Misconceptions of 9th Grade Students about Numbers

Yeliz ÇELEN
Assist. Prof. Department of Education, Amasya University, Turkey
Yeliz.celen@amasya.edu.tr
ORCID: 0000-0002-7991-4790

Abstract
Misconceptions occur as a result of learners' wrong beliefs and experiences, and because subsequent learning is built on these misconceptions, they cause new concepts to be learned incorrectly. Studies show that students have problems in making sense of many mathematical concepts in mathematics teaching processes and this situation can be encountered at different learning levels. In this study, it was tried to determine the misconceptions of 9th grade students on the subject of numbers, since the studies were carried out at the primary or university level and the students' basic knowledge about different number sets was formed in the 9th grade. This research is in the scanning model. In this study, qualitative and quantitative data analysis methods were used together, and the interview method was used to collect qualitative data. Open-ended questions prepared by the researcher were used to collect the quantitative data of the study. The sample of the research consists of 50 9th grade students studying in Ankara in the 9th grade and participating in the study on a voluntary basis through easy sampling. As a result of the research, it is seen that 9th grade students generally have misconceptions about the definitions and representations of irrational and rational numbers, students who have misconceptions see all the numbers given in the root as irrational numbers and have difficulty in showing these numbers on the number line.
Key words: Misconceptions, mathematics education, number set.

INTRODUCTION
The objectives of the mathematics education program are to train individuals who can make sense of mathematical concepts, have the power of mathematical thinking, see the relationships between mathematical symbols or definitions, and benefit from mathematics in problem solving and modeling (MEB, 2018). The main point that these goals take us to is to raise people who value mathematics and have mathematical literacy. However, raising individuals with this characteristic is very difficult due to the nature of mathematics, as students see mathematics as a difficult discipline to learn, develop negative attitudes towards mathematics or the teacher, or make false learnings (Shiland, 1998).

It is a problem-solving, answer-finding and proving activity based on basic principles and concepts, structured in a mathematical system (Çelen, 2011). Mathematics teaching is also known as relational understanding, enabling students to learn mathematical concepts and make sense of conceptual knowledge respectively (conceptual knowledge of mathematics), understanding the relationships between mathematical operations (procedural knowledge of mathematics), seeing the relationships between conceptual and procedural knowledge (connections). between conceptual and procedural knowledge) activities (Van de Wella, 1989, 6). The use of constructivist learning approach in curriculum renewal or revision studies since 2004 has also been effective in transferring this systematic structure of mathematics to students. With constructivist learning, activities such as showing sensitivity to the learner's previous learning, preventing mislearning and misconceptions, and creating multiple representations for the learner have entered the educational life (Ishii, 2003). One of the goals in constructivist teaching processes is for students to recognize and construct information with their own methods, to know where and how to use it, and to produce new information by making use of this information (Abbott & Ryan, 1999). For this reason, learning mathematical concepts correctly is of great importance in terms of structuring new knowledge. However, one of the difficulties in teaching mathematics is the misconceptions that students have gained in their previous educational experiences (Osborne, Bell, & Gilbert, 1983; Byrd, McNeil, Chesney, & Matthews, 2015).

While misconceptions are defined by Ojose (2015) as misunderstandings or evaluations caused by learners' misunderstandings caused by deficiencies in their knowledge in general, Meşeci, Tekin, and Karamustafaoğlu (2013) define it as incorrect concepts used by the learner to show the knowledge or skill that is accepted as correct. . No matter how they are defined, it is a fact that misconceptions are inconsistent with realities found to be true by people who are experts in a discipline (Koçyiğit & Zembat, 2013).

Studies show that students have problems in making sense of many mathematical concepts in mathematics teaching processes and this situation can be encountered at different learning levels (Griffiths & Preston, 1992; Zoller, 1990). The main reasons for students to have misconceptions can be counted as the fact that students come to learning environments with misconceptions mostly stemming from natural life in their previous lives, that concepts are not connected within themselves and with daily life during the teaching process, and that education and training environments are not organized in accordance with learning (Lawson & Thomson, 2008). 1988). Baykul (2005) also states that students may fall into operational misconceptions if mathematical concepts are not properly learned by the student, connections between operations are not established, or both situations occur at the same time. Since mathematics is a discipline taught with sequential and spiral teaching techniques, it is frequently seen that students have misconceptions in mathematics teaching processes. (Türkdoğan, Güler, Bülbül, & Danişman, 2015). As in other lessons, identifying and eliminating misconceptions in mathematics will make it easier for students to construct mathematical structures and learn with permanent traces, and will allow teachers to structure their mathematics teaching processes with more appropriate methods, knowing these misconceptions (Stefanich & Rokusek, 1992; Ojose, 2015).

Misconceptions occur as a result of learners' wrong beliefs and experiences, and because subsequent learning is built on these misconceptions, they cause new concepts to be learned incorrectly (Baki, 1998). The reasons for the formation of misconceptions in learners can be listed as the inability to establish the integrity of meaning during the learning of the concepts, the inadequacy of the learner in using their own ready-made knowledge, and the mistakes of the teacher in conveying the conceptual knowledge to the student (Köroğlu et al., 2003). Correction of misconceptions in mathematics is important in terms of preventing false learning at later levels.

Studies show that students have different misconceptions, especially in terms of number sets that include the concept of infinity (Fishbein, Jehiam, & Cohen, 1995; Güven, Çekmez, & Karataş, 2011). In the study conducted by Peled (1999) in order to determine the misconceptions of students about irrational numbers, it was concluded that the students could not approximately estimate the values of these numbers as rational numbers and they could not show these numbers on the number line. In this study, it was concluded that university students could not establish the relations between the number sets at the desired level. In this study, it was tried to determine the misconceptions of 9th grade students on the subject of numbers, since the studies were carried out at the primary or university level and the students' basic knowledge about different number sets was formed in the 9th grade.

NUMBER SETS IN THE CURRICULUM

In the 9th grade curriculum, the subject of number sets was included as 8 lesson hours under the Equations and Inequalities sub-learning. This class hour constitutes 4% of all 9th Grade mathematics learning time. The learning outcome and its sub-components for number sets are given in Figure 1.

9.3.1. Sayı Kümeleri

Terimler ve Kavramlar: doğal sayılar, tam sayılar, rasyonel sayılar, irrasyonel sayılar, gerçek (reel) sayılar

Sembol ve Gösterimler: $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{Q}'$, $\mathbb{R}$, $\mathbb{Z}^+$, $\mathbb{Q}^+$, $\mathbb{R}^+$, $\mathbb{Z}^-$, $\mathbb{Q}^-$, $\mathbb{R}^-$, $\mathbb{R}\times\mathbb{R}$, $\mathbb{R}^2$

9.3.1.1. Sayı kümelerini birbiriyle ilişkilendirir.

a) Doğal sayı, tam sayı, rasyonel sayı, irrasyonel sayı ve gerçek sayı kümelerinin sembolleri tanıtılarak bu sayı kümeleri arasındaki ilişki üzerinde durulur.

b) $\sqrt{2}, \sqrt{3}, \sqrt{5}$ gibi sayıların sayı doğrusundaki yeri belirlenir.

c) Gerçek sayılar kümesinde toplama ve çarpma işlemlerinin özellikleri üzerinde durulur.

ç) $\mathbb{R}$ nin geometrik temsilinin sayı doğrusu, $\mathbb{R}\times\mathbb{R}$ nin geometrik temsilinin de kartezyen koordinat sistemi olduğu vurgulanır.

9.3.1. Number Sets
Terms and Concepts: natural numbers, integers, rational numbers, irrational numbers, real numbers
9.3.1.1. Relates sets of numbers to each other.
a) By introducing the symbols of natural number, integer, rational number, irrational number and real number sets, Emphasis is placed on the relationship between sets of numbers.
b) The place of numbers such as $\sqrt{2}, \sqrt{3}, \sqrt{5}$ on the number line is determined.
c) The properties of addition and multiplication operations in the set of real numbers are emphasized.
ç) The geometric representation of $\mathbb{R}$ is the number line, and the geometric representation of $\mathbb{R}\times\mathbb{R}$ is the Cartesian coordinate system.
is highlighted.

Figure 1. Number Sets acquisition and its subcomponents

When Figure 1 is examined, it is seen that at this level, it is aimed for students to define sets of numbers and see the relationships between them, to determine the location of some irrational numbers on the number line, to show the Cartesian coordinate system as a representation, and to perform addition and multiplication operations on the set of real numbers.

There are different studies in the literature regarding the misconceptions encountered in learning number sets by students (Aztekin, 2008; Fishbein, Jehiam, & Cohen, 1995). While some of these studies show that students construct the concept of infinity as potential infinity in number sets (Aztekin, 2008), some of them are misconceptions due to not understanding that irrational numbers do not include all points in a range, despite being taught that they are dense everywhere (Fishbein, Jehiam, & Cohen, 1995). Kara and Delice (2011) also state that students know the lexical meaning of irrational numbers, but they have difficulty in understanding that these numbers are rational numbers and showing them as symbols.

METHOD

This research, which aims to identify 9th grade students' misconceptions about number sets, is in the screening model. In this study, qualitative and quantitative data analysis methods were used together, and the interview method was used to collect qualitative data. Open-ended questions prepared by the researcher were used to collect the quantitative data of the study.

Sample of the Research

The sample of the research consists of 50 9th grade students studying in Ankara in the 9th grade and participating in the study on a voluntary basis through easy sampling.

Data collection tool

In the research, open-ended questions prepared by the researcher were used as a data collection tool in order to determine the misconceptions of the students about number sets. While preparing these questions; Attention was paid to the misconception classification prepared by Güneş (2007). Güneş (2007) describes misconceptions as those stemming from non-scientific beliefs stemming from students' knowledge other than scientific reality, such as legendary speeches; Conceptual misunderstandings and misunderstandings originating from spoken language, which are caused by their previous mislearning and cause problems in their new learning, are grouped as those originating from modelling, those originating from geometric or symbolic representation, and misconceptions about definitions and properties. Efforts were made to create questions to identify misconceptions. In addition to these questions, a questionnaire consisting of five items was applied to the students in order to collect the demographic information of the students. The answers given to the open-ended questions applied to the students were evaluated by the researcher and the situations that caused the misconceptions were exemplified in the study. In order to make sense of the answers given by the students and to identify the existing misconceptions in detail, interviews were conducted with the students who had misconceptions in their answers, and a semi-structured interview form prepared by the researcher was used for the interviews. Students' misconceptions were presented as S1, S2, S3… by specifying the sequence number given to the students.

Data analysis

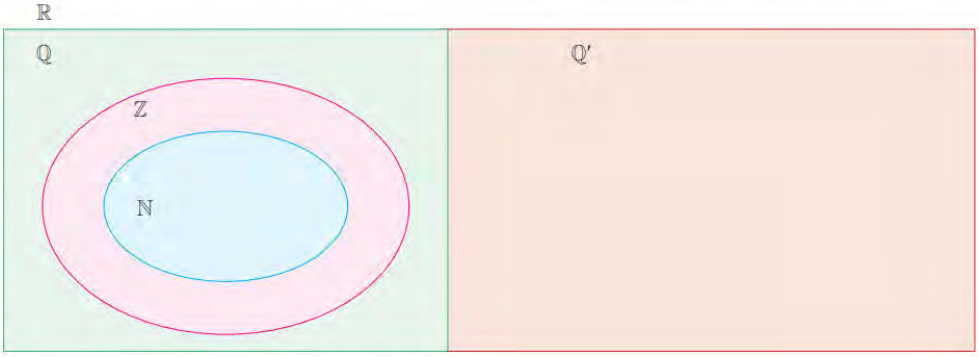
The data required for the research were obtained as a result of the questions prepared by the researcher and the interviews with the students who had misconceptions. In the analysis of the data, descriptive analysis methods and document analysis method were used.

RESULTS

The answers given by the students to the questions applied to the students in order to identify the misconceptions about the subject of number sets are explained by giving examples in this section of the research. The first question asked to identify students' misconceptions about number sets and their answers are given below:

1.Question :

1,4; – 8; $-\sqrt{9}$; 1; 0; 13; $\frac{5}{4}$; $-3,\overline{4}$; $\sqrt{5}$; $3+\sqrt{2}$; π sayılarını aşağıdaki sayı kümesi tablosunda uygun yerlere yerleştirelim.



1,4; – 8; $-\sqrt{9}$; 1; 0; 13; $\frac{5}{4}$; $-3,\overline{4}$; $\sqrt{5}$; $3+\sqrt{2}$; π Let's place the numbers in the appropriate places in the number set table below.

Answer:

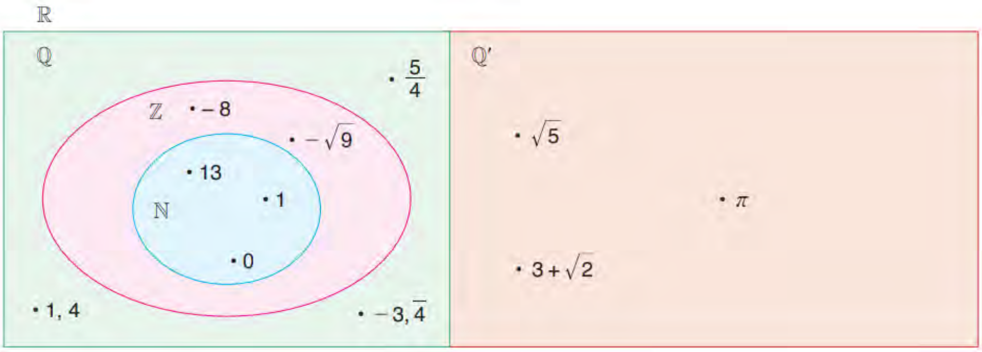


Table 1. Findings regarding the answers given by the students to Question 1

	N	Percantage (%)	Examples of Misconceptions
Number of students who answered correctly	24	48	Ö12: I think the number pi is rational. Because rational numbers can be represented as fractions. Since the number pi is written as 22/7, it is a rational number, why would it be irrational? Ö17: Periodic numbers go on forever. So these numbers are written in an interval infinite times. So this number must be in the set of irrational numbers. I think so. Doesn't the value of the root 9 number go to infinity? Shouldn't this number be irrational? Ö29: Numbers that are not rational are called irrational numbers. In other words, numbers that cannot be known exactly after the comma and go to infinity are called irrational numbers.
Number of students who gave wrong answers	17	34	
Number of students with incomplete answers	9	18	
Number of Students with Misconceptions	22	44	

When Table 1 is examined, it is seen that the students confuse the rational numbers with the irrational number sets, and they interpret the definitions and features of the irrational number sets differently due to their incorrect prior learning or evaluation. It is seen that 44% of the students have misconceptions on this subject. As a result of the

interviews with the students who made misconceptions, it is seen that the students do not have the knowledge that the decimal expansions of irrational numbers do not repeat themselves and take forever, and that some students do not know that there are no both rational and irrational numbers. It was also observed that some of the students knew that the set, which is the combination of the set of irrational numbers and the set of rational numbers, is called the set of real (real) numbers, but they confused the representation of this set with the representation of the set of rational numbers as symbols. It is also among the factors determined for this question that students have no difficulty in placing integers and natural numbers into existing sets.
The second question asked to identify students' misconceptions about number sets and their answers are given below:

2.Question:

• $\sqrt{2,5}$ • $\sqrt{\frac{16}{4}}$ • $0,\overline{16}$ • $0,02$

sayılarından hangisinin rasyonel sayı olmadığını bulalım.

Let's find out which of the numbers is not a rational number.

Answer:

• $\sqrt{\frac{16}{4}} = \frac{4}{2} = 2$

• $0,\overline{16} = \frac{16}{99}$

• $0,02 = \frac{2}{100} = \frac{1}{50}$ şeklinde yazılabilir. Ancak $\sqrt{2,5}$ bu şekilde yazılamaz. Diğerleri rasyonel sayı olduğu hâlde, $\sqrt{2,5}$ rasyonel sayı değildir.

can be written as But 2,5 cannot be written that way. While the others are rational numbers, 2,5 is not a rational number.

Table 2. Findings regarding the answers given by the students to Question 2

	N	Percantage (%)	Examples of Misconceptions
Number of students who answered correctly	28	56	Ö27: Since there is a line above 0.16, it goes on forever. Aren't such numbers, the numbers that can be at any point in a range, not irrational numbers?
Number of students who gave wrong answers	22	44	Ö43: I thought the numbers in the square root were all irrational. Since I cannot mark two options, I marked one and passed.
Number of students with incomplete answers	0	0	Ö46: Well, the only thing I'm sure of is that 0.02 is rational, I don't know about the others.
Number of Students with Misconceptions	18	36	

As can be seen from Table 2, it is seen that students have deep-rooted misconceptions about the definitions and properties of rational and irrational numbers. As a result of the individual interviews with the students, it was seen that the students did not know that cyclic decimal numbers were rational numbers, that they lacked knowledge about the repetition of the numbers in the decimal part of the rational numbers written in decimal form, and that they had misconceptions based on definition such as that these numbers are irrational numbers.

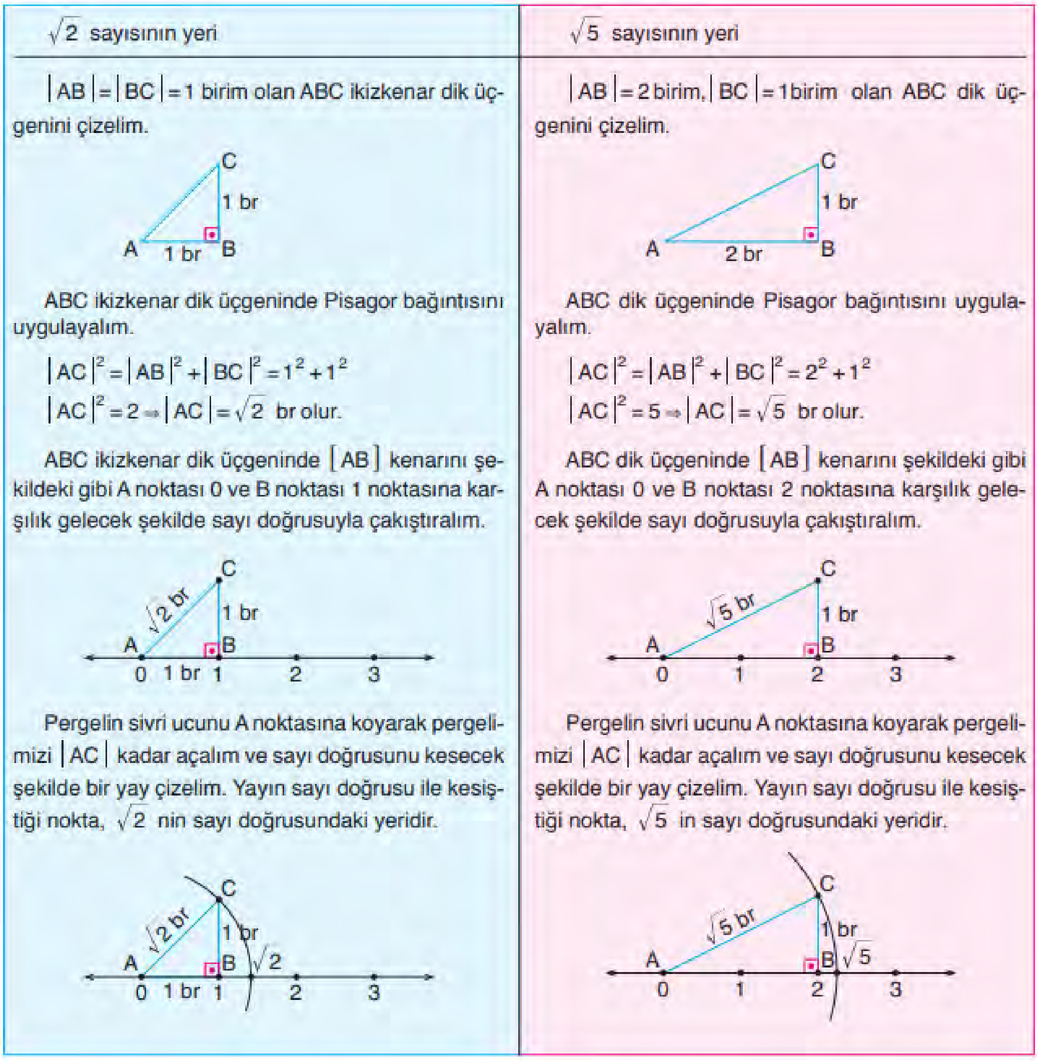
The third question asked to identify students' misconceptions about number sets and their answers are given below:

3. Question:

$\sqrt{2}$ ve $\sqrt{5}$ sayılarının sayı doğrusundaki yerlerini belirleyelim.

Let's find the places of the numbers on the number line.

Answer:



$\sqrt{2}$ sayısının yeri

$|AB| = |BC| = 1$ birim olan ABC ikizkenar dik üçgenini çizelim.

ABC ikizkenar dik üçgeninde Pisagor bağıntısını uygulayalım.

$|AC|^2 = |AB|^2 + |BC|^2 = 1^2 + 1^2$

$|AC|^2 = 2 \Rightarrow |AC| = \sqrt{2}$ br olur.

ABC ikizkenar dik üçgeninde $[AB]$ kenarını şekildeki gibi A noktası 0 ve B noktası 1 noktasına karşılık gelecek şekilde sayı doğrusuyla çakıştıralım.

Pergelin sivri ucunu A noktasına koyarak pergelimizi $|AC|$ kadar açalım ve sayı doğrusunu kesecek şekilde bir yay çizelim. Yayın sayı doğrusu ile kesiştiği nokta, $\sqrt{2}$ nin sayı doğrusundaki yeridir.

$\sqrt{5}$ sayısının yeri

$|AB| = 2$ birim, $|BC| = 1$ birim olan ABC dik üçgenini çizelim.

ABC dik üçgeninde Pisagor bağıntısını uygulayalım.

$|AC|^2 = |AB|^2 + |BC|^2 = 2^2 + 1^2$

$|AC|^2 = 5 \Rightarrow |AC| = \sqrt{5}$ br olur.

ABC dik üçgeninde $[AB]$ kenarını şekildeki gibi A noktası 0 ve B noktası 2 noktasına karşılık gelecek şekilde sayı doğrusuyla çakıştıralım.

Pergelin sivri ucunu A noktasına koyarak pergelimizi $|AC|$ kadar açalım ve sayı doğrusunu kesecek şekilde bir yay çizelim. Yayın sayı doğrusu ile kesiştiği nokta, $\sqrt{5}$ in sayı doğrusundaki yeridir.

The findings regarding the answers given by my students to the 3rd question are given in Table 3.

Table 3. Findings regarding the answers given by the students to Question 3

	N	Percantage (%)	Examples of Misconceptions
Number of students who answered correctly	12	24	Ö8: I don't know how to mark the square root of 2 on the number line. I don't know what a geometric interpretation is either.
Number of students who gave wrong answers	29	58	Ö39: I think that since square root numbers are irrational, they cannot be represented on the number line.
Number of students with incomplete answers	9	18	Ö41: I guessed that the square root numbers are between which numbers on the number line. I think the square root of 5 should be between 4 and 5.

Number of Students with Misconceptions	27	54	

When Table 3 is examined, it is seen that the students do not have a geometric interpretation of the representation of irrational numbers on the number line, they have difficulty in finding or estimating the approximate values of irrational numbers, and they think that the places of the square root numbers 5 and 5 are close to each other on the number line. It is seen that 18% of the students who make wrong analysis due to their past learning and beliefs about the representations of irrational numbers on the number line are closed to new learning.

The fourth question asked to identify students' misconceptions about number sets and their answers are given below:

4. Question:

Aşağıda verilen eşitlikleri inceleyelim. Bu eşitliklerde toplama ve çarpma işlemlerinin hangi özelliklerinin kullanıldığını belirleyelim.

a) $6+\sqrt{7}=\sqrt{7}+6$

b) $\sqrt{3}+0=0+\sqrt{3}=\sqrt{3}$

c) $7\cdot 0=0\cdot 7=0$

ç) $2\sqrt{2}\cdot\frac{1}{2\cdot\sqrt{2}}=\frac{1}{2\sqrt{2}}\cdot 2\sqrt{2}=1$

Let's examine the equations given below. Which properties of addition and multiplication operations are in these equations? Let's determine what is used.

Answer:

a) $6+\sqrt{7}=\sqrt{7}+6$ (Gerçek sayılar kümesinde toplama işleminin değişme özelliği)

b) $\sqrt{3}+0=0+\sqrt{3}=\sqrt{3}$ (Gerçek sayılar kümesinde toplama işleminin etkisiz eleman özelliği)

c) $7\cdot 0=0\cdot 7=0$ (Gerçek sayılar kümesinde çarpma işleminin yutan eleman özelliği)

ç) $2\sqrt{2}\cdot\frac{1}{2\cdot\sqrt{2}}=\frac{1}{2\sqrt{2}}\cdot 2\sqrt{2}=1$ (Gerçek sayılar kümesinde çarpma işleminin ters eleman özelliği)

The findings regarding the answers given by the students to the 4th question are given in Table 4.

Table 4. Findings regarding the answers given by the students to Question 4

	N	Percantage (%)	Examples of Misconceptions
Number of students who answered correctly	29	58	Ö5: I can only make one swallower. I don't know the others.
Number of students who gave wrong answers	11	22	Ö23: How can square root numbers be commutative? I don't know if typing before or after when adding changes the result.
Number of students with incomplete answers	10	20	

Number of Students with Misconceptions	6	12	

When Table 4 is examined, it is seen that the students' misconceptions about the subject are 12%. As a result of the interviews with the students, it was seen that the students lacked knowledge about the properties of the numbers in the real numbers set regarding addition and multiplication, and that their low level of misconceptions may have arisen from these shortcomings.

DISCUSSION AND CONCLUSION

As a result of the research, it is seen that 9th grade students generally have misconceptions about the definitions and representations of irrational and rational numbers, students who have misconceptions see all the numbers given in the root as irrational numbers and have difficulty in showing these numbers on the number line. These results coincide with the results of the research on students' misconceptions on number sets (Adıgüzel, 2013; Cengiz, 2006; Şandır, Ubuz, & Argün, 2007). In the study conducted by Adıgüzel (2013), it was seen that the students had difficulty in determining an irrational number given within the root, 72.5% of them could not state that the number pi is a real number, and that the students did not know that irrational numbers were not rational numbers at the same time. In the same study, it was concluded that 48.02% of the students could not determine that the number 9 given within the square root was also a real number or an integer, and from this, as seen in the findings of this research, the students did not know the coverage relations between the number sets exactly.

In the first question of the research, when the square root of 9 is given as -, the students' difficulties in placing them in number sets and their inability to place numbers such as this and pi in cluster shapes are in line with the results of the study conducted by Orhun (1998). In this study, it is seen that the students think that the square root of positive numbers exists, but that the square roots of negative numbers are not defined.

As a result of the research, it was seen that the students did not know the definitions of irrational and rational numbers, could not predict the approximate values of irrational numbers, and had difficulty in showing these numbers on the number line. In the study conducted by Cengiz (2006), it was observed that 9th grade students had misconceptions about rational numbers, exponential and radical numbers, and it was determined that they had problems in showing rational numbers on the number line. Şandır, Ubuz, and Argün (2007) also stated that students could not calculate the decimal representations of irrational numbers as an approximation and they could not show these numbers on the number line in their study, in which they tried to identify students' misconceptions on number sets.

Based on the few studies on this subject, it is thought that using geometric and mathematical models and cartoons in mathematics lessons in order to prevent students' misconceptions about number sets, and ensuring that the inclusion relations between number sets are fully taught to students by using activity-based worksheets will contribute to preventing misconceptions in students.

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