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Proportional and Non-Proportional Situation: How to Make Sense of Them

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Abstract: Teacher knowledge is one of the main factors in the quality of mathematics learning. Many mathematics teachers have difficulty using proportional reasoning. Proportional reasoning is one of the essential aspects of the middle school mathematics curriculum to develop students' mathematical thinking. Teachers should realize that developing proportional reasoning is not an easy task. In this study, we investigated how teachers give proportional reasoning about the concept of proportional and non-proportional situations, especially in making sense of them. The research subjects were mathematics teachers who had taught proportional-related material. Data was collected using task-based interviews outside the teacher's working hours. Data analysis and interpretation were completed using a framework meaning-based approach. The results of the data analysis showed that the teacher is careful in understanding information, is aware of multiple meanings, and knows key information in understanding the contextual structure of proportional and non-proportional situations. Furthermore, they are also able to identify additive and multiplication relationships, have flexibility in understanding proportional and non-proportional situations separately or collectively, and understand problem-solving systematics in detail.

Keywords: *Characteristics of mathematics teachers, non-proportional situations, proportional situations.*

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Introduction

Proportional reasoning is essential in elementary and secondary school students' mathematical thinking (Izsák & Jacobson, 2017; Lamm & Pugalee, 2010; Langrall & Swafford, 2000; National Council of Teachers of Mathematics [NCTM], 2000; Walle et al., 2007). According to the 2013 curriculum in Indonesia, the material on proportionality is taught in grades V and VII (Ministry of Education and Culture, 2018). The focus of proportional reasoning in grade V provides basic knowledge that can be developed when students engage in various mathematical concepts in the higher grades (NCTM, 2000). These concepts include fractions, scale, similarity, and probability (Izsák & Jacobson, 2017; NCTM, 2000; Walle et al., 2007). The failure of students to promote proportional reasoning can impact their understanding of mathematics at the next level (Langrall & Swafford, 2000).

Proportional reasoning emphasizes recognizing, interpreting, examining, clarifying, and giving proof to back up statements regarding proportional relationships (Lamon, 2011). The relationships in the proportional situation are connected to the concepts of ratio and proportion. A ratio is a multiplicative connection between two measures or quantities in a given situation. If two ratios have equality, then it is called a proportion. Proportional reasoning requires understanding the multiplicative relationship between quantity (ratio), quantity covariance, and ratio invariance (Lamon, 2007; Walle et al., 2007). Aspects of the property of proportionality include establishing two equal ratios and solving problems, including recognizing proportionally related quantities, as well as using numbers, graphs, tables, and equations to analyze their relationships and quantities ([NCTM, 2000). Reasoning uses mental processes that require analysis rather than just a standard rule or procedure (Lamon, 2011). Therefore, it can be said that proportional reasoning includes understanding the proportional relationship in a situation invariant or covariant and solving it using logical thinking rather than standard procedures.

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Learning proportions are often difficult for middle school students due to not only understanding the problem at hand but also determining how to use problem-solving strategies (Irfan et al., 2019; Jitendra et al., 2017; Nunokawa, 2012). Students also often choose solving strategies depending on the context of the problem (Park et al., 2010). However, the cross-product strategy in solving student problems is the most commonly used, even though students do not fully understand the relationship between invariance and covariation (Mahlabela & Bansilal, 2015; Nugraha et al., 2016). The teacher must provide various types of proportional problems and instill the basics of ratio more deeply (Frith & Lloyd, 2016; I et al., 2018). The teaching and usage of proportional reasoning in everyday life will become more difficult if it is not understood conceptually but algorithmically (Dooley, 2006). Not only experienced by elementary and middle students, but classroom teachers also have difficulty understanding proportional relationships (Ben-Chaim et al., 2012; Irfan et al., 2018; Jacobson & Izsák, 2014; Lamon, 2007).

One of the crucial elements of proportional reasoning is the ability to identify proportional situations (Brown et al., 2019; Izsák & Jacobson, 2017; Lobato et al., 2010). Students still often use the addition strategy in solving a comparison problem, where they should use the multiplication strategy instead (Lamon, 2011). Proportional and non-proportional situations are important components that must be considered by the teacher (Ekawati et al., 2015). Defining and representing non-proportional relations seems to be their most challenging task (Arican, 2020). The use of an addition strategy between variables in solving a problem that uses proportional reasoning is called a non-proportional problem (Van Dooren et al., 2010). One of the non-proportional relationships between variables is the additive relationship. The additive relationship has a constant difference between the two quantities, so the correct answer for the other variable can also be identified by doing the sum.

An understanding of this multiplication relationship is essential because it is an indication of the ability of proportional reasoning. Changing students' thinking paradigms from addition to multiplication is very difficult. The capability to recognize non-proportional and proportional situations is very closely related to the knowledge possessed by the teacher. One aspect of teacher knowledge related to proportional reasoning that has received little concern so far is the teacher's ability to recognize and distinguish between non-proportional and proportional situations (Weiland et al., 2019). Mathematics teachers must understand the content to be taught deeply and comprehensively (Ball et al., 2008; Sa'dijah et al., 2021). Teachers should help students recognize how and when various ways of proportional reasoning may be appropriate for solving problems (NCTM, 2000).

There are several studies related to proportional and non-proportional situations carried out by teachers, including the studies from Nagar et al. (2016), Jacobson et al. (2018), Weiland et al. (2019), and Brown et al. (2019). The study carried out by Nagar et al. (2016) explored the extent to which secondary school teachers can correctly identify proportional situations when presented with various mathematical structures, along with the relationship between teacher attributes and their ability to identify them. Weiland et al. (2019) investigate a teacher's ability to correctly recognize non-proportional and proportional situations, as well as the possible factors that may be related to those abilities. Brown et al. (2019) investigate the ability of teachers to identify appropriate situations with proportional reasoning and the factors that might influence their abilities. These studies examine aspects similar to this study relating to proportional reasoning, especially understanding proportional and non-proportional situations.

Based on the findings of these previous studies, many teachers often fail to identify proportional and non-proportional situations separately or collectively. Consequently, the researcher conducted a preliminary study by giving problems related to proportional and non-proportional situations to 15 mathematics teachers who had taught proportional-related material with different backgrounds. The provided two problem situations are similar, so the teacher must focus on understanding to solve them. Determining whether a given situation is proportional becomes the most challenging issue for the teacher (Nagar et al., 2016). After the teachers answered the problem, the researcher reviewed the answers and conducted answer-based interviews to deepen the findings. Of the 15 teachers, 14 failed to interpret the situations. The researcher found several types of failures, which are divided into two. The first type is the teacher's failure to identify proportional and non-proportional situations, and the second is that they can only identify one situation. In the first type of failure, the teacher sees a non-proportional problem as a problem related to a proportional situation and solves the problem using a multiplicative relationship. Linearly, they do not see the proportional problem as a problem in a non-proportional situation and solve it using an additive relationship. The findings align with Nagar et al. (2016) that most teachers mistakenly identify non-proportional situations as proportional.

In the second type of failure, teachers can only correctly identify the problem as one situation and are wrong in identifying other situations. That happens when there are teachers who can identify and solve problems in non-proportional situations correctly but fail to identify and solve problems in proportional situations, and vice versa. Most of the mistakes in solving problems in non-proportional situations are caused by the teacher perceiving the problem as a proportional situation. These findings align with Brown et al. (2019) that teachers sometimes appear to believe that identifying any relationship between the two quantities indicates a proportional situation. It was also found that school mathematics teachers correctly identified proportional situations more often than non-proportional situations (Nagar et al., 2016), even when faced with a non-proportional situation (Jacobson et al., 2018; Weiland et al., 2019). This explanation shows that proportional and non-proportional situations are important components that teachers must consider (Jacobson et

al., 2018). This failure does not only occur in teachers but is also experienced by students (Atabaş & Öner, 2017; Tunç, 2020; Van Dooren et al., 2010) as well as future mathematic teachers (Izsák & Jacobson, 2017).

It is known that the 14 teachers come from different backgrounds, but the backgrounds of these teachers have no impact in correctly identifying proportional and non-proportional situations. It is supported by Nagar et al. (2016) that none of the teachers' background attributes seemed to relate to their ability to identify proportional situations. This is in line with Weiland et al. (2019), which found that the number of attended teacher preparation courses focused on teaching mathematics did not significantly correlate with teachers' ability to identify proportional and non-proportional situations. However, teachers must have the capability to distinguish proportional and non-proportional situations (Izsák & Jacobson, 2017) and communicate them to students (Lobato et al., 2010). This situation happens because their learning mathematics learning is more towards memorizing without any reasoning. This type of teaching must be avoided as memorization makes learning mathematics meaningless.

Teachers must realize that the development of proportional reasoning is not an easy task. Training proportional reasoning skills can be done by understanding the situation or context regarding the comparison problem. The proportional problem is divided into two: the problem of missing values and the problem of comparing (Lamon, 2007). The purpose of the missing value problem is to find a value that does not exist in a proportion, while the purpose of the comparison problem is to compare two ratios in a proportion. Learning does not only represent a study on cross-multiplication algorithms; students need to understand that various proportion problems require different cognitive skills to solve these problems (Toluk-Ucar & Bozkus, 2018). Problem-solving is considered an important skill in learning mathematics (Hidayah et al., 2020), aiding students to get actively involved in practicing mathematical knowledge and skills (Prayitno et al., 2020). Understanding to solve comparison problems using their strategy is very important for students before being introduced to the cross-multiplication algorithm (Fazio & Siegler, 2010).

Several studies related to proportional reasoning on identifying and solving problems in proportional and non-proportional situations have been widely carried out. However, research has not yet been found that examines explicitly the thinking characteristics of teachers who can interpret these two situations separately or collectively. In contrast to previous research, this study will examine specifically the characteristics of teachers who can interpret correctly when solving problems in proportional and non-proportional situations using the Meaning Based Approach (MBA) framework. The researchers took teachers as respondents because teachers are one of the main factors in learning. Based on the preliminary studies that have been done, the researchers found one teacher who was able to recognize and solve problems in non-proportional and proportional situations correctly. It is interesting to study in depth the characteristics of teachers who can interpret and solve problems in proportional and non-proportional situations separately and collectively, where this has not been disclosed in previous studies. Researchers use a framework MBA in mathematical problem solving to reveal the teacher's thinking characteristics. Pape (2004) argued that the MBA category is characterized by transformative behavior, which has three characteristics, namely recording the information provided, using context, and providing explanations and/or justifications for mathematical operations. Students who are in the MBA type usually carry out activities such as (1) understanding the meaning of the context of the problem, (2) deriving procedures from the context of the problem, (3) connecting one context to another, and (4) understand procedures based on the background of the problem (Subanji, 2012). Assessment of characteristics was carried out following the MBA components because the components reflect activity developed by the respondents. Therefore, we will investigate the characteristics of teachers in interpreting proportional and non-proportional situations in depth, based on a meaning-based approach.

Methodology

Research Design

This research is qualitative research with a case study method. Qualitative research relates to an idea from the subject under study (Sugiyono, 2008), while the case study method is a research method that explains a particular phenomenon from an individual, process, and so forth (Gall et al., 2014). Therefore, a case study was considered appropriate to explore mathematics teachers' thinking processes in detail, primarily in interpreting proportional and non-proportional situations separately or collectively by using a meaning-based approach.

Sample and Data Collection

The subjects in this study were math teachers who could interpret proportional and non-proportional situations separately or collectively. To find teachers who could do this, the researchers gave tests to 15 math teachers who had taught comparative material in class VII from different schools. These teachers consisted of 7 males and 8 females. It was found that 14 teachers failed to interpret proportional and non-proportional situations, which had previously been described in the introduction. One teacher was found capable of interpreting proportional and non-proportional situations collectively. Therefore, researchers want to examine in more depth the characteristics of the thinking process of the teacher. The researcher provided the participant's identity code with the designation GA to guarantee the subject's privacy. The subject was a male teacher with about 4 years of teaching experience and had taught comparative material in class VII relating to non-proportional and proportional situations.

The data in this study were collected directly by the researchers and assisted by using assistance instruments in the form of test instruments and task-based interviews. The proportional situation instrument was adapted from the book adding it up (National Research Council, 2001), while the non-proportional situation instrument was adapted from Cramer et al. (1993). Furthermore, the question was translated and adapted to the surrounding situation to help the subject to understand the problem easily. The test instrument consisted of one non-proportional situation and one proportional situation, as presented in Table 1 below.

Table 1. Research Instruments

1. Motors A and B travel at the same speed around a track. Motor A starts moving first. After motor A completes 9 revolutions, motor B completes only 3 revolutions. After Motor B has made 15 revolutions, how many revolutions has Motor A completed? (non-proportional situation)	2. In 3 months, plant A grew from 2 meters to 6 meters, and plant B grew from 4 meters to 8 meters. Which plants grow more? Is plant A, plant B, or both growing the same? (proportional situation)
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Question number one aims to determine the teacher's ability to recognize non-proportional situations, but the form of the questions given resembles proportional problems. This problem is a non-proportional situation that must be solved using the addition relationship between the two given numbers (the difference between the two numbers) and being able to apply it to find the missing value in the next variable. In question number two, it is known that the form of the question looks like a non-proportional problem, but in reality, the question is a question in the form of a proportional situation. Problem number two is a proportional situation that must be solved using a multiplication relationship. The researcher deliberately chose this question to test the teacher's level of understanding regarding the meaning of the given problem context.

Data collection techniques in this study were completed using test instruments and task-based interviews, in which subjects were asked to solve the given problems. Then, based on the written answers, the researchers arranged questions in the interviews to reveal the characteristics of interpreting and solving problems in proportionality and non-proportional situations. The main topic of the questions in the task-based interview posed to the subject relates to 1) how is the understanding of the meaning of the problem in proportional and non-proportional situations, 2) how to change the context of the problem in the form of a mathematical structure, 3) how to relate the problem to the proportional and non-proportional situation, 4) what strategy is used in solving the problem, and 5) how to interpret the strategy used in solving the problem.

Analyzing of Data

The data that had been obtained was then analyzed, classified, and concluded by the researchers based on the stated research objectives. The data analysis phase was carried out with the following systematics, namely; 1) understanding the results of the subject's answers by providing important notes, 2) transcribing interview data, 3) reducing data by elaborating the subject's answers with interview data, 4) grouping data based on meaning-based approach components, 5) concluding the characteristics of the teacher's meaning-based approach in making sense proportional and non-proportional situations.

Trustworthiness

One way to check the validity of research data was to use triangulation. Creswell (2012) explained that triangulation is a process to strengthen evidence from different types of data and data collection. Data on the results of the subject's answers were then strengthened again with task-based interviews. The researcher also wrote important notes regarding the subject's behavior when answering questions and interviews. The data collection was carried out by giving tests that the subject should complete within 28 minutes and in-depth interviews within 36 minutes. In this study, data reliability was re-evaluated by experts (Emzir, 2016).

Results

Based on the test instrument results, we explored the characteristics of one subject in making sense of non-proportional and proportional situations using a meaning-based approach. The following is a presentation and analysis of data from subjects GA.

Analysis of Data from Subjects about Non- Proportional Situations

In identifying a non-proportional situation in question one, GA started by reading and observing the information in the question. GA then wrote down the information that motor A goes 9 turns, and motor B goes 3 turns, as seen in Figure 1.

Motor A = 3 9 Putaran Motor B = 3 Putaran	Translate version: Motor A = 9 laps Motor B = ? laps
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Figure 1. Writing the Information

GA realized that information from the questions could affect problem-solving. When reading the questions, GA was confused because there were two possible answers, namely, using the difference or the form of comparison. Then GA re-examined the question order before finally deciding to use the difference based on the context of the problem in a non-proportional situation, as shown in the following interview excerpt.

- GA : I am confused about this question because there are two perceptions, namely, using difference and using comparison.
- P : Then?
- GA : (The subject then paused for a moment and returned to reading the question. After a while of thinking, the subject finally said). However, after I reread it, I realized that solving this problem using differences.
- P : Why can there be two perceptions?
- GA : Similar to the class comparison material, the information is slightly different.

At this point, GA used previous knowledge possessed when teaching about proportions to understand the problem. GA was very careful in understanding the problems and did not immediately decide to use assumptions about the material that had been taught. GA's decision to use "difference" in solving the problem can be seen in Figure 2.

<p>Motor A = secepat 9 putaran } selisih 6 Motor B = 3 putaran } Karena kecepatan dan lintasanya sama</p>	<p>Translate version</p> <p>Motor A = 9 laps } Differences 6 Motor B = ? Laps } Because the speed and the laps are the same</p>
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Figure 2. Relationship of Key Information

GA can find key information from the questions given. This can be seen in the following interview.

- P : Where do these 6 differences come from?
- GA : I know, based on the information in the question, that is after motor A starts to move first. The problem command is clear that the start is not together, and the speed is the same. This means motor A has driven 6 new rounds of motor B, starting to run.

Based on this information, GA understood that the difference in the number of rotation paths of motorcycles A and B would always be 6. GA also knew that the difference would always be 6. This can be seen in the following interview.

- P : Is the difference always 6?
- GA : When motor A goes 6 laps, then B just starts driving. When motor A goes 7 laps, then motor B only gets 1 lap because the speed is the same. When motor A goes 8 laps, motor B gets 2 laps, and so on.

GA knew that if the information in the problem changes, the solution method also changes. This can be seen in the following interview excerpt.

- P : Is it possible to use a comparison form?
- GA : When the initial information related to the 'same speed' and 'going first' is omitted, it can only be solved using a comparison formula.
- P : What formula do you use?
- GA : Use cross-multiplication.

Based on this, it can be said that GA can connect one context to another, showing the subject has flexibility in understanding a non-proportional situation context that can turn into a proportional situation context. When the researcher asked about the formula used in the context turned into a proportional situation, the subject used the cross multiplication formula. This is based on practice during classroom learning which usually using cross-multiplication.

GA can write correct problem-solving and understand procedures and algorithms based on the problem. The subject's answers are illustrated in Figure 3.

<p>Karena kecepatan dan luntaran yang sama artinya 12 12: Motor B = 3 + 12 = 15 putaran Motor A = 9 + 12 = 21 putaran //</p>	<p>Translate version: Because the speed and the laps are the same That means, Motor B = 3 + 12 = 15 laps Motor A = 9 + 12 = 21 Laps</p>
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Figure 3. Troubleshooting Procedures and Algorithms

Based on the answers, it is known that GA can identify non-proportional situations and find the right solution. In determining the number of laps of motor A, GA referred to the speed of motors A and B, which were on the same path. The only difference is the start of the start. Based on this information, GA concluded that the difference in motor A and motor B lap would be the same. When the researcher asked about the number 12 in the answer, GA replied that the speed remains the same. For example, if 9 plus 1, then 3 is also added by 1, so if motor B gets 15 laps, then 3 plus how much produce 15? So the number 12 is obtained, as 3 plus 12 is 15. Since motor B is added by 12, motor A is added by 12, so $9 + 12 = 21$.

GA subjects have flexibility in problem-solving strategies. For example, GA does not refer to the previous understanding of using the difference in rotational speed of motors A and B, which is 6, but refers to the rotational speed of motor B at the beginning and end, where the difference is 12. Thus the difference between motors A also 12. This can be seen in the following interview.

P : Why not use a difference of 6?

GA : Because the information provided is the same speed, the difference between the final lap rate and the initial lap rate of the two motors will also be the same

Based on this, subjects fully understand the key information provided on the problem. Besides, GA also has flexibility in solving problems.

Analysis of Data from Subjects about Proportional Situations

In identifying the proportional situation in question two, GA started by reading the question and looking at the information in the problem. GA then wrote the information, as seen in Figure 4.

<p>Tanaman A = 2 → 6 Tanaman B = 4 → 8</p>	<p>Translate version: Plant A = 2 → 6 Plant B = 4 → 8</p>
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Figure 4. Writing the Information

GA could determine the relationship of general information to the problem posed. This is based on the following interview excerpt.

P : What does this mean (while pointing to writing $2 \rightarrow 6$ and $4 \rightarrow 8$)?

GA : Plant A grows from two to six, while plant B grows from four to eight.

GA can determine the relationship of the available information based on the problem posed. The information shows that plants A and B change is determined based on the initial growth. The subject transforms the information provided into a series to make it easier, as seen in Figure 5 below.

$$A = 2, 4, 6$$

$$B = 4, 8$$

Figure 5. Relationship of Key Information

To further explore the subject's answer, the researcher then conducted interviews. The following are excerpts from interviews related to changes in plant growth A and B.

P : What does it mean (pointing to the already written series)?

GA : Plant A grows from 2 to 6, showing the addition of the number 2 two times, while in plant B, there was a change in the number 4 one time.

P : Where is the change?

GA : 2 to 4, then to 6. From 2 to 4, it changes once, then 4 to 6 changes once, so the change from 2 to 6 is two times. While the change from 4 to 8 was only once.

This indicates that the series is the result of his thinking about changes in plant growth. Therefore, the subject can determine the relationship from the information known to the problem posed, namely knowing that changes in plant growth A and B are different where the changes in growth are based on multiplication from initial to late growth of each plant.

GA is aware of the key information in the problem and can reflect on his knowledge so that it does not change the strategy for solving the problem. Here is an excerpt from the interview.

P : What is the third level?

GA : Plant A becomes 8 while plant B becomes 12. So that the difference between plants A is 6 and B is 8. The value is greater than the difference in plant B, but still, the change is greater for A because it refers to the rate of change based on the original value. If the origin is both 4, then the change is easy to know.

GA realizes that the possible answer can be identified using the addition relationship (difference). However, GA can not use it because, based on key information, it is understood that it must use the multiplication relationship.

GA then wrote answers based on the strategies obtained using the multiples strategy. For example, GA wrote that the plant with the most growth is planted A because the increase is more than two times compared to B, which has only one increase, as shown in Figure 6. below.

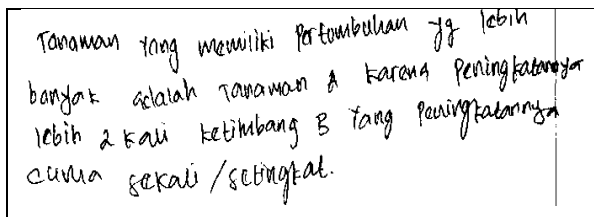
	<p>Translate version: The plant that has more growth is plant A because the increase is more than twice that of B, which only once</p>
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Figure 6. Answer Conclusion

GA subjects have flexibility in changing the context of the problem, meaning that GA can distinguish an alternative problem-solving after the context of the given problem is changed. Here is an excerpt from the interview.

P : Can the growth be the same?

GA : It can be the same if the initial values are the same. It could also be in this case, and the value is the same if the rate of increase is also the same

Discussion

Proportional reasoning is very important in learning mathematics. One of the main components of proportional reasoning is distinguishing proportional and non-proportional situations. A meaning-based approach is needed to find out the characteristics of the teacher in interpreting the two situations separately or collectively, starting from understanding the meaning of the problem context, deriving procedures from the problem context, connecting one context with another context, and understanding procedures and algorithms based on the background of the problem. Many teachers fail to distinguish between non-proportional and proportional situations. Several previous studies have found that middle school math teachers seem to frequently identify situations as proportional even when the situations are non-proportional (Atabaş & Öner, 2017; Brown et al., 2019; Nagar et al., 2016; Weiland et al., 2019). To follow up on this finding, researchers try to explore subjects who can interpret non-proportional and proportional situations separately or collectively. It was found that understanding the meaning of the problem's context requires caution in understanding information, identifying key information, and recognizing the existence of multiple meanings from the context of situations related to contextual structures. This is in line with Steinhorsdottir and Sriraman (2009) research, which explains that understanding is required regarding the contextual structure before solving the problem of proportions using proportional reasoning.

These three characteristics in understanding the meaning of the context of the problem are an early indication of the use of proportional reasoning, which significantly determines the next step, namely deriving procedures from the context of the problem. The subject's characteristics in deriving procedures from the context of the problem are by first identifying the difference (additive) relationship in a non-proportional situation and identifying the multiple (multiplicative) relationship in a proportional situation written in symbolic form. This follows the research of Brown et al. (2019) that the source of knowledge about multiplicative comparisons may be very important for identifying proportional or non-proportional situations. While writing in symbolic form has been explained by Izsák and Jacobson (2017), constructing symbolic forms will be an additional aspect of increasing the ability to understand multiplicative concepts. The

relationship between addition and multiplication closely relates to the number structure understood. This is in line with Pelen and Artut (2016) that the number structure in proportional reasoning influences the strategy used. If the teacher experiences an error in reducing the context of proportional or non-proportional problems, it will result in mistakes in the next step. After the subject can interpret the information from the problem correctly, the subject can also represent it in a mathematical form of a number structure in proportional and non-proportional situations. Therefore, teachers need to have good skills in understanding the meaning of the contextual structure to become a number structure in the problem.

The next component is connecting one context with another context. In completing this stage, characteristics of our subject include a) having flexibility in understanding proportional situations, b) having flexibility in understanding non-proportional situations, and c) having flexibility in understanding proportional and non-proportional situations collectively. This finding is similar to Berk et al. (2009), that flexibility ability is often used in activities related to proportional reasoning. In this study, it is found that this flexibility enables the subject to stay unaffected by the understanding of the learning material he had taught. Instead, the subject is able to connect the context of a situation that seemed non-proportional to a proportional situation and a proportional situation as non-proportional. The subject realizes that if the key information from a non-proportional situation changes, the problem situation can change to a proportional situation. This is different (contrast) from the findings of Brown et al. (2019) that teachers sometimes seem to believe that identifying any relation between two quantities indicates a proportional situation.

The last component is that the subject understands procedures and algorithms based on the background of the problem with a detailed understanding of the systematics of problem-solving, starting from making sense of the problem and its relationships to the strategies used in the context of non-proportional and proportional situations. The subject's systematics in solving problems has detailed steps starting from interpreting the problem in the form of a proportional or non-proportional situation, understanding the number structure based on the existing contextual structure, understanding the fundamental relationship between proportional and non-proportional situations, to being able to interpret strategy used in solving the given problem. This is supported by Ekawati et al. (2015), that the hierarchical level of mathematics content knowledge about the proportion of teachers starts with identifying the number structure in the situation, the meaning of proportional and non-proportional situations, and figural representation. It is also supported by Brown et al. (2019) that sources of knowledge about the mathematical structure of the situation are the most helpful in appropriately identifying the relationships. The strategy used by the subject in solving proportional problems is using figural representation in the form of sequences while solving non-proportional problems using subtraction arithmetic operations (additives). It is supported by Lamon (2011), who describes that someone with proportional reasoning can assess if a given situation is proportional, so they will not randomly use the procedure if the quantity is non-proportional.

When the subject realizes that the key information from a non-proportional situation changes, then the problem situation can change to a proportional situation, inducing a change in the problem-solving strategy. The strategy used in solving non-proportional or proportional problems depends on the given problem (Arıcan, 2020; Park et al., 2010). In this study, our respondent changes the problem-solving strategy into cross multiplication. It appears that even though the subject knows cross multiplication, it is not used in solving non-proportional situations problems. The source of knowledge of the cross-multiplication procedure can be useful, but it is not appropriate to apply it in a non-proportional situation (Brown et al., 2019). The findings show that the subject has flexibility in seeing an alternative problem-solving in the context of the given problem is changed. Flexibility refers to the ability to change ideas to get a variety of solution strategies (Berk et al., 2009; Subanji et al., 2021). Therefore, teachers must be flexible in implementing problem-solving strategies. Besides, Jacobson et al. (2018) revealed that the mathematical flexibility of teachers in understanding different solution methods from their own is very important.

Conclusion

In this study, we found the characteristics of teachers' proportional reasoning when faced with proportional and non-proportional situations, which in previous studies had not been thoroughly elaborated. This finding has contributed to an effort to seek answers from previous literature where many teachers were mistaken in using proportional and non-proportional situations separately or collectively. We use the help of a meaning-based approach framework to find out the characteristics of mathematics teachers in dealing with problems of proportional and non-proportional situations separately or collectively. The characteristics of these teachers include being careful in understanding information, aware of multiple meanings, and knowing key information in understanding the contextual structure of proportional and non-proportional situations. Besides, they also identify additive and multiplicative relationships in solving the problems. It takes the ability to change the contextual structure into a number structure. The next characteristic is having flexibility in understanding proportional and non-proportional situations separately or collectively. The final characteristic understands the systematics of problem-solving in detail, from interpreting the problem and its relationships to being flexible in problem-solving strategies based on the context of situations.

Recommendations

Proportional reasoning plays an important role in learning mathematics at school. One of the initial indications that a person has proportional reasoning is their capacity to distinguish between proportional and non-proportional situations. Based on the research findings, there are important aspects that teachers must have, namely understanding key information and its use which will lead to flexibility abilities, where teachers are aware of changes in understanding proportional and non-proportional situations collectively. Understanding this key information will also lead to other teacher abilities, such as flexibility in interpreting a ratio as a unit, covariation, and using various exploratory strategies. It is also hoped that teachers will be able to apply this in the learning process in class, especially when teaching material related to proportional reasoning. This can be beneficial because it helps teachers to convey learning material, transform learning becomes effective and efficient, and aid material delivery to students. We hope that other researchers will be able to dig deeper into how teachers interpret proportional reasoning from other important components, such as understanding a ratio as a unit, understanding covariation, and using various problem-solving strategies that are not based on ready-made algorithms. This research can also be used as a reference for further research.

Limitations

This research is limited to qualitative data collection by using assignments and interviews. Further research is needed regarding implementing teachers' proportional reasoning when learning in the classroom regarding the theory and this study's findings. Besides, it is also necessary to do research that involves students in learning using proportional reasoning. This research also reveals only one component of proportional reasoning. Further and in-depth research is needed regarding the meaning of proportional reasoning when faced with other components, such as understanding a ratio as a unit, understanding covariation, and exploring various problem-solving strategies.

Authorship Contribution Statement

Yandika Nugraha: Conceptualization, data collection, data analysis, drafting, and revision of the manuscript. Cholis Sa'dijah: Corresponding author, conceptualization, design of the data, critical revision of the manuscript, and final approval. Susiswo: Conceptualization, design of the data, supervision, and final approval. Tjang Daniel Chandra: Conceptualization, design of research instruments, and technical support.

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