Teaching of Topology and its Applications in Learning: A Bibliometric Meta-Analysis of the Last Years from the Scopus Database

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Abstract: In this work, a bibliometric analysis of the investigations of the last 54 years focused on the teaching of topology and its applications in the learning of other areas of knowledge was carried out. The articles that appear in the SCOPUS database were taken into account under the search criteria of the words topology and teaching, connected with the Boolean expression AND in the search field ABS. As a result, 329 articles were obtained which, based on the PRISMA methodology, were reduced to 74 papers. In them publication trends, impact of publications, citation frequencies, among others, were compared. In addition, its use was identified for learning topology at different levels of training, areas of knowledge where this discipline is most applied and strategies used to teach these applications.

INTRODUCTION

According to (Wagner et al., 2011) and (Machado et al., 2015), bibliometric analysis is a research technique that identifies trends in scientific knowledge, for which it is possible to know about the current state of research in a specific area. This technique has been used in politics, science, technology, education, etc., in order to characterize scientific production in these fields (Cui, 2018; Koseoglu, Rahimi, Okumus, & Liu, 2016). For the development of a bibliometric study, it is necessary to select a database according to the scope of the research, identify keywords, establish search criteria and set categories that allow the interpretation of the information collected (Vallejo, Huertas, & Baracaldo, 2014).

In the area of mathematics, few bibliometric analyzes have been developed, in this sense, the study proposed by Behrens & Luksch (2011) stands out, which aimed to establish and characterize scientific productivity in the mathematics field between the years 1868 and 2008, and the bibliometric analysis of the Relime journal (Machado, Fanjul, López & Povedano, 2015). In the first, the results allowed to conclude that the publications and researchers in this area presented an exponential growth, which allowed to improve the cohesion index of the publications and in the second a behavior of the publications in educational mathematics is shown.

In this way, we believe that bibliometric studies in the field of mathematics can contribute to significantly improve knowledge about the applications of some of its branches, in this case, topology from an educational approach. Therefore, this study presents a bibliometric analysis where authors, quartiles, keywords, journals, among others, are identified using the PRISMA methodology (Moher, 2010). Additionally, a qualitative analysis is presented that determines the research trends, strategies and methods implemented in the teaching of topology, the areas of knowledge in which topological concepts are applied, and types of applications that have been developed.

WHAT IS TOPOLOGY?

Topology is a branch of mathematics originating in the 18th century from the need of resolving some issues that could not be approached from Euclidean geometry, reason why, some authors identified the first findings in this area as a type of abstraction from metric geometry, in which a broader concept on the proximity between space points was held (Callender & Weingard, 1996), this more universal concept associated to the element's position in space allowed relating certain objects that were completely different from a geometrical approach, for example, finding properties that remain unchanged in convex polyhedrons and that characterize non-convex polyhedrons based on their number of holes.

A mathematics topic that evidences the importance of topology is the study of sequences and their accumulation points (Schulte & Juhl, 1996), the foregoing, because a great variety of problems in the real world and different theories of some important knowledge areas like Ecology in the case of Biology, the study of the noble gasses in Physics, the central limit theorem in Statistics, etc., depend on the concept of accumulation point to guarantee that the estimations conducted are a close approximation to reality.

This discipline originated in Euler (1741), Where the solution to a famous puzzle proposed by the citizens of Königsberg-Prusia was found, which asked if it were possible to cross all of that city's bridges without having to cross the same bridge twice, this study gave rise to the so-called geometry of position, which later came to be known as topology.

Because of these new invariants, from a topological approach, geometrical concepts that depend on some type of distance lost importance, for example, angles and areas. From a ludic mathematical perspective, this way of understanding space allowed finding a solution to some problems that would be impossible to approach from a merely geometrical perspective, for example, the problem of the 5 Yens and the puzzle of the topological entanglement (Nishiyama, 2011).

The first study considered relevant in the topology area is Euler (1758), where the first topological invariant known of in the mathematical literature was found, this invariant tells us that any convex polyhedron fulfills the equation $V\text{-}E+F=2$, where V is the number of vertices, E is the number of edges, and F is the number of faces. After these studies, some contributions arise, like the classification of non-convex polyhedrons through the Euler characteristic, the classification of closed surfaces through its curvature, and the relationship between the Euler characteristic of a closed surface and its genus g.

A few years later, in 1857, the physicist Frederick Guthrie proposed the four-color problem, which assured that every map on a plane or on a 2-sphere could be colored with a maximum of four colors. Some attempts of solving this problem were proposed in the following years, among them, the most famous published in Kempe (1879), proof that was refuted one year later by the mathematician Percy Heawood. After that, nearly a century had to go by for a solution to this conjecture to be found, which is disclosed in Appel and Haken (1977) and Appel, Haken and Koch (1977), studies that used computational tools to classify all the possible types of maps that can be found in these surfaces and thus solve this famous problem.

The following contributions that are considered relevant in the area were realized in Poincaré (1892), which introduces the connectivity of surfaces, subsequently he published Poincaré (1895), denominated Analysis Situs and considered the paper that gave rise to modern topology, followed by its five supplements (Poincaré, 1899, 1900, 1902-1, 1902-2, 1904).

In these studies, Poincaré developed important contributions in the topology area, like the generalization of the Euler characteristic to higher dimensions, the topological invariance of the fundamental group, the triangulation of closed surfaces, etc. However, the most important problem proposed by Poincaré in this conglomerate of articles, appears in the fifth and last supplement (Poincaré, 1904), and was known until the beginning of the 21st century as the Poincaré conjecture, furthermore, this conjecture was part of the seven millennium problems proposed by the Clay Institute.

This conjecture assures that every 3-manifold with a trivial fundamental group is isomorphic at a 3-sphere, in the 2-dimension case, this problem is known as the characterization of the closed 2-manifolds, and it is associated to the equivalence, in topological terms, of the surface of a coffee mug to the surface of a donut. The analogue to this conjecture in the 4-dimension case, was resolved in Freedman (1982), but the conjecture proposed in Poincaré (1904) was only resolved until the beginning of the 21st century as a particular case of a problem related to Ricci flows, this solution was found by Perelman and appears in three articles uploaded to the arXiv platform between the years 2002 and 2003.

TEACHING OF TOPOLOGY

The teaching of school mathematics faces different challenges, from academia, proposals have been made on how to take on this challenge. for Freudenthal (2002), it is important that school mathematics always include axiomatization, formalization, and schematization elements. According to this author, axiomatizing relates to organizing knowledge; formalizing is adapting and transforming the language based on symbolization; and schematizing requires generalizing the language in the form of laws and rules that are adapted to reality through abstraction. Freudenthal emphasizes that the problem is due to the fact that only formalizations appear in school mathematics.

Polya (2004) proposes a work route based on problem solving. For him it is important to follow four steps: First, start by comprehending the problem. Second, imagine a resolution plan that is related to the type of relationships described between the variables. Third, execute the plan. Fourth, check and interpret. Check the result, the reasoning used and the possible alternatives to obtain the result. Evaluate whether the result is obvious and whether the result or method can be used to solve the other problems.

This work proposal from a problem-solving perspective allows motivating the student, which is an important element in their learning, however, it is easy to find students that desist and dismiss their ability to learn mathematics. This, according to Wallrabenstein (1973), is due to the fact that the conceptual part is often rigorously worked on, rather than the problems that make the subject interesting, creating monotonous educational environments, far from motivation, leaving the educational process subject only to the formalization.

Faced with this problem, the teaching of topology appears as one of the possible solutions, since from the study of this area it is possible to start from the interests and experiences of the students (Hilton, 1971), allowing them to broaden their base of experiences and, for example, build more elaborate notions such as geometric space (Wallrabenstein, 1973). According to the school level, the experiences that students acquire in the study of topology can become significant, becoming the basis of more sophisticated knowledge that becomes indispensable for more abstract constructions that the student could encounter (Ke, Monk, & Duschl, 2002).

Another advantage, according to Piaget and García (1998), occurs because the usual notion of tridimensional space is constructed starting from topological space, followed by forecast space, and ending with Euclidean space, following an order contrary to that of the historical order in which geometry was formalized.

According to Wallrabenstein (1973) children are more easily interested in topological problems than in analytical geometry problems. Topology allows crossed connections with other mathematics disciplines, like set theory, logic, and combinatory. For Kawauchi and Yanagimoto (2012), the study of topology allows the development of spatial thinking and occurs at all levels of schooling.

Hence, if the topology is included in early school years, we will achieve an important degree of familiarity with it, making it possible for it to be used as a common thread in school mathematics, allowing students to relate different parts of mathematics from an early. According to Whitney (2017), topology has penetrated almost all other branches of mathematics. Basically, the study of topological notions in school mathematics courses opens the door to create an important symbolic conceptual base that the student uses for their own benefit.

METHODOLOGY

This research aimed to determine by means of a bibliographic study what the literature indicates about the teaching of topology, to identify methods used in the teaching of topology, and on the other hand, to detect the areas of knowledge where this is applied. To do this, the publications that relate topology and teaching are consulted in the Scopus database, the results of this search are applied the PRISMA methodology for systematic reviews, and thus obtain a selection of documents to which a Bibliometric analysis of the articles selected between 1966 and 2020.

The Scopus database, from the Elsevier publishing house, was selected as it is considered one of the most recognized bases for abstracts and citations of the scientific literature in the world. Within this database, searches can be carried out by using Boolean operators and field codes that connect the author's name, title, keywords, publication date, area of interest, among others. The words that guided the search were topology and teaching, connected with the Boolean expression AND in the search field ABS, that is, the query was focused on the articles that contained these two terms in the abstract.

The search performed identified 329 publications, these were analyzed using the PRISMA (Moher, 2010). methodology and 70 articles were selected from this first stage, which were subsequently reviewed based on the criteria described in Table 1.

Criterion	Description
Article Title	Article title
Keywords	Keyword 1, Keyword 2, Keyword 3, ...
Number of authors	Quantity of authors (1, 2, 3, …)
Authors	Name(s) of the author(s)
Institutions of the Authors	Universities or research centers of the authors
Project or grant related to the project	Type of relationship of the article with any project
Year	Year of publication of the article (1974, 1975 , 1976, …, 2018).
Country	Place of origin of the journal where the article was published.
Journal	Name of the Journal.
Journal's SJR Quartile 2017	It is an indicator that evaluates the importance that a journal has within its area.
Journal's H Index	It is a numeric value that measures the quality of the scientific production of a journal.
Number of Citations	Number of times that the article has been cited by other authors.

Table 1. Criteria for article analysis.

The analysis of the articles was conducted through a database that contains the criteria of table 1, the systematization of the information was developed from the detailed reading of each article, and the quantitative analysis of the information was performed with the Statistical Package for the Social Sciences (SPSS).

In a second part of the study, a qualitative analysis of the selected articles was carried out by classifying them into two categories: 1- teaching of topology, and 2- applications of topology in learning other areas of knowledge. These categories arose a priori since our objective was to identify methods used in the teaching of topology, and areas of knowledge where this discipline is applied. In table 2, the criteria used in each one of the categories are described.

Analysis Criterion	Teaching of Topology	Application of topology in learning
Level of Education	High school students, university students, university professors, or workers	
Area of education	Career being studied by the population in which the study was developed	
Conceptions addressed	Topological concept that appears in the article	NA
Description of the method	Method used for teaching the concept	NA
Topic of application	NA	Subject matter on which the study is developed
Type of application	NA	Software, course, theoretical work

Table 2. Criteria of the qualitative analysis. *(NA: Not Applicable*).

RESULTS

Using the steps indicated by the PRISMA methodology for the systematic analysis of documents, 70 works were selected that in some way mentioned the teaching of topology and the applications of topology in the learning of different areas of knowledge.

In the descriptive analysis, we found that 27% of the articles were written by a single author, 24.2% were written by two authors, 18.5% were written by three authors, and the remaining 30% were written by more than three authors (Figure 1), consequently, it is possible to conclude that this type of research has a low index of cohesion. In addition to this, among the 208 authors of the articles analyzed, we found that only two of them, Debao Chen and Feng Zou, possess authorship in two studies, which evidences a potential research current because of the small number of authors that conduct studies in this direction.

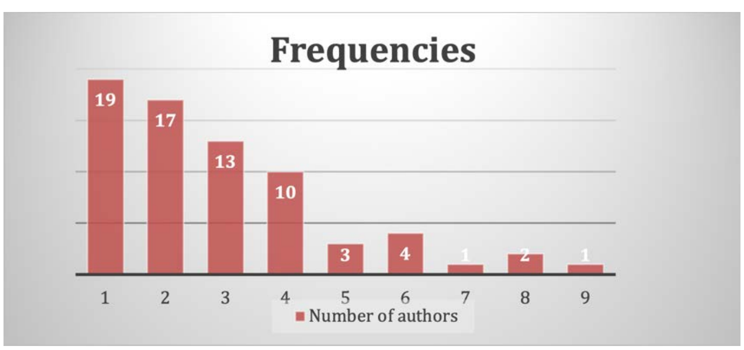

Figure 1: Number of authors per article.

On the other hand, in the articles analyzed, we found 240 different keywords, of which, only 13 are repeated in more than one article, while the remaining appear only once in any of the articles. The keywords appearing with a frequency of greater than one are: Education, Power Electronics, Project-Based-Learning (PBL), Teaching-Learning-Based-Optimization (TLBO), Topology and Virtualization (Figure 2).

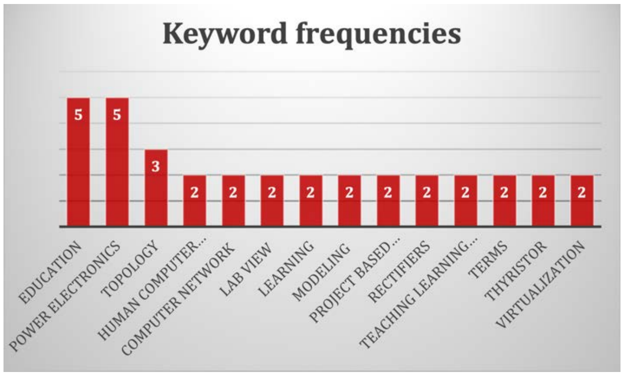

Figure 2: Keywords with Highest Frequency.

Table 3 presents the list of the 46 journals that were found in the search, as well as the country, quartile and journal's H index. The list allows establishing that there are four journals that have published the most on the topic: International Journal of Mathematical Education in Science and Technology (United States), IEEE Transactions on Education (United States), International Journal of Electrical Engineering Education (United Kingdom), and IFIP International Federation for Information Processing (Germany). The first, is a journal specialized in mathematical education at all levels of education. The second, focuses on educational methods and technology related to the theory and practice of electrical engineering and computer science. The third, publishes on learning in engineering addressing topics such as: teaching methods, curriculum design, assessment, among others. The fourth, is a journal on information systems that contributes to the knowledge on the use of computers, free information access, right to privacy, and protection of confidential data.

Journal Title	Country	Quartile SJR 2020	H index	Items
International Journal of Mathematical Education in Science and Technology	United States	Q2	33	4
IEEE Transactions on Education	United States	Q1	68	3
International Journal of Electrical Engineering Education	United Kingdom	Q3	23	3

IFIP International Federation for Information Processing	Germany	Q4	37	2
Acta Scientiarum Naturalium Universitatis Pekinensis	China	Q3	18	1
2008 IEEE Vehicle Power and Propulsion Conference	United States	NYAQ	26	1
2017 Australasian Universities Power Engineering Conference, AUPEC 2017	Australia	NYAQ	4	1
2019 5th International Conference on Control, Automation and Robotics, ICCAR 2019	United States	NYAQ	3	1
2019 Electric Power Quality and Supply Reliability Conference and 2019 Symposium on Electrical Engineering and Mechatronics, PQ and SEEM 2019	United States	NYAQ	5	1
2019 IEEE 15th Brazilian Power Electronics Conference and 5th IEEE Southern Power Electronics Conference, COBEP/SPEC 2019	Brazil	NYAQ	NYAQ	1
32nd AAAI Conference on Artificial Intelligence, AAAI 2018	United States	NYAQ	72	1
ACM International Conference Proceeding Series	United States	NYAQ	123	1
ACM Sigcse Bulletin	United States	NYAQ	47	1
Advanced Science Letters	United States	NYAQ	27	1
American Biology Teacher	United States	Q2	30	1
American Journal of Physics	United States	Q2	99	1
Applied Categorical Structures	Netherlands	Q2	31	1
ASEE Annual Conference and Exposition, Conference Proceedings	United States	NYAQ	34	1
CEUR Workshop Proceedings	United States	NYAQ	52	1
Communication Quarterly	United Kingdom	Q2	45	1
COMPEL - The International Journal for Computation and Mathematics in Electrical and Electronic Engineering	United Kingdom	Q3	31	1
Computer Applications in Engineering Education,	United States	Q2	29	1
Computer Communications	Netherlands	Q!	105	1
Computers & Geosciences	United Kingdom	Q1	123	1
Computers and Structures	United Kingdom	Q1	138	1
EDUNINE 2018 - 2nd IEEE World Engineering Education Conference: The Role of Professional Associations in Contemporaneous Engineer Careers, Proceedings	United States	NYAQ	5	1

Energies	Switzerland	Q2	93	1
ICIC Express Letters, Part B: Applications	United States	Q4	12	1
IECON Proceedings (Industrial Electronics Conference)	United States	NYAQ	71	1
IEEE International Conference on Electro Information Technology	United States	NYAQ	20	1
IEEE LATIN AMERICA TRANSACTIONS	United States	Q3	26	1
IEEE Transactions on Power Apparatus and Systems	United States	NYAQ	0	1
IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS: SYSTEMS,	United States	Q1	64	1
IET Circuits, Devices and Systems	United Kingdom	Q3	49	1
International Journal on Digital Libraries	Germany	Q2	32	1
Int. J. Advanced Media and Communication	United Kingdom	Q2	12	1
International Journal of Emerging Technologies in Learning (iJET)	United States	Q3	11	1
International Journal of Engineering Education	Ireland	Q1	50	1
International Journal of Pure and Applied Mathematics	Bulgaria	Q4	24	1
International Journal of Recent Technology and Engineering	India	NYAQ	20	1
International Review of Research in Open and Distance Learning	Canada	Q1	50	1
Journal of Molecular Modeling	Germany	Q3	69	1
Journal of Computational Chemistry	United States	Q1	188	1
Journal of Higher Education Policy and Management,	United Kingdom	Q1	42	1
Journal of Mathematical Psychology	United States	Q1	69	1
Journal of Physics: Conference Series	United Kingdom	Q4	85	1
Journal of Systems and Software	Netherlands	Q1	109	1
JournalofNortheasternUniversity	China	Q4	20	1
Mathematical Modelling of Natural Phenomena	France	Q2	36	1
Neurocomputing	Netherlands	Q1	143	1
Open Cybernetics and Systemics Journal	Netherlands	Q4	8	1
Plos One	United States	Q1	332	1
POLLACK PERIODICA	Hungary	Q3	11	1

Power System Technology	China	Q1	68	1
Proceedings - 9th International Conference on Information Technology in Medicine and Education, ITME 2018	United States	NYAQ	6	1
Proceedings of 2019 8th International Conference on Modern Power Systems, MPS 2019	United States	NYAQ	3	1
Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences	United Kingdom	Q1	135	1
Revista Facultad de Ingeniería, Universidad de Antioquia.	Colombia	Q4	12	1
Scientific Reports	United Kingdom	Q1	213	1
SIGCSE Bulletin (Association for Computing Machinery, Special Interest Group on Computer Science Education),	United States	NYAQ	47	1
Structural and Multidisciplinary Optimization	Germany	Q1	117	1
Studies in Computational Intelligence	Germany	Q4	68	1

Table 3. General Journal List. (*No yet assigned quartile 2020. NYAQ)*

Figure 3 presents the journals' quartile in which the research found in this analysis were published, which represents the importance of each publication in their corresponding area. We found that 17 journals belong to the first quartile, 10 to the second, 8 to the third, and 8 to the fourth, in addition to that, we found 19 journals that do not report a quartile since they are Conference Proceedings.

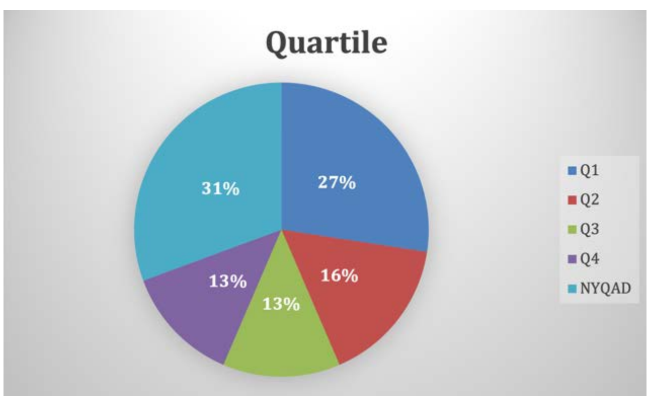

Figure 3. Journals' Quartile

Figure 4 shows each journal's country of origin, together with the number of publications. As can be observed, the country with the highest number of articles is the United States (45%), followed by the United Kingdom (17.7%) and, the latter in turn, by Germany and the Netherlands with (8%). The foregoing allows concluding that the United States is the largest generator of knowledge on teaching of topology and applications of this area in learning of other branches of knowledge.

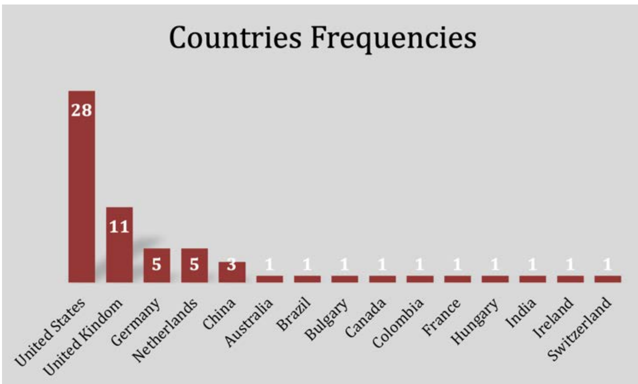

Figure 4. Journals' countries of origin.

It was possible to determine the number of citations of each article according to the Scopus database and establish that the range of citation of the articles, except for one, is between 0 and 62, table 4 shows the list of most cited articles that were found in the analysis. The atypical value corresponds to the article published by Lu and Chen (2012), cited 8321 times as of June 2021. This article describes the development of a software named Multiwfn, whose objective is to analyze wavefunctions, visualization of molecular and orbital structures, analysis of energy and density variations, generation of initial optimum conditions for wavefunctions, and other types of tasks that are performed in the area of quantum chemistry.

Article Title	**Authors**	**Year**	**Journal Name**	**Citations According to Scopus**
Multiwfn: A multifunctional wavefunction analyzer	Tian Lu and Feiwu Chen	2012	Journal of Computational Chemistry	2,684

Experiences in the application of project-based learning in a switching-mode power supplies course	Diego Lamar, Pablo Miaja, Manuel Arias, Alberto Rodríguez, Miguel Rodríguez, Aitor Vázquez, Marta Hernando y Javier Sebastián	2012	IEEE Transactions on Education	33
A new approach for teaching power electronics converter experiments	Jacinto Jiménez, Fulgencio Soto, Esther de Jódar, José Villarejo y Joaquín Roca	2005	IEEE Transactions on Education	32
An improved teaching-learning-based optimization with neighborhood search for applications of ANN	Lei Wang, Feng Zou, Xinhong Hei, Dongdong Yang, Debao Chen y Qiaoyong Jiang	2014	Neurocomputing	27
On some fundamental properties of structural topology optimization problems	Mathias Stolpe	2010	Structural and Multidisciplinary Optimization	20

Table 4. List of most cited articles.

From the number of publications, we can observe that between the years 1971 and 2005, a 34-year period, 12 articles were published, while between the years 2006 and 2020, a 14-year period, 58 articles were published (Figure 5), which shows a significant increase in the amount of research related to this topic in recent decades. Of the 70 articles analyzed, only 21 mentioned having received support from an institution or research funds, and among these institutions, the National Natural Science Foundation of China (NSFC) is noteworthy because it is the one that supported the greatest number of articles in this field of knowledge.

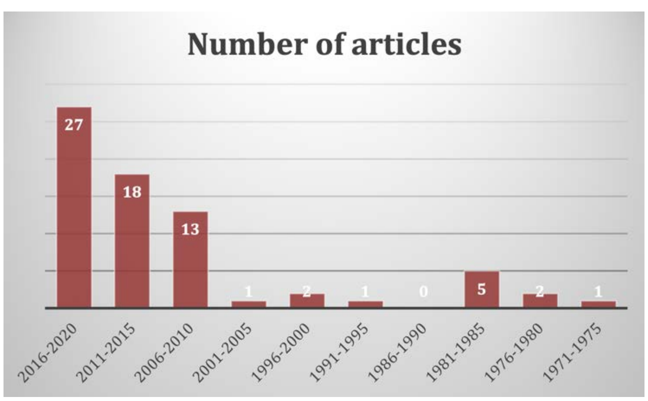

Figure 5. Number of articles in periods of five years.

QUALITATIVE ANALYSIS

The second part of the results corresponds to a qualitative analysis of the chosen articles by classifying them into two categories: the first, articles that refer to the teaching of topology, and the second, applications of topology in the learning of other areas of the knowledge. These categories arose a priori since our objective was to identify methods used in the teaching of topology, and areas of knowledge where this discipline is applied.

Based on the rigorous reading of the chosen articles, we found 7 belonging to the first category, among these, some focused in the teaching of topology for mathematics students, others seeking to bring students with basic and mid-levels of education closer to topological conceptions, and, finally, articles that seek to get students, belonging to careers in knowledge areas with no evident relationship with mathematics, to make interpretations of topological notions from the perspective of their field of expertise.

Table 5 shows the characteristics that were considered for the analysis of each one of the articles.

Article	**Conceptions addressed**	**Level of Education**	**Area of specialization**	**Method Description**
(Vukmirovic, Devetakovic, & Petrusevski, 2012)	Möbius strip; Genus of closed surfaces.	University students.	Architecture.	Building materials; Paper; Software for Image Design;
(Nishiyama, 2011)	Space continuity; Topological puzzles.	University students.	Teaching of mathematics.	Riddles; Drawings; Flexible Materials.
(Lawvere, 1996)	Continuity in space proposed by Walter Noll.	NM	NM	Theoretical approach.
(Garay, 1993)	Genus of closed surfaces; Homeomorphism; Fundamental Group.	University students.	Teaching of mathematics.	Theoretical approach; Drawings.
(Kanellopoulou, 1982)	Euler Characteristic; Nodes; Regions.	High school students.	NM	Curriculum and learning primers.
(Neubrand, 1981)	Set theory; Group theory; Topology; Category theory.	University students.	Mathematics.	Spiral curriculum.
(Baylis, 1977)	Continuity on sets.	NM	NM	Theoretical approach.

Table 5. Research on teaching of topology. (*NM: No Mention*).

In the articles focused on teaching of topology for mathematics students, the authors associate some basic topological concepts to other mathematics areas like group theory, category theory, and even unidimensional calculus, aiming for students to find the connection between these branches and understand the importance of topology in learning of mathematics in general (Neubrand, 1981).

In the articles focused on teaching topological notions to students with basic and mid-levels of education, the authors show the properties from approaches that do not use rigorous language corresponding to mathematical writing, for example, in Nishiyama (2011) the notion of continuity, probably the most important conception studied in topology, is approached performing an intuitive approximation through the use of flexible materials, drawings, and riddles, putting mathematical writing aside, that in part, because the author intends to explain this conception to a population that does not possess the necessary mathematical knowledge to understand it intricately.

In Garay (1993), using intuitive approaches and some formal definitions, the aim is to bring students closer to more advanced concepts like homeomorphism, fundamental group, Euler characteristic, genus of closed surfaces, among others. In this case, literary language and some drawings are used, with the intent of bringing the student closer to these conceptions unconsciously, subsequently the topological notion is introduced, followed by a formal definition. With this strategy, the authors seek the theoretical concepts addressed to be understood in a simple manner by students.

In Vukmirovic, Devetakovic, and Petrusevski (2012), students with careers having no evident relationship with mathematics are brought closer to some topological notions, more specifically, the objective is for some students from the career of architecture, to appropriate the conceptions that can be approached intuitively like the Möbius strip and the genus of closed surfaces, and by making use of the materials and software specialized for the design, an interpretation of these concepts is made from the perspective of their area of expertise through the design and construction of sculptures inspired in these topological surfaces.

It is possible to establish that the areas that most incorporate applications of topological concepts in their curricula are Systems Engineering, Electronic Engineering, and Electrical Engineering.

In the case of Systems Engineering, the topological application that most appears in the chosen articles is the use of networks for data management, currently this topic is of great importance because of the growing need of exchanging information between computer servers throughout the world. From a theoretical perspective, the servers' interconnection networks and their software are based on discrete topology constructions, where each connection point and each connection line are modeled through graphs (Maheswaran et al., 2009; Montero & Manzano, 2017). The schemes used in these networks are called topologies, being the most common the mesh, star, tree, bus, and ring topologies, that applied to computer networks present characteristics that allow certain benefits and limitations in information exchange, these characteristics are taken advantage of by engineers when designing networks (Wang et al., 2014).

In Electronic and Electrical Engineering, it is also possible to find the property of having elements interconnected between each other, whether to transport information or electricity. In this case, each topology allows different constructions that are taken advantage of according to the need of optimizing the performance of the circuit to be designed (Mohammed, Kawar, & Abugharbieh, 2013; Kazimierczuk, & Murthy-Bellur, 2012). Each circuit has its own topological structure, and in the cases of the power networks also a geometrical structure, which allows the construction of mathematical models that describe its behavior (Altintas, 2011; Xu et al., 2012). Since the industry and technology require more effective and reliable systems every day, new applications of topology arise in this area, showing us that for our case, a high frequency of articles in this direction.

To a lesser extent, we find articles in diverse areas like: Management, Architecture, Arts, Librarianship, Biology, Economics, Physics, Geology, Chemistry, and Environmental Engineering. In arts, an article was found for the design of a piano course (Wei, 2018), another in which Chinese characters are studied using their topological structure (Sun et al., 2014), and one of artistic creation where the students performed organized tasks from social interaction networks (Kawka, Larkin, & Danaher, 2011). In psychology, there were articles that analyzed interpersonal relationships from the topology of networks (Dudley, 1976), and development of critical thinking and analysis of learning in school environments (Hamizan, Zaid, & Noor, 2016). In most of these articles, we find that topology becomes a tool for the analysis of a structure, a class, or a method. Other applications of topology are related to the visualization of complex structures like DNA or molecular structures that can be understood from their topological properties (Halverson, 2010; Lu & Chen 2012; Robic & Jungck, 2011).

It is noteworthy to mention, how most of the applications are addressed toward the university level of education, this could be because the management of technological concept is currently carried out to a greater degree in higher education, so its application does not easily reach research in basic and middle education.

PROJECTION

This work allows us to think about the convenience of including or not including notions of topology in school. When reviewing the data, we noticed the slightly diffuse idea that it is possible to do it. This, instead of providing us with answers, introduces us to other questions about how topology should be introduced in our schools. In this regard, different works appear in the literature. Steven Greenstein (2014) mentions how studies based on Piaget show that children are able to work and understand conceptual geometry, a fact that for many means that geometric work can be oriented towards topological work.

Gavilan et al (2022) carry out work on graph theory that, although they do so with university students, work on an interesting proposal in the field of analysis of student learning processes and possible cognitive conflicts that appear, and that can be overcome through the use of different types of representation systems.

In another study, Whitney (2017) mentions how it is possible to work on topology in preschool level courses. For this author, the first objective is to show that introducing topological terms and teaching topological concepts in our schools could be improved. On the one hand, students' geometric intuition could be strengthened if we routinely encourage them to compare and contrast topological and geometric ideas. Furthermore, if topology is introduced at school stages, students can become familiar with an area of mathematics that in modern times has penetrated almost every other branch of mathematics.

This author calls on the research community to continue studying how basic education students understand topological conceptions in relation to geometric concepts. He ultimately proposes that: If the K-12 mathematics curriculum begins to incorporate some topological ideas, we might possibly observe many beneficial consequences (Whitney, 2017).

In this sense, Genevieve and Eunsook (2005) mention how the teaching of geometry in elementary school goes hand in hand with the discovery of the world, its forms and its regularities, to conceptualize a vision of space. *Overall, while not totally disproven, the topological primacy theory is not supported. It may be that children do not construct first topological and later projective and Euclidean ideas. Rather, it may be that ideas of all types develop over time, becoming increasingly integrated and synthesized. These ideas are originally intuitions, grounded in action...* (Clements & Battista, 1992,pp. 425-426). These authors do not affirm that, however, the ideas of topology will have to appear first in children and then Euclid's projectivist geometry, but rather that the two are gradually being built, interrelated and consolidated.

For Genevieve and Eunsook, when working with children in the construction of maps, it is possible to find topological relations involving proximity and separation, spatial order, enclosure and continuity; projective relationships involving perspective; and Euclidian relationships including proportion, distance, and relative size. (Genevieve & Eunsook, 2005 p 80)

Finally, Sugarman (2014) presents us in her thesis a work proposal in topology for school grades supported by the study of mathematics education reforms. Here, the author prepared a topology teaching work in grades 4, 5 and 6, applying it only in grade 5, where he concludes that it is possible to teach topology at the school level. For the Discussing Unknowns in Topology presents math as a subject in which we do not have all the answers, a viewpoint which is not commonly seen in early mathematics education.

As a complement to this bibliometric analysis work, some teaching designs of topology didactics were made to a group of future mathematics teachers. There, the theoretical part was worked on, complementing it with practical exercises on each of the topology topics covered. This type of work can be extrapolated to apply in secondary education. The sequence of proposed topics is divided into chapters, which can be worked on at different levels of the secondary education.

Unit 1: Introduction to graph theory

1. Definition of graph
2. Concepts and terminology
3. Eulerian paths and cycles
4. Fleury algorithm
5. Hamiltonian paths and cycles

Unit 2: Polyhedra and Euler characteristics

1. Regular Polyhedra
2. Graph, plane
3. Faces of a flat graph
4. Euler's formula for polyhedra
5. Triangulation of surfaces
6. Euler characteristic, topological invariant

Unit 3: The 4-color theorem

1. History of the 4 Color Theorem
2. Activity: Map painting
3. Graphs, plans
4. Double a map
5. Coloring graphics
6. The error in Kempe's proof

Unit 4: The Mobius strip and the Klein bottle

1. The Mobius Strip.
2. The Klein Bottle.
3. The projective plane as conscious spaces.
4. Activity (Triangulate and color).

Unit 5: Introduction to knot theory

1. History of knot theory.
2. Knots in topology and their classification
3. Prime knots
4. Activity (Knotplot Software)

Unit 6: Basic topological notions.

1. Topology definition
2. Open, closed sets.
3. Interior, exterior, adherence, limit, isolated points.

4. Topology on finite sets
5. Usual topology in the line and the plane.
6. Continuous functions. Homeomorphism.

Each of the proposed themes has the possibility of being introduced from the students' own conceptions and taken to active work, for example, playing at following paths in graphs, using polyhedrons of different materials to check the Euler characteristic, drawing and coloring maps, building the Mobius strip and describing characteristics of the knots that the students know how to make. Also from set theory, and using different types of representations, it is possible to understand the basic topological notions. Each experience lived by the students enables the creation of new experiences that enhance learning not only of topology but of other fields of mathematics, by requiring in each activity processes such as: serialization, hierarchy, problem solving, among others.

CONCLUSIONS

At the beginning of this study, we saw that topology originated in consequence of the need of identifying objects by their position in space, putting aside the notion of distance between thereof (Euler, 1741), this way of conceiving space began to gain importance in recent years, mainly by the methods used to exchange information. In this bibliometric study, publications focused on learning topology and the use of topology in teaching other areas were identified.

The criteria of the bibliometric analysis allowed concluding that the country where most of the articles of this type were published was the United States, followed by the two main European academic powers, United Kingdom and Germany, which can be interpreted as a consequence of tradition and strength of the educational institutions in these countries. On the other hand, the articles that were found in this search are in their majority published in Q2 and Q3 quartile journals, which demonstrates that these publications achieve a medium impact on the academic environment, this generates certain confusion if the relevance that these areas have to the development of technologies are considered, in addition to their importance in the construction of the theoretical framework of mathematics.

Regarding the number of authors per article, it was found that in most cases, they do not exceed more than four per article, this shows a low index of cohesion in these researches, which implies a potential research current because of the small number of authors that conduct studies in this direction. On the citations, it was possible to establish a low range of citation, with the exception of an atypical datum appearing in the Lu and Chen (2012) article, which is cited 8321 times by other authors. Finally, with respect to the H index, it is possible to conclude that the journals with the highest index, are in their majority, focused on applications of this branch of mathematics, while the journals specialized in teaching have slightly lower indexes.

Through the qualitative analysis, it was possible to identify some methods used by mathematics professors to encourage the learning of topology in university students, furthermore, articles were found focused toward the teaching of this area to basic and middle school students. Among the most common methodologies, is the use of intuitive approximations to topological concepts through drawings, puzzles and examples (Garay, 1993), in addition, it was possible to identify curricula constructions in which attention is given to the connection of topology with other branches of mathematics (Neubrand, 1981). From this we can conclude that topology allows intuitive approaches to its basic conceptions, which can facilitate its learning in students with elementary levels of training in mathematics (Gavilan et all 2022). The number of publications on the teaching of topology suggests that it is still a branch of work that does not receive much attention from the academic community of researchers in mathematics and mathematics teaching.

On the other hand, it was possible to find some topological applications in learning different branches of the knowledge, which were classified in area, topic, and type of application. Within these categories, there are designs of courses, software use, among others. One of the most interesting articles in design of courses is MacArthur and Anderson (1981) (2017), in which a learning program for power plant operators is designed, in these cases the adequate operation of the plant depends mainly on the interconnection with other similar plants, consequently, it becomes relevant for operators to understand the topological structure of these networks.

Among the articles that use some type of software to teach applications of any topological notion, one of the most important ones is Lu and Chen (2012), which uses, like one of its multiple functions, the topological structure of the molecules to study their properties from a quantum chemistry approach, which implies a potential learning tool in this area of knowledge. Another type of software appearing in some of the articles analyzed, for example in Altintas (2011) and Xu et al. (2012), consists in simulation programs of power plant processes, which are used to train operators and university students with less resources than those used in traditional teaching methods. Finally, it is noteworthy to name the application in the study of computer networks, which use the topological structure of the hardware and software used in the network to optimize information Exchange processes (Montero & Manzano, 2017; Restrepo et al., 2016; Xu et al., 2012).

The above verifies the relevance of topology in the evolution of the modern society, since these articles use applications of topological properties in the teaching of optimization processes and in the generation of other types of useful knowledge, mainly in the case of technological development, therefore the learning of this area in basic and middle education curricula should be encouraged, in a similar way than in other branches of mathematics like algebra, calculus, and geometry (Vizcaino & Terrazzan 2020), (Castiblanco & Nardi, 2018).

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