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Pre-service mathematics teachers' learning to notice student statistical thinking in the context of lesson study

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ABSTRACT

It is important to note that the development of pre-service teachers' noticing abilities does not happen spontaneously; hence, assistance programs are crucial. This qualitative study aimed to examine pre-service teachers' noticing of student thinking within the context of lesson study. Three pre-service teachers conducted three lesson study cycles. Lesson plans, voice and video recordings of lesson study meetings and implementations, observations, field notes, and reflective writings are used as data collection techniques. The findings indicated that the pre-service teachers' early levels of noticing were constrained. Their noticing levels increased as the lesson study progressed. Hence, the improvement of pre-service teachers' noticing abilities can be assisted by lesson study. Activities such as planning, reflection and implementation helped pre-service teachers develop their noticing levels. To enhance the development of noticing skills, it can be proposed that lesson study should be integrated into teacher training programs.

Keywords: pre-service teachers, students' statistical thinking, noticing skills, lesson study

INTRODUCTION

Teacher noticing is considered as an integral part of professional competence and expertise (Kaiser et al., 2017; König et al., 2022). Generally, it can be defined as "attending to particular events in an instructional setting" and making sense of them (Sherin et al., 2011a, p. 5). A teacher with advanced noticing skill can examine the events in the teaching processes in depth and analyze the mathematical thinking of students (van Es, 2011). Noticing skill allows the teacher to interpret the events that affect learning and plan accordingly (Jacobs et al., 2010; Sherin et al., 2011a; van Es, 2011). As a result, teachers who pay attention to student thinking are better able to plan lessons effectively before, during, and after lessons and adapt their teachings to meet the needs of their students (Jacobs et al., 2010). At this point, noticing emerges as an important skill that should be possessed by teachers (Sherin et al., 2011a). In this regard, institutions that are highly renowned in mathematics education emphasize that teachers should notice their students' mathematical thinking, difficulties, mistakes and misconceptions (e.g., National Council of Mathematics [NCTM], 2000). This study aimed to examine pre-service teachers' noticing skills within the context of lesson study.

There are differences among researchers in terms of defining teacher noticing and its components. For instance, Jacobs et al. (2010) define it as "professional noticing of children's mathematical thinking" and they conceptualize this expertise "as a set of three interrelated skills: attending to children's strategies, interpreting children's understandings and deciding how to respond to the basis of children's understandings" (p. 172). Therefore, they mainly focus on teachers' decisions, actions, or responses based on what they noticed. On the other hand, van Es and Sherin (2002) described three key aspects of teacher noticing: "(a) identifying what is important or noteworthy about a classroom situation; (b) making connections between the specifics of classroom interactions and the broader principles of teaching and learning they represent; and (c) using what one knows about the context to reason about classroom interactions" (p. 573). Hence, their focus is on whether or how teachers make connections with principles of learning and teaching based on the variety of what they noticed. Afterward, van Es (2011) described two general categories: what teachers notice and how they notice. The former category refers to the actors (e.g., whole class, a group, particular student, and teacher) and the topics (e.g., pedagogical strategies, behaviors, classroom climate) that teachers notice. The latter category refers to how teachers analyze what they noticed. That is, how teachers notice refers to the approach they take (i.e., descriptive, evaluative, or interpretive) and the specificity of their analysis (i.e., general, detailed, elaborative, and providing evidence). This framework allows researchers to examine the development of teacher noticing because elaboration of how teachers notice as well as what they notice determines their expertise on noticing (Goldsmith & Seago, 2011; Stockero, 2014; van Es, 2011).

Most of the studies aiming to examine and improve teachers' noticing used video recordings of lessons or given tasks (e.g., van Es, 2011). Another line of research investigated teacher noticing during (Jacobs et al., 2010; Sherin et al., 2011b) and after (Choy et al., 2017; Sherin et al., 2011a) the lesson. In some researcher's work, the relationship between both types of noticing (noticing in and after the lesson) is focused on and attention is drawn to the relationships between them (Sherin et al., 2011b). Furthermore, some researchers argue that notice should not be ignored during lesson planning (i.e., preparing to notice) (Choy et al., 2017). It is important to study teacher noticing at different teaching stages because each reveals different aspects of teacher noticing and its development. Indeed, the skill of noticing certain details when planning, teaching and reviewing a mathematics lesson is seen as the distinguishing feature of a competent teacher (Yang & Ricks, 2012).

Additionally, researchers argue that noticing should be studied specifically within the relevant field (Jacobs & Empson, 2016; Nickerson et al., 2017). Several studies thus far have examined teachers' or pre-service teachers' noticing of student thinking related to a specific content or skill such as pattern generalization, additive or multiplicative thinking or quadrilaterals or derivative concepts (e.g., Fernández et al., 2012; Sánchez-Matamoros et al., 2015). Overall, it has become a common opinion that teachers' content-specific noticing based on a student-centered approach is directly related to the quality of mathematics teaching (Jacobs & Spangler, 2017; Jacobs et al., 2010).

Teacher Noticing of Students' Statistical Thinking

Statistics is becoming a part of school curricula because of its growing significance in all areas of life (Franklin et al., 2007; NCTM, 2014). Hence, teachers' skills to notice students' statistical thinking and reasoning drew attention (delMas et al., 2014; Shin, 2020, 2021). Research in this area mainly focused on teachers' and pre-service teachers' skills to notice the situations in video recordings or given tasks (Nagle et al., 2020; Shin, 2020). For example, Nagle et al. (2020) asked secondary mathematics teachers to interpret and respond to student work on a task involving informally drawing a line of best fit. The results revealed that the teachers generally interpreted student work in descriptive and evaluative ways and provided responses that were less likely to promote student thinking. Shin (2021), on the other hand, examined the selective attention and knowledge-based reasoning of pre-service teachers who watched statistics lesson videos. The results showed that the pre-service teachers were more focused on the teacher's pedagogy in the video than the students and their statistical thinking. Analysis of the pre-service teachers' knowledge-based reasoning showed that they tended to use an interpretative rather than a descriptive or evaluative stance to reason about what they observed. However, rather than the statistical knowledge required to teach students to reason about statistics, they tended to choose to employ general pedagogical knowledge and knowledge unrelated to statistical reasoning. The results of these studies suggest that teachers and pre-service teachers need support in noticing students' thinking in statistics. Lesson study, where teachers plan their lessons together, observe and reflect by putting student thinking into the center (Lewis & Hurd, 2011; Takahashi et al., 2013), can be used to support pre-service teachers' noticing skills (Bakker et al., 2022; Wessels, 2018).

Learning to Notice Within the Context of Lesson Study

It is difficult for teachers to develop the noticing skills that would help students' learning and enhance the teaching process accordingly, as the classroom environment is quite complex. Many in-class activities, which are either content specific (e.g., student thinking) or non-content specific (e.g., classroom climate) occur simultaneously (Copur-Gencturk & Rodrigues, 2021; Jacobs et al., 2011). Likewise, for pre-service teachers, noticing skills do not develop automatically, and in this regard, the importance of support is emphasized (Jacobs et al., 2010; Sherin et al., 2011b; van Es, 2011). Even though teacher education programs are expected to develop pre-service teachers' noticing skills (Sherin et al., 2011a), improperly designed fieldwork may result in pre-service teachers' focus on decision-making and judgment rather than in-depth analysis of student thinking (Levin et al., 2009). In addition, due to a lack of guidance (Loughran, 2002), pre-service teachers may not develop their observation skills and pedagogical content knowledge needed for the complex analyses of teaching and learning (Hammerness et al., 2005).

According to research, teachers' perceptions of their students are primarily formed during lessons. Hence it is important to further this understanding outside of the classroom (Cai & Ding, 2017). Both teachers and pre-service teachers need productive methods to analyze students' thoughts (Hiebert et al., 2003). Although studies emphasized many ways that would support the development of student knowledge, there are a few recommendations that these studies have all agreed upon. The first suggestion is that both teachers and pre-service teachers should discuss the target concept of mathematics and reflect on how this concept is learned by students (Berk & Hiebert, 2009). Researchers also suggest that student-centered implementations in real classroom environments help pre-service teachers gain knowledge about how students think and encourage them to attach greater importance to understanding and interpreting student responses (Ball & Forzani, 2009). In addition, teachers and pre-service teachers should analyze and reflect on the conducted lessons. It has been demonstrated that teachers' and pre-service teachers' analysis and reflection of their activities had a positive impact on the development of their knowledge about the student (Crespo, 2000; Hiebert & Morris, 2012; Steinberg et al., 2004). In a study with teachers, Steinberg et al. (2004) reported that teachers' evaluation of their interaction with students supported their knowledge on students' different thinking styles and assisted them in revising their instruction accordingly. Such practices also helped teachers realize that they should reflect on their students' thoughts rather than merely assessing student responses as right and wrong (Franke & Kazemi, 2001).

As it encompasses all the points emphasized above, lesson study can provide a suitable context to develop pre-service teachers' noticing skills (Amador & Weiland, 2015; Amador & Carter, 2018; Bakker et al., 2022). In the lesson study process, teachers/pre-service teachers deliver and evaluate a lesson after designing it in line with the target objectives/problems. All the processes is analyzed according to the observations made. This cycle can be repeated as many times as needed (Dudley, 2014; Lewis & Perry, 2014). This process enables teachers and pre-service teachers to reflect on students' thinking and the effectiveness of the teaching process, which can foster their noticing skills (Bakker et al., 2022; Guner & Akyuz, 2020).

Lesson study provides the opportunity to apply the knowledge learned in the university environment; hence, it allows a more-detailed characterization of the real classroom environment (Zhang & Cheng, 2011). Focusing on students' thinking skills for both planning and teaching, making instructional decisions with other pre-service teachers and improving lessons accordingly can be considered as important for developing components of noticing (Jacobs et al., 2010; Sherin et al., 2011b). According to research, lesson study can help pre-service teachers become more aware of how their students think leading to high-quality professional development (Amador & Carter, 2018; Amador & Weiland, 2015; Guner & Akyuz, 2020; Lee, 2019). Moreover, due to its structure and nature, lesson study allows teachers/pre-service teachers to attend to student thinking on the subject before (preparing to notice), during (noticing at the moment) and after (noticing after the moment) the instruction (Bakker et al., 2022; Wessels, 2018). However, what is not yet clear is how teacher noticing skills at different phases of teaching (i.e., planning, teaching, reflecting) develop within the context of lesson study. In this study, we examined what and how pre-service teachers noticed about students' statistical thinking as they planned, implemented and reflected on the lessons they delivered. We addressed the following research question: Whether and to what extent did a group of pre-service teachers' noticing skills for each phase (i.e., planning, teaching/re-teaching and reflecting/re-reflecting) of three lesson study cycles evolve?

METHOD

This case study investigated the changes in pre-service teachers' noticing skills on students' statistical thinking in a detailed manner. The case is determined as three consecutive lesson study cycles of three pre-service teachers at a university setting and a real classroom environment.

Participants

Three pre-service teachers from the elementary mathematics teacher education program at a state university in Ankara participated in this study. After completing this four-year program, pre-service teachers receive the degree required to be employed as middle school mathematics teachers. They are then qualified to instruct math to students in grades 5 through 8 in public or private schools. In the first two years of their undergraduate studies, pre-service teachers are primarily offered content knowledge courses, and in the latter two years, pedagogical content knowledge courses. Participants in the study are three pre-service teachers who attended the micro-teaching in mathematics education course, a selective course offered for senior students. All the pre-service teachers enrolled in the course successfully completed content courses (e.g., Statistics and probability) and pedagogy courses (e.g., Principles and methods of teaching, Methods for teaching mathematics) related to teaching statistics. Although a total of 12 pre-service teachers (four groups) enrolled in the course, this study focused on the data collected from one group. This group was selected because of their willingness to participate, motivation to share, skills for working as a group and higher achievement (grade point average [GPA]) compared to other pre-service teachers enrolled in the course. The participants having the pseudonyms Suelan, Brook, and Gale are 21, 21, and 22 years old, respectively. GPA of these pre-service teachers are 3.21, 3.26 and 3.29 on a four GPA scale, respectively. The lesson study group also included the classroom teacher (mentor teacher), the academician (the instructor of the course), and the facilitator (the researcher). Since the focus of the current study is pre-service teachers' noticing skills about student thinking, these members were not included in the study. However, information is provided for them as knowledgeable others are important in the lesson study (Yoshida & Jackson, 2011). The mentor teacher, who evaluated and interpreted the lessons in the actual classroom environment, has 12 years of professional experience, holding a master's degree. She was an innovative teacher grounding her practices on constructivism and highly willing to be involved in academic studies. The academician (the second author) took the role of evaluating and interpreting the lessons delivered in the university setting. The researcher (the first author) was in the third year of her doctoral studies. She served as an observer and did not involve in the discussions conducted and decisions made by the pre-service teachers.

Context of the Study

This study consists of three lesson study cycles including two stages. Lesson study was introduced to the pre-service teachers within the context of the micro-teaching in mathematics education course. The pre-service teachers examined sample lesson plans and read about lesson study. Throughout the whole process, they benefited from resources such as textbooks, research books, and a middle school mathematics curriculum. Before starting the actual study, a pilot study was conducted with the pre-service teachers to let them experience the lesson study. Then, they carried out the processes described in **Figure 1**.

The pre-service teachers planned their lessons using four-column lesson plan template (**Table 1**).

The pre-service teachers created three lesson plans to be delivered in a university setting throughout the three lesson study. Afterward, they revised and put them into practice in a real classroom setting. About 1.5 to two hours were spent on each lesson's planning. Lesson plans for the seventh-grade objectives concerning graphs (bar, pie and line graphs) were created by pre-service teachers. Each pre-service teacher delivered two lessons throughout the lesson study cycles. After each implementation, the lesson plan was revised according to evaluations. In addition to the researcher, the instructor took part in these evaluations as an expert after the implementations in the university classroom, and the mentor teacher took part after the implementations in the actual classroom. Furthermore, group members in the real classroom environment and other pre-service teachers engaging in the session during micro-teaching evaluated the lessons and offered suggestions for improvement.

These evaluations lasted from 30 minutes to one hour. In light of these evaluations, the pre-service teachers revised their lesson plans. Their reflecting processes lasted between 60 and 90 minutes. The summary of the lesson study process is shown in **Table 2**.

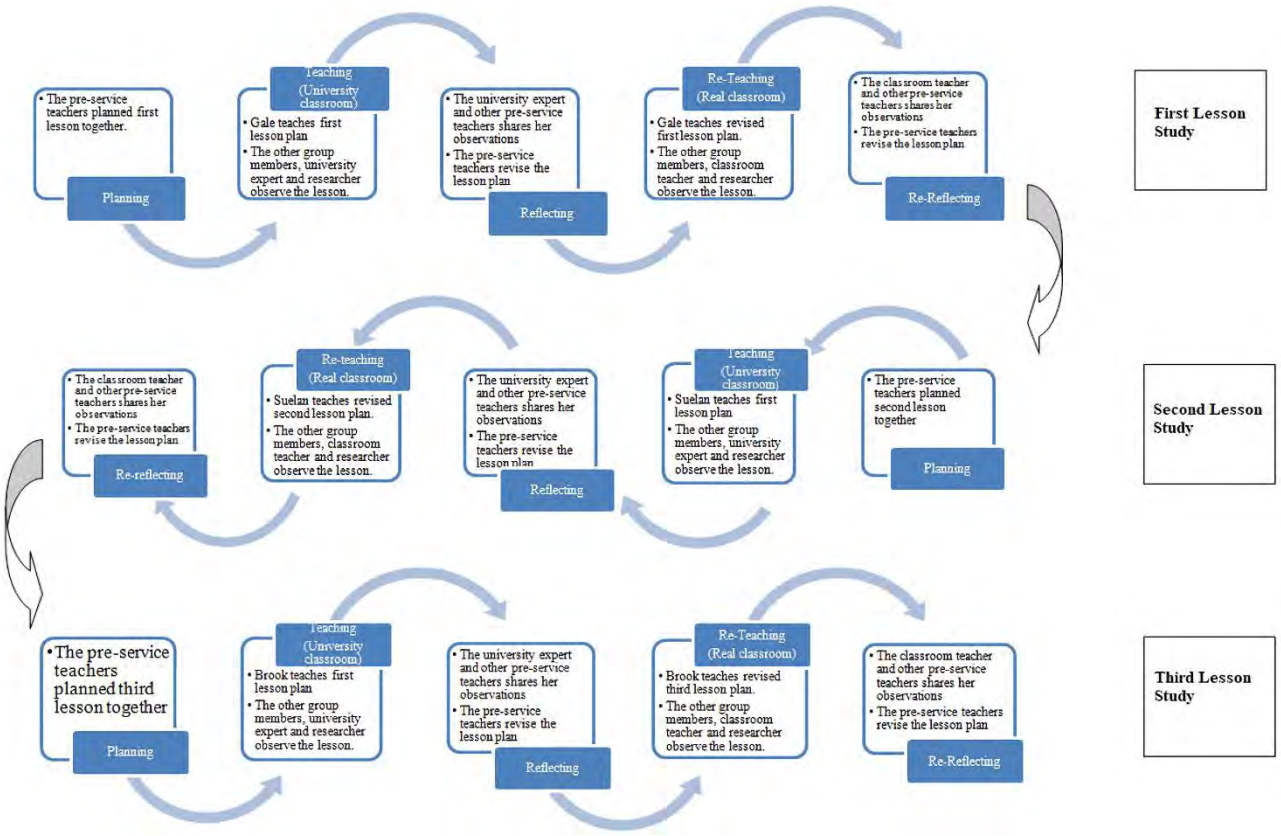

Figure 1. Implementation process (Adapted from Zhang & Cheng, 2011).

Table 1. Four-column lesson plan template (Adapted from Mathews et al., 2009, p. 506)

Steps of lesson: Learning activities & key questions	Expected student reactions or responses	Teacher's response to student reactions/things to remeber	Goals & method(s) of evaluation

Table 2. Summary of the lesson study process

	First cycle (pie graph)	Second cycle (line graph)	Third cycle (pie, line, & bar graph)
Planning	As a group	As a group	As a group
Teaching (university classroom)	Gale	Suelan	Brook
Reflecting	As a group	As a group	As a group
Re-teaching (real classroom)	Gale	Suelan	Brook
Re-reflecting	As a group	As a group	As a group

Data Collection and Analyses

Data collection process lasted for 14 weeks and consisted of three lesson study cycles with two stages (university and real classroom). Each lesson study cycle included following stages: planning, teaching (university classroom), reflecting, re-teaching (real classroom), and re-reflecting. The data collection tools were lesson plans prepared by the pre-service teachers; video and audio recordings of the meetings (planning and reflecting) and teaching in the university and real classroom environments; field notes taken by the researcher, university expert, and mentor teacher; and reflective papers written by the pre-service teachers.

To examine pre-service teachers' development of noticing skills, we analyzed data from each phase of the lesson study based on Van Es's (2011) framework, i.e., "framework for learning to notice student mathematical thinking". The video and audio recordings of the planning and reflecting meetings and of the lessons taught by the pre-service teachers in the university and in the real classroom environment were transcribed. The pre-service teachers' decisions, expressions, explanations, and interactions with students in the transcribed data were coded according to the components of "what teachers notice" and "how teachers notice" in the framework of van Es (2011) for each stage of lesson study (e.g., planning, teaching, reflecting).

In addition, lesson plans, observation reports, field notes, and reflective papers written by pre-service teachers were coded according to same framework (van Es, 2011). Representations of pre-service teachers (e.g., verbal, written, and visual), which are produced during interactions with each other or students, were considered as unit of analysis. The planning of relevant lesson was carried out as a group and each pre-service teacher implemented the lesson individually in line with the decisions taken as a group. Therefore, the individual pre-service teacher's interaction with the students was also coded and analyzed as being produced by the group.

Table 3. Framework of data analysis (van Es, 2011).

	Level 1: Baseline	Level 2: Mixed	Level 3: Focused	Level 4: Extended
What teachers notice	Attend to whole class environment, behavior & learning, & to teacher pedagogy	Primarily attend to teacher pedagogy	Attend to particular students' mathematical thinking	Attend to relationship between particular students' mathematical thinking & between teaching strategies & student mathematical thinking
		Begin to attend to particular students' mathematical thinking & behaviors		
How teachers notice	Form general impressions of what occured	Form general impressions & highlight noteworthy events	Highlight noteworthy events	Highlight noteworthy events
	Provide descriptive & evaluative comments	Provide primarily evaluative with some interpretive comments	Provide interpretive comments	Provide interpretive comments
	Provide little or no evidence to support analysis	Begin to refer to specific events & interactions as evidence	Refer to specific events & interactions as evidence	Refer to specific events & interactions as evidence
			Elaborate on events & interactions	Elaborate on events & interactions
				Make connections between events & principles of teaching & learning
				On basis interpretations, propose alternative pedagogical solutions

Noticing at the planning stage refers to what pre-service teachers attend while preparing the lesson plans. Based on van Es's (2011) framework, we examined whether and how they attend to the students and themselves as actors of the lesson as well as what kind of topics (e.g., content, tasks, materials, pedagogical strategies, and mathematical thinking) they considered in order to prepare for teaching. Noticing at teaching/re-teaching stages refers to what each pre-service teacher attends while teaching the lesson planned as a group. Hence we examined what triggered (e.g., student response, behaviors, and mathematical thinking) her actions while teaching. On the other hand, noticing at the reflecting/re-reflecting stages refers to what pre-service teachers select and bring to discuss after teaching the study lesson. Hence, we examined which specific moments of instruction they chose to discuss as well as the actors (e.g., whole class, individual student, and a group of student) and topic (e.g., concepts and procedures related to statistical thinking) for these instances. Moreover, we focused on the approaches they adopted for the interpretation of the topic noticed (descriptive, evaluative, and interpretive) as well as the details in their explanations (general, detailed, providing evidence, and elaborate) based on van Es's (2011) framework. Afterwards, for each stage of lesson study, pre-service teachers' levels of noticing were determined based on the instances coded taking into consideration two dimensions of noticing and descriptions of levels presented by the framework (**Table 3**).

Within this framework, development of pre-service teachers' levels of noticing is identified according to four levels: level 1 (baseline), level 2 (mixed), level 3 (focused), and level 4 (extended). Finally, we compared percentage of instances coded for each noticing level in order to examine the changes in levels of noticing within each phase (e.g., planning for the first, second, and third lesson) and across different phases (e.g., from planning to teaching, then reflecting, re-teaching, and re-reflecting) of lesson study.

In order for the study to meet the credibility criteria, we attached great importance to obtain data from different data sources. This also ensured the confirmability and dependability of the study. Long-term interaction with the participants throughout the study and observation of all working processes of the participants were important in terms of meeting the credibility criteria of the study. In addition, the researcher coded the relevance of the data for the themes at different time intervals (two times at three-week intervals). Elimination of the emerging inconsistencies ensured the dependability of the study. Another credibility strategy employed was expert review. 30% of the data were coded by another mathematics educator studying in this field. As a result, the inter-coder reliability was found to be 92%.

FINDINGS

For clarity, the findings are presented in detail under three sections related to the phases of lesson study (planning, teaching/re-teaching, and reflecting/re-reflecting). Additionally, changes of pre-service teachers' noticing skills observed across the phases of lesson study are highlighted within each section.

Learning to Notice at Lesson Planning

In **Figure 2**, the noticing levels of the pre-service teachers regarding the planning stage during the three lesson study are presented. Afterward, examples of the pre-service teachers' noticing levels in each lesson study process are given.

First lesson study

Pre-service teachers' noticing during the planning phase at the first lesson study demonstrated mainly the features of level 1 (77%) and to some extent level 2 (23%). The participants usually attended to the tasks and materials, and how to use them. They focused on their pedagogical moves and students' actions to follow the instructions. Below, Suelan reads an activity about making the transition from a bar graph to a pie graph presented in the teacher book:

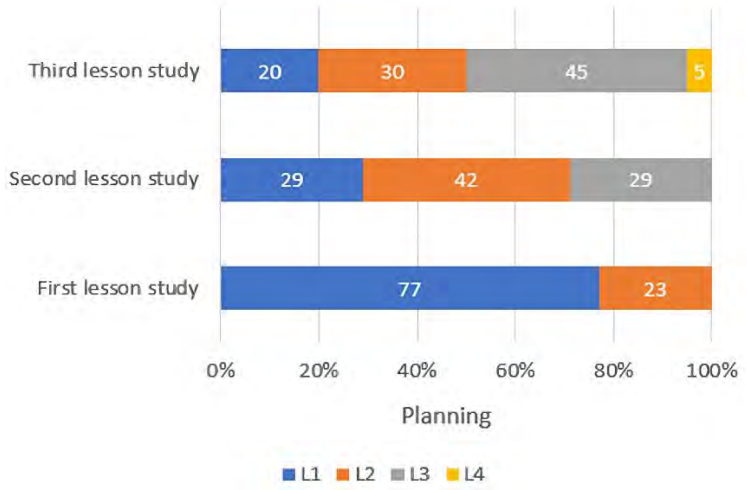

Figure 2. Noticing of the pre-service teachers in the planning stage (Source: Authors' own elaboration)

> Suelan: … students can convert bar graphs to pie charts by cutting out the bars and taping them together. Once a bar graph is complete, cut out the bars themselves and tape them together end to end. Next, tape the two ends together to form a circle. Estimate where the center of the circle is, draw lines to the points, where different bars meet, and trace around the full loop. You can estimate percentages using the rational number wheel or percent necklace (Van de Walle etal., 2016, p. 564).
>
> Gale: We can even use different colors for bars. It could be nice.
>
> Brook: We call each student to the board to form a circle. But I am not sure about how to do this.

Here, the pre-service teachers focused on how they could use the material (*different colors of paper strips for bars)* and made descriptive (*calling each student to the board to form a circle)* and evaluative comments about the implementation of the material (*it could be nice).* Even though the group planned the ways that students could be participated in the activity, the discussion was focused on general issues and did not include how students' engagement with the task could be related to their understanding of bar and pie graphs. However, from time to time, they attended to particular student thinking and behaviors and made some interpretive comments. The following statements from Brook and Gale show that they considered students' prior knowledge and possible misunderstandings related to bar and pie graphs while planning:

> Brook: The student can make a mistake like this; in the data set represented with a bar graph, a taller bar indicates that it has a larger amount of data. However, in a pie graph, they may not represent the area covered by this data set proportional to this amount.
>
> Gale: They studied the concept of the circle in the 6^{th} grade, and they learned the area of a circle in the 7^{th} grade. They learn the circle and the area of a circle in the 7^{th} grade by means of ratio and proportion. That is, students know these.

Even though their comments are mostly evaluative *(the student can make a mistake like this),* they made some interpretive comments and formed general impressions related to student thinking based on the curriculum.

Second lesson study

Pre-service teachers' noticing during the planning phase at the second lesson study demonstrated mainly the features of level 2 (42%). Fewer instances were coded as level 1 (29%) and level 3 (29%). Similar to the planning of the first lesson study, the pre-service teachers mostly focused on their pedagogical moves such as presenting the task and managing time. However, they discussed more about how students can make sense of the task and began to anticipate students' responses. Based on the evaluations of the potential student responses, they made pedagogical decisions. For instance, in the dialogue below, the group attended to students' possible understanding about role of the line graph and made decisions about the instruction accordingly.

> Suelan: Here students can confuse the role played by line graphs [they could think that] they can compare data [by using line graph]. I should also emphasize this [line graph is not only used to compare data].
>
> Gale: Our aim here is to give information about the use of line graph.
>
> Brook: Well, we use a line graph to show increases and decreases. We can see an increase or decrease. If we had shown the temperature values with dots [could we see the trend]?
>
> Suelan: Let's assume that 'they said we could see it [see the change by means of dots]. One [dot] is at the low and one [dot] is at the high. We can see [the change].

> Gale: If they would say 'yes we could see.' But there is a purpose in drawing the lines [to be emphasized]. Lines allow us to see the change [between the dots].

Here, the group attended to students' possible understandings about the use of line graphs *(to compare data)*, elaborated students' possible interactions with the task *(showing the change in temperature by means of dots)* and then made decisions about the instruction *(emphasize the role of lines between dots)*. These actions demonstrated features of noticing at level 2 and level 3.

Third lesson study

Pre-service teachers' noticing during the planning phase at the third lesson study demonstrated mainly the features of level 3 (45%) and level 2 (30%) and to some extent level 1 (20%). In addition, a few instances were coded as level 4 (5%). Similar to the planning of the second study lesson, the group made pedagogical decisions by considering students' possible understandings related to the task they planned to use. The dialogue presented below shows how they decided to choose the context of the task that requires selecting the appropriate type of graph.

> Gale: We need to address in which situations the bar, line and pie graphs will be used and how they are converted into each other in this activity.
>
> Brook: What kind of context should we use?
>
> Suelan: We can use the number of votes for the pie graph.
>
> Gale: But the students immediately know because they see it [distribution of votes represented by pie graph] everywhere.
>
> Brook: I have an idea. Let's do it this way; the students always make a generalization as if the number of votes is always represented by a circle graph. Let's try to avoid this. For example, we can create a problem situation with the number of votes, but we need to create it in such a way as to represent this problem situation with a bar graph.
>
> Suelan: Yes, it makes sense. Let's see if they can find that it would be more appropriate to represent it with a bar graph [in this problem]. Or will they overgeneralize and say a pie graph?

It was seen that the group focused on a specific situation in which students could make an overgeneralization *(distribution of votes is usually represented by pie graph),* made interpretive comments (*they see it everywhere),* and planned the instruction to reveal and overcome this generalization. These attempts allow the interpretation that the pre-service teachers' noticing level is predominantly at Level 3. In addition, it was observed that some of the dialogues between the pre-service teachers included signs indicating Level 4. They began referring to specific events they encountered during the earlier stages of lesson study and attending to the relationship between teaching strategies and particular student's understandings. For example, the dialogue below shows that the group attended to a particular student's difficulty in constructing line graphs at the previously, discussed about the relationship between the difficulty and the instructional strategies and took measures to overcome the difficulty for the next lesson.

> Suelan: The student drew the line graph incorrectly because he could not determine the intervals properly. I think we should give him either scaled or squared paper. In this way, he will not have any problems when determining the intervals.
>
> Brook: I think we can also give it [referring to the scaled/squared paper] for the bar graph not only for the line graph. The students may experience a similar problem there too.
>
> Gale: Then, let's give ready-made circles in the pie graph so that the students will not have difficulty in showing the slices of the circle and finding the angle measurements. You know, it happened in the lesson when I was teaching, one student had shown the sector of circle wrong.

When compared to the first and second lesson study cycles, the lesson planning was organized in a way that put more of an emphasis on student thinking, challenges, and finding solutions to these problems with student understanding.

Learning to Notice at Teaching and Re-Teaching

In **Figure 3**, the pre-service teachers' noticing levels regarding the teaching (university classroom-UC) and re-teaching (real classroom-RC) stages during three lesson study are presented. Then, examples of the pre-service teachers' noticing levels in each lesson study process are indicated.

First lesson study

Pre-service teachers' noticing during the teaching and re-teaching phases at the first lesson study demonstrated both characteristics of level 1 (65% for UC, 45% for RC) and level 2 (35% for UC and 55% for RC). Because the pre-service teachers mostly attended to their own pedagogical moves during the planning phase, Gale took a great deal of effort to implement the lesson as planned. She attended to students' behaviors rather than their thinking and made evaluations of the whole class by focusing on particular students who responded her questions correctly. That is, correct responses were what prompted her to act. On the other hand, there were a few instances in that she asked for elaboration. Below is a section from the teaching in the real classroom, where Gale asked students to construct a pie graph based on the data presented by the bar graph.

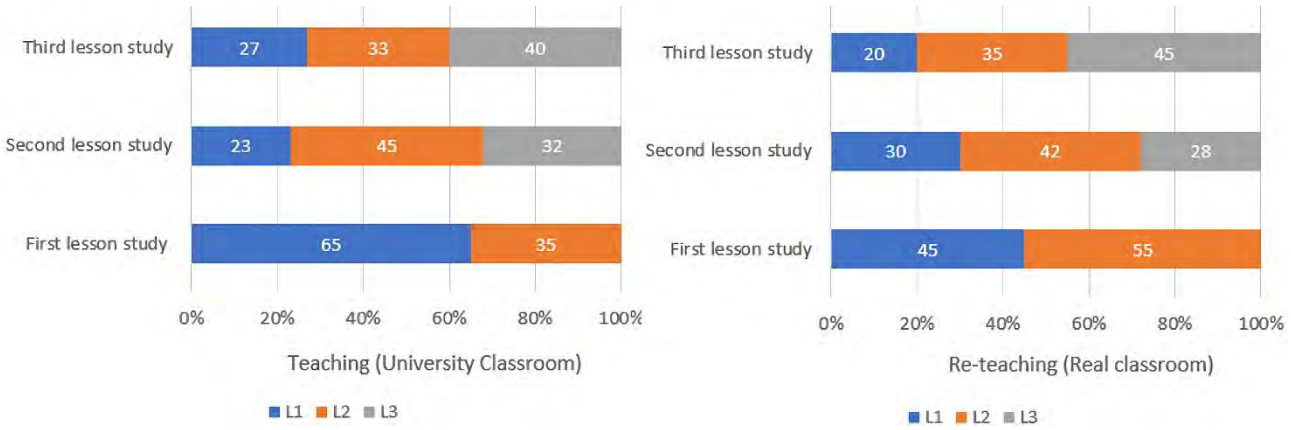

Figure 3. Noticing of the pre-service teachers in the teaching and re-teaching stages (Source: Authors' own elaboration)

Gale: Let's create a pie graph based on these data [number of siblings of 20 students]. What are the angle measures of the circle sectors according to the number of siblings? How many people were there? [Referring to the previous problem].

Student 1: 360 people correspond to 360 degrees.

Gale: How many people are there now?

Student 2: There were 20 people, so it means 20 degrees.

Student 3: 360 degrees.

Gale: Who wants to show it on the board?

Student 3: Me.

Gale: Can you explain your thought process?

Student 3: The total number of people is 20. Then, we say that if 20 people are equal to 360 degrees, we ask what the total degree covered by nine people having one sibling is. We can solve this by using proportions.

[wrote down the following on the board]

360 degrees 20 people

? degrees nine people

Thus, we can find the answer as 162 degrees. That is, the total degree covered by people having one sibling is 162.

Gale: Has everybody found 162 degrees? (re-teaching in the real classroom).

Here, Gale asked the student who responded correctly to explain her solution and made an evaluative comment. Hence, her teaching and re-teaching can be characterized as focusing on the correct answers, making evaluative comments and failing to elaborate student thinking. Thus, level 1 and level 2 can be used to describe the group's noticing skills during the teaching and re-teaching phases.

Second lesson study

Pre-service teachers' noticing during the teaching and re-teaching phases in the second lesson study demonstrated mostly the characteristics of level 2 (45% for UC and 42% for RC) and to some extent level 1 (23% for UC and 30% for RC) and level 3 (32% for UC and 28% for RC). The group's attention to maintain the lesson as planned was still getting ahead of attending to students' thinking. They demonstrated contradicting behaviors in terms of attending to students' thinking. Suelan, who was responsible for teaching the second study lesson, attended to some students' thinking while ignoring others. For example, one of the students asked why they needed to draw a line graph if they could see the changes of weekly air temperature with a bar graph. Even though the group discussed this issue as possible student thinking during the planning phase, Suelan did not attend to the student's question and said, "let's talk about this later." However, she did not attempt to explain or question this issue throughout the rest of her lecture. On the other hand, there were instances when she attended to students' thinking and responded to their questions. The following dialogue is an example of these instances:

Suelan: What do you know about line graph?

Student 1: We can see increases and decreases more easily in line graph.

Suelan: What else do you know?

Student 2: It makes it easy for us to show the changes in the data. We can interpret more easily, especially when we have a lot of data.

Suelan: What do you think of our friends' opinions?

Student 3: Yeah, when I looked at the graph, I understood why we drew the line. For example, I can say that it [the temperature] dropped from Monday to Tuesday, but not immediately.

Student 4: So we can see the change. Can I ask something? If we consider this change monthly, does it matter?

Suelan: It does not matter. While we are constructing the graph in this activity, we made the scaling on a daily basis. We could draw it on a monthly basis. From January to February, February to March. We could still see the change (teaching in the university classroom).

Here, instead of telling directly what to do, Suelan gave students the opportunity to explain their thoughts. In addition, she focused on the specific student question about scaling while constructing the graph.

Third lesson study

Pre-service teachers' noticing during the teaching and re-teaching phases at the third lesson study demonstrated predominantly the characteristics of level 3 (40% for UC and 45% for RC). Almost one third of their attention could be considered as level 2 (33% for UC and 35% for RC). Level 1 was the level with the fewest observations (27% for UC and 20% for RC). Brook, who was responsible for the implementation in both university and real classroom, focused on specific student responses, asked questions to elaborate student thinking, made interpretive comments related to student responses rather than evaluating them as true or false, and directed the instruction based on student responses. For example, below is a section from the class, where Brook made inquiries to reveal how the student thought.

Brook: Can you give me an example? What type of graph should I use? When to use these?

Student 1: I can use line graph to show temperature change or population change.

Brook: Himm. Did you consider the temperature or the change [in temperature] when deciding on which graph to use?

Student 1: Change.

Brook: Yes when we want to show "the change of anything" [we use line graph].

…

Brook: How do you decide on the appropriate type of graph?

Student 1: I use bar graph if I need to compare data and pie graph if I need to inquire about what the part means in the whole (teaching in the university classroom).

Here, Brook asked questions to elaborate the student's thinking and used her response to emphasize important ideas related to the topic. The pre-service teachers decided to use group work for real classroom implementation. This strategy allowed Brook to observe students' thinking, examine their understandings in-depth and respond accordingly. The dialogue below demonstrates how Brook asked questions to inquire about students' thinking, made them realize about their mistake and guided them to find the correct solution. Students are trying to find the angle measure of the circle sectors.

Student 1: We multiply all by five to get 20. But it is too small [the size of a circle sector].

Brook: Why do you multiply by five?

Student 1: Because there are 20 people in total here, we have to multiply by five to complete 100 [he thinks the whole should be taken as 100 rather than 360].

Brook: Okay what are you writing inside the bar graph right now, angle? What do we do while finding the angle measures [for circle graph]?

Student 1: Hıımm. The whole should be taken as 360 but I thought it would be taken as 100.

Brook: Yes, did not you notice? What did your friend do [asking to the other student in the group]?

Student 2: He concluded it smaller [indicating that the area is smaller].

Brook: Why did he (student 1) find the area smaller?

Student 2: He proportioned it to 100.

Brook: Why? Is not it 100?

Student 3: We need to take the whole as 360, not 100 (re-teaching in the real classroom).

As stated in the dialogue, Brook noticed that the students took the whole as 100 rather than 360 while finding the angle measure of the circle sector. Then she asked questions to make them realize that they made a mistake. Brook did not evaluate the answer of the student as true or false but made the student question it in depth. Focusing on the specific student difficulty during teaching and making inquiries to examine this difficulty indicates that the noticing level of the pre-service teachers has shifted to level 3.

Learning to Notice at Reflecting and Re-Reflecting

In **Figure 4**, the noticing levels of the pre-service teachers regarding the reflecting and re-reflecting stages during the three lesson study are presented. Then, examples of the pre-service teachers' noticing levels in each lesson study process are given.

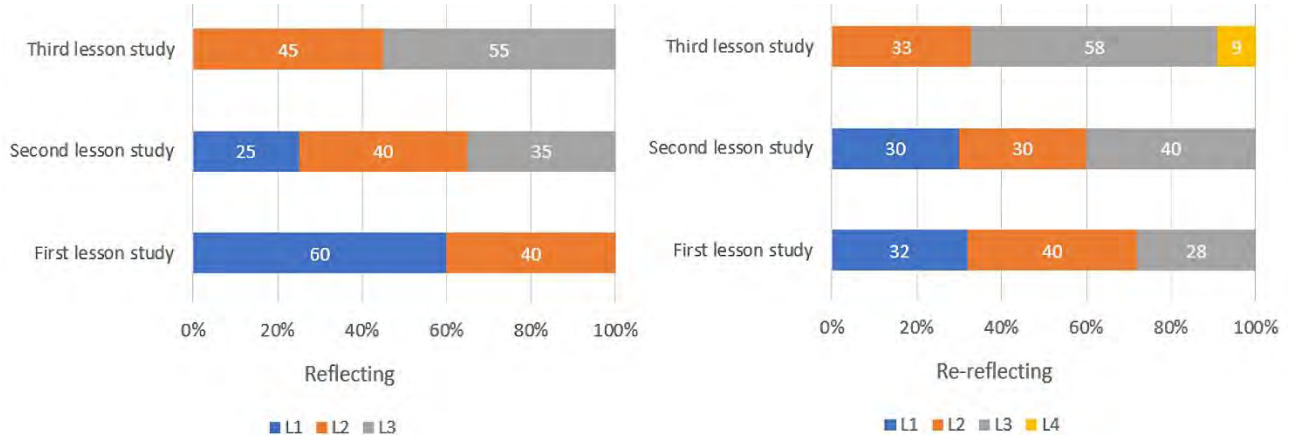

Figure 4. Noticing of the pre-service teachers in the reflecting and re-reflecting stages (Source: Authors' own elaboration)

First lesson study

Pre-service teachers' noticing during the reflecting stage demonstrated predominantly the features of level 1 (60%) and to level 2 (40%) to some extent, whereas their noticing during the re-reflecting stage demonstrated the features of level 3 (28%) as well as level 2 (40%) and level 1 (32%). The pre-service teachers focused on their own pedagogical actions (e.g., explanations, questions) and the responses of the whole class. They made general and evaluative comments about their own teaching and students (e.g., "*students did not understand why we converted from a bar graph to a pie graph*"). However, there were times; albeit highly limited when they focused on specific student responses. For instance, Suelan made the following comment on the Gale's implementation based on his/her observation notes kept during the implementation; *one of the students said, "we know the bar graph, so why do we need to learn the pie graph?*" However, there was no discussion about why the student asked this question and the underlying reasons during the reflection meeting. At the re-reflecting stage, the pre-service teachers were observed to reflect on some specific student responses and on how these students thought. During the implementation, one of the students drew the circle sectors as shown in **Figure 5**.

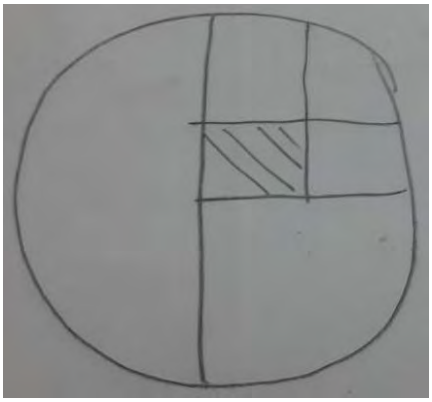

Figure 5. Student answer in the real classroom implementation (Source: Authors' own elaboration)

The pre-service teachers focused on this student's response during the reflection. Gale, who carried out the implementation, stated "*I realized that the student was doing wrong, but I panicked, I did not know what to do and what to say*". They started a discussion on the answer given by the student and the following dialogue took place.

> Gale: Why could he have given such an answer? Did I explain it wrong?
>
> Brook: I noticed it, and even wrote it in my observation notes. I think he did not know what the circle sectors mean. He was not aware of the fact that one end of the circle sectors must be on the center and the other end on the circle.
>
> Suelan: In this case, he does not know the concept of center.
>
> Gale: He may have thought of the area of the circle sector same as the area of the quadrilateral region (re-reflecting in the real classroom).

Here, the pre-service teachers focused on the reasons of student thinking. Their comments were predominantly interpretive. Since they focused on specific student responses and referred to important cases in explaining student thinking, there is evidence showing that the pre-service teachers' noticing levels are at level 3. When compared to the other stages of the first lesson study cycles, pre-service teachers' noticing about attending to a particular student's thinking process in this stage can be considered as attention grabbing.

Second lesson study

Pre-service teachers' noticing skills demonstrated mostly the characteristics of level 2 (40%) and to some extent level 3 (35%) and level 1 (25%) for the reflecting stage of the second cycle. On the other hand, their noticing skills demonstrated features of level 1 (30%), level 2 (30%), and level 3 (40%) at approximately similar ratios for re-reflecting stage. Similar to the first cycle, the pre-service teachers tended to attend to their own pedagogical actions, but they began to make connections with the understanding of whole class and made evaluations and suggestions related to the next instruction. For instance, Gale evaluated the implementation of Suelan, as follows; "*I do not think it was noticed much why you drew a line graph. It would have been better to walk around more between the groups and have them all talk*." Even though this interpretation was quite general and did not include details about why the students did not understand, it shows that the group began to focus on the specific moments of instruction.

During the re-reflecting, Suelan focused on the reactions of certain students and evaluated the reasons behind the students' responses. For example, "*he did not notice that there should be equal spacing when scaling a line graph. So he drew it wrong and misinterpreted … His misinterpretation may be due to the incorrect drawing*." Likewise, Gale and Brook shared their observation regarding the student: "*One of the students said that days should always be represented on the x-axis, and the temperature values must always be shown on the y-axis. Did this draw your attention? Here, the students overgeneralized. It may not always be like this. He always drew graphics like that; maybe it's because of this*". That is, they attended to certain students' thinking and tried to make sense of them.

Third lesson study

Pre-service teachers' noticing during the reflecting stage of third cycle demonstrated both features of level 3 (55%) and level 2 (45%). For the re-reflecting stage, the predominant level was still level 3 (58%). Fewer instances were observed related to noticing at level 2 (33%). In addition, a few instances that could be considered as noticing at level 4 (9%) were observed. During the reflecting stage of the first cycle, the pre-service teachers discussed the task and the implementation process without considering students' thinking. At the third cycle, they began to revise the tasks according to the answers and difficulties of specific students.

In the reflection phase of the first lesson study cycle, the task and the implementation process were mostly discussed without considering the student's thinking, while in the reflection of this cycle, the tasks were revised according to the answers and difficulties of specific students. Below is given a relevant section from the reflection meetings.

> Brook: It would be convenient for us to use the line graph to answer this problem situation. Well, then I should make students question how the problem situation should be if I want to draw a bar graph.
>
> Gale: We definitely should make them question this. The student should realize that he has to make changes in the structure of the problem situation. I think students have difficulty in this part.
>
> Suelan: One student even said that he (talking about the university implementation) could draw all the graphs regardless of the problem situation. So he was not aware of the difference between them.
>
> Brook: They are unaware that the process starts with the research question. I think that's the main reason why they do it wrong. If you remember, we did not know at first, either (reflecting in the university cassroom).

The above dialogue shows that the pre-service teachers focused on the difficulties experienced by students. The fact that the pre-service teachers tended to interpret these thoughts rather than define them and that they tried to highlight the important events in this process show that the pre-service teachers' level of noticing is predominantly level 3. Here, it can be seen that their reflection on the difficulties experienced by the students provided guidance for the pre-service teachers.

Likewise, during the re-reflecting, Suelan evaluated Brook's implementation: "*Did the answer of student 4 draw your attention? I took notes. He insisted to use a pie graph in a problem situation in which a bar graph would be more suitable. He could not be convinced.*" Then, Gale made an interpretive comment about the student's thinking: "*I think he had difficulty in understanding the problem situation and this was the main reason*". Similarly, it was discussed as to why the student made the mistake of taking the whole 100 instead of 360 when calculating the angle measure of the circular sector. Suelan made the following interpretation: "*he has not yet developed the concept of the whole; therefore, he makes such mistakes*". At this point, they tried to elaborate, focusing on specific student thoughts regarding the teaching. Based on these efforts, it can be said that the pre-service teachers' noticing levels are predominantly level 3. In addition, the efforts of the pre-service teachers at this stage to produce alternative solutions *(let's design an activity that will enable them to experience the statistical problem-solving process themselves; in this case, they can determine which graphs are appropriate in which situation)* to overcome the difficulties experienced by the students were remarkable.

DISCUSSION AND CONCLUSIONS

The importance of teacher noticing has been highlighted in previous research, but it has also been noted that developing this skill can be challenging, especially for pre-service and inexperienced teachers (van Es & Sherin, 2002). The findings of the current study show that lesson study can support the development of noticing levels of pre-service teachers. These results are consistent with those of other studies and suggest that noticing is a learnable practice (Amador & Carter, 2018; Amador & Weiland, 2015; Guner & Akyuz, 2020).

Overall, the results showed that the pre-service teachers' initial noticing levels were low. Level 1 was the most prevalent level of noticing for the first lesson study. The pre-service teachers mostly attended to their pedagogy and the expected responses from the whole class in general. They made evaluative comments based on general impressions related to their teaching and student learning. These results are consistent with those of other studies reporting that teachers tended to focus on instructional components (e.g., classroom management, tasks) rather than student thinking (Fernández et al., 2012; Jacobs et al., 2010; Sherin et al., 2011a, 2011b) and suggesting that focusing on student thinking was a challenging activity for teachers and pre-service teachers (van Es, 2011).

In this study, it was revealed that the pre-service teachers' noticing skills increased throughout the lesson study. The prevalent level of noticing was level 2 for the second lesson study and level 3 for the third lesson study. Also, the distribution of noticing levels for each lesson study cycle shows a gradual improvement in noticing skills. This shows that the pre-service teachers began to attend to particular students' thinking and tended to associate components of teaching (e.g., tasks, material) with student thinking. They shifted from being descriptive and evaluative in their comments to becoming more interpretative. This finding agrees with van Es and Sherin's (2002) findings, which show that there is an improvement in teachers' noticing from a descriptive and evaluative approach to an analytical and interpretive approach with support.

Furthermore, the current study contributes to the literature by examining how pre-service teachers' noticing skills at different levels of teaching (i.e., planning, teaching, and reflecting) develop within the context of lesson study. The results revealed improvements in noticing skills within each phase of lesson study (i.e., planning, teaching/re-teaching and reflecting/re-reflecting). In the planning stage (preparing to notice), what the pre-service teachers attended was changed from 'their pedagogical moves and the possible reactions from the class' to 'the students' possible understandings and engagements with the task'. Likewise, how they noticed was changed from 'general descriptions and evaluations of the tasks, instructional moves or student thinking on the basis of teacher resources and the curriculum' to 'specific interpretations of the cases they observed related to student thinking'. They gradually designed the lesson by putting the student thinking into the center as they proceeded from the first to the third lesson study. On the other hand, what triggered the pre-service teachers to act at the teaching/re-teaching stages (noticing in the moment) was changed from 'the evaluations of students' responses as right or wrong and of proceeding the instruction as planned' to 'the interpretations of specific instances related to student understandings or misunderstandings that are noteworthy to address in the class and the ways to elaborate these understanding'. That is, their focus of attention moved from 'making general evaluations of students' thinking' to 'understanding of the reasons behind their thinking'. Compared to other phases of lesson study, the pre-service teachers' noticing at the reflecting/re-reflecting stages (noticing after the moment) demonstrated the highest improvement. Throughout the three lesson study cycles, what pre-service teachers selected and brought to discuss after teaching changed from 'the general and evaluative comments about their own teaching and the responses of the whole class' to 'the interpretations of specific moments of instruction and of particular students' understanding'.

As the lesson study proceeded, the pre-service teachers began to make connections between their own teaching and the understandings or misunderstandings of specific students and made suggestions for the next instruction. Even though the pre-service teachers' noticing showed improvement within each phases of teaching, the most improvement was observed at the reflecting/re-reflecting stages. This finding can be expected because their noticing of student thinking before teaching was mostly fed by printed sources such as teacher textbook and the curriculum, whereas noticing after teaching was mostly fed by their own interactions with the students. In other words, noticing after teaching was positively affected from what they noticed and how they noticed at the planning and teaching. It can thus be suggested that the development of noticing at different stages of teaching may show a different trajectory.

The development of the pre-service teachers' noticing skills during the process shows that there may be a relationship between this development and the activities they performed during the lesson study. It can be thought that what the pre-service

teachers did during the planning was an important factor that supported this development. The title of expected student reactions in the four column lesson plan template may have triggered the pre-service teachers to attend student thinking while planning their lessons. This is also supported by other studies (e.g., Choy, 2015; Yoshida & Jackson, 2011) stating that teachers' attention to the planning contributes positively to their focusing more on student thinking and making appropriate explanations. In this study, the pre-service teachers focused on a specific topic, i.e., graphs (bar, pie and line). They examined student thinking, misunderstandings and difficulties related to graphs and studied on the instructional strategies for teaching graphs. Their focus on a specific content can also be considered as an important factor that may have affected the development of noticing skills, as suggested by prior research (Fernández et al., 2012; Harle, 2008). The pre-service teachers' engagement in reflection and discussion by focusing on student thinking and interpreting the reasons behind producing solutions can also be expressed as a factor that supports the development of noticing. Observations based on students' thoughts and reactions shape the reflection stage and contribute to the evaluation of the lesson and make necessary changes (Harle, 2008). It is also thought that the collaborative nature of the lesson study has affected the noticing skills of the pre-service teachers. The group's carrying out all the stages (e.g., planning and reflecting) together may have helped them gain different perspectives and see their deficiencies, hence triggered them to put an effort to overcome these deficiencies. Other studies in the literature (e.g., Lee, 2019; van Es & Sherin, 2002) draw attention to a similar emphasis. In addition, taking on the role of the teacher in the implementations conducted in both the university and the real classroom environment provided the opportunity for them to observe students' thoughts more closely. Similar points have been emphasized in the literature, and it is claimed that lesson study helps pre-service teachers understand students' thoughts and redesign the learning environment based on this comprehension (Amador & Carter, 2018; Amador & Weiland, 2015; Amador et al., 2016; Guner & Akyuz, 2020; Lee, 2019). It was also stated that pre-service teachers shifted from a descriptive and evaluative perspective to a more interpretative perspective in terms of how to notice over time, and this was an indicator of a development in noticing (van Es & Sherin, 2002). When the process is evaluated holistically, lesson study cycles lead the pre-service teachers to the center due to their structure (Fernández et al., 2003). This point of view is likely to affect pre-service teachers' noticing skills positively.

In previous studies it has been emphasized that pre-service teachers' noticing skills may be limited during the transition to different learning objectives (e.g., Amador & Carter, 2018). In this study, three lesson study cycles were carried out and different objectives were discussed in each cycle, albeit related to each other. When the noticing levels of the pre-service teachers were evaluated, no such limitations were observed in the lesson study transitions (for example, the transition from the first lesson study cycle to the second lesson study cycle). The results obtained in this context, including and focusing on the lesson study differed from the literature.

Implications and Limitations

The entire lesson study process was conducted as a group in this study, thus the noticing skills of the pre-service teachers as a group was examined. Even if the lessons were implemented independently, the group made the decisions, reflections, and revisions that impacted the delivery of the lesson together. The fact that the noticing skills of individual pre-service teachers were not revealed can be considered as a limitation of this study. Although studies indicate a positive relationship between individual learning and activities conducted as a group in a lesson study (e.g., Vrikki et al., 2017), it is not known whether such a relationship exists in the current study. Future studies can be designed to observe the change in the noticing skills of individual pre-service teachers.

In this study, there was little evidence of Level 4 in terms of the pre-service teachers' noticing skills about student thinking. This is confirmed by the research, which also points to the pre-service teachers' inexperience as the key factor in their inability to apply what they noticed to their teaching practices (Amador & Weiland, 2015; Lee, 2019). Developing noticing skills at higher levels may require more time and effort (Crespo, 2000). Conducting more lesson study cycles can support pre-service teachers in terms of advancing their noticing skill to Level 4. In addition, interventions to the lesson study process can contribute to the development of pre-service teachers' noticing skills. For example, Lee (2019) emphasized that revisions and additions (for example, task-based formative assessment) should be made in the lesson study in order to support pre-service teachers' noticing skills further. Another strategy to guide pre-service teachers in focusing more on noticing student thinking is integrating the framework proposed by van Es (2011) into this process. In this way, teachers/pre-service teachers can have information about what and how to focus on to develop their noticing skills (Lee & Choy, 2017).

In the current study, a group of pre-service teachers' noticing skills regarding student thinking were examined. Also, since there was no experimental design used in the current study, no causal relationship could be inferred between the pre-service teachers' noticing skills of student thinking and the effect of lesson study. The causal relationship between pre-service teachers' noticing skills and the effect of lesson study can be investigated through experimental designs in future studies. Despite the limitations mentioned above, the results obtained in this study show that the development of pre-service teachers' noticing skills can be supported by lesson study processes. From this point of view, it is thought that the integration of teaching practice courses into the undergraduate curriculum based on lesson study has the potential to contribute to the development of pre-service teachers' noticing skills.

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