Journal on Mathematics Education
Volume 14, No. 3, 2023, pp. 563-584

Onto-semiotic analysis of one teacher's and university students' mathematical connections when problem-solving about launching a projectile

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Received: 25 February 2023 | Revised: 11 July 2023 | Accepted: 16 July 2023 | Published Online: 21 July 2023

Abstract

An onto-semiotic analysis of the mathematical connections established by one in-service mathematics teachers and university students when solving a problem about launching a projectile using the derivative was carried out. Theoretically, this research was based on the articulation between the Extended Theory of Mathematical Connections and the Onto-semiotic Approach. The methodology was qualitative-descriptive where data was collected through interviews based on a task. Subsequently, following the joint analysis method of both theories, the mathematical activity of the participants when they solved the task was analyzed. The results show that, teacher and students established a system of connections of feature type, different representations, meanings, part-whole, procedural and implications in terms of practices, processes, objects, and semiotic functions that relate them. However, some students presented difficulties caused by some incorrect mathematical connection such as stating that the maximum height of the projectile is the time obtained with the critical number, errors in performing arithmetic calculations when evaluating the function, graphically representing the quadratic function as a straight line and use the general formula in an inappropriate way that prevents the procedural connection from being made.

Keywords: Derivative, Mathematical Connections, Mathematics Education, Onto-Semiotic Approach, Projectile

How to Cite: Rodríguez-Nieto, C. A., Font, V., Rodríguez-Vásquez, F. M., & Pino-Fan, L. R. (2023). Onto-semiotic analysis of one teacher's and university students' mathematical connections when problem-solving about launching a projectile. *Journal on Mathematics Education, 14*(3), 563-584. http://doi.org/10.22342/jme.v14i3.pp563-584

Mathematical connections are important because they are a marker for consistently understanding mathematical concepts and problem-solving (Arenas-Peñaloza & Rodríguez-Vásquez, 2022; Berry & Nyman, 2003; National Council of Teachers of Mathematics [NCTM], 2000; Rodríguez-Nieto et al., 2021a; Rodríguez-Nieto et al., 2022a, 2022b). Likewise, students are more likely to make mathematical connections when teachers can provide opportunities to solve everyday situations where they experience relationships between mathematics and other subjects (Rodríguez-Nieto et al., 2021b; NCTM, 2000).

Studies have been reported on mathematical connections at the primary, secondary, pre-university and university levels in the Mathematics Education research agenda. For example, in primary school, Frías and Castro (2007) investigated the influence of connections on the symbolic representation of two-step or stage arithmetic problems with fifth and sixth grade students, showing that nodes or connections influence resolution. In the eighth grade of high school, Kayhan et al. (2017) showed that the student's ability to connect mathematics with real life is not at a sufficient level and that most students can only relate real-life math to numbers and shapes. In relation to the concepts of Calculus, Dolores-Flores and García-García (2017) investigated the intra and extra-mathematical connections that occur when university students solve problems on the definite integral; Dolores-Flores et al. (2019) analyzed the mathematical connections that pre-university students established when they solved problems that involved the concept of rate of change. García-García and Dolores-Flores (2019, 2020) studied the mathematical connections established by pre-university students when solving tasks on the derivative and integral and application problems. Campo-Meneses and García-García (2020) explored the connections associated with the exponential and logarithmic function in Colombian university students.

Rodríguez-Nieto et al. (2021a) analyzed the mathematical connections on the derivative and proposed the articulation of the Extended Theory of Connections (ETC) with the Onto-semiotic Approach (OSA) as a tool to analyze the connections in terms of practices, processes, objects, and semiotic functions. Subsequently, based on this articulation, Campo-Meneses et al. (2021) focused their research lens on the mathematical connections of high school students when they solve tasks that involve exponential and logarithmic functions. Another study investigated the intra-mathematical connections developed by university students when solving tasks on the classification of prime order groups related to subgroups, cyclic groups, isomorphism, isomorphic groups, and Lagrange's theorem (Zubillaga-Guerrero et al., 2021).

Relative to pre-service mathematics teachers argued they have difficulties in teaching geometric concepts, due to the lack of connections between mathematical domains, for example, most of the pre-service teachers did not make connections between different representations of a function (Eli et al., 2011). Also, Moon et al. (2013) stated that pre-service high school teachers have cognitive difficulties in the topic of conic curves (e.g., Cartesian connection, graph as a locus of points), which limits them in making connections between representations. Yavuz-Mumcu (2018) indicated that most pre-service teachers establish less frequently connections between different representations of the derivative.

As well as important that students at different school levels and pre-service mathematics teachers establish mathematical connections, in-service teachers must promote them, therefore, mathematical connections are achieved when in-service teachers are able to provide students with opportunities to solve everyday situations, intra-mathematical or extra-mathematical where they experience relationships between mathematics and other subjects (NCTM, 2000). Also, they need to relate conceptual knowledge to procedures and equivalent representations of mathematical concepts (Coxford, 1995). Without an understanding of mathematical concepts and their functionality, teachers may lack adequate preparation to engage their students in making mathematical connections, reasoning, and problem solving as core competencies (Eli et al., 2013). In this line, Businskas (2008) analyzed the connections made by in-service high school mathematics teachers in solving tasks that involve the quadratic equation and function, from which the model emerged with five types of connections: different representations, implication, part-whole, procedural, and instruction-oriented connections. Likewise, in Mhlolo et al. (2012) they inquired about the quality of mathematical connections made by in-service teachers, focusing especially on the connections of different representations on an algebra topic and some made defective or superficial representations located in quality level 0 and 1 (no justifications) mostly. Rodríguez-Nieto et al. (2022a) carried out a theoretical reflection on the mathematical connections made by an in-service teacher when he teaches the derivative subject where the metaphorical connection was found.

After reviewing the literature on mathematical connections, it has been identified that students, pre-service mathematics teachers and some mathematics in-service teachers have problems understanding the derivative because they have difficulties connecting partial meanings and representations of the derivative concept (Amaya, 2020; Pino-Fan et al., 2015, 2017, 2018; Rodríguez-Nieto et al., 2021c; Sánchez-Matamoros et al., 2015; Sari et al., 2018; Vargas et al., 2020), make mechanized processes with the use of formulas (Antonio et al., 2019) and students find it difficult to relate the function *f* with its derivative and graph the derivative f' similar to the function *f* and difficulties in graphing f' of when the symbolic expression is not known (Fuentealba et al., 2018; Pino-Fan et al., 2018). In fact, pre-service mathematics teachers had some difficulty using the derivative as the instantaneous rate of change when solving an application problem about the speed with which a ball travel and reaches a certain height in a time determined (Pino-Fan et al., 2018), further Rodríguez-Nieto et al. (2021c) reported that pre-service teachers have difficulties in finding the equation of the tangent line, due to the inadequate meaning of the derivative. However, though the difficulties of in-service teachers are recognized, and research has already been carried out focused on what connections a teacher establishes when defining the derivative, we observed the need to emphasize application problems. Therefore, the objective of this study is to carry out an onto-semiotic analysis of the mathematical connections established by an in-service mathematics teacher and six university students when solving a problem about launching a projectile.

Extended Theory of Connections (ETC)

A mathematical connection is assumed from the perspective of the ETC articulated with the OSA, as the tip of an iceberg, in a metaphorical way, made up of a conglomerate of practices, processes, primary objects identified in the mathematical activity of a subject when solving a task and semiotic functions that relate them (Rodríguez-Nieto et al., 2021a). Mathematical connections have been classified into two groups: intra-mathematical and extra-mathematical (Dolores-Flores & García-García, 2017, p. 160-161). Next, the categories of the mathematical connections of the ETC are described (see Table 1).

Table 1. Categories of mathematical connections of the ETC

Description
1) Instruction-oriented: Refers to the understanding of concept C based on two or more previous concepts A and B, required to be understood by a subject. They are manifested in two ways: a) association of a new topic with previous knowledge, b) mathematical concepts and procedures connected to each other are considered prerequisites or skills that students must master before the development of a new concept (Businskas, 2008).
2) Modeling: They are characterized by being a link between mathematics and real life or the daily life of students and are evidenced when the subject solves non-mathematical or application problems where he has to pose a mathematical model or expression (Evitts, 2004).
3) Different representations: they are identified when the subject represents mathematical objects using equivalent and alternate representations. Equivalents are transformations of representations of the same record. Alternates refer to representations of the same object where the register in which they were formed is changed (graphical-algebraic) (Businskas, 2008).
4) Procedural: they are identified when a subject uses rules, algorithms, or formulas to solve a mathematical task. This type of connection is of the form: A is a procedure to work with a concept B (Businskas, 2008).

5) Part-whole: These connections occurs when logical relationships are established in the following two ways: 1) The generalization relationship is of the form A and is a generalization of B and B is a particular case of A. 2) The relationship of inclusion is given when a mathematical concept is contained in another (Businskas, 2008).
6) Implication: they are identified when a concept P leads to another concept Q through a logical relationship (P → Q) (Businskas, 2008).
7) Feature: it is identified when the person expresses some characteristics of the concepts or describes their properties in terms of other concepts that make them different or similar to the others (Eli et al., 2011).
8) Reversibility: they occur when a subject starts from a concept A to obtain a concept B and reverse the process, starting from B until returning to A (García-García & Dolores-Flores, 2019).
9) Meaning: it is identified when a subject "attributes a meaning to a mathematical concept, that is, what it means for him [...]. It includes those cases in which a student gives a definition that he or she has constructed for these concepts" (García-García & Dolores-Flores, 2019).
10) Metaphorical: Understood as the projection of the properties, characteristics, etc., a known domain based on bodily experiences to structure another less known or abstract domain (Rodríguez-Nieto et al., 2022a).

Onto-Semiotic Approach (OSA)

The onto-semiotic approach is an inclusive theoretical system of mathematical knowledge and instruction, which from its inception-maintained interaction with the theoretical developments of French mathematics didactics, and is concerned with the need to clarify, articulate, and improve theoretical and methodological notions of different theoretical frameworks used in Mathematics Education from a unified vision (Godino & Batanero,1994). The OSA seeks to answer questions such as: what a mathematical object is? What the meaning of a mathematical object is? What types of objects are involved in a mathematical activity? Also, in OSA, is essential to describe mathematical activity from an institutional or personal perspective, which is modeled in terms of practices and configuration of primary objects and processes that are activated in said practices (Godino et al., 2007). For Godino and Batanero (1994) mathematical practice is understood as "any situation or expression (verbal, graphic, symbolic) carried out by someone to solve mathematical problems, communicate the solution obtained to others, validate it or generalize it to other contexts and problems" (p. 334). Six primary objects are considered: problem situations, linguistic elements in their different registers, concepts/definitions, propositions/properties, procedures, and arguments. These interconnected objects form the configuration of primary objects (Godino et al., 2019). A configuration is a heterogeneous system of interrelated objects. The epistemic configuration is the system of primary objects that, from an institutional perspective, are involved in the mathematical practices carried out to solve a specific problem and the cognitive configuration is the system of primary mathematical objects that a subject mobilizes as part of the mathematical practices you develop to solve a problem (Godino et al., 2019).

In agreement with Godino et al. (2007), primary objects emerge in mathematical activity through the activation of primary mathematical processes derived from the application of the process-product perspective to these primary objects, are given together with those derived from applying the process-product duality to the five dualities mentioned above (institutional/personal, expression/content, ostensive/non-ostensive, unitary/systemic and extensive/intensive): personalization-institutionalization; synthesis-analysis; representation-meaning; materialization-idealization; generalization-particularization (Font et al., 2016; Godino et al., 2007). Another important tool in OSA is the notion of semiotic function that allows practices to be associated with the objects and processes that are activated and allows the construction of an operational notion of knowledge, meaning, understanding and competence (Godino et al., 2007). A semiotic function is a triadic relationship between an antecedent (initial expression/object) and a consequent (final content/object) established by a subject (person or institution) according to a certain criterion or correspondence code (Godino et al., 2007). In Rodríguez-Nieto et al. (2021a) it is stated that the notion of semiotic function (OSA) is more general than the notion of mathematical connection (ETC), since the connections are considered particular cases of semiotic functions of a personal or institutional nature. In the ETC, the mathematical connection may or may not be true, revealing from the perspective of OSA that, when a subject makes a correct connection, it coincides with the institutional one, and when it is incorrect, it is of a personal type.

METHODS

This research is qualitative (Cohen et al., 2018) where the mathematical connections are described through a thematic analysis of the content embodied in the written productions of the participants and developed in three stages: 1) the voluntary participation of a mathematics teacher and six university students was achieved; 2) semi-structured interviews were carried out based on a task dealing with an application problem and; 3) the data were analyzed through the analysis method that resulted from the articulation between the ETC and the OSA to characterize the mathematical activity and connections carried out by the participants. In this third stage, a temporal narrative was obtained from the transcription of the interviews (what the subject does when solving the task is explained mathematically). Based on the narrative, mathematical practices were described (see Table 3). Then, the configuration of the primary objects (see Table 4) activated in the practices and the semiotic functions that relate them to each other (data analysis method with OSA tools) is constructed; finally, parts of the mathematical activity (practices, processes, primary objects, and SF) were grouped as a proposed connection type in the ETC (see Table 5).

Participants and Context

In this research, one in-service mathematics teacher (P1) of a high school participated voluntarily, who is fifty years old and has twenty-seven years of work experience. This teacher has a degree in mathematics and physics and recently completed his master's degree in physics at a university in northern Colombia. Currently, he is a full-time professor at a Technical-Industrial Educational Institution of Atlántico, Colombia. Likewise, six students (P2-P7) participated voluntarily with an average age of 19 years enrolled in the second year of the mathematics degree at a university in southern Mexico, who had completed and passed the Differential Calculus course and developed the topic of derivatives. This is an intentional sample taken in two different contexts.

Data Collection

To collect the data, interviews (one for each participant individually) were conducted based on a task consisting of an application problem where the derivative is required to solve it. This type of interview was chosen because as the interviewed participant solves the task, he can simultaneously issue his arguments to justify each step of his resolution. Likewise, you can answer questions that the interviewer (I) asks you to deepen your procedure (Goldin, 2000). These interviews where the participants explained their written productions were videotaped and later transcribed.

***Task**: A projectile is fired vertically upwards and forwards from the ground, whose height is given by the expression $s(t) = 343t - 4.9t^2$. If friction with air is neglected. What is the maximum height reached by the projectile? How long does it take for the projectile to hit the ground again? Justify your answer.*

During the interview, a main question will be asked, for example, how did you solve the problem? Justify your answer, and secondary questions to deepen the procedures related to: What concepts, properties, definitions, representations did you use in this step of the procedure? Could you explain in detail why that is the maximum height that the projectile reaches?

Data Analysis

For the analysis of the data, the thematic analysis method of the mathematical activity that resulted from the articulation between the ETC and the OSA (Rodríguez-Nieto et al., 2021a) was used (see Table 2).

Table 2. Method to analyze the mathematical activity of the teacher and students

	Phases	Description
1	Transcript of the interviews	The task-based interview is transcribed and read in detail so that the researcher becomes familiar with the data collected and the written productions are reviewed, which corresponds to the first phase of the thematic analysis of the ETC (Braun & Clarke, 2006).
2	Temporal narrative	It is explained in mathematical terms, what the teacher and the students do when they solve the task (from the video where they explain their written productions). It contains the practices carried out by the subjects and some primary objects that, metaphorically, are the protagonists of the narrative.
3	Mathematical practices	Based on the narrative, mathematical practices are described and understood as sequenced actions, regulated by institutionally established rules, oriented toward the resolution of a problem (Font et al., 2013; Godino et al., 2007).
4	Cognitive configuration	The system of primary mathematical objects that the subjects mobilize as part of the mathematical practices developed to solve the problem is built.
5	Semiotic functions	Relationships are established between the primary objects of the identified in the cognitive configuration.

Note: Adapted from Rodríguez-Nieto et al. (2021b).

Each one of the authors carried out the analysis of the mathematical activity in terms of practices, primary objects, processes, and semiotic functions. We then carried out a triangulation procedure, comparing the different results of the analyzes carried out by each author to identify coincidences (the agreement between the codes was 90% between the authors. Any coding differences were resolved by consensus.

RESULTS AND DISCUSSION

The findings of this research are directed based on the development of the method described in Table 2 and the theoretical framework (ETC and OSA) is made operational.

Transcript of the Interviews

The interviews conducted were transcribed in Excel and read to understand the information or responses of the participants to the proposed task (see Figure 1).

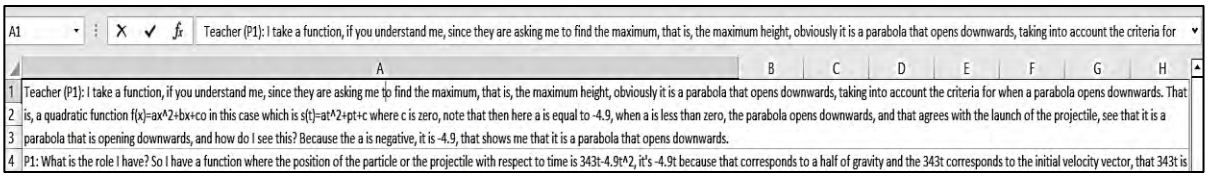


Figure 1. Transcript evidence in Excel format

Based on the interview, the temporal narratives of the participants are constructed. In this case, only the temporary narrative of teacher P1 will be presented.

P1's Temporal Narrative

Initially P1 reads and understands the first question of the problem that is asking him to find the maximum height and stated that the function given in the problem is of second degree. Then, recognize a property of the function and its graph that is a parabola that opens downwards and is an incomplete quadratic function because in its explanation it mentions "a quadratic function $f(x) = ax^2 + bx + c$ o in this case that is $s(t) = at^2 + pt + c$ where c is zero". Then P1 states that in the expression $s(t) = 343t - 4.9t^2$, $a = -4.9$ is less than zero so the parabola opens downwards, which agrees with the launch of the projectile. Later, P1 mentions that the function $s(t) = 343t - 4.9t^2$ is the position of the particle or projectile with respect to time and describes its characteristics, for example, -4.9 is a half of gravity on earth and the $343t$ corresponds to the initial velocity vector, that $343t$ is saying that the projectile is leaving with an initial velocity of $343\ m/s$, from there that value comes out and the $y = 0$ that would correspond to the c y P1 argues that this value 0 is placed because it is understands that the projectile is leaving from the origin of the Cartesian coordinate system and its position, that is, its height at that moment, is zero.

Next, P1 sketches the graph and to find the maximum height used the first derivative criterion to find the maximum of the function. To do this, implicitly using the formula to derive the power function $\left(\frac{d(t^n)}{dt} = nt^{n-1}\right)$, find the derivative of the function $s(t) = 343t - 4.9t^2$ obtaining as a result $s'(t) = \frac{ds}{dt} = 343 - 9.8t$. Then, P1 found the maximum considering that *s'(t)* must be equal to zero and this happens when $t = 35$ seconds, which he substituted in $s(t)$, $s(35) = 343(35) - 4.9(35)^2$ which resulted in 6,002.5 meters being the maximum height reached by the projectile. P1 reads and understands the second question. Then he finds the time it takes for the projectile to hit the ground. To achieve this, P1 sets $s(t)$ equal to zero $343t - 4.9t^2 = 0$, then factors $t(343 - 4.9t) = 0$ from which he deduces that at $t_1 = 0$ the projectile has not yet been fired and at $t_2 = \frac{343}{4.9} = 70$ seconds is when the projectile lands back on the ground. Finally, he corroborates the result by explaining that it must be remembered that the projectile's time of flight is twice the time of rise (35 seconds), so 2(35 seconds) = 70 seconds.

P1's Mathematical Practices

Table 3 presents the mathematical practices (Mp) in an organized manner that represent the set of sequenced actions carried out by P1 when solving and explaining the proposed task.

Table 3. Mathematical practices

Mp	Sequenced actions
Mp1	P1 read and understood the first question of the proposed task and stated that the function given in the problem is quadratic or second degree.
Mp2	P1 identified properties of the quadratic function that is incomplete and opens down because it states that in the expression $s(t) = 343t - 4.9t^2$, $a = -4.9$ is less than zero, so the parabola opens down.

Mp3	P1 described characteristics of $s(t)$, for example, -4.9 is a half of gravity on earth and the $343t$ corresponds to the initial velocity vector, that $343t$ is saying that the projectile is leaving with an initial velocity of $343\ m/s$.
Mp4	P1 stated that $y = 0$ corresponds to the c of $s(t) = at^2 + pt + c$ and argues that this value 0 is placed because it is understood that the projectile is leaving from the origin of the cartesian coordinate system and its position is say, its height there at that moment is zero.
Mp5	Find the maximum height of the projectile. To do this, he considered the criterion of the first derivative if $s'(t) = 0$ then the function there has a maximum, that is, he considers it to find the critical number where the function has a maximum, minimum or inflection point.
Mp6	P1 understands and uses the derivative as the instantaneous rate of change (see transcript excerpt). Researcher (R): What meaning do you associate with the derivative in the problem you are solving? P1: As the instantaneous rate of change.
Mp7	P1 sketched the graph of the function $s(t) = 343t - 4.9t^2$.
Mp8	P1 found the derivative of the function $s(t) = 343t - 4.9t^2$ obtaining $s'(t) = \frac{ds}{dt} = 343 - 9.8t$. To do this, he implicitly used the formula to derive the power function $\left(\frac{d(t^n)}{dt} = nt^{n-1}\right)$.
Mp9	P1 found the critical number considering that $s'(t)$ must be equal to zero and this happens when $t = 35$ seconds. To do this, he set the derivative equal to zero, $343 - 9.8t = 0$, and applying additive inverses and arithmetic operations he obtained $t = 35$.
Mp10	P1 substituted $t = 35$ in $s(t)$, $s(35) = 343(35) - 4.9(35)^2$ which, after performing arithmetic operations, resulted in 6.002.5 meters, which is the maximum of the function and, in turn, the maximum height reached by the projectile.
Mp11	P1 read and understood the second question of the proposed task, considering that to solve the problem they must find the solution $s(t)$ or the zeros.
Mp12	P1 set $s(t)$ equal to zero $343t - 4.9t^2 = 0$, then factored $t(343 - 4.9t) = 0$ from which he deduced that at $t_1 = 0$ the projectile has not yet been fired and at $t_2 = \frac{343}{4.9}$ = 70 seconds is when the projectile falls back to the ground.
Mp13	Explain that it must be remembered that the time of flight of the projectile is twice the time of rise which is 35 seconds, so 2(35 seconds) = 70 seconds.

Construction of the Cognitive P1's Configuration

Based on the mathematical practices described in section 4.3., the *cognitive configuration* of P1 is elaborated where the primary objects (PO), problem situations/Task (T), linguistic elements (L-E), concepts/definitions (C-D) are described in detail. propositions/properties (Pr), procedures (Pc) and arguments (A), see Table 4.

Table 4. Cognitive configuration of primary objects

PO	Description
T	**T:** A projectile is fired vertically upwards and forwards from the ground, whose height is given by the expression $s(t) = 343t - 4.9t^2$. If friction with air is neglected. **T1:** What is the maximum height reached by the projectile? **T2:** How long does it take for the projectile to hit the ground again? Justify your answer.
L-E	**Verbal**: point, function, quadratic function, the parabola that opens downwards, maximum, maximum height, graph, sketch, initial velocity, earth's gravity, vector, cartesian coordinate system, derivative, first derivative, instantaneous rate of change, seconds, meters, flight time, among others.
	Symbolic: $s(t)$; $s'(t)$; $\frac{ds}{dt}$; $s(t) = 343t - 4.9t^2$; $s'(t) = \frac{ds}{dt} = 343 - 9.8t$; $t = 35$; $t_1 = 0$; $t_2 = \frac{343}{4.9}$ = 70; ($\frac{d(t^n)}{dt} = nt^{n-1}$); $343t$; $343m/s$; $s'(t) = 0$; $t = 0$; -4.9.
	Graphic: see Figure 2.

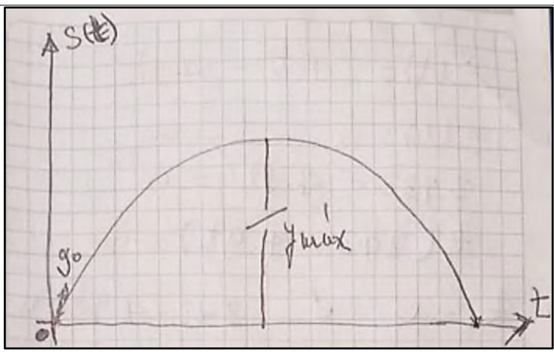


Figure 2. Sketch of the trajectory followed by the projectile

C-D

Previous concepts: point, function, quadratic function, the downward-opening parabola, maximum, maximum height, graph, initial velocity, earth's gravity, vector, cartesian coordinate system, derivative, first derivative, instantaneous rate of change, common factor, time of flight.

Definitions (D):
D1: The derivative is the instantaneous rate of change.
D2: A critical number of g is a number c in the domain of g such that $g'(c) = 0$ or $g'(c) = 0$ does not exist.
D3: Parabolic or projectile motion.

Pr

Previous propositions: Factoring cases.
Pr1: The graph of the incomplete quadratic function $s(t) = 343t - 4.9t^2$ opens down.
Pr2: -4.9 is a half of gravity on earth, $343t$ is the initial speed.
Pr3: In $s'(35)$ is the critical point where the maximum of the function $s(t)$ is.
Pr4: The maximum height that the projectile reaches is 6,002.5 meters.
Pr5: After 70 seconds the projectile falls back to the ground.

Pc

Main procedure 1 (Mpc1): Determination of the maximum height reached by the projectile.
Auxiliary procedure 1.1. (Apc1.1): Describes the features of the function $s(t) = 343t - 4.9t^2$.
Apc1.2: Sketch the graph of the function that models the trajectory of the projectile. To do this, draw a Cartesian coordinate plane and then draw the graph (Figure 3).
Apc1.3: Find the derivative of a second-degree function obtaining as a result $s'(t) = \frac{ds}{dt} = 343 - 9.8t$ (Figure 4).
Apc1.4: P1 calculates the critical number and for this he set the derivative equal to zero ($s'(t) = 0$) and did arithmetic operations obtaining $t = 35$ (Figure 4).
Apc1.5: P1 find the maximum point of the function. To achieve this, substitute the critical number in the function $s(35) = 343(35) - 4.9(35)^2$, perform arithmetic operations and find the maximum height 6,002.5 *m* (see Figure 3).

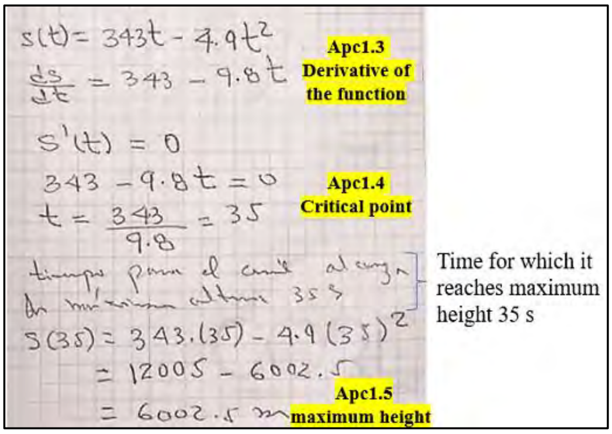


Figure 3. Procedure to find the maximum height reached by the projectile

Main procedure 2 (Mpc2): Determine the time it takes for the projectile to fall to the ground.
Apc2.1: Find the zeros or solution of the quadratic function $s(t) = 343t - 4.9t^2$. To do this, set equal to zero $343t - 4.9t^2 = 0$ (Figure 5).
Apc2.2: Apply the common factor (factorization case) $t(343 - 4.9t) = 0$ and, concludes that solutions or solutions are $t_1 = 0$ and $t_2 = \frac{343}{4.9}$ = 70 (Figure 5).
Apc2.3: Conclude that 70 seconds is the time it takes for the projectile to fall to the ground (Figure 4).

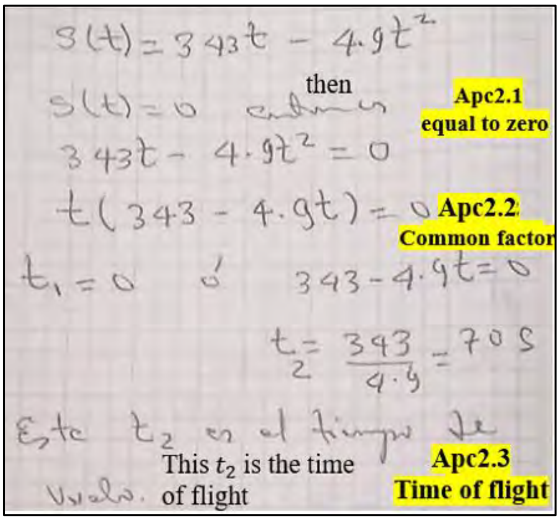


Figure 4. Procedure to find the time it takes for the projectile to fall to the ground

A

Argument 1 (A1): Thesis: The graph of the incomplete quadratic function $s(t) = 343t - 4.9t^2$ is a parabola that opens downwards.
Reason 1 (R1): The function is incomplete quadratic because in the expression for the complete quadratic function $s(t) = at^2 + pt + c, c = 0$.
R2: The function is concave down because $a = -4.9$ is less than zero.
Conclusion: The graph of the function does open down.

A2: Thesis: -4.9 is a half of gravity on earth, $343t$ is the initial speed.
R1: The function expression $s(t) = 343t - 4.9t^2$ has the same form as the formula to find the distance $d = v_0t + \frac{1}{2}at^2$.
Conclusion: -4.9 yes is a half of gravity and $343t$ yes is the initial velocity.

A3: Thesis: In $s'(35)$ is the critical point where the maximum of the function $s(t)$ is.
R1: $s'(35) = 0$.
R2: If the derivative at a point equals zero, then there is a maximum, minimum, or inflection point at that point.
Conclusion: In $s'(35) = 0$ there is the critical point and also the maximum.

A4: The maximum height that the projectile reaches is 6,002.5 meters.
R1: The critical number evaluated in the function is 6,002.5
Conclusion: The maximum reached by the projectile is 6,002.5 meters.

A5: After 70 seconds the projectile falls back to the ground.
R1: The zeros of the function $s(t) = 343t - 4.9t^2$ indicate the initial position where the projectile is fired and where it falls back to the ground.
R2: If in 35 seconds the projectile has its maximum range, in 70 seconds it falls back to the ground because the flight time is twice the rise time $\left(\frac{2v_0 * sen\theta}{g}\right)$.
Conclusion: The projectile does fall to the ground after 70 seconds.

P1's Semiotic Functions

After performing the cognitive configuration, the relationships between the primary objects are established by means of semiotic functions (SF) (see Figure 5).

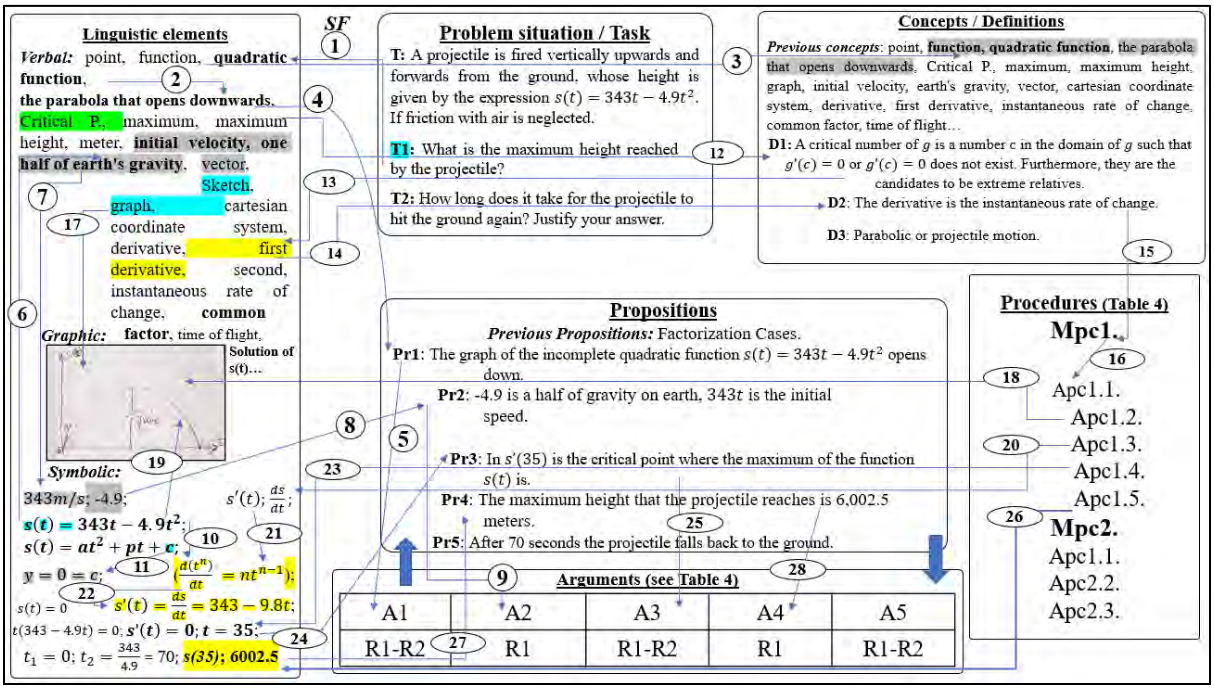


Figure 5. P1's semiotic function

Synthesis of the Onto-Semiotic Analysis of the P1's Mathematical Connections

Table 5 shows the system of mathematical connections established by P1 when he solved the task, considering the ontosemiotic analysis based on the conglomeration of practices, processes, objects (Figure 6), and semiotic functions (SF) that constitute the connection.

Table 5. Detailed analysis of the connections established by P1 when solving the task

Mp	Processes	Objects	SF	Mathematical connections
Mp1	-Signification / Understanding. -Problematization.	Explain that the function contained in the statement of the problem is quadratic or second degree.	SF1 SF2 SF3	Feature
Mp2	-Problem-solving. -Enunciation.	The quadratic function is incomplete and opens downwards because it states that in $s(t) = 343t - 4.9t^2, a = -4.9$ is less than zero, then the parabola opens down.	SF4 SF5	Feature Implication
Mp3	-Problem-solving. -Enunciation.	In $s(t)$, -4.9 it is a means of gravity on earth and the $343t$ corresponds to the initial velocity vector, that $343t$ is saying that the projectile is leaving with an initial velocity of $343m/s$.	SF6 SF7 SF8 SF9	Feature
Mp4	-Problem-solving. -Enunciation. -Representation. -Particularization.	P1 stated that s(t) has the form $s(t) = at^2 + pt + c$ and stated that c corresponds to $y = 0$ in the Cartesian coordinate system. Also, c is the independent term and the intersection point with the y-axis.	SF10 SF11	Part-whole Feature

Mp5	-Problem-solving. -Enunciation.	State the one that requires finding the critical number where $s'(t) = 0$ then the function has a maximum.	SF12	Implication
Mp6	-Signification / Understanding. -Problematization. -Problem-solving. -Enunciation.	**D1**: A critical number of g is a number *c* in the domain of g such that $g'(c)=0$ or $g'(c)=0$ does not exist. **D2**: The derivative is the instantaneous rate of change.	SF12 SF13 SF14	Meanings
Mp7	-Problem-solving. -Representation (graphic-symbolic).	Sketch of the graph of $s(t) = 343t - 4.9t^2$.	SF15 SF16 SF17 SF18 SF19	Different representations (alternate)
Mp8	-Problem-solving. -Representation (verbal-symbolic).	P1 derived $s(t) = 343t - 4.9t^2$ obtaining $s'(t) = \frac{ds}{dt} = 343 - 9.8t$. To do this, he implicitly used the formula $\left(\frac{d(t^n)}{dt} = nt^{n-1}\right)$.	SF20 SF21 SF22	Procedural
Mp9	-Problem-solving. -Representation (verbal-symbolic). -Argumentation.	Set the derivative equal to zero $343 - 9.8t = 0$ and applying additive inverses and arithmetic operations, he obtained $t = 35$. -Argument 3 (A3).	SF23 SF24 SF25	Procedural Implication
Mp10	- Problem-solving. -Representation (verbal-symbolic).	P1 substituted $t = 35$ in $s(t)$, $s(35) = 343(35) - 4.9(35)^2$ which, after performing arithmetic operations, resulted in 6,002.5 meters.	SF26 SF27 SF28	Procedural
…	*Continue…*	…	…	…

Table 5 shows the system of connections activated by P1 when solving the problem, describing in detail the practices, processes, objects, and semiotic functions, which allows visualizing what truly constitutes the connection. It should be noted that the semiotic functions (SF29 – SF37) detailed in Table 5 are not displayed in Figure 6 because space is limited, and it was impossible to locate the dates. Also, the thick blue arrows inform that the arguments support and validate the propositions and the other primary objects. Below is a schematic of a connection and its operation as the tip of an iceberg (Figure 6).

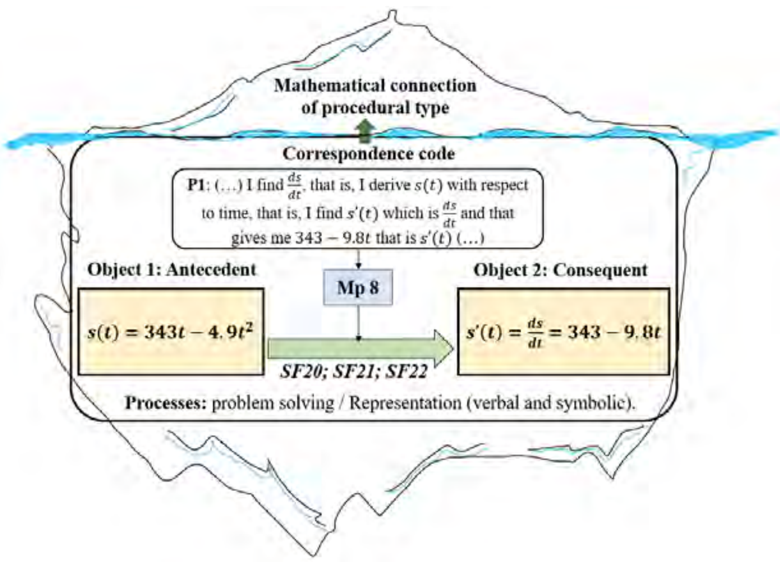


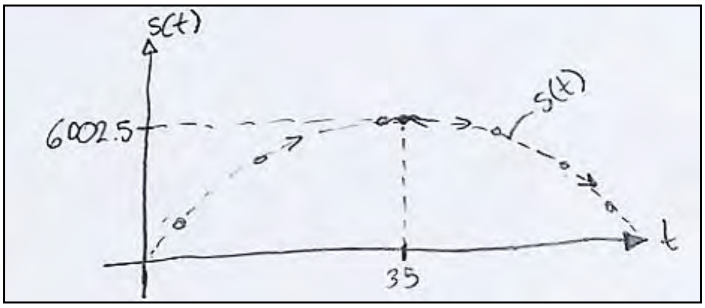
Figure 6. Scheme of the mathematical connection of procedural type

The same analysis was made to P2, P3, P4, P5, P6 and P7 are presented.

Construction of the P2's Cognitive Configuration

The cognitive configuration of P2 is presented, detailing each of the primary objects activated in the mathematical activity (see Table 6).

Table 6. Cognitive configuration when P2 solving the task

PO	Description
T	**T:** A projectile is fired vertically upwards and forwards from the ground, whose height is given by the expression $s(t) = 343t - 4.9t^2$. If friction with air is neglected. **T1:** What is the maximum height reached by the projectile? **T2:** How long does it take for the projectile to hit the ground again? Justify your answer.
L-E	**Verbal**: point, function, quadratic function, maximum, maximum height, graph, sketch, derivative, first derivative, seconds, meters, among others. **Symbolic**: $s(t)$; $s'(t)$; $s(t) = 343t - 4.9t^2$; $s'(t) = 343 - 9.8t$; $t = 35$; $\left(\frac{d(t^n)}{dt} = nt^{n-1}\right)$. **Graphic**: see Figure 7.  **Figure 7.** Sketch of the trajectory followed by the projectile
C-D	**Previous concepts**: point, function, quadratic function, maximum, maximum height, graph, sketch, derivative, first derivative, seconds, meters, among others. **Definitions (D)**: **D1**: A critical number of g is a number *c* in the domain of g such that $g'(c) = 0$ or $g'(c) = 0$ does not exist.
Pr	**Propositions (Pr):** **Pr1**: The function $s(t) = 343t - 4.9t^2$ is the formula to draw the graph and it describes the trajectory of the projectile. **Pr2**: The maximum height that the projectile will reach is the maximum point of the function, which happens if $s'(35) = 0$ is the critical point, then there is the maximum of the function $s(t)$. **Pr3**: The maximum height reached by the projectile is at $t = 35$ seconds and is 6,002.5 meters.
Pc	**Main procedure 1 (Mpc1)**: Determination of the maximum height reached by the projectile. **Auxiliar procedure 1.1. (Apc1.1)**: Sketch the graph of the function that models the trajectory of the projectile. To do this, draw a Cartesian coordinate plane and then draw the graph (see Figure 8). **Apc1.2**: Calculate the derivative of the function $s(t) = 343t - 4.9t^2$, obtaining $s'(t) = 343 - 9.8t$ (see Figure 9). To do this, he implicitly used the formula $\frac{d(t^n)}{dt} = nt^{n-1}$. **Apc1.3**: Calculate the critical number and to do so, set the derivative equal to zero ($s'(t) = 0$) and perform arithmetic operations until obtaining $t = 35$ (Figure 9). **Apc1.4**: P2 found the maximum height substituting $t = 35$ in $s(t)$, obtaining $t(35) = 343(35) - 4.9(35)^2$, performed arithmetic operations and found 6,002.5 *m* (Figure 8).

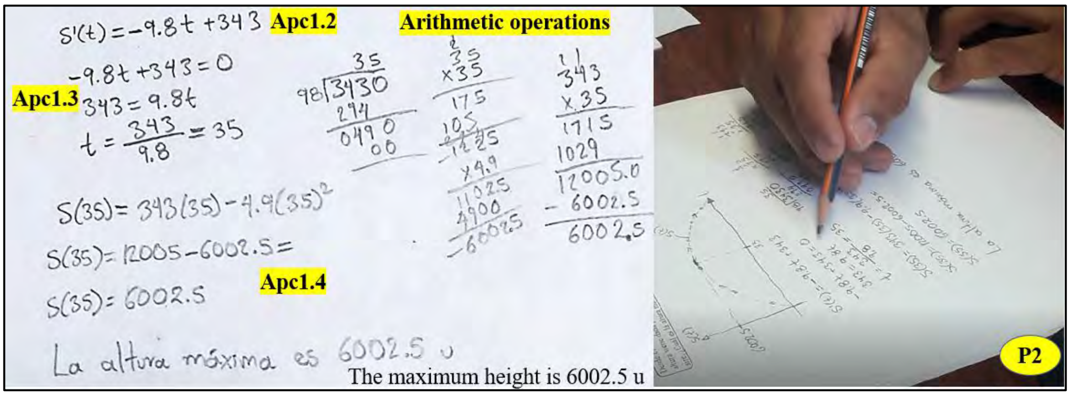


Figure 8. Procedure to find the maximum height reached by the projectile

A	**Argument 1 (A1)**: Thesis: *The function* $s(t) = 343t - 4.9t^2$ is the formula for drawing the graph and describes the trajectory of the projectile. **Reason 1 (R1)**: By means of the symbolic expression of the function $s(t)$ you can replace values of t and obtain points to graph the function. **R2**: If $s(t)$ is a function with domain in *A*, then the graph is the set of ordered pairs $\{(t, s(t))\mid t \in A\}$. In fact, the graph of s is constituted by all the points $(t, s(t))$ in a plane of Cartesian coordinates. **Conclusion**: $s(t)$ is the formula to find or draw the graph.
	A2: *Thesis*: The maximum height that the projectile will reach is the maximum point of $s(t)$, which happens if $s'(35) = 0$ is the critical point, then there is the maximum of $s(t)$. **R1**: at the critical number $t = 35$, $s'(t) = 0$ and the function has a maximum. **Conclusion**: The maximum height of the projectile itself is the maximum point of $s(t)$.
	A3: *Thesis*: The maximum height reached by the projectile is at $t = 35$ seconds and is 6,002.5 meters. **R1:** The critical number $t = 35$ evaluated in the function is 6,002.5 **Conclusion:** The maximum reached by the projectile is 6,002.5 meters.

P2's Semiotic Functions

After building the cognitive configuration, the relationships between the primary objects identified by means of semiotic functions are established (see Figure 9).

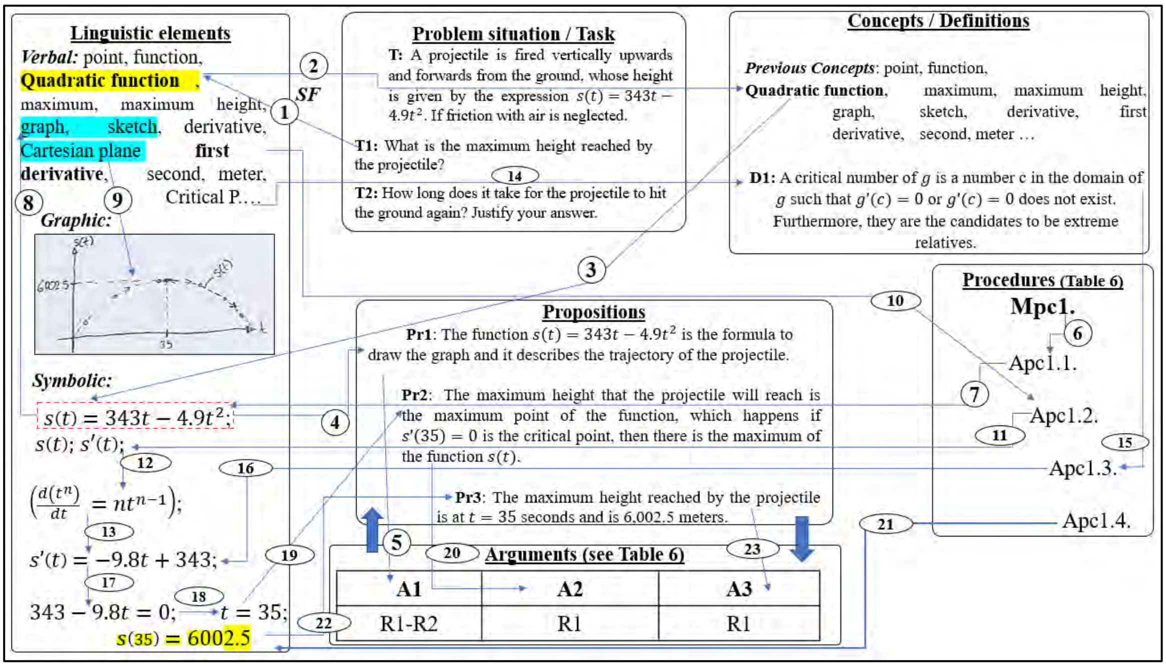


Figure 9. P2's semiotic functions

Synthesis of the Onto-Semiotic Analysis of the P2's Mathematical Connections

Table 7 shows the system of mathematical connections established by P2 when solving the task, which were analyzed from an onto-semiotic view showing the conglomeration of practices, processes, objects (Figure 9) and SF that constitute them.

Table 7. Detailed analysis of the connections established by P2 when solving the task

Mp	Processes	Objects	SF	Mathematical connections
Mp1	-Signification/ Understanding. -Problematization.	P2 reads and understands the first question of the proposed task.	SF1	
Mp2	-Enunciation -Problem-solving. -Representation (symbolic- graph)	$s(t) = 343t - 4.9t^2$ is the formula to draw the graph (Figure 7).	SF2 SF3 SF4 SF5	Feature
Mp3	- Problem-solving. -Representation (symbolic- graph)	Sketch of the graph of the function $s(t) = 343t - 4.9t^2$ (Figure 8).	SF6 SF7 SF8 SF9	Different representations (alternate)
Mp4	-Problem-solving. -Representation (verbal-symbolic).	Calculate the derivative of the function $s(t) = 343t - 4.9t^2$, obtaining $s'(t) = 343 - 9.8t$. To do this, he implicitly used the formula $\left(\frac{d(t^n)}{dt} = nt^{n-1}\right)$.	SF10 SF11 SF12 SF13	Procedural
Mp5	-Problem-solving. -Representation (verbal-symbolic). -Argumentation.	P2 set the derivative equal to zero and did arithmetic to find the critical point $t = 35$.	SF14 SF15 SF16 SF17 SF18 SF19 SF20	Procedural Implication
Mp6	- Problem-solving. -Representation (verbal-symbolic).	P2 replaced $t = 35$ in $s(t)$, obtaining $s(35) = 343(35) - 4.9(35)^2$ and got the maximum height $6.002.5\ m$.	SF21 SF22 SF23	Procedural

Students P3 and P4 proceeded in the same way as P2, but they did not find the time in which the projectile falls back to the ground. P3 made mathematical connections of meaning different from those established by the other participants, for example, they conceive the derivative as the slope of the tangent line at a point (Figure 10), however, the graphic representation of the path followed by the projectile is not consistent with the stated situation because the sketch does not start from the origin.

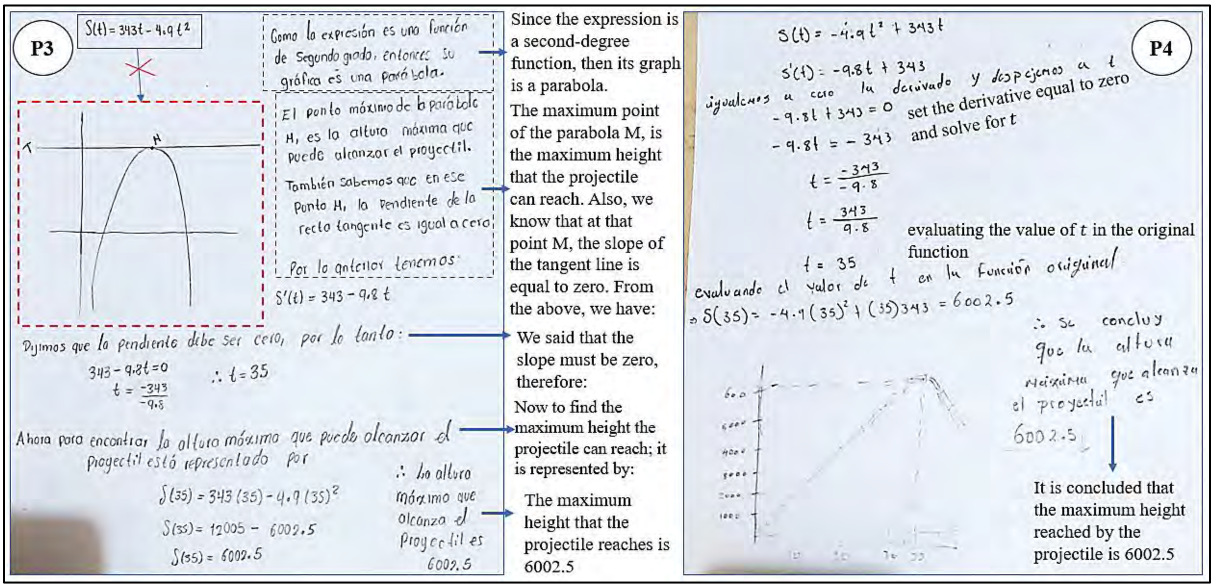


Figure 10. Evidence of procedures performed by P3 and P4

P3 made implication connections relating the maximum point of $s(t)$ to the derivative at point. On the other hand, although P5, P6 and P7 have carried out some correct steps in solving the task, they present difficulties caused by some incorrect and personal mathematical connection (red boxes in Figure 11), for example, P5 affirms that the maximum height of the projectile is the time related to the critical number (see Figure 11a), P6 made errors in performing arithmetic calculations when evaluating the function (see Figure 11b), and P7 graphed the given quadratic function as a straight line. Therefore, the connection between alternate representations (algebraic-graphic) was not achieved and the general formula $\left(x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}\right)$ was used in an inadequate way that prevented the procedural connection (Figure 11c).

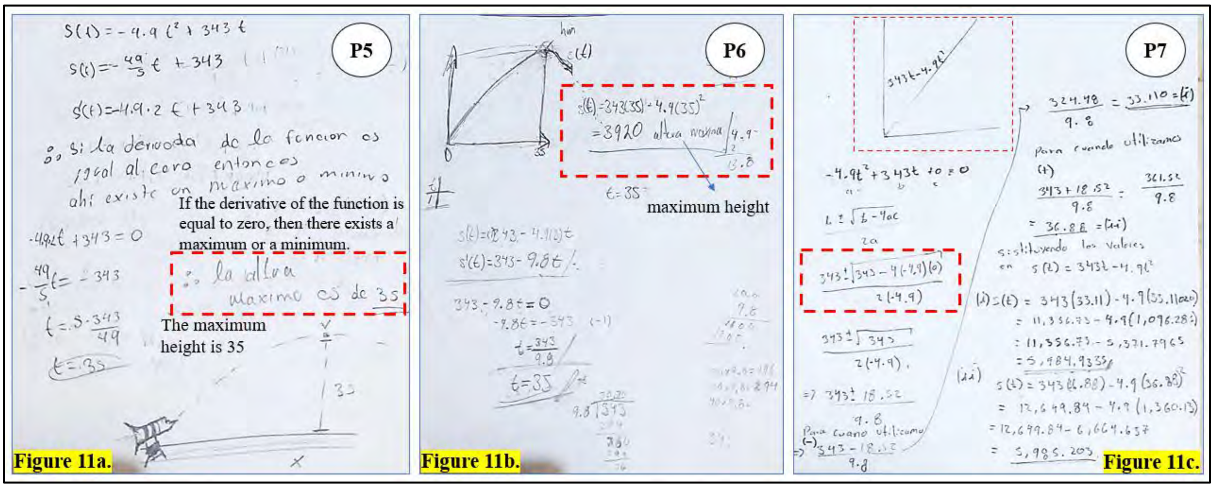


Figure 11. Evidence of the procedures of P5, P6 and P7

In this research, an onto-semiotic analysis of the system of mathematical connections established by one teacher and six students when solving a problem about launching a projectile was carried out. For example, these connections were of a significant type when conceiving the derivative as the slope of the tangent line to a curve at a point and the instantaneous rate of change. Also, procedural connections were presented when they used formulas to find the derivative of a quadratic function or the use of the general formula. The feature type connections when they mentioned properties of the quadratic function, among others. These mathematical connections were detailed from an articulated vision (ETC-OSA) in terms of practices, processes, objects, and semiotic functions that relate them as suggested in Rodríguez-Nieto et al. (2021a).

In other research on mathematical connections, the teacher, students or pre-service teachers have been considered separately as the object of study (Businskas, 2008; García-García & Dolores-Flores, 2020; Mhlolo et al., 2012; Rodríguez-Nieto et al., 2021b; Yavuz-Mumcu, 2018), however, this research analyzes one teacher and students from different countries that require the establishment of connections to solve an application problem. Although some participants consistently solved the problem, others did not because in their solving process they made incorrect or personal mathematical connections, which are the cause of the difficulties that students and teachers experience in understanding and solving problems that involve to the derivative. In this case, the in-service mathematics teacher and three students solved the problem going through different representations and meanings, but the other three students did not do so due to the inappropriate use of formulas, arithmetic errors, disconnections between the symbolic and graphical representations of the function, among others. These difficulties coincide with those reported in the literature (Fuentealba et al., 2018; Pino-Fan et al., 2017, 2018; Rodríguez-Nieto et al., 2021c; Sánchez-Matamoros et al., 2015; Sari et al., 2018). In addition, these difficulties invite in-service teachers to promote this type of mathematical connection with their students in the classroom when they develop the topic of derivatives.

On the other hand, this research reports the application of a theoretical-methodological development resulting from the articulation between the ETC and the OSA that allowed the detailed analysis of the mathematical connections of the participants and, in addition, revealed the mathematical connections that could not be established, as well as the mathematical practice that was not carried out correctly that gave rise to the disconnection. It should be noted that the established connections account for the participant's understanding of the derivative concept and application problems. In fact, the task may be simple, but the difficulty arises when the subject does not understand any of the concepts involved or is necessary for the resolution of the task.

CONCLUSION

This research shows the mathematical connections established by one Colombian secondary school mathematics teacher and six Mexican university students who were studying mathematics, which breaks the barrier of research carried out with a population located in the same sociocultural context or a same educational institution. At the same time, it allows visualizing connections or points in common between the teacher and the participants because some proceed in the same way to solve the problem or, differences since the teacher uses knowledge of physics that meets the curricular suggestions of the NCTM (2000) when they state that mathematical connections must be made between mathematical content with other subjects and with real life. We reflect that this research can be extended considering populations made up of a teacher and his students when working with the derivative concept or other mathematical concepts. Likewise, it is recommended that future research tasks rich in representations and meanings of the derivative be proposed to promote connections that are obviously not being established and that are the causes of the difficulties presented by students. Finally, it would be interesting to assess the extra-mathematical tasks where the modeling connection emerges, including situations that are typical of other subjects such as Physics, Chemistry, and Engineering, among others.

Acknowledgments

We thank the participants of this research for the information provided when they solved and explained the proposed task.

Declarations

Authors Contribution	:	CAR-N: Conceptualization, Writing - Original Draft, Editing and Visualization. VF and FMR-V: Writing - Review & Editing, Formal analysis, and Methodology. LRP-F: Review, Validation and Supervision.
Funding Statement	:	This work is part of the research project PID2021- 127104NB-I00 (MCIU/AEI/FEDER, UE) funded by MCIN/AEI/1013039/501100011033 and by "ERDF A way of making Europe." and Fondecyt 1200005 funded by Agencia Nacional de Investigación y Desarrollo (ANID) de Chile.
Conflict of Interest	:	The authors declare no conflict of interest.
Additional Information	:	Additional information is available for this paper.

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