Social Time as a Pedagogical Toll for Meaningful Mathematics Teaching and Deeper Learning

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Abstract: The study reported in this paper was conducted to examine four South African grades 7 teachers' understanding of the importance of instructional time as a teaching resource to develop learners' relational understanding in mathematics. A constructivist philosophical approach, document analysis, lesson observations and interviews were used to collect data. The findings indicate that the four teachers faced, to varying degrees, challenges in using instructional time as expected. Two did not help learners understand the relationship between the concepts taught and the examples selected to scaffold learning. They prioritised drawing learners' attention to correct responses to questions posed. Only two teachers used strategies that encouraged the learners to share experience, collectively reflect on individual taken-for granted conceptions, probe and identify how they could be used to explain mathematical concepts. The conclusion is that the teachers' lack of understanding the pedagogical significance of instructional time as social time highlighted inadequate curriculum and subject content expertise and supported the general concern about the quality of mathematics teaching within the country.

Keywords: *mathematics, instructional time; teaching; relational understanding*

INTRODUCTION

In South Africa, all schools, private and public, use the National Curriculum Statement (NCS) Grades R-12, as official curriculum policy (Department of Basic Education [DBE], 2011). The NCS comprises three policy documents that guide teaching and learning, namely, the Curriculum Assessment Policy Statement (CAPS) for each approved school subject; the National policy for the promotion requirements of the NCS for each programme and the national protocol for assessment Grades R-12. These policy documents are central in indicating, respectively, what should be taught and learnt in a particular grade, how subject content is organised for the different phases and associated grades and how learners progress to the next grade within the schooling system and finally, how the taught and learnt content should be assessed. For example, mathematics has to develop a critical awareness of how mathematical relationships are used in social, environmental, cultural, and economic relations (DBE, 2011). Problem-solving and cognitive development should be fundamental to all mathematics teaching and based on content that is grade appropriate. Teachers are expected to create environments in which learners feel free and value each other's contribution. Classrooms as judgment free zones have to facilitate learning in ways that are meaningful to cultures or concepts used routinely in the learners' lives. Learners should be treated equally irrespective of their socioeconomic status, physical conditions, and intellectual ability to ensure the attainment of the set outcomes.

It is thus reasonable to describe the teaching approach that is promoted by the NCS in mathematics and the CAPS as supporting Realistic Mathematics Education (RME) – a mathematics teaching approach that is supposed to make mathematics learning fun and a meaningful experience by introducing situational problems from contexts within the learners' experiential realm (Loc & Hao, 2016; Yet et al., 2017; Yuanita et al., 2018).

RME as a teaching approach

This approach was developed in 1971 by the Freudenthal Institute in the Netherlands (Loc & Hao, 2016; Theodora & Hidayat, 2018; Yet et al., 2017; Yuanita et al., 2018). It emphasises the need to provide learners with opportunities to construct mathematical knowledge through managing and processing real-life situational problems. For example, Yuanita et al. (2018) describe five main criteria that guide the utilisation of the RME approach, namely (i) acknowledging learners' experience in daily living, (ii) modelling reality and converting it into formal systems through the mathematical vertical process, (iii) using learners' active (learning) style, (iv) using discussion, question-and-answer teaching and learning styles and strategies to cultivate mathematical skills and knowledge, and (v) establishing relations between concepts and topics for the holistic learning of mathematics concepts. Following the criteria, RME starts with selecting problems that are relevant to learners' experience and knowledge and the teacher acting as a facilitator who guides learners as they solve contextual problems. The problems help to contextualise mathematical tasks. First, they must appeal to learners and second, they must relate the tasks or problems to their daily living and situations in ways that evoke experiences, thoughts, cultural and historical sentiments that learners can draw on to share subjectivities, perspectives, and experiences.

The teacher models a mathematical problem (task) to provide learners with opportunities or a context in which they can develop, based on experience, meaningful ways of solving issues they face in their day-to-day living and, as they do so, further develop logical thinking through vertical and horizontal mathematization processes. As a facilitator of learning, the teacher may, for example, employ question and answer strategies to provoke learners' interactions, engagement, critical thinking, creativity, and innovation in resolving the given situational problem. The interactions and engagement with each other create opportunities crucial for consensus on the processes and procedures to share experiences, perceptions and beliefs that should be used to resolve problems. The intersubjectivities that result from these activities are thus products of a willingness to engage multiple frames of reference. Through drawing on what is familiar, the learners can collectively reconstruct their mathematical knowledge as they shape, reshape, and remap their previously taken for granted knowledge to develop solutions to the given problems. As they engage and interact, they (learners) establish relationships between concepts and topics taught and develop a holistic (relational) understanding of mathematical concepts rather than seeing them as disintegrated concepts applicable to isolated topics. It is in this sense that RME may be viewed as promoting intersubjectivity as a context for meaningful learning.

Intersubjective relations as context for learning

Heidegger (1962 cited in Stroh, 2015) sees learning as a social process that depends on intersubjective relations embedded in social time. He argues that people gain their understanding of being from the communities to which they belong. Since meaning is embedded in the community into which an individual is socialised, as members of the community, individuals understand the social practices and roles within that community. However, such practices and social roles are not understood in isolation but learnt in relation to the social context in which meaning is generated; that is, in the intersubjective totality of the community. As individuals acquire knowledge by sharing their personal perspectives with others, their perspectives and those of others become the basis for taken-to-be-shared learning and understandings of social practices and roles.

As social beings, people can move within and across different social spaces, thus enabling border crossing. For example, in the classroom, interactions and engagements can enable learners to share historical contexts and embodied ways of Being and by so doing, learn from each other. For this reason, culturally responsive teachers adapt interactively (Deady, 2017), their teaching styles and strategies to suit learners' backgrounds and different levels of readiness to learn. As regards mathematics, Mogari (2014, p.3) argues that such teachers can reaffirm and restore the cultural dignity of learners. They (teachers) employ, for example, the ethno-mathematical approach in their classes, which is "an activity-oriented pedagogy that focuses on the mastery of mathematics content and induces affinity through relevant real-life activities that are familiar to learners" . It is such activities that Compton-Lilly (2015) would describe as creating social time; that is, experiential and therefore, a subjective and anthropomorphic sense of time. This is a conception of time that allows learners to make sense of themselves through anthropomorphism of the activities, experiences and relationships (Compton-Lilly, 2015, p. 4) that help them understand subject content.

Compton-Lilly (2015) asserts that when instructional time is used to create environments in which learners are free, value each other's contribution and ask questions as suggested by the CAPS, the time can be described as social because it influences how interactions and relationships are guided. Instructional time as social time provides a subjective, anthropomorphic, and experiential sense of time. The manner in which learners experience curriculum activities shape how they make sense of themselves, relationships amongst themselves and with teachers and, above all, what they are taught. It is in this sense that instructional time serves as a significant resource when devising strategies and designing activities that are aimed at facilitating learning. In Compton-Lilly's (2015,

p. 4) view, “all that people have lived and understood as well as ways they make sense of themselves, their experiences, and their relationships is dependent on social time’.

In South Africa, the Low Achievement Trap report published by the Human Sciences Research Council (HSRC) in 2012 indicates a shortage of appropriately qualified teachers, especially in mathematics, science, and foundation phase.

The Trends in International Mathematics and Science Study (TIMSS) and the Southern and Eastern Africa Consortium for Monitoring Education Quality (SACMEQ) (Spaull, 2013) also indicate knowledge gaps between South African learners and those from other similar middle-income or less developed countries and also amongst learners within South Africa based on their location. While there are studies that focus on the use of instructional tools to facilitate learning, for example, digital gaming (Keeble, 2008) or digital Game-Based Learning approach (Makri, Vlachopoulos & Martina, 2021), we could not trace any that looked specifically at instructional time as a teaching rather than regulatory tool.

The study we report on in this paper sought to examine teachers’ understanding of the importance of instructional time and how they translated it in practice to enhance meaningful or culturally relevant mathematics learning. We wished to answer the following research questions:

a) How do the grade 7 mathematics teachers in Vhembe district interpret the instructional time stipulated for specific topics in the CAPS? and
b) How do the teachers translate their conception of instructional time in practice?

METHODOLOGY

A constructivist philosophical approach (Creswell & Poth, 2018; Grover, 2015; Merriam & Grenier, 2019) or interpretivist paradigm (Kivunja & Kuyini, 2017; Lincoln & Guba, 1985; Ndlovu, 2021) of the qualitative approach was used for the study. In terms of the paradigm, people create their own meaning of social and psychological phenomena through interacting with the world and making sense of their experiences based on historical, cultural and societal perspectives (Grover, 2015).

Schools that had information-rich teachers who had been teaching mathematics as proposed by the CAPS were purposively sampled (Babbie & Mouton, 2001; Saunders, Lewis & Thornhill, 2009; Wagner, Kawulich & Garner, 2012). They (schools) had performed above 70% in grade 7 for four consecutive years despite their overcrowded grade 7classes of between 62-84 learners. Eight out of 19 schools in one circuit in the Vhembe district satisfied the criteria. The district is one of the five mega departments of education in the Limpopo Province of South Africa. Access to the schools was convenient. A manager issued a circular to all 19 schools informing them of the study the researchers wished to conduct. Once the departmental permission to conduct research was obtained, eight agreed to be involved. However only four teachers consented to participate in the study. The teachers’ experience ranged between three and 26 years.

The CAPS for mathematics was analysed to identify the instructional time and teaching strategies it suggested for the different mathematics topics that had to be taught at grade 7. Lesson observations were conducted to capture the teachers' pedagogical practices. Interviews were used to establish their understanding of (i) the prescribed instructional time and (ii) implications of the suggested teaching strategies; that is, what activities and processes were to be prioritised and how they were to be designed to help learners understand the relations between their life experiences and the subject content within the prescribed time.

Transcription of the recorded data formed part of the data analysis. As the process unfolded it was possible to establish teachers' understanding of the significance of the officially allocated instructional time, how they made sense of it and translated it during lessons. Words and phrases used to refer to these aspects and how they were related guided data coding, categories and themes (Emerson, Fretz & Shaw, 2011). Views that were related to the time and what it implied as regards the proposed teaching approach indicated teachers' understanding of mathematics as a particular type of knowledge and provided codes that were linked to categorize their curriculum expertise as mathematics teachers. The categories were afterwards used to create themes that we used to organize data presentation. To validate the research, amongst others, the four trustworthiness components; namely, credibility, dependability, transferability and confirmability (Kivunja & Kuyini, 2017; Lincoln & Guba, 1985) were used as criteria.

RESULTS

Teachers' conception of the allocated instructional time

To make sense of the CAPS for Mathematics allocation of the 45% teaching time (DBE, 2011, p. 157), the teachers seemed to rely on the recommended teaching approaches, for example, drawing on the experiences of the learners and using application, clarity/understanding and explanation/analysis questions to facilitate conceptual understanding. Opinion/synthesis questions hardly featured. Application type questions established whether the learners were able to use procedural knowledge or routine procedures accurately. Clarity seeking questions focused on learners' justifications that linked aspects in their contexts to the development of abstract-general mathematical concepts. Explanation questions focused on subject content and relevant concepts to clarify the relationship or lack thereof of the content to the abstract-general conceptions. Opinion questions required the learners to use their prior knowledge to explain new concepts and how they could be applied in a specific context. The following are examples of lessons that illustrate how teachers understood the time allocated for teaching different concepts/topics as mainly prioritising awareness of correct responses rather than the learning process.

Teacher 2's lessons

The lesson was on graphs and allocated 60 minutes. The teacher had to teach to help learners develop "the ability to analyse and interpret global graphs (with special focus on trends and features of constant, increase or decrease), and draw global graphs from given description of a problem situation" (DBE, 2011, p. 65). However, Teacher 2 facilitated/promoted learner interactions that mainly reinforced correct responses.

The lesson started just after the lunch break. The bell rang at 12h17 and the class was mostly occupied by learners at 12h20. The teacher entered the class at 12h23 together with some learners who were late. Pseudonyms are used for the learners.

EXCERPT 1: *The teacher greeted learners, …, and started the lesson by indicating that it was on graphs. Thereafter, the learners were asked to turn to page 223 of the textbook and the teacher introduced the lesson by reading the following excerpt.*

Teacher 2: *Two motorists (A and B) are travelling at a speed of 120km/h and 100km/h respectively for 5 hours, what does the word respectively mean? … this is shown on the graph. The graph shows the speed done by each motorist and relationship between the time taken and the distance travelled by the motorists (points at the graph below).*

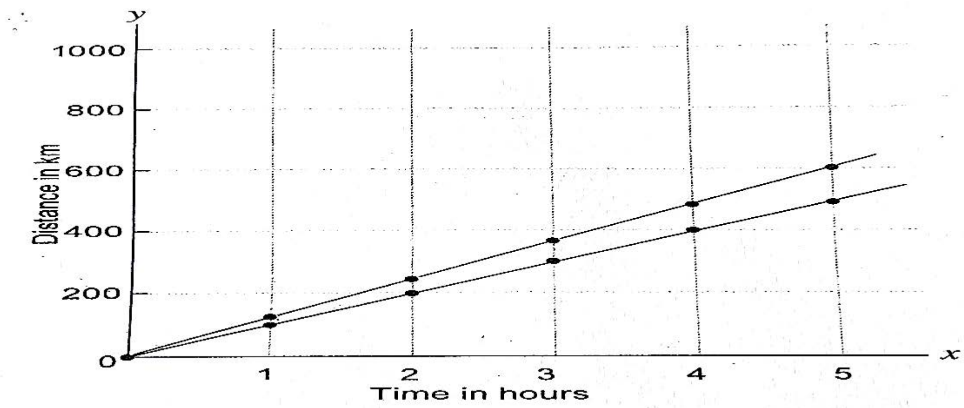

Figure 1: The graph shows time taken and distance travelled by two motorists respectively

So, you can see from the graph here, how far the motorists travelled in three hours? … how far did each motorist travel in three hours? …. remember we are comparing two motorists.

Learners: *Yes.*

Teacher 2: *The one who is travelling at 120 km/h and the other one travelling 100 km/h. How far did each travel in 3 hours …. Thapelo?*

Learner 7: *300 km and 390 km.*

Learner 8: *300 km and 600 km.*

Teacher 2: *Yes. (Pointing at another learner)*

Learner 9: *360 km and 300 km.*

Teacher 2: *360 km and 300 km. (lesson continues)*

The excerpt had to assist learners determine the correct responses by mapping and relating the provided information based on what they had been previously taught. Therefore, it was not enough to simply repeat questions until a correct response was given. The learners needed to be given opportunities to demonstrate that they understood the relevant processes and principles (mathematical representation) that applied to the subject matter knowledge (SMK) taught. To meet the requirements of the objectives of the lesson, it was important for the teacher to not only use a problem to teach the interpretation and calculation of values but also s/he needed to use the values to explain why "constant, increase or decrease" are referred to as graph features and trends. Doing so required that learners be taught how to identify the trends in the graph and, then analyse a pattern to determine whether it is constant, increasing, or decreasing (monotonically or otherwise). Pattern analysis would afford learners opportunities to use their prior knowledge of numbers, operations, and relationships (specifically counting forward, backwards, and in groups). The analysis would also enable learners to link the abstract-apart mathematical object (graph) to the abstract-general conceptions (describing rules generating the patterns that determine features of the graph). Pattern analysis would create a realistic modelling process; making it possible to imagine the lesson content in relation to their personal experiences and, thus trigger an interactive sharing of experiences, thoughts, views and understandings (Loc & Hao, 2016; Scheiner, 2016; Yet et al., 2017; Yuanita et al., 2018) to draw on and determine what is important to consider when plotting global graphs based on a given description. Similar patterns of questioning were noted in Teacher 4's lesson on common fractions.

Teacher 4's lesson

The use of teacher-posed questions by Teacher 4 was to elicit prior knowledge from short term memory of the content taught. Only one learner asked a clarity seeking question. The question sought to establish understanding whether sharing unequally would still constitute a fraction, but the teacher missed the gist of the question and left it unanswered. Again, in this instance, the teacher had missed the socio-instructional time opportunity to use engagement and interactions as scaffolding of meaningful mathematical conceptual development. The learners needed a platform for discussing the concept of a fraction, thus creating possibilities for realisation, but the teacher missed the opportunity of using the question to engage the learners and explain why unequal sharing cannot lead to a proper fraction. This could have been done by indicating that the proper procedure to share unequal wholes was to first divide equally each whole and share the parts thereof, rather than the undivided wholes and then draw on the real-life examples to explain the significance of the procedure. The missed opportunity highlighted, for example, Dhlamini et al. (2019) and Mabotja et al.'s (2018) assertions that the inadequate capacity of teachers to teach concepts in a way that facilitates meaningful understanding, compromises the recognition and actualisation rules that are important in the formation of mathematical concepts through abstraction (empirical and structural) from physical and social experiences. Perhaps it is such oversights that have persuaded Spaull (2013, 2015) to link South African learners' poor performance in mathematics to the teachers' low mathematical knowledge for teaching (MKT), which underscores lack the professional expertise to create situations and contexts in which learners can intersubjectively engage and interact, making learning anthropomorphic without compromising the essence of the subject content. In Spaull's (2013) view, teachers with limited content knowledge will not be able to simplify specific aspects of the subject content and make it relevant or interesting when relating it to real life. In contrast, the next lessons by teachers 3 and 1 might be viewed as different from the two lessons already presented, as the teachers emphasised learning-centred lessons, demonstrated a translation of instructional time as dependent on mathematics as a particular type of knowledge and created environments in which they were able to engage learners in routine and non-routine problem solving involving real-life situations. The lessons are presented below.

Translation of instructional time and mathematics as a particular type of knowledge

Teacher 3's lesson

In the lesson, the teacher deliberately took learners through the process of counting the faces (physically) of 3D objects while at the same time being assisted to conceptualise (abstractly) through imagination how the objects could be identified in the examination paper. When looking at the face of the object, it was not possible to see the opposite side of that face, so in real life, they could turn it around to see the face but in the case of mathematics, the learners had to be helped to learn how to abstractly (imagine) construct the 3D figure with its complete faces. The teacher provided them with opportunities as a platform to interactively and intersubjectively amplify mathematics content through relevant real-life activities that were familiar to them. The excerpt below is an example of what Teacher 3 did in her lesson:

> ***EXCERPT 2:*** *The teacher started by attending a staff briefing meeting which took long and the lesson that was to start at 7h30 did not start. Class started at 7h52 after the meeting. The teacher came to class with various boxes (shoe, toothpaste, washing powder soap, chalk box), prisms and pyramids as teaching aids. The class size was 62 learners, 29 boys and 33 girls. The teacher greeted learners and introduced the topic of the lesson as Geometry of 3D objects. The lesson proceeded and the following was part of it.*
>
> ***Teacher 3:*** *In the 3D objects we can find our faces as a flat surface, or we can also find our faces as curved. I will show you the examples of the faces (uses the boxes to explain the terms to learners). Here I am having the box, you can say it is a box of tools, or you can also say it is a box of sweets, or you can also say it is a box of pieces of chalk, with this box we need to identify the faces that we are talking about, do you think this box has a flat or a curved surface?*
>
> ***Learners:*** *Flat surface.*
>
> ***Teacher 3:*** *Yes, if it is a flat surface, it means we need to know how we can identify those faces. I remember I told you last time that when we talk about the face, it is like your front side as a person. So, when you look at this box, please bear with me, in your question paper, the box that you are seeing now, in a question paper you won't see it the way it is now, but it is the same box. So, when we identify the faces, we need to think, we*

use what we call our imagination. You think beyond this other part that you are not seeing. You can see that when I'm holding the box like this, you are able to see the front one only. But as a mathematician, you need to think, as you know that the box has got how many sides … How many sides do we have here?

Learners: *Six sides.*

Teacher 3: *So, this is how we are going to count the faces. …….. we also have another shape, yes, another shape, do you think this shape is different from this one? (Showing two different boxes)*

Learners: *No. (lesson continues)*

Teacher 3's pedagogic practices seemed to contradict the challenges that are highlighted in various studies (see Adler & Sfard, 2017; Brodie, 2010; Chirinda & Barmby, 2018; Dhlamini et al., 2019; Mabotja et al., 2018; Mogari, 2014; Savides, 2017; Venkat & Spaull, 2015), that some mathematics teachers fall short of teaching approaches that can facilitate the achievement of outcomes that are outlined in the CAPS. The teacher's ability to employ teaching strategies that were proper to address the recognition and realisation rules relevant to the content that they taught, revealed the expertise they had as a teacher of mathematics. Similar ability of using real-life experience of learners to facilitate understanding of concepts, dependent and independent variables, were observed in Teacher 1's lesson. The teacher used the distances that learners travel to school from their homes versus the time they spend on travelling, showing why the distance always stays the same but the time varied depending on the pace at which learners travelled.

Teacher 1's lesson

The strategies employed by Teacher 1 dealt with both the recognition and actualisation rules (Bernstein, 2003b). The recognition was addressed by encouraging the learners to describe concepts based on their experiences of travelling to school while the realisation of the concepts taught, dependent and independent, was taught by asking the learners to reflect and relate the time they spent going to school on different days – where at times they had to run to be at school on time – resulting in different times spent on the way while the distance stayed the same. According to the teacher, time had to be understood as the independent variable as its flow would be affected constantly by an event, while the distance covered would be a constant dependent on the manipulation of time. However, with the given mathematics problem, the distance was a constant while average speed was the dependent variable that changed as the independent variable was manipulated as different durations taken by learner to walk or run to school on different school days. As pointed out by Klette (2016), good questioning techniques are an attribute of high-quality teaching and optimal use of socio-instructional time. The pedagogic communications that the teacher used made it possible for learners to engage and collectively think of the concepts being taught. When the learners showed that they were struggling to grasp the meaning of dependent and independent variables, the teacher ought to have explained time in hours and minutes to illustrate that it can be manipulated (varied) and should thus be placed on the x-axis of the graph to show that it is an independent variable.

DISCUSSION

Mathematics is learnt through doing (Zhu & Simon, 1987), so the communication and activities that are meant to create learning contexts are not supposed to distort the essence (Cotton, 1988) of what is taught. Even though teachers have to facilitate meaningful learning (DBE, 2011) by encouraging the learners to share their knowledge and interact through discussing subject content in relation to their everyday knowledge, such sharing and interactions have to happen while observing the suitable rules for recontextualising the subject content within the limitations of the instructional time stipulated in the CAPS for doing so. Teachers in this study were aware and considered the instructional time as prescribed in the CAPS when presenting their lessons. However, the classroom observation revealed that they required curriculum expertise to develop pedagogic discourses that promoted logically an understanding and appreciation of relationships and patterns in the information provided to help learners develop relational understanding mathematically. Two teachers' curriculum practices (2 and 4) confirmed Dick and Dalmau's (2014) argument that there is always a gap between the espoused theories and theories-in-use. They were only able to promote recognition with their teaching and seemed unaware that it was important to aid the learners to extract concepts from the excerpts that had to be used to scaffold the understanding of how to analyse and interpret from a "… given description of a problem situation" (DBE, 2011, p. 65).

Teachers 2 and 4's strategies made the learners respond to questions during the lessons without any opportunities for sharing and discussing the experiences, ideas, and views on which responses were based. When teaching mathematics, it is always useful to provide opportunities for building meaningfully new knowledge on previous knowledge to unravel the complexity in concepts. Strong framing as suggested by (Bernstein's, 2003a) theory on the pedagogic devise, with a highly controlled classroom environment, tasks, and relationships ought to have encouraged intersubjective engagements that drew from the learners' material and social experiences and developed an understanding of mathematical concepts meaningfully through abstraction. The teachers' ability to convert the instructional time into social time through formal language and symbolic representation (Benis-Sinaceur, 2014) was important to create a classroom environment in which the learners' physical and social experiences could be drawn on within the limited time to identify and clarify both materially and abstractly mathematical concepts and thus enhance chance of relational understanding in the subject.

Abstraction takes place as learners construct mathematical knowledge through the insertion of a new discourse alongside existing concepts or images. However, the ways in which teachers 2 and 4 presented the prescribed content using real-life examples did not provide opportunities for learners to think beyond their realities and learn a new discourse of explaining the examples. The absence of more authentic mathematical problem-solving strategies (Adler & Ronda, 2014) in teachers 2 and 4's lessons was thus seen as being partly responsible for underplaying the importance of mathematical abstraction in their teaching. Makonye (2017) views such inability to adhere to the constructs in the CAPS, which include using proper approaches to teach concepts and skills and implement the curriculum as expected, as a reflection of teachers' inadequate capacity, leading to poor performance by learners. Mabotja et al. (2018) also explicitly link learners' lack of mathematical skills to the inadequate capacity of teachers to teach concepts in a way that facilitates meaningful understanding. They argue that teachers need to understand that knowledge is constructed socially, and therefore collectively and individually (see Jackson, 2014) as a cognitive enterprise; hence, the argument for the importance of using social time to guide engagement and interactions through anthropomorphic activities that promote the development and acquisition of new knowledge intersubjectively. The strategies or approaches, namely, problem-solving, investigative, project-based learning, cognitively guided instruction, and cooperative learning, are also promoted in the CAPS, Senior Phase Mathematics Participant's Orientation Manual 2013 (DBE, 2013, p. 23). For example, using a cooperative approach is described as allowing the grouping of learners for them to learn to listen to one another, share ideas and perspectives, give and receive assistance, seek ways to resolve difficulties, and actively work to construct new understanding and learning (Gillies, 2007).

Bansilal (2013) and Umugiraneza et al. (2018) argue that teachers in their studies struggled with interpreting and implementing the CAPS. They showed that teachers were comfortable with using teacher-led instruction as a method when teaching mathematics as opposed to progressive approaches advocated in the CAPS. Such a method fell short of innovative strategies that promote thinking, reasoning, and the construction of self-knowledge by learners. Many studies (see Mentz & Goosen, 2007; Mogari, 2014; Muthukrishna, 2013; Rhodes & Roux, 2004; Savides, 2017; Spaull, 2013; van der Walt & Maree, 2007; van Wyk, 2002) referred to when contextualising this study, view teacher knowledge as the main challenge to mathematics learning and teaching and the social contexts within which teaching takes place as also having a direct bearing on pedagogic discourses (Adler & Davis, 2006; Lazarides & Rubach, 2017; Mentz & Goosen, 2007; Prinsloo, 2007; Uys et al., 2007). Additionally, in this study, social contexts such as class size (Rice, 1999) and shortages of resources (Kaya et al., 2015) were also important aspects in understanding the challenges teachers faced in developing suitable curriculum practices. As Klette (2016) argues, even though social contexts such as socioeconomic background, class size, and teachers' formal education and experience contribute to the style or type of learning, teaching practices contribute significantly to such learning. In short, teachers 2 and 4 needed to improve their pedagogical content knowledge.

Modiba (2011) as well argues that even the best teachers need adequate subject matter knowledge, that is, professional knowledge and skills to develop pedagogic discourses that promote an understanding and appreciation of relationships and logical patterns in mathematical concepts. In her view, teachers with adequate subject content knowledge would be able to create instructional practices that meet situational demands. They would be responsive to learners' cultural and historical background knowledge by employing pedagogic communication or framing that addresses the potential discursive gap between the mundane and the esoteric knowledge (Bernstein, 2003a) while emphasising the importance of rules, principles, or procedures that have to be taught or are required for conceptual development. Therefore, teachers 2 and 4 could be described as having created learning environments that eased the recognition of subject content but fell short of facilitating its realisation. It was critical for learners to be helped to understand the procedures and principles (realisation rule) that were needed for abstract mathematical understanding. Inferences to real-life situations helped with similarity recognition (recognition rule) but not the development of relational understanding.

Mathematics as a knowledge area operates at an abstract level and is concerned with generalisation (Benis-Sinaceur, 2014; Mitchelmore & White, 2004; Scheiner, 2016) and this requires teaching to go beyond real-life examples and enable learners to understand the subject content conceptually. To varying degrees, Teachers 1 and 3 used learners' interactions to ease understanding of the legitimate processes of developing such understanding. The interactions focused on them and promoted thinking that probed the mundane and scientific content used in the lessons. The questioning used made intersubjective exchanges amongst the learners and teachers and the learners themselves possible. Responses were thought about and in diverse ways, the learners were helped to think about what was conceptually important in the mundane content. Teachers 1 and 3 deliberately focused on culturally relevant or responsive teaching. As Savides (2017) would argue, they seemed the better-educated teachers in the study.

As Stroh (2015) asserted, people as social beings cannot escape the process of interactions. It is in interacting that they develop the empathy that enables them to understand one another's perspectives and share experiences, which in the case of teachers 1 and 3, resulted in intersubjectivities that learners drew on to understand how to relate and solve mathematical problem situations by using their mundane experiences as a foundation or base. This enabled the learners to develop learning styles that eased sharing of experiences intersubjectively and through collective reflections on these experiences, mathematical concepts were interrogated and clarified amongst themselves. As Winfield (2014) pointed out, understanding mathematical concepts requires the availability of other learners in order to provide space or opportunities for reflecting on socially constructed knowledge and to reshape, remap, and reconstruct it as evidence of relational understanding. In contrast to teachers 2 and 4 who seemed to prioritise solipsistic individual perspectives (Stroh, 2015), teachers 1 and 3 used social time.

CONCLUDING REMARKS

Two (teachers 2 and 4) out of the four Grade 7 mathematics teachers in the study were not able to translate the CAPS' prescribed instructional time into effective teaching and learning styles. They seemed unable to understand what was needed to help the learners develop an abstract understanding of mathematical concepts and only managed to facilitate recognition of the responses to questions posed. In contrast, teachers 1 and 3, seemed to know what was necessary to translate the prescribed instructional time into social time. They used teaching styles or strategies that helped the learners develop learning styles that eased the sharing of experiences and how to collectively reflect on those experiences and identify what was conceptually significant about them that could, in turn, be used to explain mathematical concepts to each other. Therefore, the conclusion is that teachers 2 and 4 seemed unaware of the importance of the realization rules in teaching and learning mathematics. The other teachers, 3 and 1 clearly made efforts to lay the foundations that would be useful for conceptual understanding by using physical time in ways that encouraged learners to examine the subject content with the help of real-life examples and develop a relational understanding of both subject content and the examples. Admittedly, the small sample size of the study imposes limitations. A larger study in the future is recommended and likely to strengthen the conclusion.

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