	1. Understand how to determine the surface area and volume of a sphere based on observation and experimentation. 2. Can calculate the surface area and volume of a sphere using equations."
	Comparing Volume and Surface Area (7th grade, 2 pages)	1. Students can use the formula for volume and surface area of solid objects to find the volume and surface area of objects around them.

The mathematics textbook analyzed was the Mathematics Textbook for Teachers of Grade VII Junior High School. The first step was to write down the type of task **(T)** from the example given. Next, the existing technique **(τ)** was identified through the example given to solve the problem in the book. The technique **(τ)** used to solve the type of task (**T**) in the book became the technique **(τ)** that students were most likely to use in solving the given exercise problems. There are bases or reasons for the selection of the chosen technique **(τ)**, which is called Technology **(θ)**. Technology **(θ)** is here to justify the existence of Technique **(τ)**. Then, from the Technology **(θ)** there is a reference or basis for why the reason was taken and is referred to as Theory **(Θ).**

The analysis of mathematics textbooks began with the selection of books according to the chosen topic, namely the topic of spatial figure measurement. Figure 2 shows the flow of PDA. Praxis here is described as a task that must be completed. The type of task given was done with techniques, which were ways that students might be able to use to complete the given task. Logos includes two parts, namely technology and theory. After the technique was chosen, the technology was needed, which was justified by the rational reasoning shown in the mathematics textbook. The basis or reference of the technology used is called theory. Following that, an examination of three key elements: student activities, knowledge required by students, and teacher instructions in the textbook involved the results of the disclosure of praxis and logos to produce a picture of learning obstacles that resulted in recommendations for improvement. This PDA was created with the intention of continuously enhancing the book's quality. Figure 2 shows the PDA framework depicting this sequence of steps.

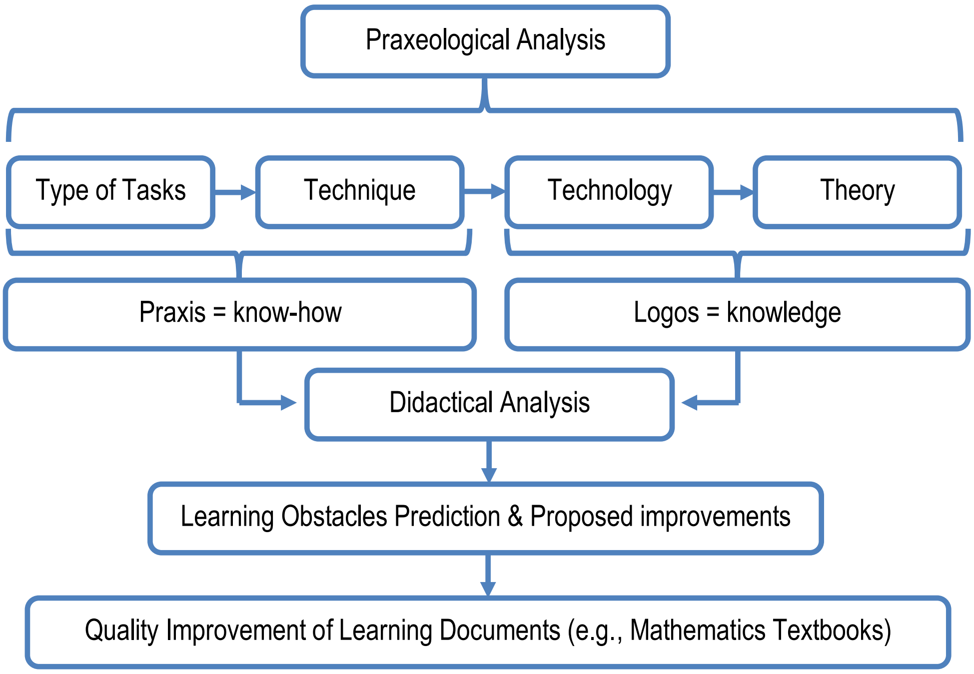


Figure 2. Praxeological-Didactical Analysis (PDA) Framework
(Adapted from Chevallard (2006) and Brousseau (2002))

Data Analysis

The data analysis procedure in this study will be conducted into two main stages. The first stage is related to the praxeological analysis. The first step of the praxeological analysis involves selecting mathematics textbook and determining the focus of the research topic. This is followed by including task design in a praxeological table by the researcher. The second step involves coding the identified findings based on the theory. The third step determines the data reliability through theoretical triangulation. Moving on to the second stage, didactical analysis is obtained as follows: the first part involves the researcher analyzing the didactic organization based on the findings of the praxeological analysis, specifically examining the organization of task type and other factors related to potential learning obstacle that may arise during mathematics learning; the second part involves studying the content and context of the textbook material, which is still linked to the first part, with the continued objective of identifying potential learning obstacles. Furthermore, to ensure the validity of all data, the data correspondence principle is employed in inter-rater analysis, and the inference process is guided by the principle of coherence.

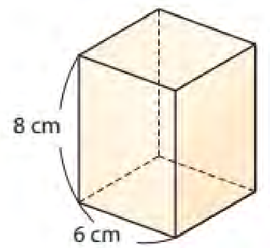
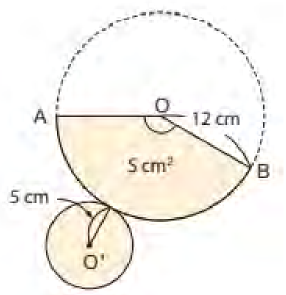
RESULTS AND DISCUSSION

The Praxis: Type of Tasks

All types of tasks listed in the mathematics textbook on measurement of spatial figures were included in this analysis. The task types here are represented by questions (problems) posed from the introductory part of the book to the exercise questions at the end of the material. The purpose of all problems that have become task types could be analyzed to see the sequence and stages of their presentation in the classroom thus identifying if there are didactical obstacles in them. The textbook chosen was the textbook that the teacher used for teaching and therefore had been shown how to complete the task. This was chosen with the aim of finding didactical situations in the math textbook.

The type of task was denoted with the **T** symbol. The labeling and coding of the type of task was made based on the data results in Table 3. All task types denoted by **T** have been sorted according to their first occurrence in the textbook. The book contained several questions that were described as tasks for the students to complete. The number of task types discovered was ten, labeled **T_1** to **T_{10}**, for a total of 57 task types, as shown in Table 3. The task types **T_1** and **T_9** were the most frequently encountered in the textbook and were, in fact, the primary topics addressed in the material on the measurement of spatial figures. According to Wijayanti and Winsløw (2017), the two dominant task types, **T_1** and **T_9**, account for most of the curriculum that is "realized" in Indonesian schools. As a result, these task types are the primary subject matter covered in the analyzed materials.

Table 3. Type and number of tasks for measurements in spatial geometry

Type of Tasks	**Number of Tasks (*n* = 57)**
T_1: Determining the surface area of a spatial figure. *Example:* "Calculate the surface area of the following spatial figure."  	15 (26,3%)
T_2: Determine the ratio of two volumes of a spatial figure	6 (10,5%)
T_3: ıdentify the parts of a spatial figure (base, lateral surface)	2 (3,5%)
T_4: Calculate the area of the base, the area of the lateral and/or surface of the lateral	8 (14%)
T_5: Calculate the circumference length and area of a circle	2 (3,5%)
T_6: Identify the area of a sector	3 (5,3%)
T_7: Calculate the area of a sector *Example:* "Calculate the area of the OAB section if the internal angle is 150°"  	5 (8,8%)
T_8: Calculate the length of arc	2 (3,5%)
T_9: Calculate the volume of spatial figure	13 (22,8%)
T_{10}: Compare Surface Area to Lateral Surface Area	1 (1,8%)

The mathematics textbook is a crucial component of education and has been given a learning path in an independent curriculum. According to Kajander and Lovric (2009), textbooks are a crucial tool that can influence how math teachers teach and how students learn the subject. In addition, Pansell and Boistrup (2018) acknowledge that the use of mathematics textbooks can affect how math is taught. Each type of the task in the textbooks does, in fact, have an impact on how the teacher teaches in the classroom. In mathematics textbooks, each task type is assigned a different weight. There is a flow that the author of the textbook has presented, as evidenced by the composition of the task types in order. The order of the composition of the task types shows that there is a flow presented in the textbook by the author. The sequence flow presented in Table 4 gives an overview of the sequence of task types given in this textbook. It flows based on the sequence of task types, with the first identified type emerging and presented in a linear form.

Table 4. Identification of task type composition sequence in textbook

No.	Topics	Pages	Sequence of the Type of Task
1.	Introduction to Measurement of Spatial Figures	213	$T_1 \rightarrow T_2$
2.	Topic 1: Surface Area of Spatial Figures	214-220	$T_3 \rightarrow T_1 \rightarrow T_4 \rightarrow T_1 \rightarrow T_5 \rightarrow T_1 \rightarrow T_5 \rightarrow T_1 \rightarrow T_6 \rightarrow T_7 \rightarrow T_8 \rightarrow T_6 \rightarrow T_7 \rightarrow T_4 \rightarrow T_1$
3.	Topic 2: Volume of Spatial Figures	221-223	T_9
4.	Topic 3: Surface Area and Volume of a Sphere	224-229	$T_1 \rightarrow T_9 \rightarrow T_8 \rightarrow T_7 \rightarrow T_4 \rightarrow T_9 \rightarrow T_1 \rightarrow T_9$
5.	Topic 4: Comparing Volume and Surface Area	230	T_2

Some task types are easier to work on because they only require one concept to be linked to be completed, whereas others require multiple concept links. Each task type is defined in terms of various levels. Each task type is given a level, with level 1 being the simplest, followed by levels 2, 3, and so on, each of which denotes a higher level. The levels identified are only up to level 3, and the explanation is as follows.

Table 5. Textbook task type load

	Level of Tasks		
	Level 1 (T_n-1)	Level 2 (T_n-2)	Level 3 (T_n-3)
T_1:	4 (27%)	11 (73%)	0
T_2:	1 (17%)	2 (33%)	3 (50%)
T_3:	2 (100%)	0	0
T_4:	6 (75%)	1 (25%)	1 (25%)
T_5:	2 (100%)	0	0
T_6:	3 (100%)	0	0
T_7:	4 (80%)	1 (20%)	0
T_8:	1 (50%)	1 (50%)	0
T_9:	7 (54%)	4 (31%)	2 (15%)
T_{10}:	0	0	1 (100%)
Total:	30 (53%)	20 (35%)	7 (12%)

The updated **T_2-1** coding indicates that this includes the task type is **T_2** Level 1, which denotes that it is the simplest type of task and only involves one concept or one step in solving the problem. **T_2-2** denotes that this is a Type of Task **T_2** Level 2 and that it links two ideas or requires two steps to complete. **T_2-3** denotes a type of task **T_2** Level 3 that links three concepts or three solution steps in completing the task. In this analysis, it is possible to find classifications of task types **T** at level 3 and above, but here, we do not find task types **T** that are at level 4 and beyond. Table 5 displays the distribution of task type load throughout the textbook.

The Praxis: Technique

Each type of task requires an appropriate technique as a solution, especially in geometry. Duval (1995) distinguishes four cognitive apprehensions techniques for the solution of geometric drawing problems: operative, perceptual, discursive, and sequential. There is no theoretical link in this case related to Duval's research, but the existing category can be considered for adaptation. On the other hand, Takeuchi and Shinno (2020) chose four techniques in solving symmetry and transformations in geometry: perceptual, operational, algebraic, and physical. The techniques chosen by Takeuchi and Shinno can also be adapted because they are relevant to geometry problems. Owen (1991) uses an algebraic approach in solving geometry problems, which uses ordinary arithmetic operations plus square roots. Solis and Isoda (2022) mentioned that there are seven techniques used to solve tasks related to length measurement topics in textbook analysis using praxeology analysis, namely: physical, transitive, iteration, partition, operational, instrumental, and conservation.

Table 6. Techniques for each type of task regarding length measurement

Task	Technique	Description of Technique
T_1:	**τ_1: Perceptual** **τ_2: Memorial** **τ_3: Operational** **τ_4: Logical** **τ_5: Instrumental** **τ_6: Partitional**	**τ_1**: Utilize visual ability to identify the size of geometric objects **τ_2**: Utilize memory skills to remember the parts of a geometric figure or the formula for measuring a geometric figure **τ_3**: Using addition, subtraction, multiplication and division operations or using calculation algorithms **τ_4**: Using logical reasoning to relate the solution to the information provided **τ_5**: Using tools to add new shapes **τ_6**: Dividing the main figure into relevant parts
T_2:	**τ_1: Perceptual** **τ_2: Memorial** **τ_3: Operational** **τ_4: Logical** **τ_7: Analytical**	**τ_1**: Utilize visual ability to identify the size of geometric objects **τ_2**: Utilize memory skills to remember the parts of a geometric figure or the formula for measuring a geometric figure **τ_3**: Using addition, subtraction, multiplication and division operations or using calculation algorithms **τ_4**: Using logical reasoning to relate the solution to the information provided **τ_7**: Using analytical skills to examine or subordinate a problem in detail with logical steps.
T_3:	**τ_2: Memorial**	**τ_2**: Utilize memory skills to remember the parts of a geometric figure or the formula for measuring a geometric figure
T_4:	**τ_1: Perceptual** **τ_2: Memorial**	**τ_1**: Utilize visual ability to identify the size of geometric objects **τ_2**: Utilize memory skills to remember the parts of a geometric figure or the formula for measuring a geometric figure

	τ_3: **Operational**	τ_3: Using addition, subtraction, multiplication and division operations or using calculation algorithms
T5:	τ_2: **Memorial** τ_3: **Operational**	τ_2: Utilize memory skills to remember the parts of a geometric figure or the formula for measuring a geometric figure τ_3: Using addition, subtraction, multiplication and division operations or using calculation algorithms
T6:	τ_2: **Memorial** τ_4: **Logical**	τ_2: Utilize memory skills to remember the parts of a geometric figure or the formula for measuring a geometric figure τ_4: Using logical reasoning to relate the solution to the information provided
T7:	τ_2: **Memorial** τ_3: **Operational**	τ_2: Utilize memory skills to remember the parts of a geometric figure or the formula for measuring a geometric figure τ_3: Using addition, subtraction, multiplication and division operations or using calculation algorithms
T8:	τ_2: **Memorial** τ_3: **Operational**	τ_2: Utilize memory skills to remember the parts of a geometric figure or the formula for measuring a geometric figure τ_3: Using addition, subtraction, multiplication and division operations or using calculation algorithms
T9:	τ_1: **Perceptual** τ_2: **Memorial** τ_3: **Operational** τ_4: **Logical** τ_6: **Partitional**	τ_1: Utilize visual ability to identify the size of geometric objects τ_2: Utilize memory skills to remember the parts of a geometric figure or the formula for measuring a geometric figure τ_3: Using addition, subtraction, multiplication and division operations or using calculation algorithms τ_4: Using logical reasoning to relate the solution to the information provided τ_6: Divide the main figure into relevant parts
T10:	τ_2: **Memorial** τ_3: **Operational** τ_4: **Logical**	τ_2: Utilize memory skills to remember the parts of a geometric figure or the formula for measuring a geometric figure τ_3: Using addition, subtraction, multiplication and division operations or using calculation algorithms τ_4: Using logical reasoning to relate the solution to the information provided

The existing identification results are the result of categorization, some of which adopt from existing references and select other techniques that are deemed suitable for use in completing the task. When the task can be completed using the five senses, particularly the visual ability to recognize the given spatial figure, perceptual techniques are used. Memorial techniques are used to recall memories stored in the mind, such as remembering the formula for the surface area of a spatial figure. The operational technique referred to here is the use of arithmetic operations in operational problem solving. Logical technique is a technique to reason logically from the type of task requested. Instrumental techniques are performed when problem solving requires instrumental tools such as rulers, geometric aids, and others. Partitional techniques are done when the problem solving must be made in the form of partitions or separations in order to make it easier to solve the problem. Analytical techniques are used when the task type must connect at least two concepts that are systematic and interrelated. Thus, the techniques used to solve the types of tasks related to the measurement of spatial geometry are perceptual, memorial, operational, logical, instrumental, partitional and analytical as indicated by the information in Table 6.

The Logos: Technology and Theory

Technology in this context refers to the justification behind the technique selected for each type of task, whereas theory in this context refers to the theoretical basis of the technique. For example, all **T₁** task type were solved using techniques $\boldsymbol{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6}$ with technologies $\boldsymbol{\theta_1}$ dan $\boldsymbol{\theta_2}$, and all of which refer to the $\boldsymbol{\Theta_1}$theory.

Table 7. Textbook praxeology analysis result

Type of Task (T)	Technique (τ)	Technology ($\boldsymbol{\theta}$)	Theory ($\boldsymbol{\Theta}$)
T_1: Determining the surface area of a spatial figure	τ_1: Perceptual τ_2: Memorial τ_3: Operational τ_4: Logical τ_5: Instrumental τ_6: Partitional	θ_1: Application of the Concept of Calculating the Area of Spatial Geometric Figures	Θ_1: Surface Area of Spatial Geometric Figures
T_2: Determine the ratio of two volumes of a spatial figure	τ_1: Perceptual τ_2: Memorial τ_3: Operational τ_4: Logical τ_7: Analytical		
T_3: Identify the parts of a spatial figure (base, lateral surface)	τ_2: Memorial		
T_4: Calculate the area of the base, the area of the lateral and/or surface of the lateral	τ_1: Perceptual τ_2: Memorial τ_3: Operational		
T_5: Calculate the circumference length and the area of a circle	τ_2: Memorial τ_3: Operational		
T_6: Identify the area of a sector	τ_2: Memorial τ_4: Logical	θ_2: Application of the Concept of Calculating the Volume of Spatial Geometric Figures	Θ_2: Volume of Spatial Geometric Figures
T_7: Calculate the area of a sector	τ_2: Memorial τ_3: Operational		
T_8: Calculate the length of arc	τ_2: Memorial τ_3: Operational		
T_9: Calculate the volume of spatial figure	τ_1: Perceptual τ_2: Memorial τ_3: Operational τ_4: Logical τ_6: Partitional	θ_3: There are systematic connections in geometric figures	
T_{10}: Compare Surface Area to Lateral Surface Area	τ_2: Memorial τ_3: Operational τ_4: Logical		

There is a link here that can be justified from what is in the textbook. When the tasks are solved, for example **T₁**, there are six techniques that can be used $(\boldsymbol{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6})$. This shows that in completing the task, there are several possible techniques used, especially in calculating the surface area of a spatial figure. Based on the existing Praxis, technology is needed as the reason for the selection of the technique. Every identified technique mentioned here is supported by technology, where the use of each technique always leads to the utilization of the corresponding technology. Furthermore, there are three possible technologies that can be used, namely θ_1, θ_2 and θ_3. Then as a basis for justification of the technology chosen for **T**$_1$, the $\Theta_1$theory is used.

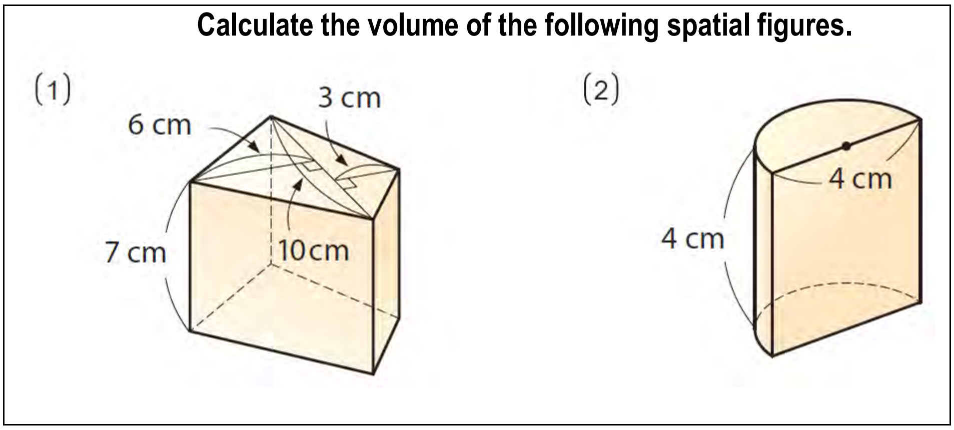


Figure 3. Overview of **T**$_9$ Task Type

As an example of a problem related to Table 6 and Table 7, task type **T**$_9$ is presented. Task type **T**$_9$ as shown in Figure 3 in the textbook is solved by the technique τ_1 by using sense of sight to observe the size of the figure provided, then using τ_2 by remembering the formula for finding the base area and the volume formula. Afterwards, proceed with τ_3 to solve it. Furthermore, the technology here is the justification of the chosen technique θ_2. The basis of the chosen justification (technology) is theory. The theory used is Θ_2. Based on this description, it is clearly shown that through praxeology it appears that the proof is epistemic.

The use of images in the type of tasks given provides students with a conscious experience of geometric objects around them. This image representation is the most obvious evidence as a psychological phenomenon to provide a fair and adequate conception (Vergnaud, 2009). In another case by Szilágyi et al. (2013), he stated that this representation is important so that students can understand the intent of the given problem, in this case, the type of task given.

This representation has a relation to the praxeology analysis that was conducted. Table 3 and Table 4 give an overview of the composition of the tasks given and their percentages. **T**$_9$ and **T**$_9$ are the most common task types that arise because of this material related to volume and surface area. Table 5 shows that the learning load on each task type varies; however, it is believed that a graded distribution of evenly distributed task type load may reduce the psychological impact on students when learning this material. This requires justification, and additional research can be conducted to determine the impact of an even distribution of task type load on students. The various learning paths that can be offered to students are shown in Table 7. Theoretical explanations are available for each Praxis as human action.

Didactical Analysis Results

According to (Bosch et al., 2017), praxeology analysis can be used to question how mathematics is taught in textbooks and how it solves didactic issues. The occurrence of didactical problems will be investigated further using didactical analysis. Suryadi (2019b) stated that the order and or stages of material presentation, as well as Brown (2008) related to the selection of instructional design, could be one of the causes of the didactic obstacles discussed here.

Three spatial figures are shown in the image to the right.

$\boxed{a}$ = A cone with a base of radius 5 cm, and a height of 10 cm.

$\boxed{b}$ = A sphere with a radius of 5 cm.

$\boxed{c}$ = A cylinder with a base of radius 5 cm and a height of 10 cm

1. The area of the entire surface of $\boxed{b}$ is equal to the spherical lateral surface area of $\boxed{c}$. Determine the surface area of $\boxed{b}$. The circumference ratio is 3,14.
2. Spatial figures $\boxed{a}$ and $\boxed{b}$ are put into $\boxed{c}$ as shown in the figure above. Determine the ratio of the volumes of each of these solid objects.

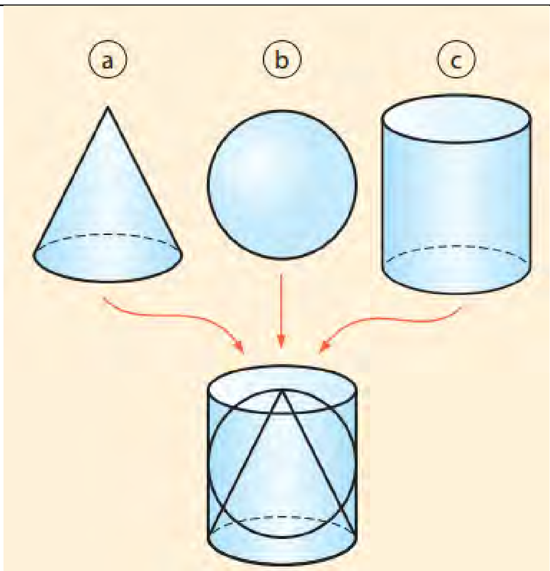


Figure 4. The first task given in the textbook on the material of measurement of spatial figures

The PDA results from Figure 4 provide a clear picture of Praxis and Logos. This task is the first task given in the introduction to the measurement of spatial figures in the textbook analyzed as shown in Figure 4. The tasks given for the first time fall into the categories of task types **T**$_1$ and **T**$_2$**.** Task type **T**$_1$ in this section is the first task with task completion techniques $\boldsymbol{\tau_1}, \boldsymbol{\tau_2},$ and $\boldsymbol{\tau_3}$. The technology used is $\boldsymbol{\theta_1}$ and the theory used is $\boldsymbol{\Theta_1}$. Then, for task type **T**$_2$ shown in Figure 4, through the purple box is solved with the techniques $\boldsymbol{\tau_1}, \boldsymbol{\tau_2}, \boldsymbol{\tau_3}, \boldsymbol{\tau_4}$ and $\boldsymbol{\tau_7}$. The technologies used are $\boldsymbol{\theta_1}$ and $\boldsymbol{\theta_3}$, while the theory chosen is $\boldsymbol{\Theta_2}$.

Didactical obstacles are indicated as shown in Figure 4 and are discussed here in more detail. First, the didactic root of this error is discussed, which is demonstrated in the textbook's teaching strategy of selecting a task that seems challenging for students, with **T**$_1$ being a level 2 task (T$_1$-2) that relates two concepts in solving, and **T**$_2$ being a level 3 task (T$_1$-3). Setting these tasks as a problem at the start of the curriculum may result in ontogenetic obstacles that affect students' readiness for learning (Prabowo et al., 2022). Students may become bored and disinterested due to the challenging order of the materials and tasks at the beginning, which can make them experience difficulties, according to (Lutfi et al., 2021). According to Suryadi (2019a), these obstacles can lower students' motivation and interest in the subject matter because they are psychological, practical, and conceptual.

The second part of this discussion is related to Figure 4, where the blue box indicating that *c* is a cylinder differs from the mention in task **T**$_1$ in the red box indicating that *c* is a sphere. This is supplemented by an explanation of the answer to the question where the task is to determine the surface area of *b*, which is a sphere, but in the textbook answer it is written how to determine the surface area of a cylinder. The red and green circles in Figure 5 is proof that the formula used with the provided picture is incorrect and that it is a cylinder. If the answer given is incorrect for the task, this can lead to an epistemological obstacle. Both the teacher and the students may be mistaken about the concepts being introduced by incorrectly typing symbols and writing answers. This can interfere with students' understanding of mathematical concepts in understanding more complex mathematics (Suryadi, 2019b) and this can be found in students who tend to follow what their teachers exemplify (Brousseau, 2002).

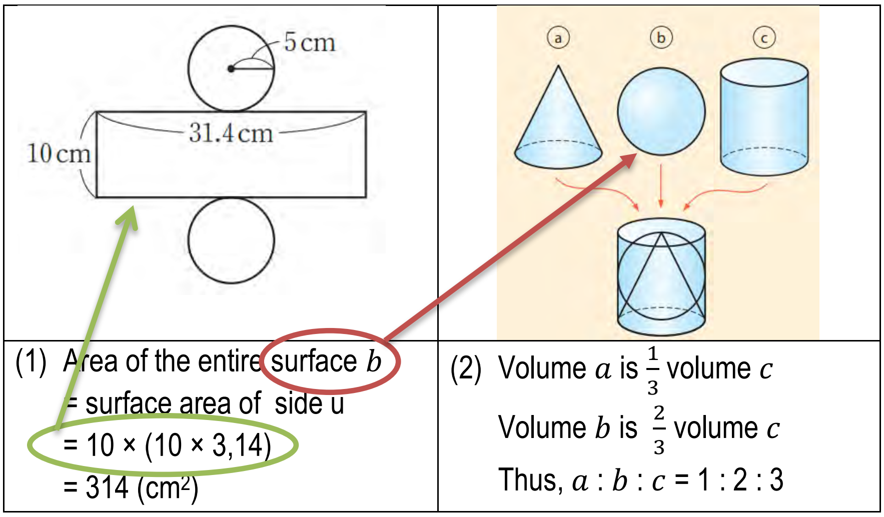
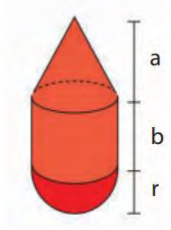


Figure 5. Answers to Problems with Task Types T_1 and T_2 according to Figure 4

The third section deals with the didactic obstacles that arise when the teaching techniques employed do not support the progression from easy to difficult in accordance with the suggested learning trajectory. According to Table 4 and Table 5, the sequence of task types demonstrates that students may struggle with a task that has a heavy load at the beginning. The proposal presented here has a consistent progression of task type distribution from the most basic to the most complex. Observing the connections, the arrangement of the learning trajectory provided as a teacher's strategy in teaching spatial figure measurement can create didactic obstacles. It was discovered here that the order of the material, starting with the most difficult part first, as well as the writing errors and answers provided, can exacerbate the didactical obstacle.

Determine the surface area and volume of the following spatial figure.



Textbook answer: $\pi r^2(\frac{2}{3}r + b + \frac{1}{3}a)$

Figure 6. Questions and answers on page 228

The findings regarding the existence of textbook answers that did not correspond to the type of task given are discussed in this section. According to Figure 6, this is a problem presented in the exercise that falls into the task types T_1 and T_9. A task type is presented, but the method for completing it is τ_3, where there are two task types but only one written response. Due to the limited context used, this may cause readers—whether teachers or students—to encounter epistemological obstacles that may interfere with their ability to understand mathematical concepts. In this context, improvements to this book's answers are required. According to Brousseau (2002), this improvement will be able to prevent students from giving incorrect answers when handling tasks. Suryadi (2019a) added that mistakes may result from the few concepts that are initially taught because students tend to mimic what has already been demonstrated.

Finally, this PDA research attempted to reveal the knowledge contained in the textbook as part of the didactic transposition process in the classroom. This study differs from that of Robutti (2018), who focuses on meta-didactical praxeologies. Another study, Giménez et al. (2013), discusses didactical analysis but is intended for the research process related to the final assignment of prospective secondary school mathematics teachers in performing their six professional tasks. This approach differs from the one used by Putra et al. (2021), who evaluated the use of mathematics comics through praxeological analysis. Additionally, this study differs from Miyakawa (2017) and Putra (2020), who were more concerned with didactic transposition and focused on analyzing the praxeology of mathematics textbooks, particularly task types and techniques. Both studies analyzed curriculum documents such as mathematics textbooks. The study can also be said to be more advanced by further examining didactical analysis compared to what has been done by Solis and Isoda (2022), Takeuchi and Shinno (2020), Wijayanti and Winsløw (2017), and Hendriyanto et al. (2023), who have only worked on analyzing the praxeology of mathematics textbooks in a particular country.

This study of mathematics textbooks reveals that a book has a distinct learning trajectory that is compiled by the author of the book as part of the anthropology of community members developed in Indonesia. However, the examined Indonesian mathematics textbooks contained ontogenetic, epistemological, and didactic obstacles. Following the Praxeological-Didactical Analysis, this research offers recommendations for improving these textbooks, but it is acknowledged that this analysis based on the limitations of the analytical skills possessed by the researchers of this study. It is possible that immaterial aspects of the word being proposed by Chevallard were not expressed in accordance with the author's intent and purpose when writing this book, either intentionally or unintentionally. Further study is necessary to provide stronger support for proposals regarding the composition of task types with gradations related to the level of tasks provided. These proposals are still at the stage of proposals based on existing references.

CONCLUSION

Through Praxeological-Didactical Analysis, this study provides a comprehensive analysis to improve the quality of mathematics textbooks. The analyzed Indonesian mathematics textbook on spatial geometry measurement includes lesson on the importance of sequential composition of task sequences with a more leveled and organized learning load level. The results of this PDA show that there are 10 types of tasks found, and 7 techniques that are predicted to be used in completing the tasks given in the mathematics textbook are perceptual, memorial, operational, logical, instrumental, partitional, and analytical. Then, there are three technologies used to justify the chosen techniques, as well as two theories that underpin the technological justification. We can derive PDA learning from Indonesian mathematics textbooks, which contain sections that can pose epistemological, ontogenetic, and didactic obstacles. Users of textbooks, particularly Indonesian math teachers, must exercise caution when implementing the tasks provided as part of didactic transposition. All the outcomes of this PDA are expected to be used for the development and improvement of existing mathematics textbooks.

Acknowledgments

The authors would like to thank Pusat Layanan Pembiayaan Pendidikan (Puslapdik) and Lembaga

Pengelola Dana Pendidikan (LPDP) for supporting and funding the publication incentive as part of the privilege of being Beasiswa Pendidikan Indonesia awardee. In addition, we would like to thank Universitas Pendidikan Indonesia for guiding and collaborating in the publication of the research results.

Declarations

Author Contribution	: TNHY: Conceptualization, Writing – Original Draft, Methodology, Visualization, Writing – Review and Editing. DS: Methodology, Formal Analysis and Supervision. DD: Conceptualization, and Formal Analysis. TH: Validation and Supervision.
Funding Statement	: This research was funded by the Education Financing Service Center of Education Fund Management Institute is a part of the Ministry of Research, Technology and Higher Education of the Republic of Indonesia for supporting and funding this research.
Conflict of Interest	: The authors declare no conflict of interest.
Additional Information	: Additional information is available for this paper.

REFERENCES

Ahl, L. M. (2016). Research Findings' Impact on the Representation of Proportional Reasoning in Swedish Mathematics Textbooks. *REDIMAT*, *5*(2), 180–204. https://doi.org/10.4471/redimat.2016.1987

AL-salahat, M. M. S. (2022). The effect of using concrete-representational-abstract sequence in teaching the perimeter of geometric shapes for students with learning disabilities. . *International Journal of Education in Mathematics, Science, and Technology (IJEMST)*, *10*(2), 477–493. https://doi.org/10.46328/ijemst.2403

Bayazit, I. (2013). Quality of the tasks in the new Turkish elementary mathematics textbooks: The case of proportional reasoning. *International Journal of Science and Mathematics Education*, *11*(3), 651–682. https://doi.org/10.1007/s10763-012-9358-8

Bittar, M. (2022). A Methodological Proposal for Textbook Analysis. *Mathematics Enthusiast*, *19*(2), 307–340. https://doi.org/10.54870/1551-3440.1555

Bosch, M., Chevallard, Y., & Gascón, J. (2005). Science or magic? The use of models and theories in didactics of mathematics. Proceedings of CERME4.

Bosch, M., & Gascón, J. (2014). Introduction to the Anthropological Theory of the Didactic (ATD). In *Networking of Theories as a Research Practice in Mathematics Education* (pp. 67–83). Springer International Publishing Switzerland. https://doi.org/10.1007/978-3-319-05389-9_5

Bosch, M., Gascón, J., & Trigueros, M. (2017). Dialogue between theories interpreted as research praxeologies: the case of APOS and the ATD. *Educational Studies in Mathematics*, *95*(1), 39–52. https://doi.org/10.1007/s10649-016-9734-3

Brousseau, G. (2002). *Theory of Didactical Situations in Mathematics*. Kluwer Academi Publisher.

Brown, S. A. (2008). Exploring Epistemological Obstacles to the Development of Mathematics Induction. *Proceedings of the 11th Conference for Research on Undergraduate Mathematics Education* , 1–19. https://www.researchgate.net/publication/254644126

Cawley, J. F., Foley, T. E., & Hayes, A. M. (2009). Geometry and Measurement: A Discussion of Status and Content Options for Elementary School Students with Learning Disabilities. *Learning Disabilities: A Contemporary Journal*, *7*(1), 21–42.

Chevallard, Y. (1989). On didactic transposition theory: Some introductory notes. *Proceedings of the International Symposium on Selected Domains of Research and Development in Mathematics Education*, 51–62.

Chevallard, Y. (1998). Analyse des pratiques enseignantes et didactique des mathématiques: l'approche anthropologique. Actes de l'UE de la Rochelle, 91-118.

Chevallard, Y. (2006). Steps towards a new epistemology in mathematics education. *Proceedings of the IV Congress of the European Society for Research in Mathematics Education*, 21–30.

Chevallard, Y. (2007). Readjusting didactics to a changing epistemiology. *European Educational Research Journal*, *6*(2), 131–134. https://doi.org/10.2304/eerj.2007.6.2.131

Chevallard, Y. (2019). Introducing the anthropological theory of the didactic: An attempt at a principled approach. *Hiroshima Journal of Mathematics Education*, *12*(1), 71–114.

Chevallard, Y., & Bosch, M. (2020). Didactic Transposition in Mathematics Education. In *Encyclopedia of Mathematics Education* (pp. 214–218). Springer International Publishing. https://doi.org/10.1007/978-3-030-15789-0_48

Dündar, S., & Gündüz, N. (2017). Justification for the subject of congruence and similarity in the context of daily life and conceptual knowledge. *Journal on Mathematics Education*, *8*(1), 35–54. https://doi.org/10.22342/jme.8.1.3256.35-54

Duschl, R., Maeng, S., & Sezen, A. (2011). Learning progressions and teaching sequences: A review and analysis. *Studies in Science Education*, *47*(2), 123–182. https://doi.org/10.1080/03057267.2011.604476

Duval, R. (1995). Geometrical Pictures: Kinds of Representation and Specific Processings. *Exploiting Mental Imagery with Computers in Mathematics Education*, 142–157. https://doi.org/10.1007/978-3-642-57771-0_10

Fan, L. (2013). Textbook research as scientific research: Towards a common ground on issues and methods of research on mathematics textbooks. *ZDM - International Journal on Mathematics Education*, *45*(5), 765–777. https://doi.org/10.1007/s11858-013-0530-6

Fan, L., Mailizar, M., Alafaleq, M., & Wang, Y. (2018). A comparative study on the presentation of geometric proof in secondary mathematics textbooks in China, Indonesia, and Saudi Arabia. In *Research on mathematics textbooks and teachers' resources: Advances and issues* (pp. 53–65). https://doi.org/10.1007/978-3-319-73253-4

Formosinho, J., & Formosinho, J. O. (2012). Towards a social science of the social: The contribution of praxeological research. *European Early Childhood Education Research Journal*, *20*(4), 591–606. https://doi.org/10.1080/1350293X.2012.737237

Giménez, J., Font, V., & Vanegas, Y. M. (2013). Designing Professional Tasks for Didactical Analysis as a research process. *Proceedings of ICMI Study 22*, 579–587.

Hendriyanto, A., Suryadi, D., Dahlan, J. A., & Juandi, D. (2023). Praxeology review: Comparing Singaporean and Indonesian textbooks in introducing the concept of sets. *Eurasia Journal of Mathematics, Science and Technology Education*, *19*(2), em2229. https://doi.org/10.29333/ejmste/12953

Kajander, A., & Lovric, M. (2009). Mathematics textbooks and their potential role in supporting misconceptions. *International Journal of Mathematical Education in Science and Technology*, *40*(2), 173–181. https://doi.org/10.1080/00207390701691558

Kang, W., & Kilpatrick, J. (1992). Didactic transposition in mathematics textbooks. *For the Learning of Mathematics*, *12*(1), 2–7.

Kemendikbudristek. (2022). *Pedoman Penerapan Kurikulum dalam Rangka Pemulihan Pembelajaran* (56/M/2022).

Kementerian Pendidikan dan Kebudayaan (2019). Laporan Hasil Ujian Nasional Pusat Penilaian Pendidikan. Https://Hasilun.Pusmenjar.Kemdikbud.go.id/. Retrieved October 3, 2022, from https://hasilun.pusmenjar.kemdikbud.go.id/#2019!smp!daya_serap!99&99&999!T&03&1&T&1&!1!&

Kleeberg, B. (2019). Post Post-Truth: Epistemologies of Disintegration and the Praxeology of Truth. *Stan Rzeczy*, *2*(17), 25–52. https://doi.org/10.51196/srz.17.2/26

Löwenhielm, A., Marschall, G., Sayers, J., & Andrews, P. (2017). Opportunities to acquire foundational number sense: A quantitative comparison of popular English and Swedish textbooks. *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education (CERME10)*, 371–378. https://hal.archives-ouvertes.fr/hal-01873468

Lutfi, M. K., Juandi, D., & Jupri, A. (2021). Students' ontogenic obstacle on the topic of triangle and quadrilateral. *Journal of Physics: Conference Series*, *1806*(1), 1–6. https://doi.org/10.1088/1742-6596/1806/1/012108

Miyakawa, T. (2017). Comparative analysis on the nature of proof to be taught in geometry: the cases of French and Japanese lower secondary schools. *Educational Studies in Mathematics*, *94*(1), 37–54. https://doi.org/10.1007/s10649-016-9711-x

NCTM. (2000). *Principles and Standards for School Mathematics*. The National Council of Teachers of Mathematics, Inc.

Owen, J. C. (1991). Algebraic Solution for Geometry from Dimensional Constraints. *Proceedings of the First ACM Symposium on Solid Modeling Foundations and CAD/CAM Applications* , 397–407.

Pansell, A., & Boistrup, L. B. (2018). Mathematics teachers' teaching practices in relation to textbooks: Exploring praxeologies. *Mathematics Enthusiast*, *15*(3), 541–562. https://doi.org/10.54870/1551-3440.1444

Pepin, B., Haggarty, L., & Keynes, M. (2001). Mathematics textbooks and their use in English, French and German classrooms: a way to understand teaching and learning cultures. *ZDM – Mathematics Education*, *33*(5), 158–175. https://doi.org/10.1007/BF02656616

Prabowo, A., Suryadi, D., Dasari, D., Juandi, D., & Junaedi, I. (2022). Learning Obstacles in the Making of Lesson Plans by Prospective Mathematics Teacher Students. *Education Research International*, *2022*, 1–15. https://doi.org/10.1155/2022/2896860

Pritchard, D. (2013). *What is this thing called knowledge?* Routledge.

Purnomo, Y. W., Mastura, F. S., & Perbowo, K. S. (2019). Contextual Features of Geometrical Problems in Indonesian Mathematics Textbooks. *Journal of Physics: Conference Series*, *1315*(1). https://doi.org/10.1088/1742-6596/1315/1/012048

Putra, Z. H. (2020). Didactic transposition of rational numbers: A case from a textbook analysis and prospective elementary teachers' mathematical and didactic knowledge. *Journal of Elementary Education*, *13*(4), 365–394. https://doi.org/10.18690/rei.13.4.365-394.2020

Putra, Z. H., Dahnilsyah, & Aljarrah, A. (2021). A Praxeological Analysis of Pre-Service Elementary Teacher-Designed Mathematics Comics. *Journal on Mathematics Education*, *12*(3), 563–580. https://doi.org/10.22342/jme.12.3.14143.563-580

Ramelan, M., & Wijaya, A. (2019). A Comparative Analysis of Indonesian and Singaporean Mathematics Textbooks from the Perspective of Mathematical Creativity: A Case Statistics and Probability. *Journal of Physics: Conference Series*, *1320*(1). https://doi.org/10.1088/1742-6596/1320/1/012037

Robutti, O. (2018). Meta-didactical transposition. In *Encyclopedia of mathematics education* (pp. 1–10). Springer International Publishing AG, part of Springer Nature 2018.

Sievert, H., van den Ham, A. K., Niedermeyer, I., & Heinze, A. (2019). Effects of mathematics textbooks on the development of primary school children's adaptive expertise in arithmetic. *Learning and Individual Differences*, *74*. https://doi.org/10.1016/j.lindif.2019.02.006

Solis, D., & Isoda, M. (2022). Comparing elementary school textbooks of China, Japan, and Malaysia: a praxeological and developmental progression analysis regarding length measurement. *Research in Mathematics Education*, 1–20. https://doi.org/10.1080/14794802.2022.2103022

Staples, M., & Truxaw, M. P. (2009). A Journey with Justification: Issues Arising from the Implementation and evaluation of the Math ACCESS Project. *Proceedings of the Thirty-First Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, 827–835. https://www.researchgate.net/publication/228466040

Suryadi, D. (2016). Didactical design research (DDR): upaya membangun kemandirian berpikir melalui penelitian pembelajaran. *Prosiding Seminar Nasional Matematika dan Pendidikan Matematika*.

Suryadi, D. (2019a). *Penelitian Desain Didaktis (DDR) dan Implementasinya*. Gapura Press.

Suryadi, D. (2019b). *Philosophical Foundation of Didactical Design Research (DDR)*. Gapura Press.

Szilágyi, J., Clements, D. H., & Sarama, J. (2013). Young children's understandings of length measurement: Evaluating a learning trajectory. *Journal for Research in Mathematics Education*, *44*(3), 581–620. https://doi.org/10.5951/jresematheduc.44.3.0581

Takeuchi, H., & Shinno, Y. (2020). Comparing the Lower Secondary Textbooks of Japan and England: a Praxeological Analysis of Symmetry and Transformations in Geometry. *International Journal of Science and Mathematics Education*, *18*(4), 791–810. https://doi.org/10.1007/s10763-019-09982-3

Tosho, G. (2021). *Buku Panduan Guru Matematika untuk Sekolah Menengah Pertama Kelas VII “Mathematics for Junior High School 1st Level.”* Pusat Kurikulum dan Perbukuan Kementerian Pendidikan, Kebudayaan, Riset, dan Teknologi.

Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H., & Houang, R. T. (2002). *According to the book : using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Kluwer Academic Publisher. https://doi.org/10.10071978-94-007-0844-0

Vergnaud, G. (2009). The theory of conceptual fields. *Human Development*, *52*(2), 83–94. https://doi.org/10.1159/000202727

Wang, Y., Barmby, P., & Bolden, D. (2017). Understanding Linear Function: a Comparison of Selected Textbooks from England and Shanghai. *International Journal of Science and Mathematics Education*, *15*(1), 131–153. https://doi.org/10.1007/s10763-015-9674-x

Weinberg, A., & Wiesner, E. (2011). Understanding mathematics textbooks through reader-oriented theory. *Educational Studies in Mathematics*, *76*(1), 49–63. https://doi.org/10.1007/s10649-010-9264-3

Wijaya, A., van den Heuvel-Panhuizen, M., & Doorman, M. (2015). Opportunity-to-learn context-based tasks provided by mathematics textbooks. *Educational Studies in Mathematics*, *89*(1), 41–65. https://doi.org/10.1007/s10649-015-9595-1

Wijayanti, D., & Winsløw, C. (2017). Mathematical Practice in Textbooks Analysis: Praxeological Reference Models, the Case of Proportion. *REDIMAT*, *6*(3), 307–330. https://doi.org/10.1783/redimat.2017.2078

Yang, D. C., & Sianturi, I. A. (2017). An Analysis of Singaporean versus Indonesian textbooks based on trigonometry content. *Eurasia Journal of Mathematics, Science and Technology Education*, *13*(7), 3829–3848. https://doi.org/10.12973/eurasia.2017.00760a