

SUBJECT MATTER KNOWLEDGE OF MALAYSIAN PRE-SERVICE MATHEMATICS TEACHER
IN RATE OF CHANGE: A CASE STUDY

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ABSTRACT

A recent review of literature in topic of rate of change mostly highlighted students **and teachers' difficulties in handling conceptual questions**. This initiated the researcher to investigate the Malaysian pre-service mathematics teacher's subject matter knowledge in rate of change through a case study design. A task-based interview was conducted with Chong, a pre-service mathematics teacher that majored in mathematics. There are 6 tasks given, comprising topics of rate of change and derivatives. The result shows that Chong has difficulties dealing with derivatives caused by his lacking knowledge in rate of change conception. His interpretation of rate of change under multiple contexts were also inaccurate, even for motion graphical context. His weakness in graphical representation also reflected his insufficient knowledge in rate of change as he was unable to convey the first principle accurately. The overall result suggested that **Chong's subject matter knowledge of rate of change is insufficient**, especially in meaning of rate of change, describing the occurrence of rate of change, and distinguishing between rate of change and instantaneous rate of change.

Keywords: *Rate of Change, Instantaneous Rate of Change, Derivatives, Pre-Service Mathematics Teacher, Subject Matter Knowledge*

INTRODUCTION

Rate of change is one of the most fundamental concepts in calculus (Thompson, 1994b) and is used to describe the relationship between the changing quantities (Tyne, 2016). Since the world is dynamic in nature, thus everything is always changing. All these changes can be explained using the notion of rate of change. Some of them may alter their behaviour dynamically and some may display cyclic patterns (Marsitin, 2019). Calculus is used to help us understand this changing behaviour and their complex characteristics. Using this advantage, a lot of things could be predicted to produce informative data which is beneficial for the sake of human life.

Ironically, learning this concept is an important matter to all secondary students even to those who are not majoring in mathematics (Amit & Vinner, 1990) or do not intend to pursue STEM field. This is because calculus is integrated in many fields, acting as a requirement for students before going any deeper into their respective fields. There have been numerous studies conducted in both levels, secondary and tertiary, **in order to figure out students' performance in calculus particularly in derivatives** (e.g. Brijlall & Ndlovu, 2013; Hashemi et al., 2015; Hashemi et al., 2014; Lee, 2020; Makgakga & Maknakwa, 2016). Unsurprisingly, they recommended and urged for future studies to focus more on conceptual part and balance out with procedural-based approach since most mistakes made in calculus-introductory course sprung from their weak mastery in the concept of calculus (Brijlall & Ndlovu, 2013; Hashemi et al., 2014; Lee, 2020).

This entails that students need a teacher with great subject matter knowledge to convey the ideas of calculus comprehensively. However, past studies (e.g. Byerley & Thompson, 2017; Desfitri, 2016; Lam, 2009) disclosed the **in-service teachers' lacking understanding which reflected their weak subject matter** knowledge in derivatives and rate of change. This brought the researchers' attention to the importance of pre-service mathematics teacher's subject matter knowledge in rate of change. The fundamental concept in calculus is the idea of rate of change (Thompson, 1994b). It is crucial for high school students to have a strong understanding of the basics of calculus before they learn more advanced calculus at tertiary level. Pre-service mathematics teachers are also future teachers, who need to be sufficiently knowledgeable in their subject area. Their subject matter knowledge will play an important role in guiding instructional practice in their future classroom. It also navigates the pedagogy knowledge which is the integration between subject expertise and the teaching skill (Shulman, 1986). Hence, this raises an important question on pre-service mathematics **teachers' subject matter knowledge in rate of change**.

LITERATURE REVIEW

The idea of rate of change as stated by Thompson (1994b) is an underpinning concept in studying calculus. It deals with changing quantities and helps to describe the happening phenomena such as in context of engineering, economy and sciences (Tyne, 2016). Therefore, the understanding of calculus understanding needs to be developed simultaneously with understanding the rate of change. The secondary school curriculum has taken the initiative to include the elementary calculus learning which include limit, differential and integral in the syllabus. The evolution in learning calculus should start from basic learning where the students learn calculus in their secondary school (Tall, 2013).

However, recent studies on high school students' performance in calculus mainly highlighted students lack understanding in derivative notation, preferred rules and formula to solve problems, and an insufficient understanding of calculus-related topics such as derivative and slope (e.g. Brijlall & Ndlovu, 2013; Makonye & Luneta, 2014). Indeed, local studies also highlighted Malaysian students low understanding in derivatives as they misunderstood the function notion and caused them to produce wrong calculation (Nasir et al., 2013; Tarmizi, 2010). Both studies focused to investigate students' **performance in algebraic manipulation and yet, discovered the root problem of students' miscalculation** is that they were confused between independent and dependent variables, reflecting their weak conceptual understanding. This may be due to the teaching approach that overemphasised on procedural aspects as well as disregarding the theoretical sides, thus, contributing **to students'**

difficulties in calculus (Nasir et al., 2013; Tarmizi, 2010). The conceptual part that was left behind leads **to the issue of teachers' subject matter knowledge** in calculus. The studies carried out in the university also suggested high school teacher's **role to instil students' understanding of basic concepts in calculus** (Ayebo et al., 2017; Estonanto & Dio, 2019; Pitt, 2015). Hence, the importance of this understanding which was first imparted in school is **situated under teachers' content knowledge** (Ayebo et al., 2017; Wade et al., 2017).

However, past studies conducted among teachers displayed issues in understanding the meaning of derivatives and rate of change (Byerley & Thompson, 2017; Desfitri, 2016; Lam, 2009). These **circumstances led to the speculation of teacher's knowledge**, specifically in rate of change. Since teacher's content knowledge highly contributes to **students' learning** (Shulman, 1986), hence, it is reasonable for the research to be conducted among in-service teachers. For instance, the studies concentrated on **in-service teachers' content knowledge in derivatives revealed the teachers were usually turned to procedural method in handling the task** (Lam, 2009) and they also admitted that they were not really sure or understood derivatives concepts (Desfitri, 2016). The issue arising from this **matter was that teachers' mastery** in this concept is actually a crucial matter since it could become a **"bridge" for their students** in the first year of university (Desfitri, 2016). This problem affects students' long-term learning since the understanding acquired from their teacher will limit their conceptualization on rate of change as they delve more into procedural aspects. Due to the importance of this matter, attention should also be given and brought towards mathematics teachers' educational program.

A local study has disclosed Malaysian **pre-service mathematics teachers' performance** in Teacher Education and Development Study in Mathematics (TEDS-M) where their score means rank lower than the international mean (Leong et al., 2015). 57.1% of them have lower content knowledge while only 6.9% at a higher level (Leong et al., 2015). A similar result obtained by Adnan et al. (2012) also shows **pre-service mathematics teachers' knowledge about mathematical concepts were considered only as good**. This phenomenon is worrisome since these pre-service mathematics teachers will be the ones who steer students' understanding and yet, their content knowledge is still at low level. Pre-service mathematics teachers may somehow have limited knowledge in rate of change which originated from their schooling experiences, but their knowledge could be varied after they attend the course in teacher education program (Fothergill, 2011). However, assuming all pre-service mathematics teachers are well-equipped with subject matter knowledge as they enter the teaching profession might not be true (Ball & McDiarmid, 1990; Even, 1993). Their varying knowledge becomes an ambiguous matter and needs to be inquired into. Due to that, a study on pre-service mathematics teachers is important to be carried out in order to investigate their subject matter knowledge, particularly in rate of change. This is also based on the literature and the problem gathered in rate of change had led researchers to pose the following research question;

what types of subject matter knowledge of rate of change do pre-service mathematics teachers possess?

METHODOLOGY

In this study, case study research design was used since the focus is to explore pre-service mathematics teacher's subject matter knowledge in rate of change. This case study is bounded to a pre-service mathematics teacher as a case study unit. Qualitative research method was employed since the interest of this study is to seek understanding of meaning by people, which is pre-service mathematics teacher's subject matter knowledge of rate of change. Knowledge, epistemologically, is believed to be gained differently by different people which served as a naturalistic aspect by the interpretivist. This matter is referred to as a distinctive event and led to the use of inductive approach (Creswell, 2013). Hence, the best method to collect the data is comprised of interview, document analysis and observation techniques. These techniques help to enable the **pre-service mathematics teacher's knowledge through verbal explanation and non-verbal behaviour**, which were stored in the form of words, behaviour and written answers such as video and audio recordings, and documents. It is a direct access into **participant's knowledge and helps** to understand subject matter knowledge thoroughly and comprehensively.

Interview is found to be the most appropriate technique to be used since this study is interested in accessing pre-service mathematics **teachers' knowledge**. The type of clinical interview used in this study is task-based interview. This type of clinical interview is most suitable and fits into this present study since it is aligned with the aim of this study. In comparison, the conventional method of paper-and-pencil based test in answering mathematical tasks will only limit the knowledge investigation and certainly not address the actual knowledge had by the individual (Goldin, 2000). This is why the adaptation of task-based interview was used because it well-describes pre-service mathematics teacher's knowledge by providing an insight into their mathematical solution and making it possible to focus onto their knowledge comprehensively rather than just on the pattern of correct and incorrect answers produced.

The pre-service mathematics teacher was selected through purposive sampling, targeting participants who can provide the most valuable insights into the central issue at hand. The selection criteria comprised the following: 1) pre-service teacher majoring in mathematics; 2) pre-service teacher who completed all courses related to calculus content; 3) pre-service teacher who agreed to participate in all interview sessions as required; 4) pre-service teacher who consented to being video and audio recorded for the study. A pre-service mathematics teacher, pseudonymously named Chong, was selected for the study. Chong majored in Mathematics with a minor in Chinese Language, achieved an A- in calculus and an A in pre-calculus courses. He attained a CGPA of 3.86. He scored an A+ in both Mathematics and Additional Mathematics in the national examination (SPM). The findings of his subject matter knowledge of rate of change are presented in each task and it contains descriptions of how Chong responded to the given task followed by an **analysis of Chong' answers and responses**. Each description consists of **excerpts of Chong's responses and Chong's written calculations**. In the excerpts, Chong is represented by "C", whereas researcher by "R".

FINDINGS

In Task 1 (see Appendix), Chong defined rate of change as an element or a variable, where it is affected by another small element and gave example of volume influenced by radius, dv per dr . He said that it required differentiation to obtain the change. He explained that rate of change is more specific like dv per dr and it is not merely just v (volume) with r (radius), but also related with time and mentioned that there was a concept he learnt where dv per dt is multiplied with dt over dr .

C: ... an element or a variable... volume or what influenced by small elements, it will be placed above while elements which affects it will be placed below.

:

C: ... like volume influenced by radius, dv per dr ... it needs to go through differentiation, then will get its change...

:

C: if rate of change, it relates with time.

:

C: yes, and its more specific, it seems like the example that I took, dv per dr and its not just relation v with r that influenced, but it like v divided with r but it seems to relate with time, for me.

:

C: one more, the concept that I remember, like dv over dr , it can be dv over dt multiply with dt over dr .

In Task 2 (see Appendix), Chong answered 5.8 as rate of change of the function. When he was asked on why he chose differentiation, he replied he did not know why but he knew that to find rate of change, differentiation need to be used. He explained 5.8 as the dollar for each increment of x gallon cooking oil.

C: ... I think 5.8.

:

C: directly do differentiation.

R: ... why you choose to do differentiation to find rate of change for this function?

C: ... **this, I do not** know how to answer.

R: ... meaning that if rate of change is mentioned, you directly do differentiation, like that?

C: yes, that is true.

:

C: ... that 5.8 represents dollar, for me, for each x gallon cooking oil if it is added.

In Task 3 (see Appendix), Chong gave an incorrect answer for intervals where displacement increase constantly and in describing velocity for [4,5]. He answered [0,2] as interval that displacement increases constantly due to constant increment of graph line. This shows that he is referring to **the "displacement increases constantly" as slope of the graph that increases constantly**. Chong correctly described the displacement and velocity in interval [6,8] as he conceived it moves backward and the velocity increases due to numerical value that increases from 0 to 2. As for interval [4,5], he described the displacement correctly as it moves backward, but described the velocity incorrectly although he initially described it correctly. He also expressed his confusion as he decided for the velocity in interval [4,5].

C: increase constantly for me maybe 0 until 2, because 0 until 1 increase one, 1 until 2 increase another one.

:

C: ...at 4 second, in that interval, the displacement for me seems to reduce, it may not move or it moves very slow, so it reduces, causes velocity to reduce and hence rate of change in displacement definitely decrease.

:

C: it decreases like just now constantly, for me it decreases constantly.

:

C: yes [6,8].

:

C: ... it actually seems to increase constantly, it increases constantly too but it moves backward...

:

R: ... the velocity increase or decrease?

C: backward means increase, maybe for me.

:

C: meaning that [6,8] it is already back to origin but it moves further backward.

:

R: ... velocity for [4,5]?

C: [4,5] velocity definitely decrease, eh no, velocity very high, actually velocity is very high, I also confuse.

In Task 4 (see Appendix), Chong answered [0,1.5] and [7,8] as intervals where rate of **bacteria culture's** mass increases. He answered interval [3,7] in which **bacteria culture's mass** increases, saying that mass increasing when rate of change decreasing. In describing interval [1.5,3], he elaborated that rate of change **of bacteria culture's** mass will follow its change in mass, hence, he gave the answer as decreasing for both. However, for interval [3,5], he expressed his confusion for the **bacteria culture's** mass since he observed rate of change as increasing, but eventually said that the mass decreasing since the graph is on negative side.

C: zero until 1.5, then 7 until 8.

:

C: because the line seems show...

:

C: ... it showing increment.

:

C: I think 3 until 7.

:

C: ok, let say the time from origin like mass already fix, then time after that maybe the mass is reduced then its rate of change will increase. After that the time when mass increase, the rate of change will reduce.

R: does it means when time **increase**...

C: when time increase, the mass reduces, when mass reduce, rate of change will get higher.

:

R: ... for 1.5 until 3, what can you say about mass and its rate of change?

C: err the mass maybe reduce, mass reduce, and it causes rate of change to reduced.

:

C: the mass I think because it is back to origin, so I think it reduce, maybe.

...

R: ok, then how about interval [3,5]?

C: [3,5] ... the mass maybe increases.

:

C: its rate of change increase... the mass, I think will increase first, after that, rate of change will increase.

:

C: because mass increase, because... eh, not increase, it reduces, right?

:

C: ... for mass I think it decrease, maybe.

:

R: the reason?

C: then its rate of change increase, the reason... because slope is on negative side, so I think it reduces, the mass.

In Task 5 (see Appendix), he answered 40 as rate of change of population. He computed derivative of $P(t)$ and substituted t with 1990 and 2000, before subtracting both of them as shown in Figure 2 below. When he was asked about term rate of population that changes, he replied, it was a difference and also a rate, which required differentiation method.

C: 40

⋮

R: ... what do you understand from rate of population that changing?

C: for me, the difference maybe.

R: difference?

C: subtract.

R: why differentiate?

⋮

C: because if want to look for rate, normally I directly differentiate.

① $P(t) = 2t^2 + 10t + 200$
 (a) $P'(t) = 4t + 10$
 $t = 2000, P'(2000) = 4(2000) + 10 = 8010$
 $t = 1990, P'(1990) = 4(1990) + 10 = 7970$
 changing rate = $8010 - 7970 = 40$

Figure 1. Chong's Calculation

In Task 6 (see Appendix), Chong answered first principles can act as measurement for rate of change. He explained that the result of y_2 subtract y_1 give a change and h limit is like a range, dividing both of them can see how much they change. However, Chong answered both points P and Q in which derivative is measured using first principles, due to h limit serving point P as initial point and point Q as last point. He said that points P and Q need to be considered in the derivative.

C: ... I think P and Q.

⋮

C: because of the limit, we have to see the limit, so P the initial and Q is the last point, so, need to see both P and Q and domain in that.

⋮

C: can, because ... I definitely have to use this to find rate of change but I forgot about the concept.

⋮

C: ... like assume y_1 eh, not y_1 , assume y_1 is the result for an original function, y_2 is the change, so we take y_2 subtract y_1 can get its change, then when we divided with h , the h is like a range... so, when we divide like that, so, supposedly can see how much its change or how much the decrement.

DISCUSSION

Briefly, the findings displayed Chong's notion of rate of change are composed of several ways. He first defined rate of change as a variable that influences another variable under time context, stating volume influenced by radius, though his example of dv per dr in Task 1. This may be due to his understanding in cross multiplication concept, in which he mentioned $\frac{dv}{dt}$ multiplied by $\frac{dt}{dr}$, indicating the need of time as an implicit independent variable. The notion of rate of change has a relationship between variables, instead of relationship between changing variables. Chong also conceived rate of change is similar as amount of change when interpreting rate of change tasks under graphical form. Nevertheless, it also shows that he understood derivative as rate of change that caused him to provide derivative notation, $\frac{dv}{dt}$ as example of rate of change. It also proved in other tasks as he used derivatives to find rate of change for the respective functions. However, though Chong was able to conceive the idea of rate of change through derivative, he was still unable to conceive derivative as instantaneous rate of change and derivative as slope of a tangent line.

Firstly, definition of rate of change that Chong expressed at the beginning of Task 1 is as a relationship between variables, instead of between the changing variables. He also defined rate of change of function $f(x)$ in Task 2 only by including the change in input variable. He, apparently, mentioned the relation between x gallon cooking oil and production cost by relating the relative size between input and output for function $f(x)$. He stated 5.8 dollar is charged due to increment of each x gallon cooking oil, indicating change in input impacting the output. Despite that, he missed to include the production cost which increased by 5.8 dollars due increment of each x gallon cooking oil. His interpretation in this context indicates the rate of change conception he has is insufficient. From those two tasks, it shows that Chong is able to conceive the rate of change as a relationship between variables but is unaware about changing variables happening in the relationship. Past studies have presented different notions of rate of change had by students, especially conceiving rate of change as amount of change (Thompson, 1994a; Zandieh & Knapp, 2006). This may be due to teaching approach that is inclined **towards student's procedural understanding in rate of change, rather than conceptual understanding** (e.g. Borji et al., 2019; Brijlall & Ndlovu, 2013; Makonye & Luneta, 2014; Tall, 1993). In Malaysia, the algebraic manipulation has been emphasised regularly especially in secondary level (Nasir et al., 2013; Tarmizi, 2010), where it was supposed to be a good starting point to develop a conceptual knowledge that will be helpful in university level (Desfitri, 2016).

Meanwhile, in Task 5, Chong understood rate of change of population as difference between rate of change since he subtracted derivatives between year 1990 and year 2000. He is unable to conceive the rate of change of population which was an instantaneous rate of change $t = 10$. He provided the reason he chose to subtract was because rate of change is a difference. However, his computation indicates his lack of knowledge of derivative as instantaneous rate of change, as he considered both methods of subtraction (difference) and derivative. This shows that Chong is unable to comprehend rate of change under different contexts, as in Task 2 that was discussed earlier. These, again, might originate from his insufficient general understanding which he displayed in Task 1. This kind of problem in interpreting rate of change is common among students even for university students (Borji et al., 2019; Desfitri, 2016; Haghjoo & Reyhani, 2021; Lam, 2009). Indeed, in study by Kertil (2021), he emphasized covariation and relative size in learning rate of change or precisely derivative concept, since he found out that school syllabus focused on derivative as optimization problem with priority in algebraic manipulation. **This will cut off students' opportunity to conceive meaning and use of derivatives through optimization problem** (Kertil, 2021).

Other notion of rate of change that Chong has was found when he interpreted rate of change under graphical contexts. Although one of the contexts is about motion linear graph of velocity against time (Task 3), he still faced difficulties to interpret displacement and velocity of the graph. He expressed his confusion if velocity will change the same way as its displacement. This shows that motion context can still cause problems to pre-service mathematics teacher though it is the most common application being used while teaching and learning the topic of rate or rate of change (Bezuidenhout, 1998; Zandieh & Knapp, 2006). This problem is common since other studies also disclosed that motion context such as velocity conception was found to be underdeveloped and being regarded as intuitive concept (Bezuidenhout, 1998; Jones, 2017; Thompson, 1994a). This may be due to familiarity that students have about velocity as directly proportional to the displacement, hence, increment of velocity is due to increment of displacement and vice versa as mentioned by Chong when he expressed his confusion between velocity and displacement. He was unable to grasp that increment in displacement can be caused by either increment or decrement of positive value velocity. This is because positive value of velocity indicates an increment in displacement over time travelled, while negative value of velocity indicates a decrement in displacement time travelled because the object moves to its origin, which means it moves in opposite direction. This knowledge requires robust understanding of rate of change when attending into the motion context, so that students are able to understand that rate of change is not the same as its amount of change of independent variable (Jones, 2017). However, a frequent use of motion context (e.g. displacement, velocity, acceleration) as preliminary context in introducing calculus application may hinder **student's knowledge about rate of change in non-motion context** since there were a range of numbers of rate of change application that are not based on motion context (Jones, 2017).

Indeed, the explanation Chong provided in another graphical calculation (Task 4) also exhibited a similar result, where he conceived amount of change in output is same as its rate of change. Similar to the motion context, he expressed his confusion between mass of bacteria culture and its rate of change while answering for this non-linear graphical context. This further clarifies **Chong's lack of knowledge to distinguish between amount of change and its rate of change**. It also shows that Chong unfamiliarity in non-motion context since he unable to make connection between mass of bacteria culture and its rate of change. Unsurprisingly, this issue has been debated by other researchers (Bezuidenhout, 1998; Herbert & Pierce, 2012; Zandieh & Knapp, 2006), where they presented the problem is due to frequent attending to motion context. As a result, these students with limited knowledge of rate of change unable to project their understanding into non-motion context (Bezuidenhout, 1998; Zandieh & Knapp, 2006). This will become a problem especially for future teachers who still cannot comprehend motion application. If they did not comprehend the motion context, how could they be expected to comprehend other non-motion contexts. Rate of change, clearly, has diverse meaning in real-life situations such as flow rate of water draining in a dam for amount of water drain-in as a function of time and cost rate for production price as a function of amount product produced, which definitely requires a conceptual understanding in rate of change.

Nevertheless, Chong is able to comprehend derivatives as operator to obtain rate of change, where it was exhibited in Task 2 and Task 5. While Chong is able to compute derivative correctly in Task 2, he faced difficulties in Task 5. Based on his calculation, he was unable to interpret variable t as number of years since year 1990 that led him to answer incorrectly for rate of change of population. This indicates his lack of knowledge in finding instantaneous rate of change, especially involving non-linear function since he did not encounter any sort of problem when finding rate of change of linear function in Task 2. This shows that he is unable to conceive derivative as instantaneous rate of change at a marked point as he subtracted the two derivatives together. It may also be caused by the misunderstanding he has for variable t . Regardless of that, the computation he made in Task 5 by subtracting two derivatives together, implies his lack of knowledge about derivative concept. Other researchers also have voiced out derivative conception is a problematic concept among school students and university students (e.g. Desfitri, 2016; Lam, 2009; Orton, 1983; Tyne, 2016; Zandieh, 2000). One of the issues is students are unable to tell apart between rate of change and instantaneous rate of change. For instance, in a study by Kertil (2014), the students were required to find instantaneous rate of change from linear function.

Several participating students that were unable to find it came out with unreasonable answers such as providing the corresponded coordinate- y . Indeed, it is also evident in Task 6 as Chong too was unable to conceive how the limit of first principles works which results in instantaneous rate of change. He expressed first principles formula is a resemblance of general formula of rate of change, hence, it can be considered as measurement for rate of change, instead as instantaneous rate of change. The source of pattern, again, was recognized to be sprung from too many algebraic manipulations while attending the topic of derivative (e.g. Desfitri, 2016; Lam, 2009; Orton, 1983; Tyne, 2016; Zandieh, 2000). Instead of balancing between conceptual aspect of derivative that gives meaning as instantaneous rate of change, the procedural aspect is heavily being addressed. This will refrain **students' opportunity to understand first principles as a derivative conception.**

Moreover, Chong was unable to comprehend that first principles formula represents a slope of tangent line at a point, since he considered the formula was only measuring a slope of secant line between two points. Hence, this becomes another reason on why he was unable to conceive derivative as instantaneous rate of change at a marked point. He explained that the formula measures derivative at both points P and Q since he understood range of limit h required both points for the computation. This shows that he was unable to conceive how limit h works in first principles and how the transition happened from the initials points of P and Q to the measured point P. Unsurprisingly, this was consistent with other studies (e.g. Herbert & Pierce, 2012; Kertil, 2014; Ozaltun-Celik, 2021; Ubuz, 2007), where they presented students' difficulties in understanding derivative concept. One of it was the limit concept that became a cause on why students have difficulty to comprehend derivatives and eventually hindered students to understand the instantaneous rate of change (Ozaltun-Celik, 2021; Ubuz, 2007). The main problem identified is in transferring the knowledge from graphical part to symbolic part as mentioned in study by Ozaltun-Celik (2021). He explained that the **student's unattended part in understanding the first principles** was pointed out by hardly using graphical representation in the learning process.

CONCLUSION

The findings indicate that **Chong's knowledge in rate of change varies** insufficiently in different ways. His knowledge, especially, in interpreting rate of change based on the context was inaccurate for both linear and non-linear function. This problem was anticipated since he also provided insufficient meaning for rate of change when he was asked what is meant by rate of change in Task 1. Due to that, he also hesitated between rate of change and its amount of change as he expressed his confusion when interpreting graphical tasks, even for motion velocity-time graph. Plus, his computation in finding rate of change for non-linear function also displayed his inadequate knowledge of derivatives since he was **unable to distinguish between "instantaneous rate of change" and "rate of change"**. Due to that, Chong was also unable to conceive first principles formula as an instantaneous rate of change. This also became another reason on why he was unable to comprehend first principles represented by slope of tangent line at a point due to presence of limit in the formula. He, instead, expressed slope of secant line as a measurement by first principles. Hence, this study's findings indicate **Chong's knowledge in rate of change is lacking** as he only understood derivatives as operator for rate of change and in finding slope of a graph. His elaboration shows that his knowledge inclined towards procedural understanding, without conceptual understanding. As stated earlier, Chong is a future mathematics teacher, who is supposed to be sufficiently knowledgeable in order to deliver teaching content to his students. Conversely, a teacher cannot teach students something that they do not know (Shulman, 1986). Hence, this study intends to convey the importance of future teachers to have deep knowledge of rate of change, so that knowledge transferring between teacher and student can take place efficiently, resulting **in students' clear and deep understanding of calculus**, especially rate of change. Since this study is limited to one case study of a pre-service mathematics teacher, thus, researchers did not make any claim by generalising it to other Malaysian pre-service mathematics teachers. This study, instead, gave a picture on difficulties or weaknesses that were found lacking in this future **teacher's knowledge, which** may help to inform other Malaysian educators or researchers about the enhancement of calculus knowledge, particularly in rate of change. However, the result of this study suggested teacher training

programs to provide a necessary approach or module to promote future teachers' knowledge in rate of change.

This problem can be encountered by looking up into modules for both topics of rate of change and derivative, which may help to strengthen student teachers' subject matter knowledge in rate of change. For instance, meaning between terms "rate of change" and "instantaneous rate of change" can be introduced diligently using different representation as encouraged by Zandieh (2000) and more engagement for real-life application should be incorporated, besides attending rigorously into motion context. This will help students to conceive how the idea of rate of change is utilised differently in different contexts and how rate of change idea functions in the real world. Indeed, learning through real-life application will improve student's knowledge in rate of change and broaden their mind, instead focusing on algebraic manipulation. Moreover, relation between slope, rate of change and derivative need to be fostered since these relations are crucial to form a full understanding of rate of change. This is consistent with Thompson (2008)'s ideas in his paper on the meaning of significant and coherence learning. He asserted the "significant" term as the ideas that are carried through instructional sequence which should be in manner that are foundational for learning other subsequent ideas. He also added by quoting the importance of coherence, the issue that has been raised numerous times in mathematics education publication and followed by its definition as an "effective, logical progressions from earlier, less sophisticated topics into later, more sophisticated ones" (Thompson, 2008, p. 32). Tall (1993) stated that students' difficulties to understand calculus were caused by the way they handle the conflicting elements separately and never considered to understand them in general or as a whole. Thus, to avoid this misunderstanding, teachers, especially pre-service mathematics teachers should acknowledge the connection made by how it is related and interrelated to avoid any confusion or mental conflict among students.

ACKNOWLEDGEMENT

The study was sponsored by Ministry of Higher Education (MOHE) Malaysia under Skim Latihan Akademik Bumiputera (SLAB) sponsorship. The author would like to express appreciation to Universiti Utara Malaysia (UUM) who granted the SLAB sponsorship, hence, allowing this study to be accomplished.

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APPENDIX

Task 1: What is meant by rate of change?

Task 2: Given $f(x) = 5.8x + 2.5$. Let say $f(x)$ represent the production cost in dollar to produce x gallon of cooking oil.

- Find rate of change for the function.
- Describe the meaning of your answer in (a) based on the given context.

Task 3: Consider the following rate of change in displacement versus time graph below.

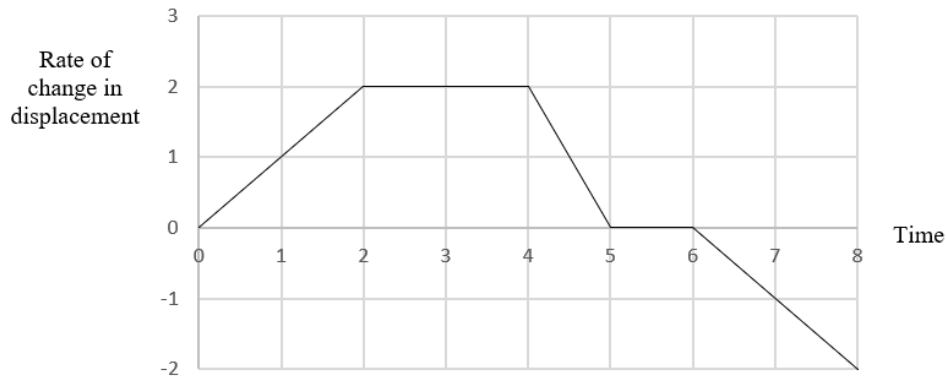


Figure 2. Rate of change in displacement versus time graph

- a. In which interval(s), does displacement increase constantly? Why?
- b. What can you say about displacement and velocity in intervals [4,5] and [6,8]?

Task 4: The graph below represents rate of changes of bacteria culture’s mass each per hour.

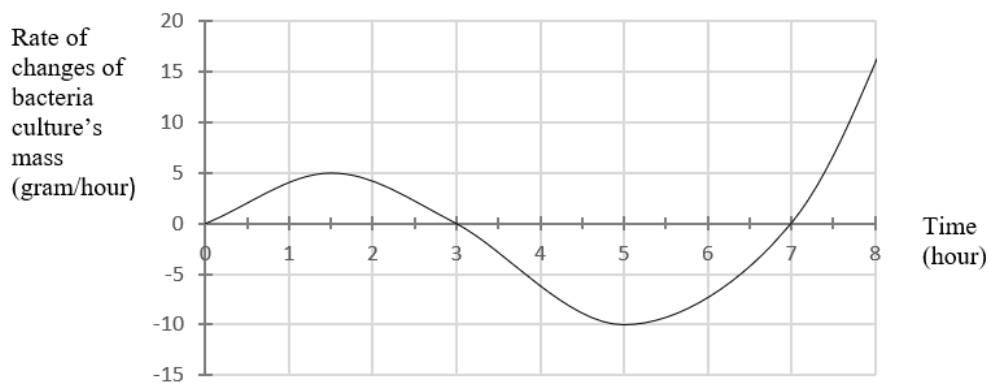


Figure 3. Rate of changes of bacteria culture’s mass each versus time graph

- a. In which interval(s), within first 8 hours, the rate of change bacteria culture’s mass increasing? Why?
- b. In which interval(s), within first 8 hours, the bacteria culture’s mass increasing? Explain your answer.
- c. What can you say about the mass of bacteria culture and its rate of change in interval [1.5,3] and [3,5].

[Task 4 was modified from Byerley and Thompson (2017)]

Task 5: The population in province A is modelled as $P(t) = 2t^2 + 10t + 200$ where t is the number of years since 1990. What is the rate of population that changed from year 1990 until 2000?

Task 6: The graph in figure illustrated on first principles formula shown below which represents the definition for derivative.

$$\frac{dy}{dx} = \frac{y_2 - y_1}{h}$$

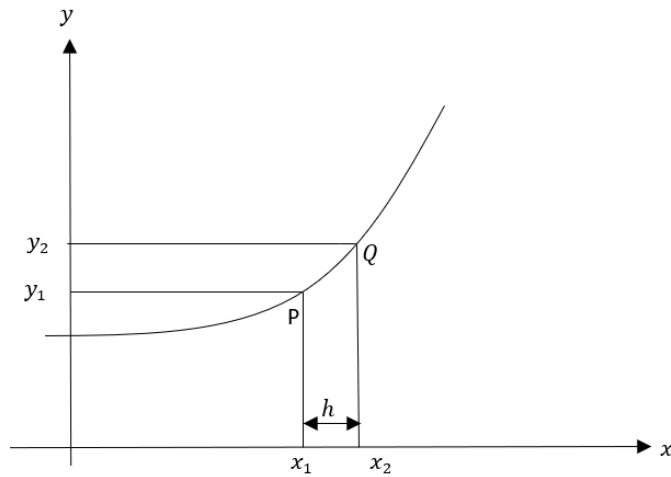


Figure 4

- Which point(s) on the graph does measure derivative as stated by the formula above? Why?
- Do you think the stated formula above can be defined as measurement for rate of change? Why?

[Task 6 was modified from Orton (1983)]