Pythagoras - Journal of the Association for Mathematics Education of South Africa
ISSN: (Online) 2223-7895, (Print) 1012-2346

Discourse-based mathematics instruction on Grade 11 learners' mathematical proficiency in algebra topics

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Dates:
Received: 14 Mar. 2022
Accepted: 28 Apr. 2023
Published: 07 July 2023

How to cite this article:
Luneta, K., & Legesse, M.Y. (2023). Discourse-based mathematics instruction on Grade 11 learners' mathematical proficiency in algebra topics, *44*(1), a686. https://doi.org/10.4102/pythagoras.v44i1.686

School algebra serves as the language of mathematics and a foundational subject for learning advanced mathematics courses. This makes developing learners' proficiency in algebra the most desirable instructional goal of school mathematics. Despite having such importance emphasis, however, studies indicate that the vast majority of learners are characterised by inadequate mathematics proficiency levels in general and in the algebra syllabus topics in particular. Consequently, this quasi-experimental study attempted to investigate the efficacy of using discourse-based instruction as an instructional approach to developing proficiency in algebra unit topics. One hundred and six (N = 106) Grade 11 learners participated in the study and were randomly grouped into an experimental group (n = 52) and a control group (n = 54). Using a test instrument that consisted of 24 Rasch-validated items, both pre-test and post-test data were collected from both groups under similar conditions. The Mann-Whitney U statistical analysis of the pre-test data revealed no significant difference between the control and experimental groups. The Mann-Whitney U analysis performed on the post-test data demonstrated that the experimental group scored significantly higher in the post-test scores when compared to the control group after the intervention. The study findings provided evidence of the efficacy of discourse-based instruction over teacher-centred instruction for developing learners' algebra proficiency.

Contribution: The study has contributed to the conceptual and practical understanding of how discourse-based instruction can be used to concretise learners' proficiency in basic algebra.

Keywords: Classroom discourse; discourse-based approach; mathematical proficiency; experimental study; teaching method.

Introduction

This manuscript reports the findings of a doctoral study (Legesse, 2022) that investigated the effectiveness of discourse-based instruction of algebra and function syllabus topics. In this dynamically changing world, mathematics plays an essential role in scientific investigation, technological progress, and every walk of life (Suh & Seshaiyer, 2013). Mathematics educators and researchers advocate the teaching of mathematics for developing mathematically proficient citizens (Suh, 2007; Suh & Seshaiyer, 2013). For a learner to be mathematically proficient in a domain of mathematics, the learner must demonstrate conceptual understanding (comprehension of mathematical concepts, operations, and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), strategic competence (the ability to formulate, represent, and solve mathematical problems), adaptive reasoning (the capacity for logical thought, reflection, explanation, and justification), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile), coupled with a belief in diligence and one's efficacy (Kilpatrick et al., 2001, p. 116).

Like many other countries around the world, Ethiopia has envisioned the teaching and learning of mathematics to enable all learners to be mathematically competent (Ethiopian Ministry of Education [MOE], 2010a). Despite this vision, the findings of the Ethiopian National Assessment and Examination Agency (ENAEA, 2010) showed that the majority of Grade 10 and Grade 12 learners scored below the average passing mark of 50% in the mathematics assessment examination. The assessment results further revealed that inadequate mathematical proficiency across different content domains of school mathematics has remained a persistent problem for the vast majority of learners (Legesse et al., 2020).

To alleviate this problem, research literature in mathematics education recommends different forms of instructional approach for enabling learners to develop mathematical understanding, problem-solving, and thinking (Bennett, 2014; Bradford, 2007; Cross, 2009; Manouchehri & Enderson, 1999; National Council of Teachers of Mathematics [NCTM], 1991, 2014). Among the recommendations was the use of classroom discourse orchestrated around mathematical tasks that foster learners' understanding of mathematical ideas, communication, and problem-solving skills (Bennett, 2014; Cross, 2009; Legesse et al., 2020; NCTM, 1991; Smith & Stein, 2011; Walshaw & Anthony, 2008). Moreover, the literature suggests the engagement of learners in mathematical discursive practices of explaining, justifying, listening to, sharing, comparing, evaluating, and interpreting each other's mathematical ideas and reasons and constructing convincing arguments (Bennett, 2014; Rumsey & Langrall, 2016; Smith, 2018; Smith & Stein, 2011) to enhance their mathematical proficiency and achievement results (Anthony & Hunter, 2017; Bennett, 2014; Bradford, 2007; Smith, 2018). However, evaluation studies of the effectiveness of discourse-based mathematics teaching and learning on targeted academic outcomes in different cultural contexts are scant (Bradford, 2007; Rumsey & Langrall, 2016). In particular, there is limited practical effort in using mathematical discourse as a teaching strategy in Ethiopian school classrooms.

In an Ethiopian context, poor performance in mathematics is often attributed to teacher-dominated classroom teaching practices (Dhoj & Verspoor, 2013). Deressa (2004) characterised teacher-dominated instruction as being associated with poor social interactions and communication between teachers and learners and among learners over the learning content. One approach to improving learners' mathematics proficiency learning outcomes could be exploring the potential of research-informed teaching approaches, namely the discourse-based approach. Consequently, the purpose of this study was to investigate the effectiveness of discourse-based instruction of algebra and function topics on Grade 11 learners' mathematical proficiency compared to traditional lecture-based instruction.

School algebra serves as the language of mathematics and a foundational subject for learning higher mathematics, science subjects, and engineering courses (Grønmo, 2018). Improving the classroom instruction of algebra enables learners to pursue learning advanced mathematics; hence, proficiency in algebra plays an important role in college and university mathematics courses (Grønmo, 2018; Kilpatrick et al., 2001). The present study, therefore, attempted to determine if discourse-based instruction enables learners to enhance their proficiency in algebra unit topics based on the Ethiopian Grade 11 mathematics syllabus. The research question was: Are there differences in mathematical proficiency scores between Grade 11 learners who were taught algebra unit topics using discourse-based instruction and those learners who were taught the same content using the traditional lecture method before and after the treatment?

This study used the term *discourse* to refer to dialogic talk that promotes the teaching and learning of mathematics through engaging in sociocultural practices, such as questioning, reasoning, listening, sharing, explaining, and justifying (Alexander, 2008; Brown & Hirst, 2007; Steele, 2001). A discourse counts as mathematical if it is about mathematical objects and involves the use of symbols, notations, representations, and definitions (Moschkovich, 2007; Sfard, 2008). Mathematical discourse includes the communication of definitions, rules, procedures, ideas, theorems, and proofs, verbal explanations of problem-solving strategies, and justification of reasons (Shilo & Kramarski, 2018).

Theoretical background and literature review

The present study is guided by a sociocultural perspective. A sociocultural perspective underlines the role of dialogic talk in the construction of knowledge in a social setting (Alexander, 2008). Through engaging in dialogic talk that promotes social interactions, learners develop mathematical understanding and construct meanings by explaining how mathematical tasks can be accomplished or how procedures work, and by challenging each other's ideas, comparing different solution strategies, sharing ideas and reasoning (Bradford, 2007; Gravemeijer & Cobb, 2006; Hiebert & Wearne, 1993; Lampert, 1990; Steele, 2002).

Mathematical discourse as a vehicle for learning promotes the construction of mathematical understanding by

> helping learners to clarify and organize their thoughts, facilitating personal and collective sense-making, supporting building connections between representations and multiple strategies, enabling learners to use others as a resource of ideas to challenge and broaden understanding, and helping learners learn the mathematical language. (Anthony & Hunter, 2017, p. 101)

Hoyles (1985) describes mathematical understanding as the ability to

> form a view of the mathematical idea, step back and reflect upon it, use it appropriately and flexibly, communicate it effectively to another, reflect on another's perspective of the idea, and incorporate another's perspective into one's own [*schema*] or challenge and logically reject this alternative view. (p. 212)

This also involves asking clarification questions, sharing ideas and reasoning, agreeing and disagreeing with each other's ideas, and respecting and listening to each other's ideas to promote the construction of mathematical knowledge (Chapin et al., 2003).

Engaging learners in collaborative classroom discourse that incorporates mathematical discursive activities such as agreeing and disagreeing with others' ideas, explaining their reasoning and thinking, and discussing and comparing solution methods promotes the development of the understanding of mathematical topics (Anthony & Hunter, 2017; Kazemi, 2008). To create a classroom environment that invites all learners to participate, the teacher should build confidence in learners that everyone can contribute to the classroom lesson by informing all learners to respect each other's responses, share, and exchange their ideas, listen to each other's responses, appreciate making mistakes as an opportunity for learning, and to question each other (Cobb, 1994; Maguire & Neill, 2006; Yackel & Cobb, 1996).

Sociomathematical norms refer to what counts as an acceptable mathematical explanation and justification of the 'why' and 'how' aspects of teachers' and learners' activities that are specific to the mathematical discussion (Cobb, 1994; Maguire & Neill, 2006; Stephan, 2014; Yackel & Cobb, 1996). The teacher and learners may set criteria for deciding what is regarded as an acceptable explanation, or a different solution method (Stephan, 2014). The teacher is responsible for establishing a '*psychological safety*' learning environment in which learners feel free to respectfully express their thinking and reasoning during group discussions of mathematical ideas (Cobb et al., 1990). The teacher should create a classroom culture in which learners are expected to explain and justify their solution strategies and reasoning (Cobb et al., 1990).

Regardless of the method of instruction, sociomathematical norms are present in all classrooms; however, what counts as an acceptable mathematical explanation and solution in traditional classrooms may not be a mathematically elegant and sophisticated explanation and solution in learner-centred classrooms (Stephan, 2014; Yackel & Cobb, 1996). For instance, just describing the solution procedure for solving a quadratic equation might be counted as an acceptable explanation in traditional classrooms (Stephan, 2014). Setting the use of accurate mathematical terms, symbols, representations, syntax, rules, and notations as sociomathematical norms can help learners develop mathematical language proficiency (Pourdavood & Wachira, 2015; Summers, 2012). Questions that can be used to establish sociomathematical norms include: How can you prove that your answer is right? Can you prove it in more than one way? How is your solution strategy different from that of another learner's solution strategy? Do you agree or disagree with another learner's solution? Why? Why does strategy A work? Why does strategy B not work? (Kazemi, 2008, p. 414).

This study conceived discourse-based mathematics instruction as the teaching and learning of mathematics through engaging in dialogic talk orchestrated around carefully designed tasks in which the topics to be learned are embedded. Discourse-based instruction can be characterised by the engagement of learners in dialogue-elicited tasks wherein the learner-teacher, learner-learner, and learner-group interactions are anchored on discourse practices of challenging each other's ideas, sharing ideas, agreeing and disagreeing with solution strategies and ideas, comparing solution procedures, and explaining problem-solving strategies (Legesse et al., 2020, 2021). In such classroom learning environments, mathematics learning is more than 'appending' new knowledge to existing knowledge; it involves the reconstruction of understanding and building a web of interconnected conceptual understanding that fosters the transfer of learning to new contexts (Hiebert et al., 1997).

Some evaluation studies indicate that discourse-oriented forms of instruction positively influenced mathematics learning outcomes (Bradford, 2007; Cross, 2009; Legesse et al., 2020; Sepeng & Webb, 2012; Smith, 2018). Bradford (2007) found that discourse-oriented instruction in pre-algebra classes helped low-achieving learners improve their mathematics achievement and problem-solving skills. A quasi-experimental study by Legesse et al. (2020) examined the effects of discourse-based mathematics instruction on Grade 11 learners' acquisition of conceptual and procedural knowledge of probability and statistics topics in an Ethiopian secondary school. Legesse et al. found that discourse-based instruction of probability and statistics increased learners' knowledge of concepts and procedures. Moreover, the discussion-based teaching strategy improved the experimental group of Grade 9 learners' word problem-solving performance when compared to the control group taught with the traditional lecture method (Sepeng & Webb, 2012).

Research material and methods

The present study employed a quasi-experimental design with a pre-test and post-test control group (Shadish et al., 2002). A quasi-experimental design with a control group is an appropriate method of inquiry when the primary intent of the study is to evaluate the effectiveness of an instructional intervention (Goodwin, 2009). Such a quasi-experimental design is also appropriate when the randomisation of each participant is impossible for practical reasons (Gall et al., 2003). Thus, a quasi-experimental study with a pre-test and post-test control group established a *cause-effect* relationship between discourse-based instruction and proficiency in algebra unit topics.

Participant selection

This study was conducted with Grade 11 learners in a randomly selected public school in Bahir Dar, Ethiopia. The school principal provided permission to conduct the study with the intended groups of learners. The study lasted for about 10 class weeks. During each class week, there were five lessons and each lesson was 42 minutes.

In collaboration with the head of the mathematics department at the participating school, two comparable Grade 11 mathematics teachers, in terms of professional and academic qualifications, who were willing to be involved in the study were recruited. The learner population in the participating school has similar ethnicity and socioeconomic status. All Grade 11 learners enrolled in the natural science stream were allocated into 14 sections. After this, each section was assigned a unique natural number. Fourteen equal-size pieces of paper were prepared and each piece was numbered 1 through 14. Then each piece was folded and placed in a carton. After shaking the box and mixing the pieces well, an independent person randomly picked two pieces of paper one after the other without replacement, representing two sections of Grade 11 learners. The two sections were assigned to either the control group (n = 54) or the experimental group (n = 52). One hundred and six Grade 11 learners (N = 106) participated in this study. After being briefed about the study, all learners in the selected classes offered their verbal consent for participation. Names and details of participants were kept confidential while analysing the data and reporting the results.

Based on prior studies in a similar context (e.g. Legesse et al., 2020), there was an anticipation that the data would violate the assumptions for the parametric tests. Accordingly, the Mann-Whitney U test was chosen for analysing and comparing differences in the pre-test and post-test scores between the control and experimental groups. Using the evidence from the literature (e.g. Happ et al., 2019; WMWssp, n.d.), a sample size calculation based on the expected medium effect size (r = 0.349) for the Mann-Whitney U test (Mangiafico, 2016), power ($1–\beta$):0.80, and two-sided type I error (α: 0.05) produced an estimated total sample size of 112 participants. This power analysis indicated that the sample size of 106 was sufficient to detect a statistically significant difference between the control and experimental groups on the continuous dependent variable.

Locating the control and experimental groups in the same school controls differences in the physical conditions (Fraenkel et al., 2012) and avoids possible threats due to differential selection (Gall et al., 2003). On the other hand, it might cause treatment contamination. Danga and Korb (2014) examined the effect of treatment diffusion using an educational experimental design in a Nigerian secondary school where learners had maximum opportunities to exchange information and talk about their classroom teaching and teach each other. The study found that placing the control group and the experimental group in the same school did not result in a potential treatment diffusion to affect learners in the control group (Danga & Korb, 2014). In this study, the authors made efforts to prevent or reduce potential treatment diffusion (Shadish et al., 2002) between the control group and the experimental group by making the groups blind to one another and by taking the groups to different classes in physically separated buildings (Rhoads, 2011). Under such circumstances, it was most unlikely there would be information leakage about the nature of treatment between the control and experimental groups.

Implementation of discourse-based mathematics instruction

Two comparable mathematics teachers (both male) in terms of academic profiles and professional qualifications were both chosen from the Grade 11 classes involved in the study. After recruitment, the two teachers received two different training sessions of four hours each on how to design and implement discourse-based lessons. The training involved the design of discourse-elicited tasks, crafting different questioning strategies, facilitation strategies of learners' classroom discourse, and setting up and maintaining social and sociomathematical norms. During the training session, the teachers were maintained blind to the hypotheses of the study (Rhoads, 2011). The training was offered in such a way that the teachers did not disclose information to their learners about the new method of teaching.

After the training, using a coin toss, the teachers were assigned to teach either the experimental group or the control group. Both the control and experimental groups were taught the same unit topics outlined in the first two chapters of the Grade 11 mathematics syllabus (MOE, 2009). The unit topics include relation and function, inverse relation, graphs of relation and inverse relation, even and odd functions, one-to-one and onto functions, absolute value and signum functions, inverse functions and their graphs, simplification of rational expressions, arithmetic operations on rational expressions, rational functions and their graphs, solving rational equations and inequalities, and some applications of rational equations and inequalities as word problems.

The experimenter teacher was provided with a lesson plan format, prototype lessons, and an intervention guide, that assist in discourse-based lesson preparation and implementation while the other teacher was strictly informed not to implement discourse-based instruction in the control group but to use the traditional lecture method. The classroom discourse in the experimental class was structured in individual work, small-group discourse, and whole-class discourse. The experimental group underwent discourse-based mathematics instruction where learners actively engaged with mathematics learning through explaining, justifying, conjecturing, comparing, sharing, and questioning.

Learners in the experimental group were grouped into heterogeneous small groups (Webb, 1991) of four or five learners. The experimenter teacher was responsible for establishing classroom rules that equally entertain and respect all learners' ideas and contributions (Bennett, 2014) and creating discourse-elicited mathematical tasks (Lampert, 1990). The teacher briefed his learners on how to respectfully listen to and share ideas, communicate, agree or disagree with each other's ideas, work in groups, and maintain classroom norms (Bennett, 2014; Legesse et al., 2021). The experimenter teacher followed the implementation process illustrated in Table 1.

As outlined in Table 1, the design and implementation of discourse-based instruction goes through planning and creating of tasks that embed the learning topics in the stages of task presentation: individual work (each learner thinks about the given task), small-group discussion, whole-class discussion, and reflection. The task implementation requires the active involvement of learners in individual work (give learners the task and allow each learner to think about it individually for some minutes), small-group discourse (encourage learners to work together in small groups to talk about the task, discuss, and listen to each other's ideas), whole-class discourse (allow learners to present and share their ideas and strategies), and reflection (allow learners to reflect on the task through open-ended questions, such as 'What solution strategies did you find very useful for solving rational equations?' 'What challenges have you faced in today's lesson?' 'What concepts have you understood well and what concepts do still you want to understand?').

The teacher is responsible for establishing classroom rules that equally entertain and respect all learners' ideas and contributions (Bennett, 2014) and creating discourse-elicited mathematical tasks (Lampert, 1990). Learners should respectfully challenge, listen to, and share each others' ideas, work in groups, and be accountable for their learning (Bennett, 2014).

By applying the five practices model – anticipating, monitoring, selecting, sequencing, and connecting (Smith & Stein, 2011, p. 8) – the experimenter teacher (A) facilitated and guided learners' discourse of conversations about the given tasks. The experimental group of learners engaged in discursive activities that include generating examples and non-examples, agreeing and disagreeing, comparing equation-solving strategies, exploring different simplification strategies for rational expressions, explaining rational equation-solving strategies, describing relations using multiple representations, and evaluating mathematical statements. From the leading author's classroom observations, a sample lesson on identifying functions graphically in the experimental class is presented below.

TABLE 1: The process of implementing discourse-based mathematics instruction (Legesse et al., 2020).

Component	Description of activities
Planning for instruction	The teacher identifies and communicates the topic to be taught and articulates the goal of the lesson. The teacher plans and crafts a different set of questions for promoting learners' discourse.
Task choice or design	The teacher chooses or selects discourse-elicited tasks aligned with the learning goals.
	The teacher presents tasks to learners and consistently probes their thinking.
	The teacher crafts task-specific questions that invite learners to engage in the discourse practices of explaining, conjecturing, comparing, questioning, and sharing ideas.
	The teacher anticipates learners' possible responses for each task.
Individual work	The teacher initiates the discourse by reviewing core concepts through questioning and briefing the topic of the current lesson.
	The teacher probe learners' thinking about the task.
	The teacher encourages learners to work on the task individually and elicits their thinking through probing and clarifying questions.
	Each learner thinks about the tasks to generate ideas and figure out solution strategies to be explained and shared (Lampert, 1990).
Small-group discourse	The teacher uses the five practices model (Smith & Stein, 2011, p. 8): anticipating, monitoring, selecting, sequencing, and making connections for facilitating social interactions.
	The teacher promotes the mastery of mathematical terms when learners talk, discuss, and explain their ideas and reasoning.
	Small groups of learners work on the given task. Learners explain their ideas to others, express their thinking, and agree and disagree with each other's ideas (Franke et al., 2015).
	Learners compare solutions, share ideas, listen to each other's ideas, ask, and answer questions (Franke et al., 2015).
Whole-class discourse	Selected responses or solutions will be presented or shared.
	Learners make connections by sharing ideas and comparing multiple solution strategies.

Planning for instruction: The teacher articulated the goal of the lesson and asked learners to open their mathematics textbooks. Then the teacher wrote the task on the chalkboard.

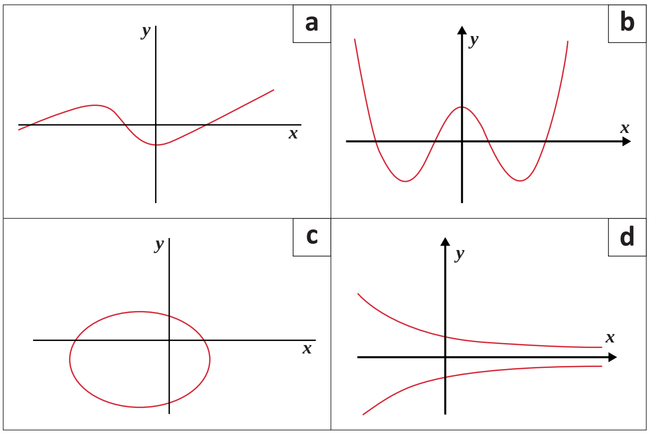
Task choice or design: The teacher read and presented the task shown in Figure 1. The questions asked were 'Which of the following graphs represent functions?' and 'Which graph does not represent a function? Explain your answer.'

Individual work: The teacher allowed learners to think about the task independently for some minutes.

Small group discourse: The teacher encouraged learners to discuss and exchange their ideas in small groups for about six minutes. After ensuring all groups completed the task, the teacher asked the class to share their ideas and explanations.

Whole-class discourse: The teacher selected a learner from a group. The learner answered 'the graphs in (a) and (b) represent functions'. The teacher posed the question 'How did you know that?' The same learner verbally explained that 'if we draw a vertical line, it crosses exactly at one point. So, the graphs represent functions by a vertical line test'. The teacher asked the class 'Do you all agree with this explanation?' The class approved the explanation by saying loudly 'yes'. The teacher asked, 'What about the graphs in (c) and (d)?'. Some other selected learners answered that the graphs in (c) and (d) do not represent a function. The teacher asked the class 'Who would like to explain why these graphs do not represent functions?' A learner explained to the class by saying 'a vertical line crosses the graphs at two points'. The teacher asked 'do you agree?' The class agreed with the learners' explanations. The teacher said 'But what does it mean when the vertical line crosses the graph at two points?' A learner explained by saying that 'two numbers in the domain are related to the same number in the range'.

The control group was taught the same unit topics with the traditional lecture method and attended the same class hours per week in similar shifts as the experimental group. The duration of each lesson was 42 minutes. The traditional lecture method can be characterised as the 'chalk and talk'



Source: Ministry of Education (MOE). (2010b). *Mathematics: Student textbook grade 11*. Ministry of Education

FIGURE 1: A discourse-elicited task presented to the experimental class: By a vertical line test, Blocks (a) and (b) represent functions, whereas Blocks (c) and (d) do not.

method where the teacher mainly dominates the classroom talk, and the majority of learners are passive listeners to the teacher's lecture with a lack of opportunities for challenging ideas, or discussing with peers to construct an understanding of concepts (Rosenthal, 1995). The lesson observations showed that the classroom instruction in the control group was dominated by the teacher's demonstration of rules and procedures for arithmetic operations of rational expressions, simplification strategies, equation solving, and graphing procedures for rational functions. Learners often engaged with procedural and computation tasks that did not provide opportunities for learners to be actively engaged in discourse practices of explaining, justifying, questioning and answering, agreeing and disagreeing, and comparing solution strategies. Learners copied notes and worked on examples and were passive listeners to what was explained by the teacher.

Findings from the observations were consistent with the traditional teacher-centred forms of teaching practices (Deressa, 2004; Lampert, 1990). In an Ethiopian context, the traditional teaching method was found to manifest poor social interactions and communication between teacher and learners and among learners over the learning content (Deressa, 2004). The implementation of the tasks is based on teacher-led demonstration where learners are accustomed to rehearsing facts and mimicking formulas, solution procedures, and rules to do similar tasks (Hsu, 2013). The instruction focuses on teaching procedures and computational skills. The teacher-dominated way of teaching mathematics restricts opportunities for learners to explain, share, and discuss their ideas and thinking (Hsu, 2013). For instance, the teaching of simplification of rational expressions in a teacher-dominated classroom focuses on enabling learners to memorise and practise procedural rules by assigning a set of routine tasks (Kooloos et al., 2019).

Data collection instrument

The variables involved in the present study were methods of teaching as the independent variable and mathematical proficiency scores as the dependent variable. Test scores as quantitative and classroom observations as qualitative data were collected. The total test score (total sum of scores in each strand) was used as a measure of participants' proficiency in algebra. The maximum possible score assumed was 48 while the minimum was 0. During the progression of the experiment, the researcher as a neutral observer made unannounced classroom observations twice per week in both groups. Accordingly, the researcher observed and documented teachers' actions, learners' participation, and classroom activities.

Anchoring to the characterisations of the four strands of mathematical proficiency (see Table 2), the authors developed 24 test items from reviewing different literature sources including national mathematics examination papers. The constructs to be measured were operationalised through the test items that covered a variety of algebra and function topics outlined in the Grade 11 mathematics syllabus (MOE, 2009). More specifically, the Rasch partial credit model (Wilson, 2005) was applied to validate the test items using Winsteps software (version 4.4.2). The Rasch analysis provided psychometric evidence of item reliability (0.98), person reliability (0.79), item separation (7.66), and person separation (1.97), item fit, item difficulty, item polarity, local independence, and unidimensionality that made up the test instrument. Measures of item difficulty were found to be within the range from –1.96 logits to 1.92 logits; which is located inside the normal range from –3 logits to 3 logits (Timofte & Siminiciuc, 2018). Items with difficulty measures below –2.0 or above 2.0 are described as easy and hard items (De Ayala, 2009).

TABLE 2: Characterisations of the four strands of mathematics proficiency (Legesse, 2022).

Strand	Description of the strand
Procedural fluency	Knowing rules, facts, or algorithms and being able to execute procedures accurately (Allen et al., 2009), knowing when and how to use a procedure (Danley, 2002), and accurate use of procedures or formulas (Schoenfeld, 2007).
Conceptual understanding	'Being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes' (Kilpatrick et al., 2001, p. 119); being able to make connections between concepts and between concepts and procedures (Kilpatrick et al., 2001).
Strategic competence	Recognising which solution strategy is more appropriate and useful than others and formulating, representing, and solving problem situations (Kilpatrick et al., 2001).
Adaptive reasoning	Being able to prove mathematical proofs and the ability to make a conjecture and justify the choice of a particular solution strategy as appropriate.

TABLE 3: Sample test items (Legesse, 2022).

Sample of test tasks	Task-specific manifestations
Given the functions $f(x) = 2x + b$ and $g(x) = 3x + 4$, find the value of b such that $f(g(x)) = g(f(x))$. Show your work.	Knowledge of procedures for solving equations involving composite functions.
Classify the function $f : R \rightarrow R$ defined $f(x) = 2^x$ as one-to-one, onto, or neither. Justify your answer.	It demands a conceptual explanation of whether the given function is one-to-one, onto, or neither.
Prove that the sum of two odd functions (with the same domain) is an odd function.	It demands deductive proof with detailed explanations and justifications.
A ball is thrown vertically upward from the ground level with an initial velocity of 80 meter per second. Its height at the time t is given $h(t) = -16t^2 + 80t$ where the height is in metres and the time is in seconds. In which time interval will the ball be at a height more than 64 metres above the ground level?	A word problem that involves the application of quadratic equations and inequalities.

Before the start of the experiment, pre-test data were collected using Rasch-validated test items (see Table 3) on the dependent variable from both groups. The time permitted to complete the test was 2 hours. After the completion of the implementation phase, using the same test instrument, post-test data were collected from both groups under similar conditions. The same classroom teachers and two doctoral students invigilated the administration of the pre test and post test. The researcher attended the invigilation processes as a neutral observer. Learners' test performance was marked using an item-specific scoring rubric (0/1/2). An initial format of the scoring rubric was created after analysing each item for its consistency with the content and characterisations of the proficiency strands. It was then reviewed by doctoral students and senior

TABLE 4: Descriptive statistics for pre-test scores and post-test scores.

Group	Mean	Standard deviation	Skewness			Kurtosis		
	Statistics	Statistics	Statistics	Standard error	*Z* ratio	Statistics	Standard error	*Z* ratio
Pre-test control (n = 54)	5.85	2.76	0.631	0.325	1.94	-0.366	0.639	0.57
Pre-test experimental (n = 52)	5.38	2.31	0.699	0.330	2.12	-0.076	0.650	0.12
Post-test control (n = 54)	15.20	7.79	0.856	0.325	2.63	0.367	0.639	0.57
Post-test experimental (n = 52)	17.38	5.62	0.315	0.330	0.95	-0.735	0.650	1.13

TABLE 5: Normality test for pre-test scores and post-test scores.

Outcome	Group	*n*	Shapiro-Wilk test	
			Statistic	Significance
Pre-test scores	Control	54	0.930	0.004
	Experimental	52	0.926	0.003
Post-test scores	Control	54	0.936	0.006
	Experimental	52	0.967	0.152

TABLE 6: Results of the Mann-Whitney *U* test for the pre-test scores.

Group	*n*	Mean rank	Sum of rank	*U*	*Z*	*p*
Control	54	55.78	3012	1281	-0.786	0.432
Experimental	52	51.13	2659			

TABLE 7: Results of the Mann-Whitney U test on the post-test scores.

Group	*n*	Mean rank	Sum of ranks	*U*	*Z*	*p*
Control	54	47.29	2553.5	1068.5	-2.124	0.034
Experimental	52	59.95	3117.5			

mathematics teachers. The modified version of the rubric was used for scoring learners' test performance.

Data analysis results

Statistical data analyses were performed using the Statistical Package for the Social Sciences (SPSS, version 23). The normality of the test data was examined by considering the absolute sizes of the *Z*-ratios for skewness and kurtosis (Orcan, 2020). The *Z*-ratio is determined by dividing the value of skewness and kurtosis by their standard errors (Orcan, 2020). For a small sample size, when the absolute sizes of the *Z*-ratios for skewness and kurtosis are greater than 1.96, the data would be non-normally distributed (Orcan, 2020).

As shown in Table 4, the absolute size of the *Z* ratio for the skewness of pre-test scores in the experimental group and that of post-test scores in the control group are greater than 1.96. The values of skewness and kurtosis indicated that the pre-test scores and post-test scores were non-normally distributed for each sample (Orcan, 2020). Results of the Shapiro-Wilk test supported that the raw scores on the dependent variable failed to meet the normality assumption for each sample.

Davison (1999) recommended non-parametric tests when the violation of the assumptions for parametric tests becomes evident. Consequently, the authors performed Mann-Whitney U test analyses on rank-converted pre-test and post-test raw scores (Feys, 2016; Pallant, 2016).

Pre-test and post-test data analysis

To determine if there was a significant difference in the pre-test scores between the control group and the experimental group (see Table 5) before the treatment, the pre-test scores were analysed using the Mann-Whitney *U* test.

The result of the Mann-Whitney *U* test indicated no statistically significant difference in ranked pre-test scores between the control group and the experimental group (U = 1281, z = –0.786, p = 0.432) (see Table 6), which showed that both groups were comparable at the start of the experiment.

The result of the Mann-Whitney test on the post-test scores showed that the ranked post-test scores in the experimental group were significantly higher compared to the ranked post-test scores in the control group (U = 1068.50, Z = –2.124, p = 0.034) after the intervention (see Table 7), which revealed that the experimental group scored statistically higher in the post-test scores than the control group.

Calculation of the effect size for the Mann-Whitney U test (r = 0.21) using the formula $r = \frac{Z}{\sqrt{N}}$, where Z refers to the absolute standardised test statistic Z and N is the total number of participants (Pallant, 2016; Tomczak & Tomczak, 2014), showed that the degree to which the experimental group had post-test scores with higher ranks than the control group was small (Mangiafico, 2016). This value of effect size might suggest that learners should be exposed to discourse-based instruction for a relatively longer period of intervention for having a bigger outcome difference.

Description of discourse types observed in the experimental class

The analysis of discourse types focused on capturing and describing the types of discourse practices enacted in the experimental class. Using the discourse observation protocol (Weaver et al., 2005), learners' actions and teachers' actions were observed for the potential discursive indicators during classroom instruction. A total of eight classroom lesson observations were conducted during the implementation phase, apart from the first and last weeks of the experiment period. Themes of potential discursive indicators were identified by reviewing the literature (e.g. Moschkovich, 2007; Weaver et al., 2005). The discourse of conversation utterances made between learners and teacher and among learners along the segments of each observed lesson were coded and categorised for the following discourse types: sharing of ideas, solution procedures and strategies (S), conceptual explanation (E), exemplifying (EX), questioning (Q), justification (J), comparison and contrast (CC), agreeing and disagreeing (AD), and generalising (G) (Weaver et al., 2005). The discourse types extracted from the observed lessons are summarised and presented in Table 8.

Descriptive analysis of classroom instruction in the control group

A total of six classroom observations were made for the control group. The classroom instruction predominantly focused on the teacher's demonstration of rules and procedures for arithmetic operations of rational expressions, simplification strategies, rational equation solving, and graphing procedures for rational functions.

TABLE 8: Discourse types extracted from lessons observed in the experimental class with discursive activities (Legesse, 2022).

Discourse type	Description of discursive activities from the observed lessons
Explanation	A learner in a small group explained the teacher's question about the core idea of the lesson. A learner explained to their peers in a small group how the graph of a relation can be constructed from the graph of its inverse. Selected learner explained how they can classify a given set of functions as even, odd, or neither to the class. Learners explained their understanding of definitions, rules, and concepts to one another or the class. Learners explained rational equations and inequalities solving procedures and strategies. Learners explained simplifying strategies and procedures for rational expressions; teacher questioned learners to explain ideas and concepts. Learners compared and elaborated on their solution procedures, challenged each other's ideas, explained problem-solving strategies, and described graphing procedures. Learners explained how rational expressions can be simplified and graphing procedures of rational functions.
Questioning	The teacher probed learners, thinking through task clarification questions. A learner asked another learner about generating a relation that is not a function. Learners asked questions of each other for an explanation of concepts and procedures. A learner asked for clarifications about the symmetry of an even function. A learner challenged a solution procedure shared by another learner for solving a rational inequality. Learners asked their teacher to elaborate on how to determine whether a given function is onto.
Comparison and contrast	The teacher encouraged learners to compare and contrast their solution procedures and strategies. The learners compared the domain and range of a relation and its inverse. Learners compared and contrasted the algebraic properties of rational functions. Learners compared how the addition of rational fractions is similar to the addition of rational expressions. The learners compared and contrasted properties of rational functions in terms of domains, asymptotes, intercepts, and end-behaviours.
Justification, verification, and exemplification	Learners applied definitions to evaluate the truth of mathematical statements, verified solutions of rational equations and inequalities, justified if a given graph represents a function, and generated an example that meets the stated property. A learner verified that the signum function is odd. A learner showed that the absolute function $f(x) = \lvert x \rvert$ is onto. A learner justified whether the graph of a rational function crosses the x-axis. Learners justified whether two functions are inverses of each other.
Generalising, challenging other's ideas, agreeing, and disagreeing	The teacher probed learners' reasoning to arrive at a conclusion or to challenge each other's solution procedures. Learners were challenged with each other's simplification and solution procedures. A learner challenged a classmate to give an example of an onto function that is not one-to-one. Learners generalised sum properties of odd functions.
Sharing of solution procedures and strategies	The teacher encouraged learners to share ideas and answers. The teacher asked learners to define the key mathematical terms. The teacher asked learners to construct and share examples of relations in real life. Learners exchanged and challenged each other's ideas and reasoning when solving rational equations and sketching graphs of rational functions. Learners shared multiple representations of a given relation. Learners shared examples and agreed and disagreed with each other's examples, simplification strategies, rational equations, and word problems solving strategies.

Learners often engaged with procedural and computation tasks that did not provide opportunities for learners to foster the construction of mathematical ideas and concepts through engaging in discourse practices of explaining, justifying, questioning and answering, agreeing and disagreeing, and comparing solutions strategies. Descriptive analysis of the observed lessons revealed that learners in the control group lacked opportunities to compare and explain different solution strategies and graphing procedures and to express their thoughts in the process of simplifying rational expressions and solving equations and inequalities.

The teacher often interrogated learners' understanding by gathering information about their mastery of the rules, definitions, and algorithms, using questions such as 'What did you recall about yesterday's lesson?', 'What is the definition of relation?', and 'What are the graphing procedures for rational functions?'. For instance, the teaching of simplification of rational expressions focuses on enabling learners to memorise and practise procedural rules by assigning a set of routine tasks (Kooloos et al., 2019). Such procedural-oriented mathematics teaching restricts opportunities for learners to explain, share, and discuss their ideas and thinking (Hsu, 2013).

Classroom observations in the control group revealed that the instruction was characterised by the absence of questions that probed learners' justification and reflection. The teacher questioning strategy mainly manifested a characteristic of *teacher asking-learner salient-teacher answering* for definitions, procedures, and rules. The nature of classroom discourse was dominated by the unidirectional flow of ideas from the teacher to learners, less frequent interactions among learners and between the teacher and learners, asking closed questions, lack of opportunities for learners to conjecture, discuss, and share their ideas and solution strategies. The teacher was the sole responsible person to approve learners' responses and to state rules and procedures to be followed without soliciting learners' thinking (Lampert, 1990).

Limitations of the study

Unwanted researcher-related bias and teacher-related variables such as teaching experience, teaching style, difference in their views about mathematics and its teaching, lack of discourse-oriented learning materials, and relatively large class size might influence the implementation of discourse-based teaching as was witnessed by Bradford (2007). The study did not provide enough information on which strands of proficiency either group might perform better relative to the other one. A separate analysis of groups' scores in easy and difficult items was also interesting. Furthermore, the study did not examine the interaction of the mathematical tasks, questions, and discourse types to influence learners' learning and this limitation was also observed in other similar studies (Hiebert & Wearne, 1993). After the completion of the experiment, the researcher planned to treat the control group with discourse-based instruction. However, about two months after the completion of the experiment, it was impossible to expose the control group to the treatment in a similar manner to the experimental group because schools were closed due to the COVID-19 pandemic. To circumvent this unexpected situation, the control group learners received classroom activities that were enacted in the experimental class in the form of handouts and group assignments via their mathematics teacher after schools reopened.

Conclusion and recommendation

The study examined the effectiveness of discourse-based mathematics instruction on Grade 11 learners' proficiency in the syllabus topics of algebra and function. Results of the data analyses showed that learners who engaged in discourse-based instruction of algebra topics demonstrated better performance in mathematics proficiency than those who were taught the same syllabus topics through teacher-centred instruction. It appeared that discourse-based instruction was more effective in developing learners' proficiency than the teacher-centred method for teaching mathematics. That is, learners in the experimental group gained a better understanding of mathematical concepts and procedures than those learners in the control group. This was most likely the result of their regular engagement in the discourse-based learning activities that promote individual thinking and discussions of mathematical ideas where learners compared solution methods, explained and posed questions for clarification, and shared and listened to one another's ideas and reasoning. The findings in this study were consistent with theoretical and empirical literature (e.g. Bradford, 2007; Cross, 2009; Franke et al., 2015; Legesse et al., 2020; Sepeng & Webb, 2012; Smith, 2018; Stein, 2007). For instance, a sociocultural learning perspective contends that a classroom environment that creates a learning platform for social interactions and cultural contexts fosters the construction of knowledge and understanding.

Overall, the study findings provided innovative ideas on how school mathematics should be taught in Ethiopian classroom contexts and that assist mathematics teachers in being aware of the potential benefits of discourse-based mathematics teaching for enhanced conceptual learning outcomes. Based on these findings, the study recommended discourse-based instruction for teaching mathematics at upper secondary and university levels. It proposed discourse-based education as a component of mathematics teachers' professional development programme. For these findings to be highly informative, replication studies should be conducted across different grade levels and other mathematical content domains with similar or mixed research designs.

Acknowledgements

This manuscript is based on a doctoral thesis submitted to the University of Johannesburg for which the university provided funding.

Competing interests

The authors declare that they have no financial or personal relationships that may have inappropriately influenced them in writing this article.

Authors' contributions

K.L., conceived the research and conceptualised the research design and methodology as well as the initial analysis of the data. M.Y.L., collected the data and developed the data analysis tools, finalised the analysis of the data, and the initial writing of the manuscript.

Ethical considerations

Ethical clearance to conduct this study was obtained from the University of Johannessburg Faculty of Education Research Ethics Committee (no. Sem 2-2019-031).

Funding information

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Data availability

Data sharing is not applicable to this article as the participants did not authorise the authors to share the data collected and analysed in this study.

Disclaimer

The opinions, findings, conclusions, and recommendations communicated in this manuscript are those of the authors and do not necessarily reflect the views of other persons or any institution.

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