Journal of Theoretical Educational Science, 16(3), 617-640, July 2023
Kuramsal Eğitimbilim Dergisi, 16(3), 617-640, Temmuz 2023
[Online]: http://dergipark.org.tr/akukeg
DOI number: http://doi.org/10.30831/akukeg.1238865

An Analysis of The Questions on Mathematical Literacy Designed by Mathematics Teachers with A Postgraduate Degree

Lisansüstü Eğitim Gören Matematik Öğretmenlerinin Tasarladıkları Matematik Okuryazarlığı Sorularının İncelenmesi

Ebru SAKA*

Received: 18 January 2023 **Research Article** **Accepted:** 14 June 2023

ABSTRACT: This study aimed to examine mathematical literacy questions designed by mathematics teachers with graduate-level in mathematical literacy concerning the categories of mathematical content, real-life context, and mathematical processes, which are dimensions of the PISA mathematical literacy framework. Another aim was to investigate the appropriateness of the classifications made by mathematics teachers for the mathematical literacy questions they designed. The study design was a qualitative case study. The participants of the study consist of six secondary school mathematics teachers who are attending a postgraduate mathematical literacy course at a state university in Turkey. The data of the research were obtained from three mathematical literacy questions designed by each of the participants and the classifications made for these questions. The study found that the majority of the questions designed by the participants fell into the category of "change and relationships," and questions related to real-life situations were mostly designed in the "personal" context, with the least number of questions designed in the "scientific" and "societal" contexts. Most questions included all three categories of mathematical processes when examined in terms of mathematical processes. In light of the results of this study, it can be suggested that the training to be provided in the future for writing questions related to mathematical literacy will greatly contribute to teachers.

Keywords: Mathematical literacy, mathematics teachers, question design.

ÖZ: Bu çalışmada lisansüstü matematik okuryazarlığı eğitimi almış matematik öğretmenlerinin tasarlamış oldukları matematik okuryazarlığı sorularını; PISA matematik okuryazarlığı çerçevesinin matematiksel içerik, gerçek yaşam kategorileri ve matematiksel süreçler boyutları açısından incelemek amaçlanmıştır. Bununla birlikte matematik öğretmenlerinin tasarladıkları matematik okuryazarlığı sorularına yönelik matematiksel içerik, gerçek yaşam kategorileri ve matematiksel süreçler açısından yaptıkları sınıflandırmaların uygunluğunun araştırılması çalışmanın bir diğer amacıdır. Çalışmada nitel araştırma desenlerinden durum çalışması yöntemi kullanılmıştır. Araştırmanın katılımcıları Türkiye'de bir devlet üniversitesinde lisansüstü matematik okuryazarlığı dersine devam eden altı ortaokul matematik öğretmeninden oluşmaktadır. Araştırmanın verileri katılımcıların tasarladığı üçer matematik okuryazarlığı sorusundan ve bu sorulara yönelik yaptıkları sınıflandırmalardan elde edilmiştir. Araştırmanın sonucunda katılımcıların tasarladıkları soruların büyük kısmının matematiksel içerik alanı bakımından değişim ve ilişkiler kategorisinde yer aldığı, gerçek yaşam kategorisi açısından soruların çoğunlukla kişisel bağlam kategorisinde tasarlandığı, en az sorunun ise bilimsel ve toplumsal bağlamda tasarlandığı tespit edilmiştir. Tasarlanan sorular içerdikleri matematiksel süreçler açısından incelendiğinde soruların büyük kısmının her üç matematiksel süreç kategorisini içerdiği belirlenmiştir. Çalışmadan elde edilen sonuçlar ışığında gelecekte matematik okuryazarlığı ile ilgili soru yazmaya yönelik verilecek eğitimlerin öğretmenlere büyük katkı sağlayacağı düşünülmektedir.

Anahtar kelimeler: Matematik okuryazarlığı, matematik öğretmenleri, soru tasarlama.

* Asst. Prof. Dr., Kafkas University, Kars, Türkiye, ebrudmirci@gmail.com, https://orcid.org/0000-0003-1975-3160

Citation Information
Saka, E. (2023). An analysis of the questions on mathematical literacy designed by mathematics teachers with a postgraduate degree. *Kuramsal Eğitimbilim Dergisi [Journal of Theoretical Educational Science], 16*(3), 617-640.

ISSN: 1308-1659

Considering the importance of mathematics in today's world, mathematics is inevitably one of the main areas covered in international studies. Standing out among many international assessment studies aimed at evaluating students' achievement in mathematics, the PISA (Programme for International Student Assessment) is one of the assessment tests in which Turkey also participates. PISA distinguishes itself from other international assessment studies because it focuses on the extent to which students who have completed compulsory education are able to apply the information in their everyday lives, as opposed to merely assessing their ability to recall curriculum-based information. When the literature is examined, many definitions of ML appear. In addition, PISA considers various dimensions when evaluating ML. First, the ML concept and the dimensions of the PISA ML framework are explained below. Then, in light of the literature, the role of teachers in ML and, finally, the research problems are included.

Mathematical Literacy and PISA

In PISA, students' mathematical skills to be measured are based on "mathematical literacy." Although the concept of Mathematical Literacy (ML) first appeared in the written sources of the National Council of Teachers of Mathematics (NCTM, 1989), the concept gained international recognition owing to PISA applications. ML is defined by the OECD (Organization for Economic Co-Operation and Development) as an individual's capacity to use mathematics in various ways for formulating, employing, and interpreting mathematics in real-life contexts (OECD, 2013). ML also helps individuals to understand the role of mathematics in the world and make the well-founded judgments and decisions that constructive, sensitive, and reflective citizens need (OECD, 2019).

By definition of ML, questions are presented in context to measure students' capacity to use mathematical skills in real life. Questions on ML in PISA tests include four different real-life contexts: personal, occupational, societal, and scientific contexts (OECD, 2019). The questions in the category of personal context focus on activities related to individuals, their families, and peers, while questions in the occupational context are business-oriented problems. The questions in the category of societal context are related to the communities in which people live, while the questions in the category of scientific context focus on mathematical practices related to science and technology. While real-life contexts are equally distributed in PISA-style questions on ML, the aim is to provide students with items that cover a wide range of individual interests and various situations they may encounter in their lives (OECD, 2019).

In addition, questions on ML include four content categories: change and relations, space and shape, quantity, uncertainty, and data, which consider the situation of students who have completed their compulsory education to measure basic competencies (OECD, 2019). By and large, the category of change and relationships includes the capacity to understand the variables in the data, understand and interpret them along with their relationships, and evaluate and transform them if necessary. This category includes perspective drawings, map drawings, drawing and transforming shapes, three-dimensional views, and the representation of figures. The quantity category includes determining the measurable properties of objects based on quantity, understanding, interpreting, and proving different numerical representations. In contrast, the uncertainty and data category focus on understanding, interpreting, and evaluating various ambiguous and probabilistic situations.

ML focuses on the mathematical processes experienced by an individual who solves a problem and classifies the questions in the sub-dimensions of mathematical content and framework. Consequently, three mathematical processes are defined as an individual's capacity to formulate, employ, and interpret-evaluate mathematics within the PISA ML measurement and evaluation (OECD, 2013, 2016, 2019). Individuals are able to create a mathematical structure for a problem presented in the theoretical content after becoming aware of and recognizing the situations in which they can apply their mathematical knowledge and skills. The employing process refers to the situation where individuals employ mathematical concepts, facts, procedures, and reasoning to solve mathematically formulated problems and make mathematical decisions. On the other hand, the process of interpreting/evaluating refers to the interpretation and evaluation of the outcome or solutions by transferring them to a real-life context.

Teachers' Role in ML

The knowledge and skills required for ML are often included in existing mathematics curricula at schools (Genç & Erbaş, 2019). The learning areas of numbers and operations, algebra, geometry and measurement, and data processing and probability, which are instructed in five sections in the secondary school mathematics curriculum in Turkey, largely overlap with the subject areas determined by the PISA project in ML (Ministry of National Education [MoNE], 2009). Achieving success in any curriculum reform mainly depends on teachers giving due importance to the concepts of the reform or innovation movement (Handal & Herrington, 2003). Teachers are, therefore, expected to have an adequate understanding of ML and to know how to incorporate this understanding into their teaching practice whenever and wherever needed (Milton et al., 2007). However, raising mathematically literate students is necessary to meet the increasing expectations of the modern world, and this need is growing by the day (Edge, 2009). To meet this need, the biggest task falls to the teachers, who are responsible for preparing the necessary environment for the mathematical processes in the classroom. Nevertheless, the major challenge in bringing ML to the classroom is that teachers must first comprehend what ML is (Baştürk-Şahin, 2021). Besides that, it is also important for mathematics teachers to know the features that questions on ML should include and to bring these questions into the classroom environment when necessary to raise mathematically literate individuals (Kozaklı-Ülger et al., 2022; Mosher, 2015).

Studies conducted on teacher qualification, which is one of the components of ML teaching, emphasize the impact of teacher qualification on student performance and, accordingly, the fact that teacher qualifications exert a positive impact on students concerning the skills necessary for more effective learning (Akbaşlı et al., 2017; Demir & Altun, 2018; Mammadov & Çimen, 2019). In recent years, studies that provide mathematics literacy training to mathematics teachers and prospective teachers have become remarkable (Bansilal et al., 2015; Bozkurt, 2019: Canbazoğlu & Tarım, 2021; Kozaklı-Ülger et al., 2022; Özgen, 2019). In some of these studies, mathematics literacy questions designed by mathematics teachers and prospective mathematics teachers were examined in various dimensions. For instance, Kozaklı-Ülger et al. (2022) conducted a ten-week mathematics literacy course to 28 secondary school mathematics teachers. They examined the participants' ability to pose ML problems in the context of problem posing. Similarly, Özgen (2019) examined the ML problem-posing skills of five mathematics teachers and 13 prospective mathematics teachers who had received ML education. In the study, ML problems developed by the participants were evaluated in terms of difficulty level, real-life context, mathematical content areas, and processes. Canbazoğlu and Tarım (2021) explored the ML problem-posing skills of 61 primary school teacher candidates and analyzed the problems regarding mathematical content areas, real-life contexts, and mathematical processes. In the first dimension of the present study, the problems designed by mathematics teachers will be examined in terms of ML dimensions, as Canbazoğlu and Tarım (2021) did. However, unlike Canbazoğlu and Tarım's (2021) study, the second dimension of this research will involve asking teachers to categorize the ML questions they designed according to ML dimensions and the appropriateness of these classifications will be assessed. Thus, the second dimension of the study aims to facilitate teachers' self-evaluation by having them categorize the questions they designed. Examining the appropriateness of the classifications was intended to reveal the extent to which the teachers internalized the ML process as a result of the provided course and reflected it on the ML questions.

Research Problem

This study will examine how mathematics teachers trained in ML reflect the ML framework in the questions they produce in line with the education they have received by focusing on their question-writing processes for ML. Within this framework, it is believed that teachers' way of carrying out the process of writing questions about ML is highly likely to contribute to their consideration of the dimensions that make up ML, to realize the relationship between mathematics and real life, and to help their students realize this relationship in the future. In addition, it is expected that this study will instruct teachers on how to incorporate ML-improving questions into future mathematics lessons.

From this perspective, the goal of this study is to look at the ML questions created by math teachers with graduate-level ML training regarding the PISA ML framework's three dimensions: mathematical content areas, real-world contexts, and mathematical processes. Another aim is to investigate the appropriateness of the classifications made by mathematics teachers in terms of mathematical content areas, real-life contexts, and mathematical processes for the ML questions they designed. From this standpoint, answers were sought to the following questions:

1. What is the distribution of the questions designed by mathematics teachers in terms of mathematical content areas, real-life contexts, and mathematical processes?
2. To what extent do the questions developed by mathematics teachers align with mathematical content areas, real-life contexts, and mathematical processes in assessing ML?

Method

Since the study aimed to examine the questions on ML designed by mathematics teachers in-depth, the case study method was used as one of the qualitative research design methods. Descriptive analysis and content analysis techniques were used to examine the questions created by mathematics teachers with graduate-level ML training and the classifications assigned to these questions.

Participants

This study consisted of six secondary school mathematics teachers who had attended a postgraduate ML course at a public university in Turkey in the spring semester of the 2020/21 academic year. The research participants were determined by criterion sampling, one of the purposive sampling methods. In the criterion sampling method, the participants are formed from people, events, objects, or situations with the qualifications determined for the problem (Büyüköztürk et al., 2013). The criterion is created by the researcher, or a previously prepared criteria list can be used (Marshall & Rossman, 2014). In this study, it was a prerequisite for the participants to have completed postgraduate ML education. The participants were coded as T1, T2, …, and T6. Table 1 presents the relevant information on the participants' gender and professional experience.

Table 1
Demographic Information of the Participants

Participant	Gender	Years of work experience
T1	Female	2
T2	Female	10
T3	Female	2
T4	Female	5
T5	Female	3
T6	Female	3

As seen in Table 1, all of the teachers participating in this study were women whose professional experience ranged from two to ten years. All participating educators were employed in public schools. None of the participants had prior experience with ML training. Even if three participants were familiar with ML, their understanding was superficial. The other three participants did not know this concept before the training. In addition, the participants completed Mathematical Modeling and Activity-Based Learning in Mathematics Education courses during their graduate education. Participants worked on designing activities for the content of these two courses. In this sense, the participants had experience designing activities.

Data Collection

Data were collected from a total of 18 ML questions, which were designed by six mathematics teachers. Each teacher contributed three questions, and all of them had attended the graduate ML course during the spring semester of the 2020/21 academic year. The participants had received training for 12 weeks on the general framework of PISA on ML, ML levels, basic competencies of ML, questions on ML, and the basic features that the questions should encompass within the scope of the graduate ML course. The ML question design part of the course took six weeks. First, teachers were informed about Altun's (2020) ML question design references, which were explained in detail in the data analysis section. Then, sample ML questions were examined regarding relevant references and dimensions of ML. Afterward, the teachers were given one week for each question design task and asked to design their questions. The designed questions were examined in the next week, feedback was given, and the teachers were asked to revise the questions.

Data Analysis

The initial focus of the data analysis was to determine whether or not the questions created by the teachers matched the nature of the questions on ML. Altun (2020) states that some reference information should be considered when writing an ML question. These are language issues, objecting to problems and questions in textbooks, considering basic concepts related to ML, and evaluating opportunities in daily life (Altun, 2020). In the study, the researchers gave the teachers one week to design each question, and they primarily looked at the designed questions using Altun's (2020) references for creating an ML question. In this direction, the designed questions were primarily examined regarding language. For this, the questions were examined in terms of whether they contain long and complex sentences, considering the level of the student and whether the active expression is used in the sentences. Then, it was checked whether the designed questions differed from those in traditional textbooks and whether they included verbs and actions that describe people's and society's needs. After that, whether the questions considered ML's basic concepts were examined in terms of mathematical processes. Finally, it was examined whether the questions reflected real-life situations. As a result of these examinations, the designed questions were discussed with the teachers, and feedback was provided to address any missing or incorrect aspects of the questions. In line with the feedback, the teachers revised their questions. For example, the first question designed by one of the participants (T3) for the first week was evaluated as weak in terms of real-life context and only required routine procedures. Consequently, T3 was requested to redesign a question based on the provided feedback regarding its apparent weaknesses. After the revisions, the questions prepared by the teachers were evaluated following the references for designing an ML question.

Then, the designed questions were analyzed using the descriptive analysis method regarding mathematical content areas, real-life contexts, and mathematical processes. Data collected in the descriptive analysis method are organized and interpreted according to predetermined themes (Miles & Huberman, 1994). In this situation, the OECD (2019) determined the components of ML and used them to analyze the teacher-designed ML questions descriptively. For this purpose, the ML questions designed by the teachers were divided into categories according to the dimensions of "mathematical content areas, real-life contexts, and mathematical processes." The obtained data are presented in tables and graphs. The analysis revealed that the questions designed by the teachers were categorized under a single context. It is almost impossible for a problem to completely contain all behaviors belonging to a process. The dominant phase in the solution is examined, and the problem is included in that process (Temel & Altun, 2020).

For this reason, in the analysis of mathematical processes, in cases where some of the questions designed by the teachers simultaneously included two different process categories, the question was classified under the dominant mathematical process category. A sample analysis is presented below to analyze the questions on ML designed by the teachers in terms of such categories as mathematical content areas, real-life content, and mathematical processes and to determine whether or not the classifications made by the teachers are appropriate. A sample analysis is explained through the 1st question on ML designed by T2, presented in Figure 1.

Figure 1

The 1st Question on ML Designed by T2

COVID-19 can cause lung disease with various symptoms, such as dry cough, fever, and fatigue, by settling in the respiratory tract in the human body and further leading to diseases such as severe acute respiratory failure syndrome when not treated, thereby scaring the whole world since 2019. Those with severe co-morbidities and conditions may require inpatient treatment. Moreover, the death toll reported has been considerably high since the day the pandemic started. As a solution to such a problem, different countries started to produce and sell their vaccines in different ways. Turkey first decided to supply vaccines by BioNTech, produced in Germany, and CoronaVac, produced in China.

1)

aving ordered 50 million CoronaVac vaccines together with 1 million BioNTech vaccines in the first batch and then 25 million more, Turkey planned to administer the vaccines in 2 doses at three-month intervals. With the aim of vaccinating 1.5 million people a day, in how many days would the vaccine run out for the first stage?

2)

ince Turkey needed to vaccinate approximately 50 million people for the first dose, the vaccine cost created serious problems for the country's economy. Considering that the BioNTech vaccine was available for 19.5 dollars and the CoronaVac vaccine for 13.5 dollars each and that 2 doses of each vaccine were required, Turkey needed to allocate a substantial budget for obtaining the vaccines only. Generally speaking, to administer a newly produced vaccine, it must go through 3 phases. However, it also became inevitable to consider administering the domestic vaccine that Turkey would produce without implementing phase-3. The CoronaVac vaccine had also been received before phase-3 was completed. In this connection, phase-3 of the domestic vaccine was planned to be completed from April to September. When the domestic vaccine started to be administered, Turkey's vaccine cost was estimated to decline by at least 50%. If you were a decision maker for Turkey, how would you obtain the vaccine at the most affordable cost during the 6 months? Please present your calculations to support your answer.

3)

eams of 4 people were formed in the health institutions assigned to vaccinate, and each employee was to vaccinate 60 people daily. For instance, 300,000 first-dose vaccines sent to the province of Iğdır had to be stored in cabinets at -70 degrees, but since there was no such storage area, the vaccination process had to be completed within 5 days. Based on this information, how many teams should be assigned to finish the process without contaminating the vaccines?

When the question presented in Figure 1 as designed by T2 is examined in terms of the mathematical content area category, the actions in the question are evaluated within the scope of the "quantity" category since they are mainly aimed at executing numerical operations, estimating with numbers, and making sense of numerical results. When the designed question is examined in terms of the real-life context category, it appears that the context of the problem is related to the health policies in the society in which the individual is included and to a certain social problem. For this reason, the real-life context category of the problem has been classified under the "social" category. Finally, given the mathematical durations in the 1st item of the question, the student solving the problem is expected to calculate how many days the vaccines will run out in the first place by dividing the total number of vaccines available by the number of vaccines administered per day. The category of mathematical processes has been rated as the employment process because this question item involves performing the necessary actions and achieving the desired result. Moreover, in the 2nd item, the student solves the problem and is provided with different options for Turkey's vaccine supply and some information about their costs. This question requires the problem solver to develop different mathematical models and decide on the most cost-effective option for vaccine supply with the help of these models. The category of the particular mathematical process is interpreted as an evaluation since the question item requires discussing the significance of the mathematical results obtained in the real-life context and reaching a decision most appropriately. Like the case in the 1st item, the 3rd question item requires the problem solver to conduct incremental numerical operations to conclude. For this reason, the mathematical process category of this question item is also considered the employing process.

In the second stage of the data analysis, the appropriateness of the classifications was examined by comparing the classification made by the OECD (2019) according to the ML components used for the first research problem and the classification made by the teachers for the questions they had designed. When the classification made by the teachers for the mathematical content area category was consistent with the classification made by the OECD (2019), it was considered correct. When the content classification made by the teachers was given under the wrong category, it was considered incorrect. However, some of the questions designed by the teachers included two different content categories simultaneously, in which case, if the participants identified only one of the two different content categories correctly and failed to determine the other category, the classification was considered incomplete. Similar to the analysis for the appropriateness of content categories, the classification made by the teachers for the questions they designed was compared to that made by the OECD (2019) for real-life context to analyze the appropriateness of teachers' classifications for real-life situations. When the classification made by the teachers for the real-life context category was consistent with the classification made by the OECD (2019), it was deemed correct.

In contrast, the classification was deemed incorrect if it was assigned to the incorrect category. Finally, an examination of the teachers' categorizations of the mathematical process categories for the questions they developed. When the classifications made by the teachers were consistent with the definitions of mathematical processes made by the OECD (2019), they were considered correct. If participants did not fully identify the mathematical process in the designed question, their response was incomplete. If some of the mathematical processes included in the items of the designed question were classified correctly, and some were classified incorrectly or incompletely, they were considered partially appropriate. Table 2 illustrates the classifications T2 made for the question in Figure 1.

Table 2

The Classification for The First Question Designed by T2

Question Items	Mathematical Process	Content Category	Real-life context
1.	Employing	Change and Relations-(Misclassification)	Societal
2.	Interpreting and evaluating		
3.	Employing		

As shown in Table 2, although T2's first question falls under the quantity category, T2 has categorized this question under the change and relations category. For this reason, T2's classification for the content category is considered incorrect, while the classification T2 has made for the real-life context category is deemed correct. When the designed question is examined in terms of its mathematical processes, as stated in the first stage of the data analysis, the 1st and 3rd question items include the employing process. In contrast, the 2nd question item includes the interpretation-evaluation process. Table 2 demonstrates that T2 can correctly categorize the mathematical content categories of the question. As another example, although T5 determined the mathematical content area of the second question as quantity, the question also included the categories of change and relationships. Therefore, the classification made by T5 under the mathematical content category was considered incomplete. Besides, if the participants stated that the question included all three categories of mathematical processes, although the question did not include some of the categories, it was considered partially appropriate. For instance, although T1 stated that the third question included all three categories of mathematical processes, it turned out that it included the processes of formulating and employing, not the interpretation-evaluation process. For this reason, the classification made by T1 for the 3rd question was considered partially appropriate.

Validity and Reliability

Another expert in the field examined the data on hand independently and classified the questions in terms of content, context, and mathematical operations before determining whether the designed questions were appropriate for the teachers' classification. Then, the analyses made by both researchers were compared. The agreement rate between the researchers' analyses was determined as 89% by using the Miles & Huberman formula. The researchers discussed the points of disagreement in the analyses and reached a consensus for the points of disagreement. For example, in the analysis carried out by two independent researchers, the formulation process of the second question designed by T2 was included in the classification of the mathematical process category by the researcher at the beginning. The other field researcher, on the other hand, did not classify the question during its formulation in her analysis. Two field researchers reached a consensus after discussing the designed question that it includes the employing, interpreting, and evaluating processes but not the formulation process. The results were reported in accordance with scientific and ethical standards.

Ethical Procedures

Ethical approval and written permission were obtained from the Kafkas University Social and Human Sciences Ethics Committee, with the decision dated July 12, 2021, and numbered 21/11. The research was carried out following ethical rules at every stage. The participation of the candidates in the research took place voluntarily.

Results

This study investigated the questions on ML that mathematics teachers designed with a graduate-level education in ML, as well as the appropriateness of the classifications made by the teachers for the questions they designed. The results were presented according to the sub-problems of the study.

Distribution of Questions on ML Designed by Mathematics Teachers in terms of Mathematical Content, Real-Life Context, and Mathematical Processes

Table 3 presents the distribution of eighteen questions on ML designed by six mathematics teachers according to mathematical content categories.

Table 3

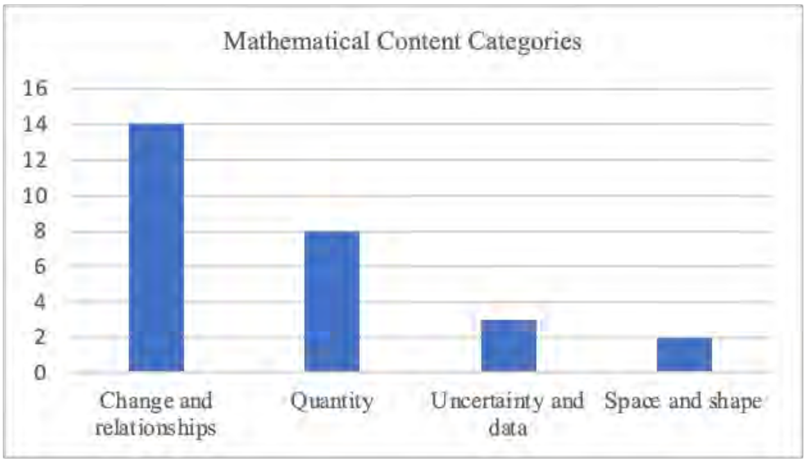
Mathematical Content Categories of Questions Designed by Mathematics Teachers

Participant	Question 1	Question 2	Question 3
T1	Uncertainty and Data Change and Relationships	Change and Relationships Quantity	Change and Relationships
T2	Quantity	Change and Relationships Space and Shape	Quantity
T3	Change and Relationships	Change and Relationships	Change and Relationships Quantity
T4	Change and Relationships Quantity	Change and Relationships Quantity	Change and Relationships Quantity
T5	Change and Relationships	Change and Relationships Quantity	Space and Shape Change and Relationships
T6	Change and Relationships	Uncertainty and Data	Uncertainty and Data

As seen from Table 3, most of the questions designed by mathematics teachers were in the category of change and relationships. However, nine of the problems designed by teachers included two different content categories at the same time. Figure 2 illustrates the distribution of the designed questions according to the mathematical content categories.

Figure 2

Distribution of Questions Designed by Mathematics Teachers According to Mathematical Content Categories



As shown in Figure 2, 14 (78%) of 18 questions on ML designed by mathematics teachers fell in the category of change and relationships. Eight of the designed questions fell in the category of quantity, three in the category of uncertainty and data, and two in the category of space and shape. It appeared that the participants designed most of the questions under the sub-learning areas of percentages and equality-equation under the category of change and relationships. When it came to the category of quantity, it seemed that they mostly included questions requiring arithmetic solutions and questions that included estimation. Furthermore, the analysis of the questions indicated that the sub-learning fields of area measurement and liquid measurement were included in two questions designed for the category of space and shape. The three questions designed for uncertainty and data were created for data analysis, creating tables, and the probability of simple events.

Table 4 shows the distribution of ML questions that the mathematics teachers created following real-world categories.

Table 4

Classification of Questions in Conformity with Real-Life Situations

Participant	Question 1	Question 2	Question 3
T1	Societal	Personal	Occupational
T2	Societal	Personal	Personal
T3	Occupational	Personal	Personal
T4	Occupational	Occupational	Personal
T5	Scientific	Occupational	Personal
T6	Scientific	Personal	Personal

As shown in Table 4, mathematics teachers appeared to have included different contexts in their questions on ML. For example, T1 and T5 included different contexts in all three questions they designed.

Figure 3

Distribution of Questions Designed by Mathematics Teachers According to the Category of Real-Life Situations

Figure 3 illustrates the distribution of the designed questions according to real-life categories. As shown in Figure 3, half of the designed questions are in the personal context category, five are in the occupational context, and two are in the scientific and social categories. For example, the activity named "Coronavirus," the 1st question on ML designed by T1, was evaluated in the societal category in terms of real-life context. In the activity, T1 was asked to predict when the virus spread would decrease according to the conditions given in the activity by using the map that showed the coronavirus risk status of cities published by the Ministry of Health every 15 days in Turkey. Since the activity included a situation related to society, it was evaluated under the category of societal context. In another activity named "Parallax Method," which was the 1st question on ML designed by T5 and evaluated in the scientific category (See Appendix 1), a mathematical method was mentioned to be used to determine the distances of objects, requiring a formula to be developed for finding the distance between any object measured by this method and the person making the measurement. In this respect, the designed activity was evaluated in the scientific category since it included a situation related to the nature of mathematics itself.

Table 5 shows the distribution of questions on ML designed according to the mathematical process categories.

Table 5

Mathematical Processes Included in The Questions Designed by Mathematics Teachers

Participant	Question 1	Question 2	Question 3
T1	Formulating Employing	Formulating Employing	Formulating Employing
T2	Employing Interpreting and evaluating	Formulating Employing Interpreting and evaluating	Formulating Employing Interpreting and evaluating
T3	Formulating Employing	Formulating Employing	Formulating Employing Interpreting and

		Interpreting and evaluating	evaluating
T4	Formulating Employing Interpreting and evaluating	Formulating Employing Interpreting and evaluating	Formulating Employing
T5	Formulating Employing Interpreting and evaluating	Formulating Employing Interpreting and evaluating	Formulating Employing Interpreting and evaluating
T6	Formulating Employing Interpreting and evaluating	Formulating Employing Interpreting and evaluating	Formulating Employing Interpreting and evaluating

As seen in Table 5, most of the questions on ML designed by mathematics teachers, except for T1, include all three mathematical process categories. In contrast, those designed by T1 were limited to the processes of formulating and employing and did not include the process of interpretation and evaluation. The first question designed by T2 did not include the formulation process. In addition, one of the three questions designed by T3 and T4 did not include the interpretation and evaluation process.

Figure 4

Distribution of Questions Designed by Mathematics Teachers According to Mathematical Processes

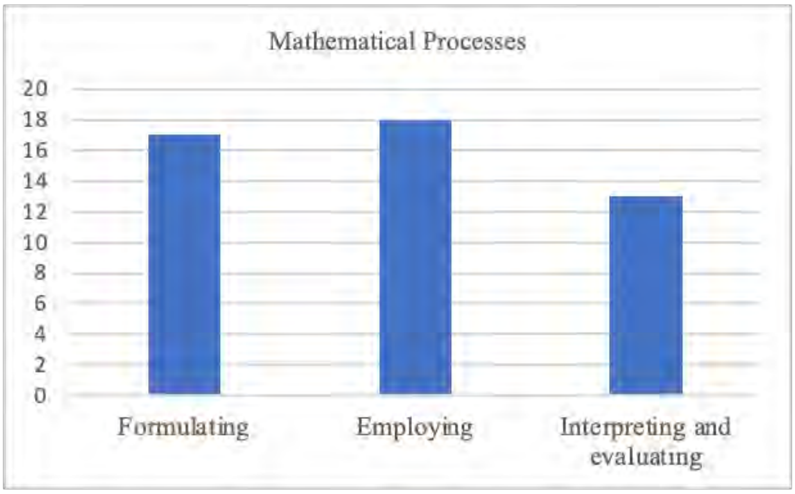


Figure 4 illustrates the distribution of the designed questions according to mathematical processes. As shown in Figure 4, all the designed questions include the employing process. Only one of the designed questions did not include the formulation process. Five (%28) of 18 questions designed by mathematics teachers did not include the interpretation and evaluation process.

The Appropriateness of the Classifications Made by The Mathematics Teachers for the Questions They Designed on ML

This section focused on examining the extent of the appropriateness of the classifications made by six mathematics teachers in terms of mathematical content, real-life context, and mathematical processes for each of the three questions on ML. Table 6 presents the findings obtained from the analysis of the category of mathematical content determined by the mathematics teachers for the three questions designed.

Table 6

The Appropriateness of The Classifications Made by The Mathematics Teachers According to The Mathematical Content Categories of The Questions They Designed

Participant	Question 1	Question 2	Question 3
T1	Incomplete	Incomplete	Correct
T2	Incorrect	Correct	Correct
T3	Incorrect	Incorrect	Incomplete
T4	Correct	Correct	Correct
T5	Correct	Incomplete	Correct
T6	Correct	Correct	Incorrect

As seen in Table 6, only T4 could correctly determine the mathematical content category of all three questions she designed. T2 and T6 were able to classify two of the three questions they designed correctly, but they classified one question under the wrong category. Although T2 intended for the question to fall under the category of quantity, she placed it under the category of change and relationships. Similarly, T6 classified her third question on ML under the category of change and relationships; however, when the content area of the question was examined, it turned out that it was indeed related to the category of uncertainty and data. The analysis of T5's classifications for the questions revealed that she had classified the first and third of her questions according to the appropriate mathematical content area yet failed to fully determine the second question in conformity with the relevant content area since she specified the mathematical content area of that question as to quantity only, though the question should have been included in the category of change and relationships as well. Another participant, T1, on the other hand, classified only one of the three questions she designed under the appropriate content category and underdetermined the content area of the other two questions. Although T1 classified the first question she designed under the category of change and relationships, as a matter of fact, the question also belonged to the category of uncertainty and data. T3, on the other hand, determined the content category of two of the three questions she designed incorrectly, and the other one was incomplete. It seemed clear that the first two questions had been designed in conformity with the category of change and relationships; however, T3 classified them under the category of quantity. Moreover, although T3 classified the third question she designed under the category of quantity when the question was examined, it turned out that it should also have been included in the category of change and relationships.

Table 7 presents the results obtained from the analysis of the classifications made by the mathematics teachers participating in this study regarding the "real-life context" category for the three questions designed.

Table 7

The Appropriateness of the Classifications Made by The Mathematics Teachers for The Questions They Designed According to The Category of Real-Life Situations

Participant	Question 1	Question 2	Question 3
T1	Correct	Incorrect	Incorrect
T2	Correct	Correct	Correct.
T3	Incorrect	Correct	Correct
T4	Correct	Correct	Correct
T5	Correct	Correct	Correct
T6	Correct	Correct	Correct

As seen from Table 7, four of the six mathematics teachers determined the questions appropriately in conformity with real-life situations. When the first question designed by T3 was examined, it seemed that the content of the question was about the cheese production process and was designed to require ratio and percentage calculations for the nutritional elements in the cheese content. T3 classified the question under the societal category in terms of real-life context; however, the question should have been evaluated under the category of occupational context since it included a context concerning a profession. Therefore, T3's classification of the first question was considered incorrect. While T1 could determine the real-life category of the first of her three questions, she misclassified the second and third questions. The content of the second question included situations such as choosing the most appropriate route for a journey by using navigation, how long the journey would take according to the variables given in the question, and what the average speed should be. Although T1 classified the question under the societal category in terms of real-life context, it seemed appropriate to classify it under the personal category since it was related to an event that the individual may have experienced and was not a society-related problem.

As a result, T1's classification was determined to be incorrect. The third question, on the other hand, was designed to calculate the amount of yarn obtained from spider webs on a farm and the number of bulletproof vests produced from that yarn. T1 classified that question under the category of societal context from among real-life situations. However, when the content of the question was examined, it seemed clear that it fell under the occupational context category since it was designed to concern a production process belonging to a particular profession. It was thus determined that the classification made by T1 was incorrect.

Table 8 presents the findings obtained from the analysis of the classifications made by the mathematics teachers in terms of the mathematical process category for the three questions they had designed.

Table 8

The Appropriateness of The Classifications Made by The Mathematics Teachers According to The Mathematical Process Categories for The Questions Designed

Participant	Question 1	Question 2	Question 3
T1	Incomplete	Partially appropriate	Partially appropriate
T2	Appropriate	Appropriate	Appropriate
T3	Appropriate	Appropriate	Appropriate
T4	Appropriate	Appropriate	Appropriate
T5	Appropriate	Appropriate	Appropriate
T6	Appropriate	Appropriate	Appropriate

As seen from Table 8, five of the six mathematics teachers could determine the mathematical process categories appropriately included in all the questions they designed. However, T1 specified the mathematical process category of the first question she designed incompletely. In contrast, her classification according to the mathematical process category of the other two questions was partially appropriate. T1 only classified her first question's mathematical content area as the formulating process. However, when the question was examined, it turned out that, besides the formulation process, it required processes such as finding information based on a given map, developing a strategy to come up with mathematical results, or employing mathematical rules. Thus, it appeared that the mathematical process of the problem designed by T1 also included the process of employing.

For this reason, the classification made by T1 for her first question was considered incomplete. As regards her classification for the second question, T1 stated that the question included all three categories of mathematical processes. However, when the content of the question was examined, it was found that although it included the processes of formulating and employing, it did not include the interpretation-evaluation process. From this standpoint, the mathematical process classification that T1 made for the second question was partially appropriate. Similarly, although T1 stated that Question 3 included all three categories of mathematical processes, the question turned out to be included in the interpretation-evaluation process only. The classification made by T1 for Question 3 was also considered partially appropriate.

Discussion and Conclusion

As a result of this study, most of the questions designed by the participating mathematics teachers were found to be in the category of change and relationships, followed by the category of quantity in terms of the mathematical content area. The least number of problems were found in space and shape, uncertainty, and data categories. This study's results were in parallel with those obtained from the study in which Kohar et al. (2019) examined the PISA-like questions designed by 14 pre-service mathematics teachers. The analysis of the distribution of ML questions according to real-world categories revealed that the majority of questions were designed for the personal context, followed by the occupational context, and the fewest questions were designed for the scientific and societal contexts. These results align with those reported by Canbazoğlu and Tarım (2021), who examined the problem-posing skills of prospective classroom teachers regarding ML. Additionally, given the results of the studies focusing on designing questions on ML, it frequently appears that most questions are likely to be designed in a personal context (Baran-Saraç, 2021; Kohar et al., 2019; Özgen, 2019; Şahin & Başgül, 2018). It can be assumed that this situation derives from the fact that teachers commonly associate mathematics with personal contexts such as shopping, travel, nutrition, and housing. The result obtained from the study conducted by Suharta and Suarjana (2018) with 12 pre-service teachers indicates that pre-service teachers can associate mathematics with the problems presented in a personal context more easily, supporting this result of the present study. In a study by Kohar et al. (2019), while pre-service teachers mostly designed questions in a personal context, they designed only a few questions in a scientific context. The researchers stated that the reason for very few questions designed in a scientific context was the nature of scientific knowledge or the difficulties of searching for accurate data in sources and finding reliable sources. However, in this study, although the teachers encountered questions related to all types of contexts in graduate courses, they could freely determine the contexts of the questions they would design in the practice part of the present study. However, research has shown that using various contexts in questions on ML increases the likelihood of students associating the problems with the situations they are dealing with in the 21st century (OECD, 2013). In light of this, teachers should be inspired and encouraged to create questions with varying contexts by analyzing the types of questions on ML, which contain a variety of contexts.

Examining the questions on ML that the mathematics teachers created in terms of the mathematical processes reveals that, with the exception of T1, the majority of the questions included all three categories of mathematical processes. However, the questions on ML designed by T1 were limited to the processes of formulating and employing and did not include the processes of interpretation and evaluation. The first question designed by T2 did not include the formulation process. In addition, two of the three questions, T3 and T4, failed to include the interpretation and evaluation process. Although the teachers were free to determine the content and context types of the questions they would design, the participants were asked to include all three mathematical processes in the questions.

Consequently, teachers seemed to try to consider all mathematical processes in the questions they would design. In a related study by Baran-Saraç (2021), it was found that while the pre-service teachers initially designed questions at the level of formulating and employing, it turned out that they expanded the diversity of the mathematical processes of the problems on ML and that the problems they created covered more than one process after the teaching experience. Similarly, the effectiveness of postgraduate education can be inferred from the fact that teachers tended to include multiple processes in their questions. However, it is also noteworthy that three questions designed by T1, and one question each designed by T3 and T4, did not include the interpretation and evaluation process. Similarly, Kohar et al. (2019) reported that most PISA-like questions designed by pre-service teachers involve the hiring process. Baştürk-Şahin and Altun (2019) and Gürbüz (2014) concluded that pre-service teachers generally have difficulties producing questions that require interpretation-evaluation skills. It is believed that the reason for this is that teachers are most familiar with the multiple-choice questions in the textbooks and central exams in

Turkey, which include the hiring process. Öztürk (2020) investigated the PISA ML proficiency levels of the mathematics questions in the central exam of the High School Transition System (LGS) and concluded that the questions did not cover all the levels but were concentrated at the 2nd level in general. Similarly, Ekinci and Bal (2019) classified the mathematics questions in the LGS exam held in 2018 according to the Revised Bloom Taxonomy by including only the questions in the "application" and "analysis" steps. They stated that no questions were found in "recalling, understanding, evaluating, and synthesizing." Another study that examined the questions in the 8th-grade mathematics textbook based on PISA ML proficiency levels reported that the questions in the first four levels were generally included in the book. In contrast, the questions in the 5th and 6th levels were never encountered (Aydoğdu-İskenderoğlu & Baki, 2011). The results of the studies support those found in the present study. However, creating a ML question is not an easy activity. Studies conducted with teachers and teacher candidates on ML revealed that participants had difficulties creating ML questions (Demir & Altun, 2018; Özgen, 2019; Saenz, 2009). The fact that teachers create fewer questions about the interpretation and evaluation process can be seen as a reflection of the beliefs about mathematics education, as the participants perceive mathematics more dominantly in the operational dimension, as Özgen (2019) states. Based on these results, it is suggested that in future training on ML, more emphasis should be placed on the interpretation and evaluation process, both in problem-solving and problem design.

The second part of the study aimed to examine the appropriateness of the classifications in terms of mathematical content categories, real-life categories, and mathematical processes for the three questions. According to the results, while the content categories of 10 of the 18 questions designed by six teachers were determined correctly, four were deemed incomplete, and the other four were incorrect. The incompleteness of the content category of the questions seemed to have stemmed from the fact that they contained two different contents simultaneously. While PISA emphasizes that classification by content category is important for item development and selection, it is also essential to note that certain content issues may develop in more than one content category (OECD, 2013). The reason for this lies in the nature of the PISA task, which is based on the contextual environment in which more than one branch of mathematics is used (Kohar et al., 2019). The analysis of the classifications made by the mathematics teachers for the questions they designed in terms of the real-life category revealed that only three of the 18 questions were determined incorrectly. In this regard, the results indicated that teachers were more adept at identifying the real-world categories of the questions than the content categories. Finally, five of the six mathematics teachers participating in the present study could determine the mathematical process categories appropriately in all the questions they designed. At the same time, T1 made an incomplete specification for the mathematical process category of the first question. Moreover, her classification according to the mathematical process category of the other two questions was partially appropriate. Furthermore, Baştürk-Şahin and Altun (2019) examined the questions prepared by 66 teacher candidates about ML and concluded that 36% of the pre-service teachers could identify all mathematical processes and correctly produce appropriate questions. Compared with the results of this study, it is believed that the postgraduate course provided to the teachers proved effective in determining mathematical processing skills. Gürbüz (2014), on the other hand, stated that pre-service teachers were more successful in classifying PISA questions than writing. The researcher explained this situation as the participants' lack of experience in writing questions. In this study, however, teachers were largely successful in both ML question design and classification. In addition, it is thought that the effects of courses such as Mathematical Modeling and Activity-Based Learning in Mathematics Education that they take in graduate education were also important in the overall success of the ML questions designed by the teachers. Studies on ML question design generally seem to have positive effects on participants. Kozaklı-Ülger et al. (2022) stated that teachers could pose ML problems, but this potential can be developed by spending more time on such problems. For this reason, it is recommended to carry out longitudinal studies on ML to develop the potential of teachers.

Implications

The mathematics literacy course is believed to yield positive outcomes regarding the products the teachers created. In light of the results of this study, it can be suggested that the training to be provided to write questions on ML in the future is highly likely to benefit the teachers. For that reason, increasing the number of relevant training sessions on writing ML-related questions can be recommended. For this, in-service training can be organized for teachers, and workshops and projects can be carried out by field experts on ML. In ML training, it is suggested that teachers should be inspired and encouraged to create questions with varying contexts by analyzing the types of questions on ML, which contain a variety of contexts. Also, it is suggested that in future training on ML, more emphasis should be placed on the interpretation and evaluation process, both in problem-solving and problem design. In this study, after 12 weeks of ML training, the ML design process took 6 weeks. In future studies, it is recommended to devote more time to the ML question design process and to emphasize the interpretation and evaluation process. This study has limitations in some respects, such as the lack of interviews and reflective thinking reports. Considering these limitations, more comprehensive studies can be done in the future.

Author Bio:

Ebru SAKA is Asst. Prof. Dr. at Kafkas University, Department of Mathematics and Science Education. She has a PhD degree in mathematical modelling in mathematics education. She also studies problem solving, problem posing, mathematical literacy and teacher education.

References

Akbaşlı, S., Şahin, M., & Yaykiran, P. S. Z. (2017). Teacher qualification influence on teaching cross-curricular skills: A study based on PISA 2015 assessment. *Journal of Education and Practice, 8*(18), 21-29.

Altun, M. (2020). *Matematik okuryazarlığı el kitabı*. Aktüel Alfa Akademi Yayıncılık.

Aydoğdu İskenderoglu, T., & Baki, A. (2011). Quantitative analysis of pre-service elementary mathematics teachers' opinions about doing mathematical proof. *Educational Sciences: Theory and Practice*, *11*(4), 2285-2290.

Bansilal, S., James, A., & Webb, L. (2015). Teacher training for mathematical literacy: A case study taking the past into the future. *South African Journal of Education*, *35*(1), 1-10. https://hdl.handle.net/10520/EJC167028

Baran-Saraç, A. B. (2021). *A study on development of prospective middle school 'teachers' competence of posing mathematical literacy problems* [Unpublished Master's thesis]. Middle East Technical University.

Baştürk-Şahin, B. N. (2021). *Analysing mathematics teachers' professional development on mathematical literacy within the framework of documentational approach* [Unpublished Master's thesis]. Bursa Uludağ University.

Baştürk-Şahin, B. N., & Altun, M. (2019). Investigating mathematical literacy problems that pre-service mathematics teachers posed by means of mathematical processes. *Western Anatolia Journal of Educational Sciences, 10*(2), 146-161.

Bozkurt, I. (2019). *The investigation of the development of students' mathematical literacy who are educated by mathematic teachers trained about mathematical literacy* [Unpublished doctoral dissertation]. Bursa Uludag University.

Büyüköztürk, Ş., Akgün, Ö. E., Karadeniz, Ş., Demirel, F., & Kılıç, E. (2013). *Bilimsel araştırma yöntemleri.* Pegem Akademi.

Canbazoglu, H. B., & Tarım, K. (2021). Mathematical literacy problem posing skills and mathematical activity development processes of primary school teacher candidates. *Journal of National Education, 50*(231), 147-172. https://doi.org/10.37669/milliegitim.743434

Demir, F., & Altun, M. (2018). Development of mathematical literacy question writing process and skills *Education and Science, 43*(194), 19-41. http://dx.doi.org/10.15390/EB.2018.7111

Edge, D. L. (2009). Math literacy: *The relationship of algebra, gender, ethnicity, socioeconomic status and avid enrollment with high scholl math course completion and college readiness* [Unpublished doctoral dissertation]. University of North Texas.

Ekinci, O., & Bal, A. P. (2019). Evaluation of high school entrance exam (LGS) 2018 in terms of mathematics learning field and revised bloom taxonomy. *Journal of Social Sciences of Mus Alparslan University, 7*(3), 9-18. https://doi.org/10.18506/anemon.462717

Genç, M., & Erbaş, A. K. (2019). Secondary mathematics 'teachers' conceptions of mathematical literacy. *International Journal of Education in Mathematics, Science and Technology (IJEMST), 7*(3), 222-237. https://ijemst.org/index.php/ijemst/article/view/611

Gürbüz, M. Ç. (2014). *The effect of PISA mathematical literacy teaching on writing PISA questions and mathematical literacy* [Unpublished Master's thesis]. Bursa Uludağ University.

Handal, B., & Herrington, A. (2003). Mathematics 'teachers' beliefs and curriculum reform. *Mathematics Education Research Journal*, *15*(1), 59-69.

Kohar, A. W., Wardani, A. K., & Fachrudin, A. D. (2019). Profiling context-based mathematics tasks developed by novice PISA-like task designers. *Journal of*

*Physics Conference Series, 1200(*1), 012014. https://doi.org/10.1088/1742-6596/1200/1/012014

Kozaklı-Ülger, T., Bozkurt, I., & Altun, M. (2022). Analyzing in-service 'teachers' process of mathematical literacy problem posing. *International Electronic Journal of Mathematics Education, 17*(3), em0687. https://doi.org/10.29333/iejme/11985

Mammadov, R., & Çimen, I. (2019). Optimizing teacher quality based on student performance: A data envelopment analysis on PISA and TALIS. *International Journal of Instruction*, *12*(4), 767-788.

Marshall, C., & Rossman, G. B. (2014). *Designing qualitative research*. Sage.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook.* Sage.

Milton, M., Rohl, M., & House, H. (2007). Secondary beginning teacher's preparedness to teach literacy and numeracy: A survey. *Australian Journal of Teacher Education (Online)*, *32*(2), 37-56.

Ministry of Education [MONE]. (2019). *Ministry of Education [MONE]. (2019). PISA 2018 Turkey preliminary report.* Education Analysis and Evaluation Reports Series.

Mosher, C. A. (2015). *Curriculum redesign with an emphasis on mathematical literacy in an 8th grade function unit* [Unpublished Master's thesis]. The College at Brockport: State University of New York.

National Council of Teachers of Mathematics (NCTM). (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

Organisation for Economic Co-operation and Development. OECD. (2013). *PISA 2012 assessment and analytical framework. Mathematics, reading, science, problem solving and financial literacy.* OECD Publishing.

Organisation for Economic Co-operation and Development. OECD. (2016). *PISA 2015 assessment and analytical framework. Science, reading, mathematics and financial literacy*. OECD Publishing.

Organisation for Economic Co-operation and Development. OECD. (2019). *PISA 2018 assessment and analytical framework*. OECD Publishing.

Özgen, K. (2019). Problem-posing skills for mathematical literacy: The sample of teachers and pre-service teachers. *Eurasian Journal of Educational Research, 19*(84), 179-212. https://doi.org/10.14689/ejer.2019.84.9

Öztürk, N. (2020). *Classification of the high school central exam math questions according to the PISA mathematical literacy levels* [Unpublished Master's thesis]. Sakarya University.

Saenz, C. (2009). The role of contextual, conceptual and procedural knowledge in activating mathematical competencies (PISA). *Educational Studies in Mathematics, 71*(2), 123-143.

Suharta, I., & Suarjana, I. (2018). A case study on mathematical literacy of prospective elementary school teachers. *International Journal of Instruction*, *11*(2), 413-424. https://files.eric.ed.gov/fulltext/EJ1174910.pdf

Şahin, O., & Başgül, M. (2018). Examination of pre-service mathematics 'teachers' ability to pose PISA problems. *International Journal of Field Education, 4*(2), 128-148. https://doi.org/10.32570/ijofe.483163

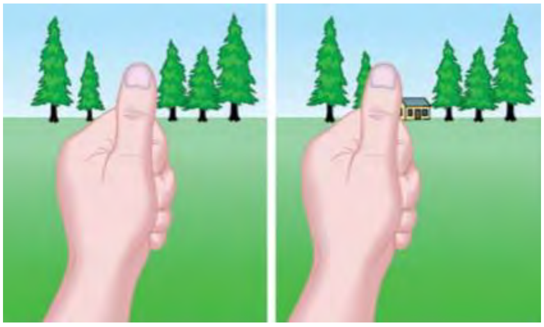
Temel, H., & Altun, M. (2020). The effect of problem-solving strategies education on developing mathematical literacy. *Necatibey Eğitim Fakültesi Elektronik Fen ve Matematik Eğitimi Dergisi*, *16*(2), 275-303. https://doi.org/10.17522/balikesirnef.1164584

Appendix

Appendix 1. The Question on ML Designed by T5 in a Scientific Context

PARALLAX (JUMPY FINGERS) METHOD

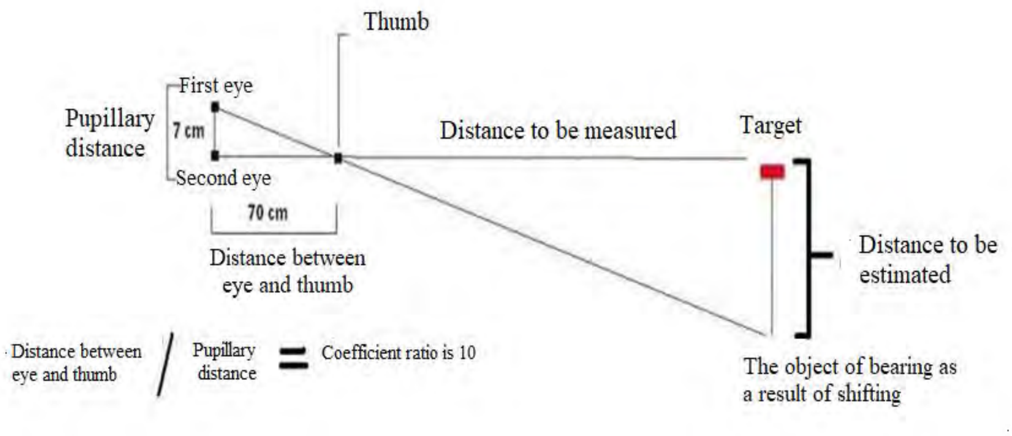
Parallax is the displacement in the position of an object from the observer's point of view. Here, we can make distance estimations using this feature. For example, we need to estimate the distance of a distant house on a plain. We turn our front towards the house and extend one arm parallel to the ground and perpendicular to our torso. Then we close one eye and target the house with our open eyes and thumbs. Then, without moving our hands or heads, we close the open eye, open the closed eye, and look with that eye.



In this case, the image will shift, as seen in the picture above. We estimate how far our thumb has shifted (this distance is easier to estimate) and multiply that by 10. The resulting number will indicate the distance of the house from us. In the example above, if we assume that our finger has shifted 20 meters to the left of the house, then, in that case, the distance of the house is 20X10 = 200 meters.

Question 1: According to the method given above, the difference in the distance between the thumbs in the two images should be shown with the letter "f." We then write the formula that gives the distance of any object we measure from us. (Formulation process)

Question 2:



The parallax method is based on the fact that the base/side ratio of both triangles given above is the same and that the ratio between the arm and the pupil of a normal person is 10 times (The distance between the eyes is 7 cm, and the distance between the eyes and the thumb is 70 cm.) People who want healthier results can find their ratio by measuring the distance between their pupils and their arm's length. Based on this, if people considering this information measure the pupillary distance as 6 cm, the length of his arm as 70 cm, and the estimated result by moving away the thumb as 85 cm, then what will be the distance when the parallax method is applied? Please show your calculations. (Employing process)

Question 3: If you were making the above measurement for your pupillary distance and arm length, and the distance to be estimated was 40m, how would you find the distance? Explain by drawing triangles (Interpretation-evaluation process)

The pictures used in this question were obtained from: https://dogayakacis.com/2014/05/11/mesafe-tahmin-yontemleri/