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Research Article

Discussion contents during a lesson study conducted with knowledgeable others

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Over the past two centuries, lesson study has been applied in different countries with different participant profiles. Lesson study is defined as a dynamic research-learning cycle of planning, applying, observing, and analyzing a lesson. Although the participants' discussions in lesson study practices were examined from different perspectives, they have not been analyzed in depth in terms of their contributions to teachers' and preservice teachers' professional development. In this study, the content and frequency of participants' discussions are examined. Six weeks of research were conducted by an academician, a teacher, and three mathematics pre-service teachers. The data were collected through field notes, video recordings, interviews, and focus group interviews. A total of eleven major headings were identified in the lesson study, concerning pedagogical and mathematical issues. The study also concluded that while classroom management was the most widely discussed topic, group and individual instructional techniques received less attention.

Keywords: Lesson study; Discussion content; Knowledgeable others

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1. Introduction

Raised in Japan in the late 19th century, Lesson Study (hereafter LS) has first migrated to China in the 1950s and then to the rest of the world following its meaningful contributions for the mathematics achievements of Japanese students in international examinations (especially in Trends in International Mathematics and Science Study results of 1995) (Stigler & Hiebert, 1999). Since then, LS model has been the subject of tremendous educational research. The preferred style of LS in in-service (Cooper & Karsenty, 2018; Elkomy & Elkhaial, 2022; Fernandez, 2002; Metelerkamp, & Kraft, 2023; Ylonen & Norwich, 2012) and pre-service (Leavy & Hourigan, 2016; Lewis, 2019; Shelton et al., 2023) teacher education is defined as a dynamic research-learning cycle (Kanbolat, 2015). LS is a collaborative professional development model that involves a group of teachers over a long-term period, planning, applying, and observing a lesson, and then analyzing their observations and sharing their experiences with each other (Amador & Galindo, 2021; Iksan et al., 2014; Lewis, 2000; Murata & Kattubadi, 2012). This model could be used to analyze the

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teaching process (Amador, & Galindo, 2021; Fernandez, 2002) and its effectiveness (Allan et al., 2020; Subadi, Khotimah, & Sutarni, 2013), to evaluate the quality of teaching (Chassels & Melville, 2009; Laoli et al., 2022; Perry & Lewis, 2011), and to allow teachers to learn from their practices (McGrane & Lofthouse, 2010; Winsløw, 2021; Wong &Wong, 2009).

LS is a *long-term and continuous development model* for teachers, which spans a specific time period (Abdella, & Reddy, 2022; Lewis, 2002; Ronda & Danipog, 2022; Stigler & Hiebret, 1999). LS requires teachers to *work together* for a common purpose and on a subject matter that is more difficult to teach (Lewis, 2002). Hence it gives *teachers the opportunity to work in collaboration with their colleagues* (Ronda & Danipog, 2022; Stigler & Hiebret, 1999), and therefore encourages them to reflect on the conclusions they derive from the sharing of their own practices (McGrane & Lofthouse, 2010; Wong & Wong, 2009), to improve educational activities, and even to contribute to the curriculum (Felux & Snowdy, 2006; Perry & Lewis, 2011). Also, LS allows teachers to observe the lesson they designed together (Iksan et al., 2014; Murata & Kattubadi, 2012) and to focus specifically on students' learning processes (Fernandez, 2005; Ronda & Danipog, 2022; Stigler & Hiebret, 1999; Yarema, 2010). So, LS allows teachers to develop their teaching skills (McGrane & Lofthouse, 2010; Stigler & Hiebret, 1999; Wong & Wong, 2009). In this respect, teachers discuss students' prior knowledge and understanding and exchange information and ideas about how to teach new content (Baki, 2012; Ronda & Danipog, 2022). They also discuss students' learning situations (Pektas, 2014), their difficulties, and how feedback should be provided (Fernandez, 2005).

In addition to the strengths of the LS, it is a model that has just started to be applied in the international arena; Difficulties experienced in establishing a common work schedule and ensuring participation of LS group members in meetings and observations; teachers' hard work and fatigue in the LS process; There are also weak sides due to reasons such as the existence of different teacher working cultures in different cultures. In their research, Abdella and Reddy (2022) expressed this as lack of time, the newness of the model, and a resource-poor context posed a threat to teachers' participation in LS.

LS has originally been used to contribute to teachers' professional development. Having observed its efficiency on teachers, researchers have begun to use LS in pre-service teachers training and many researchers demonstrated that it also contribute to pre-service teachers' professional development (Baki, 2012; Lewis, 2019; Murata & Pothen, 2011; Shelton et al., 2023; Vermunt et al., 2019). For example, in their study with 25 pre-service teachers, Leavy and Hourigan (2016) focused on the types of pedagogical content knowledge (PCK) developed by pre-service teachers as a result of engaging in LS. They stated that pre-service teachers' knowledge of student was improved through LS and that they became more skilled in identifying the source and nature of student errors. The pre-service teachers stated that they had the opportunity to closely examine the mathematically incorrect answers of the children at the LS meetings, and that this was a very important opportunity. Appova (2018) also reported that LS contributed to the development of both content and pedagogical content knowledge of pre-service teachers.

In addition, the professional identities of pre-service teachers are shaped in the process of applying this model (Gunnarsdóttir & Pálsdóttir, 2011). When they undergo this process while planning, applying, or evaluating a course, acting as a teacher, they begin to truly feel like teachers before they graduate (Burroughs & Luebeck, 2010).

On the other hand, researchers also stated that LS process helped to develop self-assessment skills of pre-service teachers (Baki, 2012; Lee, 2019; Suh & Parker, 2010). Baki (2012) found that this model enabled pre-service teachers to evaluate their classroom practices by focusing on their own competences and shortcomings as teachers. Thus, in the process, pre-service teachers can evaluate their own lessons or another pre-service teacher's lessons from the perspective of a researcher (Burroughs and Luebeck, 2010). In addition, it can be seen that LS process develops the cooperation skills of pre-service teachers and makes them a part of a learning community

(Gunnarsdóttir & Pálsdóttir, 2011). Because LS is carried out in collaboration, lesson plans are shaped by the ideas of all group members to take their final forms.

More specifically, LS contributes to metacognitive skills, in terms of preparing a lesson plan (Burroughs & Luebeck, 2010; Lee, 2019; Pektas, 2014), designing a learning environment (Pektas, 2014), focusing on students' learning (Burroughs & Luebeck, 2010; Fernandez, 2005; Gunnarsdóttir & Pálsdóttir, 2011; Pektas, 2014), providing meaningful learning (Pektas, 2014), and supporting material use (Burroughs & Luebeck, 2010; Ishii, 2014), mathematical knowledge (Appova, 2018; Fernandez & Zilliox, 2011; Gunnarsdóttir & Pálsdóttir, 2011; Suh & Parker, 2010), and professional development process of pre-service teachers. In short, pre-service teachers who learn and experience a lot with each other and learn lesson plans in real teaching environments learn to teach (Fernandez, 2005; Gunnarsdóttir & Pálsdóttir, 2011; Murata & Pothen, 2011).

The use of LS in different cultures and countries has led to differences in LS applications and the emergence of different LS models, especially over the last decades. For example, depending on the profile of LS participants and the purpose a LS serves for; different types of LS are available such as in-school, general, inter-schools, and with pre-service teachers' LS (Fernandez & Yoshida, 2004; Takahashi & McDougal, 2016; Takahashi & Yoshida, 2004), with knowledgeable others or without knowledgeable others (Kanbolat & Arslan, 2022). When LS groups are categorized according to the purpose they focus on, there are lesson study types such as in-school, general, inter-schools, and with pre-service teachers (Fernandez & Yoshida, 2004; Takahashi & McDougal, 2016; Takahashi & Yoshida, 2004). On the other hand, the types of LS to participant profiles are LS with knowledgeable others and LS without knowledgeable others. In some LS groups, knowledgeable others may also be involved in the process in order to support the LS of teachers and/or pre-service teachers (Appova, 2018; Baki, 2012; Burroughs & Luebeck, 2010; Fernandez & Yoshida, 2004; Kanbolat, 2015; Suh & Parker, 2010; Watanable, 2005). In the without knowledgeable others LS, the participants of LS are the same profile (consisting of only pre-service teachers or in service teachers). It is easier to create and process such lesson study groups than other types. For example, it will be easier to determine a common time period in the lesson study held with teachers or pre-service teachers than in applications where participants from different profiles take part (Kanbolat & Arslan, 2022).

In work on the LS model, knowledgeable others are involved in the process to support the work of teachers or pre-service teachers, enriching the process with the knowledge and experiences they have. Knowledgeable others who possess a strong knowledge of the subject, pedagogy, LS applications (Fernandez, 2002; Gutiérrez, 2015; Seino & Foster, 2021; Stepanek et al., 2006; Takahashi & McDougal, 2016), and curriculum are expected to contribute to discussions in the LS process that will allow students to pursue mathematical thinking (Amador & Weiland, 2015). Knowledgeable others can also be instrumental in providing other participants with access to theoretical knowledge or current research findings (Fernandez, 2002). Bjuland and Mosvold (2015) observed that the efficiency increased at the end of the LS process with knowledgeable others and commented that knowledgeable others would guide the pre-service teachers. Wood and Sithamparam (2014) stated that knowledgeable others are of vital importance in LS model and that they increase the quality of the discussions.

In this context, the LS discussed in the research is LS conducted with pre-service teachers according to its purpose and with the participation of knowledgeable others according to the participant profile.

1.1. The Aim

The number of studies on LS with pre-service teachers continues to increase day by day, and as a result, there are many studies conducted on various research topics in this field (Ponte, 2017). While some of the research are aimed at describing the LS process with pre-service teachers (Kannellopoulou & Darra, 2019; Lamb & Aldous, 2016; Shelton et al., 2023), some of them focus on the professional development processes of LS pre-service teachers (Amador & Galindo, 2021;

Appova, 2018; Leavy & Hourigan, 2016). In addition, when the LS studies conducted with knowledgeable others are examined, the focus is on the characteristics of knowledgeable others and contribution of knowledgeable others participation to the LS process (Bjuland & Mosvold, 2015; Seino & Foster, 2021; Wood & Sithamparam, 2014). The research focuses on the sharing of the members of the LS group during the LS with the pre-service teachers with the participation of knowledgeable others. The research carried out in this context is important in that it points to a different focus in the literature of both types of LS. It is thought that presenting the sharing content of the participants and the frequency of sharing of these contents in the LS process, which includes academicians and teachers as knowledgeable others in the research, which is carried out to support the professional development of pre-service teachers, is important in order to describe what kind of support pre-service teachers need in the context of professional development. In addition to this, it is thought that the research is important in terms of defining the characteristics that the knowledgeable others who will take part in the process should have. Moreover by analyzing these contributions, it is expected that readers will gain a better understanding of how participants benefit from the support of others who are knowledgeable about the topic at hand, and thus a gap in the literature will be filled.

2. Method

This qualitative research was conducted with a case study method (Creswell, 2013) and without any intervention in the LS process. In the case study, the environment, individuals and processes in which the research is carried out are examined holistically and the relationships and interactions between them are focused (Meriam, 1998). In this research, the case study design was preferred because the LS process, environment, group members and the relationships and interactions between them were presented to the reader holistically.

2.1. Participants

The research was carried out at a public university in Türkiye within the scope of a "Teaching Practice" course, with the participation of one academician, one teacher, and three pre-service teachers. The academician had worked as a mathematics teacher for one year, after completing undergraduate education in an elementary mathematics teaching program, and then completed graduate and doctoral studies and has taught for 1.5 years at the university where this research was conducted. Besides conducting academic studies in the fields of mathematics and mathematics education, he also teaches courses such as General Mathematics, Linear Algebra, Instructional Technologies and Material Design.

The teacher graduated from an elementary mathematics teaching program and has been working for three years. He has been preparing a thesis on developing problem-solving skills of students as part of his master's degree. In observations made in order to select an appropriate teacher before the research began, this individual attracted the attention of the researcher and was deemed appropriate to take part in the research. The teacher also gained experience by taking part in the pilot study. The academician and teacher who participated in the research as knowledgeable others will be coded as K1 and K2 in the article.

The participant pre-service teachers (1 female, 2 males) were selected from an elementary mathematics teacher education program in the 4th year and from teaching practice lesson groups on a voluntary basis. These pre-service teachers were given pseudonyms taking the ethical rules of the study into consideration: Esma, Hakan, and Erkan.

2.2. Pilot Study

In the pilot study, the researcher focused on the LS process with a mathematics teacher, whom she determined on a voluntary basis, and four volunteer pre-service teachers attending the school experience course under her supervision. The pilot study provided significant experience in terms of the researcher's rehearsal of the research process, data collection and data analysis process, and the preparation of the appropriate environment for the actual implementation to be realized close

to the ideal. In addition, it was decided to include an academician working in the field of mathematics education, in addition to teachers and teacher candidates, in order to contribute to the sharing environments that occur in the context of mathematical knowledge and teaching knowledge in mathematics in real practice, thanks to the pilot study.

2.3. Process

Three cycles were carried out in this research, with planning, implementation, reflection and re-planning, re-application, reflection, and finalization of the plan performed in each cycle (Lewis, 2000; Stigler & Hiebert, 1999; Yoshida, 1999). During the planning meetings, the participants shared their views on how to implement the application, and then they designed and implemented a common lesson plan. In the reflection and re-planning meetings that were carried out after implementation, the participants evaluated the application of the lesson plan and shared their opinions on what could be done to eliminate deficiencies, and the plan was revised. After the second application, the final evaluation of the participants shaped the final version of the plan and a new lesson plan was prepared. The applications of the research were carried out in the 5th grade of a middle school for 6 weeks and each pre-service teacher performed a cycle for a different learning outcome (length measurement units, calculating the perimeter of polygons or the area of rectangles, etc.).

Information about the learning outcomes, which were the focus of attention in a specific cycle of the LS, the order of cycles, the related activities and the pre-service teachers who were responsible for the classroom applications of the learning outcomes are given in the Table 1.

Table 1
Lesson study process

Cycles and activities	*Activity codes*	*Learning outcomes*	*PSTs*
Cycle 1			
First meeting	1M1C	• Recognizes units of length measurement; converts meters-kilometres, meters-centimetres-millimetres into each other and solves related problems.	Esma
First application	1A1C		
Second meeting	2M1C		Hakan
Second application	2A1C		
Third meeting	3M1C	• Calculates the perimeter of polygons; creates different shapes with a given circumference.	
Cycle 2			
First meeting	1M2C	• Recognizes units of time measure, converts them into each other and solves related problems.	Erkan
First application	1A2C		
Second meeting	2M2C		Esma
Second application	2A2M	• Calculates the area of the rectangle; It uses square centimetres and square meters.	
Third meeting	3M2C		
		• Estimates a specified area in units of square centimetres and square meters.	
Cycle 3			
First meeting	1M3C	• Creates different rectangles with a given area.	Hakan
First application	1A3C		
Second meeting	2M3C	• Solves problems that require calculating the area of the rectangle.	Erkan
Second application	2A3C		
Third meeting	3M3C		

Note. PSTs: Pre-service teachers.

As can be seen from the Table 1, in the first cycle of the LS, group members focused on two learning outcomes related to length. The implementation of the first lesson plan, which was the common product of the group, was carried out by Esma, and this lesson was observed by all group members. In the meeting held after the first application, the lesson plan, which was rearranged,

was applied by Hakan in a different classroom and was observed by other members of the group again. After the second application, the group members came together and gave the final version of the lesson plan of the first cycle, and thus the first cycle was completed.

In the second cycle, group members focused on three learning outcomes for time measurement units and the area of the rectangle. Erkan performed the first application of this cycle and Esma performed the second application. They completed the second cycle by following a similar process as above.

In the third cycle, group members discussed two learning outcomes related to the area of a rectangle. Hakan performed the first application and Erkan performed the second application, and they completed the cycle by following a similar process as above.

Resarch lesson were carried out in two different 5th grades. During the research, each pre-service teacher carried out 2 applications lasting for 4 lesson hours. In the research, the pre-service teachers performed in-class practices totally for 12 hours, which took 2 hours a week for 6 weeks.

In order to present the meetings and application activities during the LS process in a more understandable way to the readers, the order of LS cycles was coded regarding the activity type and the activity order. For example, the event '1M2C' stands for the 1st meeting held in the 2nd Cycle; the activity expressed as '2A2C' represents the 2nd application that took place in the 2nd Cycle.

2.4. Data Collection Tools

During the course of the study, the in-class practices of the three pre-service teachers were also observed for a total of 300 minutes during 12 lessons and also with the participation of the academician, teacher, and other pre-service teachers. The data of the research were collected by field notes, video recordings, interviews, and focus group interviews. In this paper, excerpts from the transcripts of the video recordings will be presented to the readers.

2.4.1. Field notes

The important parts of the meetings and classroom applications and the comments about them were noted. With the help of the notes taken in the meetings, it was possible to describe and illustrate the interactions of the participants in detail, and with the help of the notes taken on classroom applications, the ability to describe and illustrate parts of the lectures in more detail was enhanced.

2.4.2. Video recordings

LS meetings lasted for 300 minutes in total and were video-recorded. These records were analyzed with the help of field notes; and then the findings regarding the content and the frequencies of the participants' exchanging information were analyzed.

2.4.3. Interviews

Semi-structured interviews were conducted about each part of the LS. In these interviews, which were held with all participants after each of the plan meetings and in-class practices, questions were asked to the participants about how the relevant meeting or in-class practice affected their professional development processes. Each interview was tape-recorded and lasted for about 15-30 minutes.

2.4.4. Focus group interviews

The focus group interview method was preferred because it is a complement to other data collection methods in a short interval of time. In order to evaluate the LS process, focus group interviews were conducted under the guidance of the focus group interview report prepared at the end of the LS process in order to reveal the effects on the participants' professional development processes and to learn their opinions about the roles of the others. These interviews lasted 86 minutes in total and were video-recorded.

2.5. The Role of the Researcher

The first author, who prepared the environment, avoided intervening in the process and observed the environment while taking notes.

In the focus group meetings, she tried to create an environment in which all participants could present their ideas equally and managed the process accordingly. In addition, in face-to-face interviews, she became an interviewer who avoided interpreting or directing the participants (Johnson & Christensen, 2010).

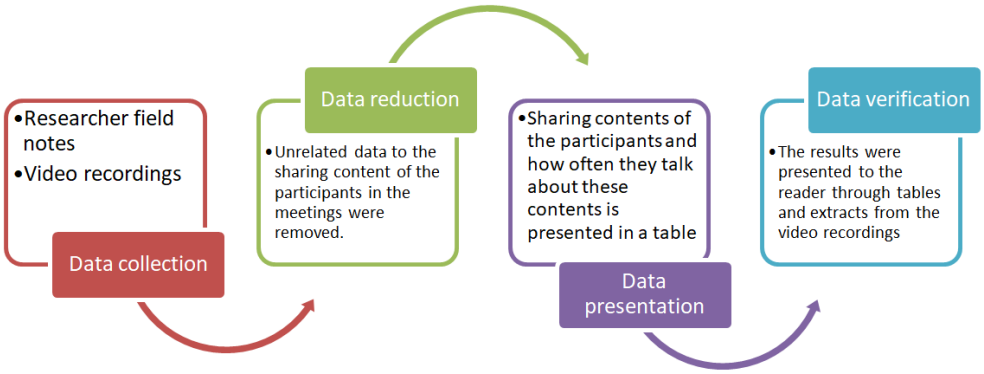
2.6. Data Analysis

Data were analyzed with the help of content analysis. Firstly, video recordings of the LS meetings were analyzed with the help of field notes; and then the findings regarding the content and the frequencies of the participants' sharing of their expertise, knowledge or suggestions were analyzed. The findings were supported by interviews, focus group interviews, and extracts from field notes taken by both the researcher and the participants. In this study, extracts from the LS meeting video recordings transcripts are presented in the findings section in dialogs.

In the data analysis, three-step analysis process of Miles and Huberman's (1994) was used to determine what the participants' sharing contents were and how often they talked about these contents. The data analysis process is summarized in Figure 2.

Figure 2

Three-Step Analysis Process (Miles and Huberman, 1994, p. 23)



First of all, primary codes for LS meeting sharing contents were created with the help of researcher field notes. Next, video recordings of the LS meetings were reviewed again and again with the guidance of field notes and primary codes were revised With the data reduction step, data unrelated to the sharing content of the participants in the LS meetings were removed.

In the data display step, similar codes were categorized together under common themes (Yıldırım & Şimşek, 2013) and final checks were done. In addition, the researcher obtained the number of video sections relevant to each code and theme with the help of the NVivo qualitative data analysis program. During the meetings, the participants were informed about the topics that were shared in the LS process as well as how often they shared the relevant content. In the data display step, the data is presented in an understandable way for the reader. For this purpose, information about the sharing contents of the participants and how often they talk about these contents is presented in a table (Miles & Huberman,1994).

Finally, in order to ensure the verifiability of the research, the research process was tried to be explained to the reader with all its transparency, and the results were presented to the reader through tables and extracts from the video recordings. In order to ensure the reliability of the research, the research questions and the role of the researcher were explained. In addition, the researcher repeatedly coded the data and received support for the coding process from a researcher who is an expert in qualitative research. This process continued until the obtained codes and themes were satisfied. In the process of obtaining the results, the researcher was also confirmed by making use of the statements in the field notes, video recordings of the meetings,

focus group interviews and interviews with the participants. The findings of the study were presented to the reader only with the help of meeting records.

2.7. Validity and Reliability of the Study

According to Guba and Lincoln (1982), validity and reliability for qualitative research are divided into four components: credibility (internal validity), transferability (external validity), reliability, and confirmability (objectivity).

In order to ensure the credibility (internal validity) of the research, the researcher had a long-term interaction with the pre-service teachers through observation (field notes and video recordings) and interviews (focus group and semi-structured interview) throughout the LS process. Since the first researcher conducted the research with pre-service teachers studying at the university where she worked, an academician working at this university and a teacher who completed her graduate education in this university, she observed the events in their natural environment and for a long time. The fact that the participants and the first researcher have known each other since previous years has increased the confidence in the answers given. As the focus group interviews and semi-structured interviews conducted within the scope of the research were conducted in a friendly environment, the participants had the opportunity to express themselves comfortably. Since the researcher did the LS process at the university where she worked, she stayed in the environment for a long time, constantly interviewed when necessary, examined his findings with a holistic view, and tried to obtain consistent results by comparing them with each other.

After the determination of suitable participants for the research, purposive sampling was done through voluntary participation in order to ensure its transferability. The LS process has been presented to the reader in detail by making use of the video recordings, field notes and interviews of the LS meetings.

The research design, data collection tools, research process and the role of the researcher are presented to the reader in detail in order to ensure that other researchers who examine the findings and results obtained from the research can also infer meaning from the data and find it consistent and reliable. The analysis of the data and the results were carried out comparatively by a researcher and an expert in the field of mathematics teacher training and qualitative research. In order to reach a common conclusion by the researcher, the raw data were analyzed in line with the feedback of expert opinion in each examination, and the data obtained at each stage were recorded regularly. Thus, the reliability of the study was increased by ensuring the consistency of the study as well as confirmability.

3. Results

In this section, discussion contents of the participants, which are grouped under eleven themes, are presented in Table 2.

Table 2
Discussions Contents

Content	*f*	%
1. Classroom Management	71	14.31
2. Knowledge of Students	69	13.91
3. Feedback	60	12.10
4. Mathematical Knowledge	56	11.29
5. Unexpected Situations	55	11.09
6. Teaching Problem-Solving	51	10.28
7. Material Usage	48	9.68
8. Assessment	24	4.84
9. Context Information	24	4.84
10. Achievements	22	4.44
11. Group/Individual Instructional Techniques	11	2.22

In the following, detailed information on each type of shared content presented in Table 2 is provided. During LS meetings, the most often discussed topic (14.31%) was *classroom management*. In the context of *classroom management*, *time management* was most often discussed. For example, Dialogue 1, which contains participants' views about how much time should be spent on an activity, is presented below (K1 and K2 stand respectively for the academician and the teacher).

Dialogue 1
Contribution on Classroom Management

K1: What do you think? How should we start [the course]?
K2: I think we should start with the repetition of the previous lesson.
Erkan: I agree, let's start with this field [i.e. the measurement of area]. Let it be a review.
K2: Let's give brief information about the topic and make up for missing parts. In the first twenty minutes we can deal with this […]
Hakan: For previous lessons, twenty minutes was excessive. Ten minutes is enough; in fact, it is up to the atmosphere in the classroom. Let's say twenty minutes and maybe we'll cut it off early depending on how it goes.
K2: Let's agree on fifteen [minutes]. In twenty-five, you can form a rectangle. You draw their shapes on the board and finish the lesson. You sum up those things.

Note. The dialogue is excerpted from the transcript of 2M2C.

In addition to time management, participants talked about different dimensions of classroom management. Participants shared views and opinions on subjects including the following: how to make students participate in the course, how to keep students' attention while using the board, how to get answers to the questions of the students, what kind of flexibility can be allowed in the prepared course plan, how to make write notes in the students' notebooks, how to communicate with students during the course, how to achieve simultaneous and collaborative work, and how to respond to the behaviour of special students in the classroom.

During the LS process, the topic of *knowledge of students* was frequently mentioned (13.91%). During the process, in relation to knowledge of students, students' *learning difficulties* were most often discussed. An example is presented in Dialogue 2.

Dialogue 2
Contribution on Student Recognition

Esma: They asked about the difference between the perimeter and area, but I guess Hasan asked because they couldn't reconcile it. For example, because they could not make a correlation, for the last question [they answered] 25 times 25, 50.
K2: When you made them solve the example on the board […] Ümit said later that he performed addition. They still couldn't think of multiplication. And as you know, Ümit is one of the best pupils [but] he said he did addition instead of multiplication. I think you should have cleared it up.

Note. The dialogue is excerpted from the transcript of 1M2C.

Participants also discussed student knowledge from different angles. For example, the learning processes, and the pre-learning experiences and willingness of the students. Participants were able to exchange ideas about feedback in LS meetings (12.10%). When these points are taken into consideration, it can be stated that the participants mostly shared information about *inadequate feedback*. The K2's expressions in Dialogue 3 are an example of such a sharing environment.

Participants shared feedback on subjects such as replying to the answers or questions of students in a timely manner and giving feedback in a way that students can understand; avoiding ambiguous, contradictory, or false statements while responding; rewarding students' correct behaviours; and so on. Almost all of the group members commented on the feedback given to the students in a critical sense and contributed to the process. The comments on the feedback to students were mostly made in the revision meetings. It can be said that the pre-service teachers

Dialogue 3

Contribution on Giving Feedback

K2: [Showing the question on the planning paper] For example, you had this solved on the board. [...] Then you told them to write it down in their notebooks. The solution is not fully understood here, I think, because [showing the shape] they made it a different shape. They tried to solve it from there. They made different patches on the shape, and you haven't explained that patch over again. You cut it short, just by explaining it on the board. Yusuf made this shape using a geometry board. For example, you saw him; the solution there was very nice. If it were me, I would show the areas between those unit squares. Yusuf solved this on the geometry board. I think it was very nice. […] You could have shown the solution to the class and made them calculate it by geometry board. It would have been nice, you know, instead of using a pen.

Note. The dialogue is excerpted from the transcript of 1M3C.

were inexperienced on the basis of the mistakes they made in the course related to feedback. In addition, the pre-service teachers' efforts to minimize the risk of unexpected situations and anxiety about sticking to the course plan caused problems related to the provision of feedback.

There was sharing about mathematical knowledge (11.29%) during the LS meetings. Regarding the learning outcome "recognizes the measurement units of length; converts between meter-kilometre, meter-centimetre-millimetre, and solves related problems", it can be taken as an illustrative example that the K1 suggested that students work on measurement values obtained within the classroom environment instead of studying pre-tailored measurements given by the K2.

Dialogue 4

Contribution on Mathematical Knowledge

K1: Let them measure the lengths of their desks. I think they should measure something directly themselves, with their rulers.

K2: So, what do we do after the students measure their desks?

K1: No, no. The students need to measure concrete things by themselves. Let the student measure his desk, his friend´s height, and so on. If they want to add up something, let them add the measurements of the length of the desk and their friend's height. They should face something concrete.

Note. The dialogue is excerpted from the transcript of 1M1C.

In this context, participants described the concepts related to the subject, how formulas can be presented to the students, demonstrations that can be used in the course, points to be taken into consideration in geometrical drawings, mathematical language, problems related to the subject, and issues related to daily life. In addition, it was observed that they shared ideas about terms, formulas, and drawings related to the subject.

There was some discussion about *unexpected situations* (11.09%) during the LS meetings. In such environments, it was observed that they mostly shared ideas about the "intervention process". The statements in Dialogue 5 are an example of sharing environments that occur in the context of unexpected situations.

Dialogue 5

Contribution on Unexpected Situations

K2: We have said 0.85 dm, sorry, here we also said meter. Then we said 85 dm. Here [writing on a piece of paper] the students come up with 85 dm instead of meters. Not 8.5. This is what they did at first; they didn't find anything else. Sema found this at first. Then Hakan´s suspicions started. "Could it be 85?" he said. "Do it again". He did it again, and this time he found something that was nonsense. "Do it again", he said. In the end, he wrote 8.5, and here he wrote 85. Sema gave up. Then Yusuf came up. You asked Yusuf: "Which is bigger?" He said, "They are equal".

Note. The dialogue is excerpted from the transcript of 3M1C.

Besides intervention process participants talked about many different dimensions of unexpected situations. The participants shared unexpected and surprising situations in the classroom, unexpected reactions from students, and questions about how to respond to these situations. Sharing about unexpected situations was mostly done in revision meetings. Because pre-service teachers encounter their first experiences during classroom practice.

The participants shared information about teaching problem-solving (10.28%) during LS meetings. Almost half of this kind of sharing seemed to be about the *problem-solving process*. The statements in Dialogue 6 are examples of such a sharing environment.

Dialogue 6
Contribution on Problem-Solving

Esma: For example, write three problems on a piece of paper, copy them, and give them to the students […]
K2: It's up to you. Do as you wish.
Erkan: Write more on the board instead of that.
Esma: Solve them one by one. One by one?
Erkan: Make them write in their notebooks and provide solutions.
Esma: One by one?
K2: Sure, one by one.
Erkan: One by one. When one question ends, you move on to another.
K2: Move from easy towards difficult.
Esma: OK.
K2: Don´t let all three be at the same level.

Note. The dialogue is excerpted from the transcript of 1M1C.

Besides, the participants discussed the problem-posing process, and the problems to be used in the course. The participants shared information and ideas about the teaching of problem-solving or every detail of the problem-solving activities they perform. For example, they made common decisions on what kinds of problems will be used in the lesson, how many problem-solving activities will be done, and how the problem will be presented to the students. The participants also shared their opinions about the problem-solving activity process and the use of problem-solving activities in some classroom applications.

The participants shared about *material usage* during LS meetings (9.68%). Dialogue 7 represents an example of such a sharing environment.

Dialogue 7
Contribution on Material Usage

Esma: Using a unit cube for the area has misled [the students].
Hakan: I already said that.
Esma: Yes...
Hakan: You know, I was going to create a high ground with geometry strips. When they put it in, the surface would be zeroed. I thought of doing it later, and then I couldn't go back to it. We've already discussed this together. I couldn't go back to it, so it's past. There were other achievements. I thought I couldn´t wrap it up […]
Erkan: Everyone said that. The children had difficulties using the unit cube. Some of them said "It doesn´t fit" […]
K1: […] I didn´t interfere with Erkan while he was structuring an area with unit cubes. It would have been a mistake. Unit squares could have been used.

Note. The dialogue is excerpted from the transcript of 3M2C.

In the context of material usage, the participants talked about determining the material to be used in the course and how to use that material. While the participants shared knowledge during the preparation process on subjects including which materials could be used during the lesson and how, which materials would be more proper to utilize, how much/how many materials are needed, and where the materials could be obtained, they also shared comments on how to use materials effectively in the classroom and how to ensure classroom management while using these materials.

In the LS meetings, input related to assessment (4.44%) was also shared. In this context, the participants talked about the assessment process. Dialogue 8 may be given as an example of this type of sharing of the participants, who mostly discussed the measurement tool.

Dialogue 8

Contribution on Assessment

K1: […] How can we fill in the contents of this [measurement tool]? How many questions do we need? We need 6 questions.
Erkan: For example, you gave a rectangle, and the short side. Let's combine two educational outcomes. Like this: Give a rectangle. Give the length of the short side. Then the long side. Then we ask them to calculate the perimeter of the rectangle. The result is given in meters. We can then ask for the result in centimeters.

Note. The dialogue is excerpted from the transcript of 1M1C.

The members of the group shared ideas with each other about the content of the measurement tool, the formal and technical characteristics, and how to manage the assessment process. For example, they made common decisions on how many questions would be present in the measurement tool or what kinds of materials could be used in the measurement process. After making those joint decisions about the measurement process, the participants then shared their opinions on how the evaluation would be carried out. For example, they made common decisions on how to implement the awards and what awards would be given. The participants were informed about the different approaches of other participants while discussing the assessment process. They also realized that the assessment and evaluation process is a part of the course that needs to be prepared critically and carefully.

In the lesson study meetings, exchanges of suggestions about *context information* (4.84%) were observed. Dialogue 9 is an example of such a sharing environment.

Dialogue 9

Contribution on Context Information

K2: As the fifth grade becomes the new secondary school, math teachers have difficulty adapting to this fact. Fifth graders are still at the elementary level. The more games we play, the more similar the examples are to the language they know. They like the lessons much more, they understand more, and they participate in the lessons more.

Note. The dialogue is excerpted from the transcript of 2M1C.

The participants shared input about the classroom environment, students' willingness, students' learning situations, students' pre-learning and some students' characteristics, and mostly about the classroom environment.

Among the things that the participants focused on when preparing their plans for the course were the expected educational *achievements* of the course. In the LS meetings, achievements were recorded (4.84%). In this context, the participants mostly focused on the limits of the achievements and rarely on the content of the achievements. An example of this type of sharing is given via the statements of the K2 in Dialogue 10, which is related to the learning outcomes of the lesson.

Dialogue 10

Contribution on Achievements

K2: "Recognizes units of time measurement, converts them to each other". You just prepare questions about this subject. Make students solve problems. You can also ask them, "How many are there in an hour?" or "How many seconds are there in a minute?" or you can divide the day. You can do things like this.

Note. The dialogue is excerpted from the transcript of 3M2C.

After interpreting the curriculum together, the participants made a decision about the achievements and the content of the course, and then they made a common decision about sharing the knowledge of the subject and sharing with the students.

In LS meetings, the topic that was least discussed was *group/individual teaching techniques* (2.22%). *Group/individual instructional technique* was mostly discussed in terms of which technique would be preferred. The conversation among Esma, Hakan, and K2 in Dialogue 11 is an example of such a sharing environment.

Dialogue 11
Contribution on Group/Individual Teaching

Esma: A rectangle was formed with counting stamps. Some students remained very abstract from the group. If they had created an individual [rectangle], it could have been better.
Hakan: It takes so much time individually.
Esma: Yes, it takes so much time.
Hakan: We decided to form a group in a previous example. That's why I also formed a group here; otherwise, I could have done it individually by giving them materials in twos or threes. That was an option, but then it would be necessary to get an answer from each student.
K2: It was nice with a group. They communicate with each other, exchanging ideas. I like group studies.

Note. The dialogue is excerpted from the transcript of 3M2C.

In general, the participants' contributions on this topic can be categorized as *group/individual teaching technique preference* and *group/individual teaching process.* During the meetings, depending on variables such as the structure, the atmosphere of the class, and the content and duration of the activity to be carried out, the participants discussed whether the activities were more appropriate to be carried out in group or individually. The participants also shared views on how the process should be managed after deciding which teaching technique to apply. For example, if a group teaching technique was preferred for an event, details were determined together by the participants such as how groups should be formed, what steps should be followed during the event, etc. While talking about how to choose the teaching technique, as well as how to manage the process, the participants had the opportunity to evaluate the positive and negative aspects of both techniques (i.e. group versus individual).

4. Discussion and Conclusion

The aim of this study is to describe a mathematics LS model creating an environment of discussions with the participation of an academician, a teacher, and pre-service teachers. For this purpose, we focused on the content and the frequencies of participants' discussions in a LS.

The topics of the discussions in the LS can be grouped under eleven main headings: *classroom management, knowledge of students, feedback, mathematical knowledge, unexpected situations, the teaching of problem-solving, material usage, context information, learning outcomes, assessment process, and group/individual teaching*. In the LS, it is quite natural that the participants focus on teaching and learning, given the fact that the aim is to improve the knowledge and skills of teaching and learning (Takahashi, 2010). The participants, who came together to create the most ideal learning environment for the stated learning outcomes, conducted an in-depth evaluation of each component of the learning-teaching process, both while planning the course and while evaluating the lessons that they observed.

In the present research, it was observed that group members mostly discussed *classroom management*. During the process, procedures such as *time management, student participation, board use or writing, students' attention, students' participation, flexibility of the prepared plan, communication with students, concurrent material work,* and *individual communication with students* were discussed as aspects of classroom management. The participants mostly discussed classroom management in the meetings that took place after the observations. Similar to the results of this study, some other studies also provided conclusions about the discussion of classroom management in LS models (Suratno, 2013; Widjaja et al., 2017). Besides Schipper et al. (2018), stated that LS contributes to teachers' effective classroom management skills. Classroom management is frequently mentioned in the discussions because pre-service teachers who had their first teaching experience needed support in this context. Classroom management is a complex skill that is affected by many dynamics, from student behaviour to teacher actions, and it changes and develops with experience. In this context, LS offers important opportunities for the development of this skill.

Another issue frequently discussed in the LS is the *knowledge of students*. Discussions were held about *the students' possible or known learning difficulties*, *learning processes*, and *pre-learning experiences* or *readiness.* Vrikki et al. (2019) in describing the lesson study model, members of the group stated

that they developed their lesson plans by focusing on the learning processes of the students. Similarly, Suhali et al. (2014) stated that teachers engaging in lesson study had the opportunity to observe their students and thus were more sensitive to their students' learning processes and learning difficulties. By combining the results of both studies, it is possible to express LS both in terms of course observations and sharing in the meetings, giving the participants experience and inferences in the context of understanding their students.

Participants also discussed about topics such as *feedback* on the *answers of students or the questions they ask in a timely manner* and *in a way that the students can understand, giving feedback in general,* and *rewarding students' correct behaviour*. Therefore, it can be said that in LS, participants have an opportunity to examine how the students interpreted their statements and how they gave feedback (Olson et al., 2011). Participants mostly commented on the feedback given to students in critical terms during revision meetings. In addition, while planning the lesson, they also talked about how to answer possible student questions or answers. As a matter of fact, Pang (2016) stated that participants discovered the importance of giving adequate feedback during the lesson planning stage. And giving positive feedback to students also talked in the LS meetings (Agricola et al., 2020). It can be said that inexperience could be listed as the main reason why pre-service teachers made mistakes in the course related to feedback. In addition, the pre-service teachers' efforts to minimize the risk of unexpected situations and apply the plan in the best way also caused problems with feedback. For this reason, the topic, feedback was mentioned frequently during the process.

LS meetings often provide a basis for conducting class-based discussions on topics such as *classroom management, knowledge of students*, and *feedback* so that prospective teachers who are having the first experiences of their professional lives can obtain the support they need for their professional development. Although pre-service teachers have taken many theoretical and practical courses to acquire knowledge of the teaching field during the four-year undergraduate education process, they have problems in applying this knowledge in a real classroom environment.

Participants also shared *mathematical context* related to the subject they discussed. While talking about the concepts or terms related to the subject and how they could be presented to the students, they had the opportunity to examine these mathematical concepts in depth. Reviewing the literature, it is seen that shifts were found to occur in the subject information of participants in LS (Appova, 2018; Sims & Walsh, 2009). In the LS process, participants *described the concepts related to the subject, how formulas can be presented to the students, demonstrations that can be used in the course, points to be taken into consideration in geometrical drawings, mathematical language, problems related to the subject, and issues related to daily life*. It is thought that these shares are important in terms of deepening the mathematical knowledge of the participants.

Unexpected and *surprising situations* in the classroom, *unforeseen questions,* or *reactions from students,* and *how to intervene in such situations* were all points that were discussed within the category of "*unexpected situations*" in this research. In the LS, the ways in which the teacher should intervene in such unexpected situations were primarily discussed. Hervas and Medina (2020) explaned that in LS, one part of the challenge is the unexpected situations. During this research process, especially in the meetings held after the LS applications and observations, it was frequently mentioned about unexpected situations. Over time, participants even started talking about unexpected situations in plan meetings. Similarly, Fernandez (2005) stated that, LS offered the opportunity for participants to discuss the unexpected events. Additionally, Meyer and Wilkerson (2011) also reported that LS participants predicted students' answers and possible questions from time to time. During the planning meetings, with the help of predictions about the students' surprising questions or comments helped prevent moments of crisis during the lessons.

In the sharing environment related to *problem-solving*, which is one of the main purposes of mathematics teaching, participants talked about the *problem-solving process*, *problem-posing process*, and *problems to be used in the course*. They discussed what kinds of problems and how many

problems should be used in the course, and how to manage the problem-solving and problem-posing processes. Haydar and Zolkower (2010) stated that, during application of the LS, teachers conducted activities such as researching math problems, placing these problems in lesson plans, and analyzing students' solutions. Similarly, Bradshaw and Hazell (2017) said that LS enabled teachers to reflect upon problem solving practices and how they can further develop the skills students need to become efficient and fluent problem solvers. Hence, it can be stated that LS provides an environment for participants to concentrate on problem solving in teaching.

Another issue that participants focus on when planning or evaluating a lesson is *using material*. Ishii (2014) found that some information is shared in LS about the use of concrete materials. Participants who share their knowledge, experience, and opinions about how to use materials in the course are able to join forces in such environments. Participants talk about *what kinds of materials are available* and *how they can be used*, *what might be the best materials for the class,* and *how much material is needed*. Murata and Takahashi (2002) said that, LS plays an important role in improving teaching and learning materials in Japan. In some LS practices, it can even turn into an environment where the participants produce classroom material together (Lewis et al., 2009).

The participants discussed about *assessment* which is important parts of the course and is effective in determining how successful the teaching and learning activities will be. In addition to *the kinds of measurement materials to be used* and *how to use those materials in order to determine the level of students' learning outcomes*, they focused more rarely on *the evaluation process*. Suratno (2013) also stated that the lesson study model allows for the exchange of experience and information about the different measurement and evaluation processes of the participants. Similarly, Setijowati (2018) states that LS is important for the development of teacher's ability to evaluate learning and teaching process. Additionally, the importance of assessment sharing in LS was also emphasized by Yenmez et al. (2017). They said that the participants of LS had the opportunity to express, test, revise, refine, and extend their assessment process.

In this study, participants also shared information and ideas about the *classroom environment, students' readiness, students' learning situations, pre-learning*, and *the characteristics of some students*. In such environments, they talk about the common rules of the class addressed by the lesson plan or what the students are like. However, thanks to LS, participants have the opportunity to focus on the characteristics of the class and the learning environment of the students (Subadi et al., 2013). Amador and Weiland (2015) state that via LS, pre-service teachers first realized the elements related to the classroom environment. In this context, it is thought that the practices and sharings made in the meetings held after the LS are important.

Another topic discussed by the participants was the *learning outcomes*. In such environments, *the limits of the learning outcomes* usually receive more attention than *the content of the learning outcomes*. Group members started by reading and studying the planned *learning outcomes* in the meetings enabled the pre-service teachers to evaluate the curriculum and outcomes. Similarly, Cheah (2010) stated that teachers who participated in LS paid significant attention to the *learning outcomes* process. Likewise Nilvus (2020) claimed that group members could analyze and concentrate on the learning outcomes of the students during LS meetings. In addition, thanks to LS, pre-service teachers realized that the answer to the question of "What should I teach and how much should I teach?" lies within the issue of the desired educational outcomes.

The LS has rarely been applied to discuss the realization of in-class activities by *groups or individually*. Following the agenda of each LS meeting in the present work, participants first discussed *what kind of technique they would prefer*. Depending on variables such as the subject, class atmosphere, physical facilities of the class, and content and duration of the activity, the technical preferences for the course may change. The details of *how to manage the process* after determining which techniques will be used in the lesson are discussed (e.g., how many classes should be divided into groups and how groups should be formed). Meyer and Wilkerson (2011) stated in their study that during LS meetings, participants had similar debates. During discussion about group and individual teaching techniques, the participants had the opportunity to evaluate what

these techniques are, how they are used, and the positive and negative aspects of these techniques. In this process, for example, participants mentioned topics such as peer learning through collaborative learning, communication between students, the background of some students in the classroom, positive situations such as interaction, and negative situations such as noise and chaos.

5. Suggestions

In the lesson study process applied in this research, the participants shared their knowledge about *classroom management, student recognition, feedback, mathematical knowledge, unexpected situations, problem-solving instruction, material usage, context knowledge, achievements, measurement and evaluation, and group/individual teaching techniques*. It can be stated that pre-service teachers need support in their professional development, particularly in these areas. In this context, it is important to support pre-service teachers in all subjects, and especially in school practice courses.

In this study, LS meetings focused on the discussion contents of both the pre-service teachers and the knowledgeable others. Studies in which the roles of the knowledgeable others are defined and discussed in depth can be suggested for future research in LS practices and related literature.

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