Examining the Effect of Elementary School 5th Grade Subject of Extraction on Readiness for Integers

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Abstract

The concept of number sense first appeared theoretically in the report published by NCTM in 1989. Although it is an important concept that includes the meaning of numbers and the relationship between numbers, it is very difficult to make a clear definition of it. This could also be true for the concept of integer, which we cannot place in our world of meaning in daily life or which we have difficulty in finding an equivalent for. For integers that have an abstract world of meaning, a misconception may occur between the integer sign and the sign used for operation. In this respect, this study aimed to examine the effect of subtraction, which can create pre-learning, on the concept of integers. Moreover, it was requested to demonstrate that the operational knowledge used by students in subtraction can be utilized in teaching integers. A total of 17 students studying in the 5th grade of a secondary school in Turkey participated in the study. As the research design, the action research method, one of qualitative research methods, was used, and the data were collected with the trilogy technique. The data obtained were analyzed using descriptive analysis and content analysis. The findings revealed that the teaching technique of subtraction opened the door for students to form and make sense of the concept of minus. In this context, it is recommended to provide mathematical skills in a cumulative manner.

Keywords: education, mathematics education, integer, integer teaching, subtraction, integer's concepts

1. Introduction

The concept of number sense first appeared theoretically in the report published by NCTM in 1989 (NCTM, 1989). Although this concept is a very important concept that includes the meaning of numbers and the relationship between numbers, it is very difficult to make a clear definition of it. The concept of number sense, which we can observe on the basis of many mathematical perceptions, can be seen as the basis of the perception that mathematics is difficult (Dunphy, 2007). If we dwell on this situation, we can undoubtedly encounter deficiencies in understanding numbers. This is followed by the ability to compare numbers because although Yang (2003) states that comparison of numbers helps order numbers correctly, the relationship between numbers in terms of quantity could be said to be necessary as well. This leads us to the semantic intensities of the numbers, and though indirectly, it basically leads us to a concept based on "meaning". Therefore, we can get down to the basis of numbers and consider the messages they want to give us theoretically and practically.

It is also possible to consider integer in this way, a concept which we cannot place in our world of meaning in daily life or which we have difficulty in finding an equivalent for. For integers that have an abstract world of meaning, a misconception may occur between the integer sign and the sign used for operation (MNE, 2009). The difference between the two can lead to the shaping of the semantic world, and, in this respect, it results in narrowing or expanding the context of the subject. However, despite all, it is necessary to examine the relationship between the concepts of "negative" and "minus". For example, let's consider the subtraction of a negative integer from a positive integer, which is included in the teaching process in subtraction of integers. When the operation of 3-(-4) is examined, it is seen that there are two signs of -. In this case, it is possible to conclude that the meanings of the signs are different. It could be stated that the first sign of - that we come across belongs to the subtraction and the second sign of - belongs to the number indicating the concept of direction (Erdem, Başıbüyük, Gökkurt, Şahin and Soylu, 2015). As a matter of fact, the student's pre-learning of these concepts during the teaching phase is also of great importance. The reason is that readiness constitutes one of the basic building blocks in the teaching process (Harman and Çelikler,

2012). This study examined how to create knowledge about integers in the teaching of subtraction, which is one of the topics that will support the level of readiness for the teaching process of integers. It is thought that the subject of integers, first taught in the 6th grade (Zengin, 2014), can be based on subtraction because the first encounter with this sign is again by subtraction although the meaning of the sign of '–' changes. The subject that needs to be investigated is how the meaning of the sign of –, which will have different meanings in integers in the teaching process of subtraction, can be introduced (Bell, 1983).

When the literature was examined, no information was found about the concepts which allowed the transition to the concept of integer and the preliminary learning of the concept. In this respect, this study aimed to investigate the effect of subtraction, which can create preliminary learning, on the concept of integers. In the first stage of the study, based on the outcome of "can do addition and subtraction with maximum five-digit numbers", which was included in the 5th grade curriculum, the concept of integer and the concept of – (minus) were tried to be understood together with the method of subtraction that the students focused on. The second step in the study would be the interpretations to be made in the process where the students would examine the meaning of integers at the 6th grade level.

1.1 Research Sample

The emergence of the concept of numbers brought with it the concept of natural numbers. However, as the process progressed, needs such as debt and decrease emerged, and the concept of integer was needed (Zengin, 2014). The first encounter with negative numbers appeared in Chinese sources between 100-50 BC (Zengin, 2014). Since the concept of integer has a negative orientation on the basis of it, it can bring along a new idea of conceptualization because almost any expression expressing deficiency would have been interpreted with negativity and in particular with the symbol of "-" in the future.

The idea of the emergence of this subject is the difficulties experienced in the subtraction of integers (Erdem, Başıbüyük, Gökkurt, Şahin, and Soylu, 2015). The definition of plural "-" brought by integers can be difficult to make sense of when combined. In order to prevent this situation, it is thought that it is important to lay the foundation of the concept of "- (minus)" and to reveal the relationship between the concepts. In this respect, the purpose of the study was to provide students who had not encountered the concept of integer yet to acquire the concept of integer and to form a basis for the concept of integer.

1.2 Examining the Subject of Integers in the Education System of Turkey

The term integer (MNE, 2018), which was first conceptually encountered in the secondary school 6th grade curriculum in the Turkish education system, then started the relational learning process in 7th grade with the integration of operational and conceptual learning. The subject of "teaching integers", which was included in the sub-learning area of Numbers and Operations, was intended to be gained through various outcomes. In general, these outcomes are as follows:

- Recognizes integers and displays them on the number line
- Compares integers and puts them in order
- Determines and makes sense of the absolute value of an integer
- Performs four arithmetic operations with integers
- Writes repeated multiplication with integers as an exponential quantity.
- Solves problems that require operations with integers (MNE, 2018).

In this respect, the definitions of the concept of integer are also important. The concept of integer is expressed as a directional number in the textbook by the Ministry of National Education (Bektaş et al., 2019). Then, the signs of "+" and "-" were said to determine the direction of the number. The important point here is to understand the meaning of the symbol of "-" because here, it is possible to make sense of it with two meanings such as direction and subtraction (Kumar et al., 2017; Sfard, 2000; Marthe, 1979; Gallardo, 2002; Işıksal-Bostan, 2010).

2. Method

2.1 Identify Subsections

This study aimed to evaluate the activities to be carried out in relation to the fact that the concept of "-", which students first encounter within the subject of integers, was based on the concept of "-", which is used in subtraction. The study is a long-term one consisting of 2 stages. The first stage and the information and analyses of the data obtained will be given here, and the data for the second stage will be given after the teaching of the subject of

integers to be included in the instructional process. In this respect, the design of the study was determined as action research, one of qualitative research methods because action research also allows the researcher to function as a data collection tool (Yıldırım and Şimşek, 2018).

2.2 Participant (Subject) Characteristics

The study group was made up of 17 5th grade students studying at a secondary school in Turkey. Maximum diversity sampling was used in order to get more than one opinion and to determine the approach (Yıldırım & Şimşek, 2018) in the selection process of the students. The names of the students were coded as S1, S2. S3, … and S17.

2.3 Data Collection Tools

All kinds of studies carried out in the teaching process were conducted with the data triangulation technique (interview forms, video-audio recordings, observation). This technique was used in order to achieve accurate and consistent results and to control reliability (Flick, 2004). In addition, for the efficiency of the study and in order to determine whether the new conceptualizations obtained were temporary or not, the students were requested at the end of the study to fill in a form which asked them to write down the concepts which they had added to them or which they had heard for the first time thanks to their friends. After the study was completed, one-on-one interviews were held with some of the students. The students, who were interviewed one-on-one, were selected based on the maximum diversity sampling.

2.4 Data Analysis

The data obtained in the study were analyzed using descriptive analysis and content analysis, which were qualitative research methods. The main goal in content analysis is to create codes, themes and categories that can explain concepts according to the results of the data (Marshall and Rossman, 2006; Yıldırım and Şimşek, 2018). On the other hand, the descriptive analysis method can be defined as the process of transferring data in a meaningful way (Merriam, 2009). The codes of the concepts obtained in the study were created, and then descriptions were made by giving place to the visuals supported by the students' operations. The codes generated were carefully selected to help us establish a relationship between the concepts of negative, minus, and subtraction. Moreover, some metaphorical codes were obtained as a result of the analysis of the interview reports obtained via the voice recorders. Various categories were created as a result of the evaluation of the metaphoric codes.

3. Results

In this section, the data obtained during the study will be given. First of all, the codes obtained will be included, and then supportive descriptions will be presented.

Table 1. The Explanations Used by the Students in the Solution Phase of the Subtraction Process

Codes	Frequency	Sample Explanations
Sign/Direction	4	As we will do subtraction here, we have to move in the opposite direction first.
Minus/Subtraction	13	As we will do subtraction here, I put the sign of minus in front of it.
Subtraction/Decrease	9	Each time we subtract, the number decreases
Difference	13	When I found the difference between them, I subtracted the curran from the largest.
Decrease	11	When we do subtraction between numbers, there should be a decrease

When Table 1 was examined, it was seen that the codes for subtraction shifted to negative numbers. Although the sign/direction conceptualization was observed in a small percentage of students, this rate corresponded to approximately 24%. This situation, which was of considerable importance, could be associated with the students' advanced thinking skills or with their ability to form preconceptions. These concepts, which continued to be used in the rest of the application process, also revealed the butterfly effect that occurred in the classroom. The increase in the familiarity in line with the increase in the use of these concepts could be an important factor in concept learning.

The concepts striking in the subtitle of the Sign/Direction code are given below.

Table 2. Concepts Belonging to the Content/Direction Code

Code	Concepts	Sample Explanations
Content/Direction	Negative	*"…We left a minus because we would do subtraction. So it was minus 20 here …"*
	Increase	*There is an increase here so I added.*
	Decrease	*"We can see that there will be a decrease…"*

At the first stage, it was predicted that the students might be biased towards subtraction in the division into hundreds, tens and ones, which they were asked to use while doing subtraction. In order to avoid this situation, a step was taken to analyze the subtraction that did not require "decimal break" and then to solve the operation.

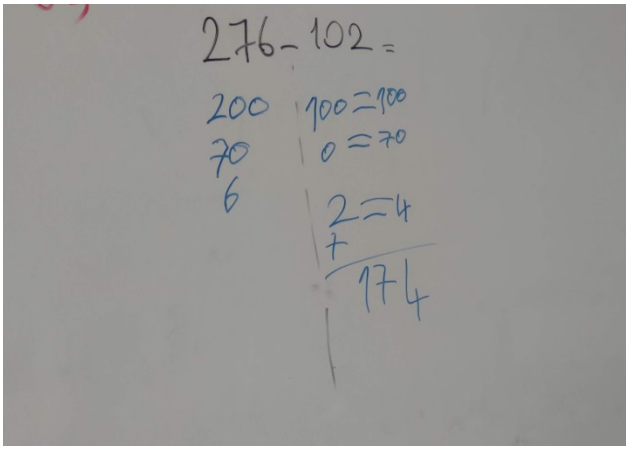

Figure 1. Solution Steps Taken by the Student Coded S13 for Subtraction That Did Not Require Decimal Break

The students' prejudices against decimal breaking also existed in the basis of subtraction. In order to prevent this situation, "subtracting the curran from the larger number", which is relatively easy for the student to understand, and the use of multiples of 10, which is seen as the easiest operation, can be used. Here, the student said "was that all..." when s/he analyzed the number, directly performed the operations and completed the solution.

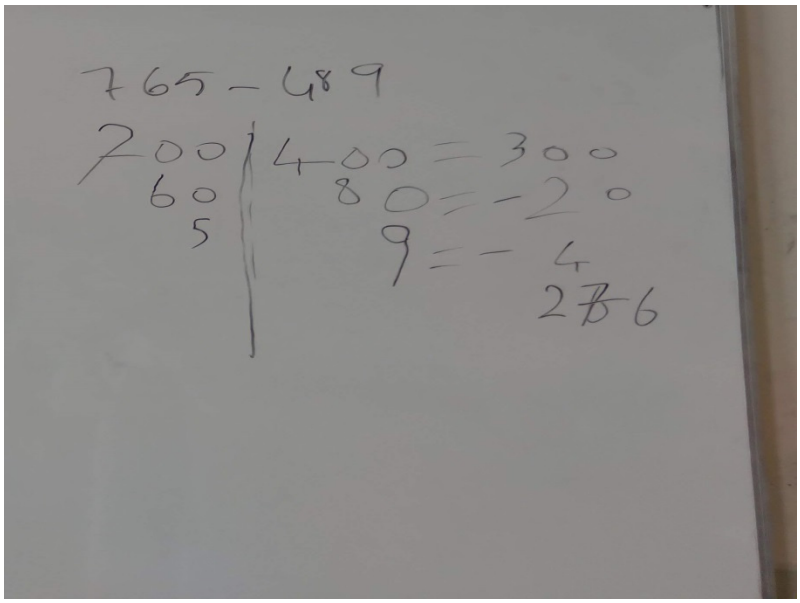

Figure 2. The Solution Made by the Student Coded S7

When the solution made by the student was examined, it was seen that s/he found -20 by placing the minus sign in front of the number where the decrease would be made. Here, it was observed that the student who did not theoretically have the concept of negative numbers developed his/her knowledge through practice. In addition, as revealed by the interview, the student drew conclusions regarding the difference code as follows:

S7: We can already perform subtraction in the first operation. There is no problem here.

R: So, where do you see a problem in the others?

S7: Actually, it's not a problem, but it seems like there is an inverse operation. I mean let's subtract the curran from the larger first, but we must make it clear that we need to do subtraction because the places changed in them. Otherwise, we cannot find the result.

R: Why do you think we couldn't subtract the larger from the curran?

S7: It's very easy teacher, that's why the larger cannot exist in the current ☺

R: Hmm well done, let's continue then...

It was also seen that the students often tried to produce metaphors in order to better express the situation in the interviews. These will be included in the following parts.

Another situation observed specifically was that the students tried to use the same method while performing mental operations. However, they stated that while doing this, what they found confusing occurred in their minds. The solution and comment put forward by S9 regarding the situation was as follows:

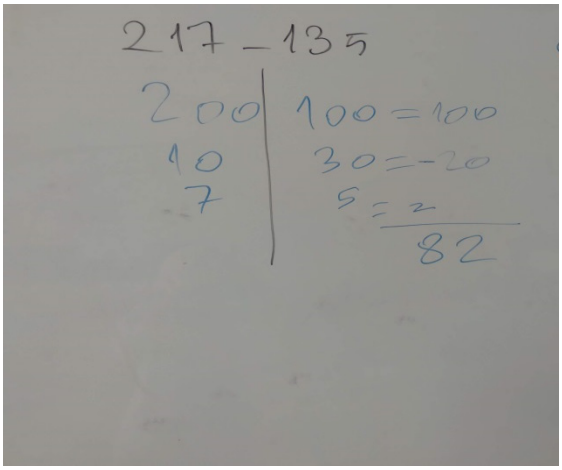

Figure 3. The Solution Suggested by the Student Coded as S9

R: …What kind of solution did you want to pursue first?

S9: Actually, I wanted to do it in mind, but I had to increase it again after I had to decrease it, and this confused me. I thus gave up.

R: So, do you have any ideas you can suggest to avoid confusion?

S9: Now, it is easier to solve it by writing, I can see where to decrease and where to increase.

R: Well, if you are allowed to choose between solving by writing and solving in mind, which one would you prefer right now?

S9: I think I can solve it better by writing.

R: Do you believe you will change your mind in the future?

S9: Yes, I can definitely solve it faster...

This dialogue between the student and the researcher showed that the student's confusion might have resulted from the initial learning and that the problem could be overcome in later stages. In the second step of the study, this situation will be checked, and a more realistic interpretation will be made.

3.1 Metaphoric Codes Emerged During the Application

Table 3. Metaphors Obtained Through the Application

Codes	Frequency	Sample Explanations
Shopping	9	…Here, as in shopping, currans are subtracted from larger ones. If the larger is subtracted from the curran, then something undesirable will happen.
Building	11	We do continuous subtraction while going down and up in stairs, and sometimes we go to the wrong floor when we go down again, a negative situation arises.
Supermarket	6	When we get change in the market, the things we buy cost less, the money we pay is much more. We'll do the larger-curran then.

When the metaphorical codes were examined, it was seen that in addition to concepts such as increase and decrease, which reveal the concept of integers, the metaphors of shopping and market, which will necessitate doing operation in integers such as increase and decrease, were obtained as well. In addition, another metaphor that was not included as multiples but was still outstanding was the metaphor of the elevator. This metaphor, which was necessary both for the meaning of the integer concept and for the transfer of the operation interpretation to the real plane, also provided relational learning. The metaphors of shopping, supermarket and building, which were observed intensely during the study, paved the way for the concept of contrast, which would constitute the basis of operational learning, and led to positive observations to be made during the production of ideas.

4. Conclusion, Discussion and Suggestions

4.1 Conclusion

In this section, the findings obtained as a result of the study will be evaluated and compared with the literature, finally, the related suggestions will be put forward.

In the first phase of the study, in order to prepare the basis for the teaching of integers, we tried to make students familiar with the concepts of integers using a method that required analysis of units, tens and hundreds used in subtraction, and the feedback we received from the students allowed us to have a partial idea. For example, the students started to use the concept of – (minus) spontaneously, even though no definition was made, and they did not find this situation strange at all. The fact that the subtraction process of integers taught in 7^{th} grade, the students used the statement of "Does it become minus when the larger is subtracted from the curran?" showed that this basic learning should be achieved in 5^{th} grade. In addition, as the students were more prone to the metaphorical approach than the older students, it was concluded that the sub-grounds of the conceptual learnings regarding the analogies obtained were better designed. Similarly, it was observed in the conversations between the students that the foundation of operational learning was laid and that the students did the addition and subtraction operations of integers without any prior knowledge. As it can be understood from here, it is possible for students to make new inferences with the knowledge they already have and to put these inferences into action.

4.2 Discussion

In addition, studies reporting that negative numbers are relatively more difficult to teach than positive numbers (Kullberg, 2007) may lead us to think that the problem can be solved by developing conceptual knowledge at an early age. However, Kullberg (2006) reported in her study that four features emerged in the teaching of integers and that the appropriate time for teaching it should be 7^{th} grade and 8^{th} grade. One reason for this could be the fact that the sign of integers and the operation sign are considered in the same context (MNE, 2009; Van de Walle, J. A., Karp, K. S. and Bay-Williams, J. M., 2010). However, it was observed that the teaching of integers on an operational basis was handled more intensively (Roby, 1981; Köroğlu and Yeşildere, 2004; Peterson, 1972). Focusing on operational knowledge may cause the teaching of negative numbers to continue without being realized and may result in making the process more confusing (Altun, 2002). In this respect, a more real interpretation can be made about the processing of a concept when the Procept (Process/Object) relationship is considered. On the contrary, the 3 components (operation, concept/object obtained when the process is completed, symbols) indicate that an expression

can have more than one expansion. In other words, a number can represent both subtraction, reverse displacement on the number line and a negative number (Gray and Tall, 1991).

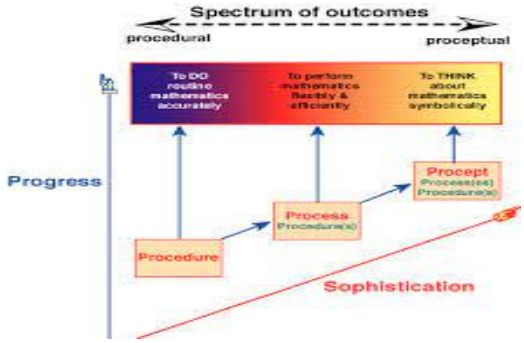

Figure 4. Procept Process (Mutodi, 2016)

When these processes are examined, it is seen that the elements or concepts to form the basis can be included in the teaching phase to make it more meaningful. Concept confusion can be determined according to the approach style at the first encounter. There is another obvious situation that in the technique applied, the student also prepares the basis for the sub-title of subtracting a positive number from a positive number, which is included in the outcome of "Can make four operations on integers". In this case, after the students discover the meaning and the adequacy of the negative number in the next step, they will be able to demonstrate the operation skill of adding a positive integer and a negative integer and observe the relationship between them. When the general evaluations are examined, the concepts that the students reached and the expressions they used confirm these. The second stage of the study will be carried out with the subject of integers that the same group will learn in the 6th grade. At this stage, these activities will be presented to the students again as a reminder and the transition between the concepts will be ensured.

4.3 Suggestions

The main point in this study is that students' world of meaning is wide and that their concept generation or concept matching skills are quite wide and developed. In future studies, students who suggest the concept of minus can be allowed to explain this concept and to make a more in-depth presentation by arranging a discussion group if necessary. In addition, by establishing cooperation between classes, that is, by removing the walls, peer learning can be taken as a basis in defining the concepts learned.

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