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Knowledge Analysis of Chemistry Students' Reasoning about the Double-slit Experiment

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ABSTRACT

Previous work has highlighted the difficulties students have when explaining wave behavior. We present an investigation of chemistry students' understanding of the double-slit experiment, where students were asked to explain a series of PhET simulations illustrating a single continuous light source, single-slit diffraction, and double-slit interference. We observed a variation in student reasoning and students were categorized into groups based on their ability to explain and generate a mechanism for the double-slit experiment. Some students struggled to explain the features of waves which impacted their reasoning about interference and caused them to rely on intuition to generate explanations. Other students were able to productively incorporate their previous knowledge about wave behavior, with their observations from the simulations, to build a robust mechanism for wave interference. However, students generally exhibited a limited understanding of interference, and specifically attending to the key features of waves during instruction can promote more sophisticated reasoning about this phenomenon.

Keywords: light-matter interactions, double-slit experiment, knowledge analysis, knowledge-in-pieces, postsecondary chemistry, scientific reasoning

Introduction

In first-year chemistry courses, the double-slit experiment is introduced to illustrate the wave nature of light and is further built upon to introduce the wave nature of quantum particles. Students are asked to extend the dual nature of light to the dual nature of matter, where matter can exhibit both wave and particle behavior. Understanding how light behaves in the double-slit experiment is a necessary first step in understanding the wave nature of matter. Specifically, students need to have a basic understanding of wave phenomena like diffraction or interference (Vokos et al., 2000). Henriksen et al. (2018) showed that students' productive explanations of the dual nature of light rely on interference patterns to explain wave behavior.

Research in physics education has investigated how physics students understand and interpret the double-slit experiment. In one qualitative investigation, three broad difficulties were identified: misapplication of geometrical optics, reliance on algebraic formulas without a conceptual understanding, and difficulties with understanding light as photons and electrons as waves (Ambrose et al., 1999). High school and university level students also displayed difficulties in meaningfully interpreting interference and diffraction patterns for a single and a double-slit during an eye-tracking study (Susac et al., 2021). Difficulties with these concepts highlight the intricate nature of understanding the double-slit experiment, and ultimately how the experiment has implications for understanding duality. In a study investigating students' understanding of the wave nature of matter, students had difficulties with basic wave behaviors such as interference, which further impacted their ability to extend wave behavior to matter (Vokos et al., 2000). In a mixed-methods study describing student performance on a light interference assessment, students performed better on assessment items targeting phase differences and interference patterns than items targeting changes in wavelength and changes in the direction of propagation (Dai et al., 2019). In this same study, the qualitative investigation revealed that novice students memorized equations without demonstrating a conceptual understanding. Dai and colleagues (2019) observed students struggle to apply interference to novel or atypical problems, such as changing wavelength or direction of propagation, and possessed a limited conceptual understanding of interference. Further, advanced physics students showed evidence of employing phenomenological primitives (p-prims) while explaining the double-slit experiment of single particles (Sayer et al., 2020). This was evident when students explained that if two particles had the same wavelength, they would have the same amount of kinetic energy regardless of particle size. This study showed that the extension of wave behavior to particles is difficult and resulted in a reliance on intuitive reasoning. Extending wave properties to matter can be further exacerbated when a student's understanding of wave behavior is limited.

While this topic has been investigated in a physics context and provides important insights into how students understand light, it remains important to investigate in a chemistry context because of how differently chemists use and approach light in chemistry instruction. Additionally, many studies in physics have focused more on investigating students' alternative conceptions regarding the double-slit experiment (Ambrose et al., 1999; Yalcin. et al., 2009). This study focuses on how students understand and reason about this experiment and the variation in their knowledge structures. Here we describe our investigation of chemistry students' understanding of the double-slit experiment. This is part of a larger project looking at how chemistry students understand the nature of light and light-matter interactions using a developmental perspective (Balabanoff et al., 2020). Specifically, this investigation was framed by the following research question:

> **RQ:** How do postsecondary chemistry students reason about light behavior in the double-slit experiment?

Theoretical Framework

This study of students' understanding of light and the double-slit experiment is framed by Knowledge in Pieces (KiP). This framework describes learner's knowledge as fragmented, where fragments of knowledge are considered the finest grain of cognitive units and can be activated in a range of contexts (diSessa, 1993, 2018; Hammer et al., 2005). KiP provides a framework for investigating how students reason in the context of the double-slit experiment and a way to evaluate the variation in students' knowledge structures.

Knowledge in Pieces

The KiP framework is grounded in the constructivist paradigm, where students' knowledge is considered *rich* and *productive*. This is because as students learn, new pieces of knowledge or information are integrated with previous knowledge. New fragments can be added to generate more complex and organized systems of knowledge. KiP considers knowledge to be multi-scaled in nature where smaller knowledge pieces are added, displaced, connected, or isolated within a larger knowledge system (diSessa, 2018). As such, the goal of our analysis focused on how students combined fragments, and whether the incorporation or displacement of specific pieces of information supported or hindered their explanations of the double-slit experiment.

Students' knowledge is described as small fragments that are often context dependent within the KiP framework (diSessa, 1993, 2018; Hammer et al., 2005). *Contextuality* is the idea that students' fragments are neither fixed nor stable, with some explanations appearing in specific contexts and not others (diSessa, 2018). Phenomenological primitives (p-prims) are abstract and intuitive ideas about how things work. Structurally, p-prims are small in nature and often isolated from other pieces of information. They are irreducible knowledge elements in that they typically cannot be further explained. One example of a p-prim is "more is more", or Ohm's Law, where more of a cause is connected to more of some effect. These intuitive elements often guide a student in the sense-making process without the student recognizing the p-prims are doing so because they are deeply ingrained (diSessa, 1993; Hammer, 1996). Students may rely heavily on *intuitive knowledge,* or p-prims, when relevant prior knowledge is inaccessible.

Coordination Classes

Within the KiP framework, a second model of cognition describes and defines the properties associated with expert-level thinking. In contrast to p-prims and fragmented knowledge, coordination classes are structurally distinct in that they consist of a complex system of knowledge elements. Coordination classes are reliable across contexts, unlike the fragmented and context-dependent nature of p-prims. The function of coordination classes is to extract some "class" or network of information from the world that is characteristic of a particular concept (diSessa & Wagner, 2005; Thaden-Koch et al., 2006).

Coordination class theory has distinct structural and architectural features to categorize students' knowledge. The two main features are *extractions* and *inferences* (diSessa & Wagner, 2005; Levrini & diSessa, 2008). Extractions correspond to observations of the world where one coordination class may use multiple observations within one single situation. Inferences are the part of the knowledge system that draws conclusions about the extractions, also referred to as the *causal* or *inferential net*. One key aspect of students making inferences is that they must first determine which extractions or observations are relevant for that particular situation (Thaden-Koch et al., 2006).

Within coordination class theory, there are two processes for how learners determine how prior knowledge is used. The first is *incorporation,* where prior knowledge and a new conceptualization is merged. The second is *displacement*, where prior knowledge is dismissed from a new conceptualization. Both of these processes involve the learner determining if the prior knowledge is relevant to the new conceptualization (Barth-Cohen & Wittmann, 2017). Other architectural features of coordination classes describe how learners consider knowledge across contexts. For instance, *span* refers to the ability to recognize and access relevant knowledge across a range of contexts. In addition, *alignment* refers to learners determining which information from different situations is actually the same information and relevant. Those who have an advanced coordination class surrounding a concept demonstrate both span and alignment when generating inferences (Barth-Cohen & Wittmann, 2017; diSessa & Wagner, 2005; Thaden-Koch et al., 2006).

A coordination class has distinct architectural specifications. In some cases, a student may possess a coordination class for only certain concepts and not others. There may also be situations where a student has a knowledge system that does not meet the strict requirements of an advanced coordination class. For example, a student could have accurately coordinated extractions with relevant prior knowledge, which indicates alignment. However, if that student inconsistently applies the coordinated extraction and prior knowledge, they would not demonstrate span. Examples such as these can be described as *developmental coordination classes.* Learners with developmental coordination classes are often in the beginning stages of generating a more complex class with limited success (diSessa, 2002; Thaden-Koch et al., 2006).

Method

Knowledge Analysis

This study is guided by the Knowledge Analysis methodological framework focusing on modeling students' knowledge. The guiding principles of this methodological framework are: (1) that the aim is to model students' thinking and learning, (2) developed models are content specific, (3) intuitive knowledge is important, (4) analysis requires capturing the thinking and learning processes, and (5) intellectual performance is context dependent (diSessa et al., 2016).

Participants

Participants in this study were recruited from multiple chemistry classes ranging from introductory chemistry through quantum chemistry during the Fall 2019 semester. Students were recruited from multiple chemistry courses because it was important to capture a variation of understanding. Students were recruited from general chemistry (GC, N=10), general chemistry for chemistry majors (GCM, N=10), organic chemistry (OC, N=11), and physical chemistry (PC, N=1). General chemistry surveys chemistry very broadly, including atomic structure, molecules, structure-property relationships, and chemical reactions (including thermodynamics, equilibrium, and kinetics). Where general chemistry serves all STEM majors, general chemistry for majors serves primarily chemistry, biochemistry, chemical engineering, and physics majors. Organic chemistry surveys molecular structure, reactivity, chemical reactions, and reaction mechanisms. In the laboratory component of organic chemistry, students encounter common techniques for characterizing and identifying molecules, many of which rely on interacting electromagnetic radiation with matter. Finally, physical chemistry introduces students to quantum mechanics, including the dual nature of light and matter, the hydrogen atom, multiple quantum mechanical models, and an introduction to spectroscopy. Physical chemistry represents the most advanced treatment of the nature of light and its interaction with matter in the undergraduate chemistry curriculum.

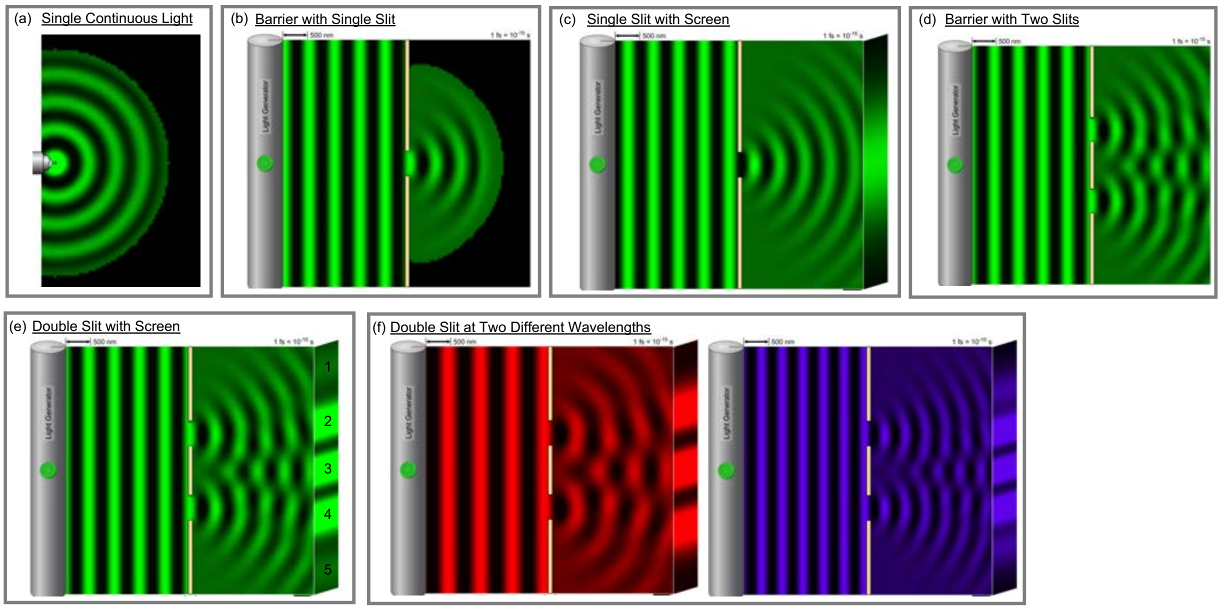
Data Collection

Data was collected via semi-structured interviews which allowed for an in-depth investigation of students' understanding of the double-slit experiment and their reasoning. Audio and video recordings were collected for each interview, which ranged from 25 to 73 minutes. Student drawings and notes were collected after each interview and scanned. The study was approved by the Institutional Review Board.

During the interview, students were shown a series of PhET simulations (Figure 1) that depicted light waves traveling without any barriers (Figure. 1a), and light waves traveling through barriers with single (Figure 1b and c) and double-slits (Figure 1d - f). The interview consisted of four parts: (1) describe the light behavior from a single continuous light source (Figure 1a), (2) make predictions and describe a single light source that is shining on a barrier with one slit (Figure 1b and c), (3) make predictions and describe a single light source that is shining on a barrier with two slits (Figure 1d - f), and (4) draw conclusions about the nature of light. Students were asked to make predictions prior to observing simulations and asked to provide explanations after observing simulations. Parts 1 and 2 of the interviews were designed to elicit students' understanding of how a single light wave travels and interacts with a barrier. Part 3 was designed to elicit students' understanding of interference caused by two sources of light. Part 4 was designed to elicit students' ideas about how light behaves and their conclusions about the way light behaves based on observations of the simulations.

Figure 1

Sequence of Simulations Shown to Students During the Interview



Note: Students first observed a single continuous light source (a), barrier with a single slit (b), the single slit with a screen (c), barrier with two slits (d), the double-slit with a screen (e), and finally, observed the double-slit experiment with red and violet wavelengths (f).

Data analysis

Interviews were transcribed verbatim using a transcription service. Gestures captured from the videos were added to the transcript and images of student work were embedded at the appropriate time points. The transcripts were open-coded using a constant comparison approach (diSessa et al., 2016; Strauss & Corbin, 1990) where codes were refined by multiple analysts (first and second author). Trustworthiness was established through an iterative series of applying and refining codes. All authors were involved throughout the entire process to ensure that the first author's codes were applied consistently and meaningfully interpreted (Golafshani, 2003).

The coding took place in two stages where transcripts were initially open coded to develop content-based codes and the second stage of coding focused on students' reasoning. The codes focusing on students' content knowledge were organized into the following categories: general light properties, light behavior, and deductions from simulations. General light properties included codes describing properties that were scientifically accepted such as light having no mass, or the amplitude corresponding to the intensity. This category also included non-normative scientific ideas such as the number of photons corresponding to the intensity of light and brightness relating to the energy of light. The light behavior category codes described students' overall ideas of how light behaves, including both normative and non-normative ideas. Some examples are light travels linearly, the wavelength of light changes as it radiates outward, and a change in frequency changes how quickly light travels. The last category of the content-based codes, deductions from simulations, described the explanations students generated based on extractions from the simulations and inferences. The deductions were further categorized to align with the four parts of the interview, as outlined in the Data Collection section. Again, the deductions from simulations coding during analysis included both normative and non-normative ideas. Some examples include interacting with the barrier results in light scattering, interference causes light to change direction, and illuminated regions on the screen represent instances of constructive interference.

The second stage of coding used the KiP framework to code students' explanations and reasoning. Because of the architecturally strict nature of this framework and explicit definitions, the codes used in the second stage were directly developed from this framework. These codes included p-prim, intuitive reasoning, reasoning grounded in experiences, contextuality, extraction, inference, incorporation, displacement, span, and alignment. Definitions for these codes can be found in Table 1.

Table 1

Codes and Associated Definitions for the Second Stage of Coding Using the Knowledge in Pieces Framework

Code	Definition
Phenomenological primitive (p-prim)	Abstract or intuitive idea about how things work, often structurally small and isolated from other pieces of information
Intuitive reasoning	Explanation where a student does not know exactly where it comes from or why
Reasoning grounded in experience	Explanation built on an experience with the physical world
Contextuality	Fragment that is not fixed nor stable, an explanation that appears in some contexts and not others
Extraction	Observation of the world
Inference	Conclusion about an extraction or observation
Incorporation	Prior knowledge and a new conceptualization that is merged
Displacement	Prior knowledge is dismissed from a new conceptualization
Span	Recognizing and accessing relevant knowledge across a range of contexts
Alignment	Determining which information from different situations is the same information and relevant

Students were then grouped based on how they drew conclusions, whether they relied more on intuitive reasoning and p-prims or the degree to which they coordinated their extractions and inferences. The grouping of students was based on the qualitatively different ways in which students reasoned about the double-slit experiment. Specifically, the analysis centered around the consistency or lack thereof across the explanations provided for the range of simulations. For instance, we looked at how students' explanations changed with the introduction of each simulation or how they built upon explanations generated from previous simulations. This resulted in three categories (Table 2): primarily fragmented (*N*=13), developmental (*N*=13), and coordinated (*N*=6).

Table 2

Distribution of Students in Assigned Levels

	Fragmented	Developmental	Coordinated
GC	7	2	1
GCM	1	6	3
OC	5	4	2
PC	0	1	0

Using some examples from Table 1, if a student's interview transcript consistently highlighted their use of p-prims, intuitive reasoning, or reasoning based on experiences, this student was categorized as primarily fragmented. In contrast, a student's transcript that was frequently coded with span, alignment, and inferences built upon prior knowledge and earlier observations if the simulation elicited a coordinated classification. In the developmental category, these students exhibited a mixture of both fragmented-type codes (e.g., p-prims) and coordinated codes (e.g., span and alignment). Each student was grouped based on their overall reasoning structure surrounding the double slit experiment.

Results

Each category (fragmented, developmental, and coordinated) will be described and explained through vignettes from an exemplary participant in each category. The generation of vignettes for exemplary students was informed by our methodological framework with the aim of modeling students' knowledge and to capture students' understanding over the course of the interview. We have selected three students because they are representative of their category and enable a detailed discussion of the variation in student reasoning across categories. Below in Table 3, the general trends and features associated with each category are outlined.

Table 3

General Trends Described for Each Eategory: Fragmented, Developmental, and Coordinated

Fragmented	Developmental	Coordinated
• Limited or absent prior knowledge • Inconsistent use of prior knowledge • Focused on visible light or shadows • Inconsistent explanations throughout the interview • Absence of mechanism	• Relied on correct relationships • More comfortable with constructive interference than destructive • Correct predictions with limited mechanistic understanding	• Easily accessed relevant prior knowledge • Comfortable with constructive and destructive interference • Detailed and accurate mechanism of double-slit experiment

Fragmented – Ramona

Students categorized as fragmented typically had limited prior knowledge relating to the double-slit experiment and inconsistently applied that prior knowledge. Students in this category heavily relied on their experiences when reasoning about the phenomenon and tended to use intuition when their understanding was limited. This limited their ability to generate explanations for their predictions or observations. In addition, the limited prior knowledge resulted in inconsistent explanations which indicated a lack of alignment and span. Ultimately students were categorized as fragmented due to the inconsistent application of prior knowledge, overreliance on intuitive reasoning in the absence of prior conceptual knowledge, and not incorporating mechanistic reasoning of the experiment. Some students were able to recall the term interference upon observing the double-slit experiment, but were not able to further explain the details of interference or which observations from the simulations corresponded to evidence of interference.

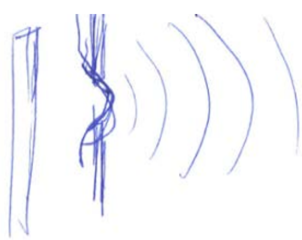
One student who exemplified the fragmented category was Ramona, a student in the general chemistry course for chemistry majors. Throughout the course of the interview, she relied on her physical experiences to explain her observations of the simulation. Because she relied on experiences, Ramona's reasoning was fragmented and grounded in intuition. Additionally, her explanations of the simulations lacked any mechanism of interference or light interactions.

After showing Ramona the simulation depicting a Single Continuous Light (Figure 1a), she was asked to predict what would happen if the single light source were shined on a barrier with a single slit. She used what she previously observed from the simulation shown in Figure 1a to inform her prediction and drew Figure 2:

> It'll probably still keep moving out, but it'll be bent because of the obstacle… Sort of how I pictured in my head, which might be wrong, it's like the lines (of light) are flat, so it's going… and [at] the barrier, it kind of gets pushed a little bit.

Figure 2

Ramona's Drawing Illustrating How Light Bends When It Meets the Barrier



Ramona explained that she expected the light to continue to move out from the source and upon hitting an obstacle, the light will bend. Ramona generated a prediction that aligned with her previous observation of light traveling with no barriers. When Ramona was shown the Barrier with Single Slight simulation (Figure 1b), she remarked:

> Yeah, looks [the same]. The fuzzy regions, I think it kind of loses its intensity as it goes out because the barrier kind of blocks some of it.

Ramona indicated that the simulation looked as she predicted and explained that because the light is going through a single slit, the barrier blocks some of the light, which caused a decrease in intensity.

Ramona constructed a prediction based on her previous observations of light continuously moving outward and subsequently generated an explanation of the simulation that highlighted a loss of intensity.

Immediately following the Barrier with Single Slit simulation in Figure 1b, Ramona was asked to predict what she expected to see once a screen was added to the simulation. She offered a prediction for two types of light behavior: light acting as particles and light particles traveling in waves. Ramona first brought up the idea that light is made up of particles. She explained that if light particles did not move like waves and traveled in a straight trajectory, she would expect to see individual dots on the screen. However, she further explained she did not expect to see dots because the light particles are in fact moving like waves.

> I know we used to think that light was just a bunch of particles, but it's particles that move like waves. So, if that were true [light being made up of particles] …the screen would just show a bunch of dots because it would just be particles and they would be individual and in different spots, **but it's not**.

This prediction indicated that Ramona considered both wave and particle behavior and ultimately decided that wave behavior was more appropriate in this context. She affirmed that she did not expect the light to behave like particles, rather the light particles are moving like a wave.

After her prediction, Ramona observed the Single Slit with Screen simulation in Figure 1c and agreed with her previous prediction that light particles move like waves because she did not observe individual dots on the screen. She then described her observations of the intensity of light on the screen (Figure 1c):

> Yeah. It shows a little bit more of the intensity and how it blurs out on the sides I guess… because there's going to be a little bit of shadow, kind of blurring out from the obstacles. There's still a little bit of light showing, but it's not going to be as strong.

When describing the intensity of the light on the screen, Ramona grounded her explanations in physical experiences by the invocation of shadows. Ramona described the barrier as creating a shadow on the screen, with some light passing through the single slit that is not as strong. Ramona connected her experiences with shadows to her observations of the simulation and explained that the blurred regions on the screen are a shadow of the barrier.

Later in the interview, Ramona was asked to make a prediction about what she expected to see on the screen now that the barrier had two slits (Figure 1e). She based this prediction on what she observed on the screen when the barrier had a single slit. She explained that she expected to see three separate regions on the screen:

> Something similar as before but it's going to be stronger right here and stronger right here and here (pointing at Regions 2, 3, and 4 of Figure. 1e). But a little blurry here and here, and here and here (pointing between Regions 2 and 3 and between Regions 3 and 4 of Figure 1e). Because the same as before, because of the obstacles in the middle. They're still going to have some effect on putting a shadow in the middle of the light. But because it's the two waves of light crashing into each other, they're going to have that spot in the middle.

Ramona used her previous observations of the Single Slit with Screen simulation and predicted to see three regions illuminated on the screen. Two of the illuminated regions were a direct result of light shining through the slits, just as she had observed with the Single Slit with Screen simulation. Unique to the Double-slit simulation, she explained that there will be a third "middle" region of light because the light waves will crash into each other. Ramona made a productive prediction by drawing on the idea of light radiating outwards. However, in this prediction she did not further explain why the waves crashing into each other results in an illuminated region. Ramona's invocation of the shadow to explain darkness and blurriness on the single slit simulation also informed her prediction for the double-slit simulation. She explained that there would be a "shadow in the middle of the light" due to the barriers.

After making her prediction, she was shown the Double-slit with Screen simulation (Figure 1e). She explained:

> These two parts (Regions 2 and 4) are still strong, bright, because that's exactly where they're showing right through in the middle of the slits... But right there (Region 3) they're going to be just as strong because that's where they meet. That's like kind of the peak of where they meet. [The gaps] are because since the waves still move out but meet in the middle, that's kind of like the remainder of the shadow from that middle part. If the waves didn't move the way they [do], that whole part (Region 3) would be shadow. But since they still like move out and hit each other, it's just going to kind of show the edges of what the shadow would be if it was a particle.

Ramona explained the illuminated regions across from the slit openings (Regions 2 and 4) are a result of light traveling unobstructed through the slit to the screen, thereby confirming her prediction. She also explained that the middle region is just as strong as the other regions because that is where the two sources of light meet, also confirming her prediction. She used prior knowledge of light radiating outward to help her explain how the light waves meet in the middle. Her explanation of the gaps between the illuminated regions was rooted in shadows, similar to her explanation for the single slit. She expected Region 3 of the screen to be a shadow if the light behaved only like a particle because she associated light radiating outward with wave behavior. But because the particles move like waves, Ramona explained that we see the region in the middle illuminated and the "edges" of the shadow.

Despite making a correct prediction, Ramona displayed a limited understanding of why her prediction was correct. Her explanation of the interference pattern relied on the intuitive expectation that if light waves continued to expand outward, they would eventually meet. Ramona's explanation also lacked any kind of mechanism or ideas about *how* the waves were combining. Many other students within the fragmented category used water waves to explain waves joining together, where students built off their experiences and connected them to light waves. This could be related to the p-prim "more is more" where two waves adding together can create a brighter region of light than a single wave. Further, we observed where her limited prior knowledge of interference impacted her explanation of the dark regions on the screen (Figure 1e) and caused her to rely on her experiences with shadows, even though shadows did not serve her very well. That is, she struggled to use shadows beyond attributing them to the barriers. Additionally, she did not connect wave behavior to the gaps on the screen. Rather, she only incorporated wave behavior to explain the illuminated regions on the screen.

Throughout the interview with Ramona, she focused on observations that aligned with wave-like behavior. For instance, she first activated wave behavior when she spoke about light bending around obstacles and light radiating outward. She continued to think about waves when she explained light radiating outward and meeting in the middle of the double-slit experiment. However, in the absence of an explanation of interference, it was evident that Ramona used intuition to predict the peak on the screen in between the slits. The reliance on intuition was further evidenced by her reasoning about dark regions on the screen; that is, dark regions were caused by shadows from the barrier. In this case, intuition was specifically grounded in physical experiences.

While the *predictions* Ramona generated of the later simulations (e.g. Double-slit with Screen) were relatively productive and built on her previous observations and extractions of earlier simulations, her *explanations* of the simulations were fragmented and intuitive and lacked any incorporation of wave behavior when explaining the dark regions on the screen (Figure 1e). Ramona's consistent predictions, but inconsistent explanations of the simulations, indicated a limited understanding of interference. Ramona, like many other students in this category, focused on her previous experiences with visible light and shadows. As the interview progressed, Ramona became more dependent on her intuition, and specifically shadows, even though shadows failed to explain the patterns observed on the screen. This reliance on intuition and her limited prior knowledge resulted in Ramona generating explanations that lacked any kind of mechanism of interference. Generally, students in this category could rely on intuition to make predictions regarding constructive interference, perhaps relying on a "more is more" p-prim to do so, but intuition failed to predict or explain destructive interference. The distinction between students in the fragmented category and the more sophisticated categories is that students did not engage in generating a mechanism. Specifically, students in the fragmented category either provided an explanation based on intuition or experiences without providing details with some students remarking *"this is just how it is."*

Developmental – Arthur

The developmental category describes students who provided accurate predictions of the simulations and who were able to generate more mechanistic explanations of light interactions, even if some knowledge elements were incorrect. Students in this group were also generally more comfortable with describing how constructive interference occurs compared to destructive interference. These students often considered fundamental wave behaviors and relied on relationships between frequency and wavelength, however, often lacked some relevant prior knowledge that would result in a detailed and accurate description of the double-slit experiment.

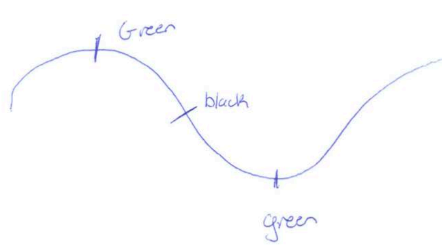
One student in the developmental category, organic chemistry student Arthur, relied on particle behavior to explain the single and double-slit experiment despite demonstrating a sophisticated understanding of wave features in the context of a single light source. Throughout his interview, Arthur tried to fit his ideas about light particles into his ideas about light waves and his observations of the simulations. He used light particles to explain interference and frequently made attempts to organize his ideas about particle behavior into his ideas about wave behavior.

At the beginning of the interview, Arthur explained the Single Continuous Light (Figure 1a) and attributed the pattern he observed to the nodes and antinodes of a wave. Drawing Figure 3, Arthur says:

> Green is the top of the peak. And then the black, I'm assuming would be the region between the two peaks.

Figure 3

Arthur's Drawing Depicting the Features of a Wave



Note. Arthur labeled the green and black regions of the Single Continuous Light simulation.

For the first simulation (Figure 1a), Arthur demonstrated a sophisticated understanding of wave features by assigning the green regions to the antinode and the black regions to the node (Figure 3).

After discussing Figure 1a, Arthur was asked to make a prediction about light passing through a single slit (Figure 1b). Arthur explained that as the light exits the slit, it will expand and emanate outward because light particles are energized and do not want to be next to another particle:

> It'll fan out in half circles again, sort of emanating from that slit and then expand out because light is energy. If you have all of these particles that are energized in one small space, they obviously don't want to be next to each other.

Arthur was correct in thinking about light as energy, however, he further explained that light radiates outward due to the light particles being energized. Here, he seemed to be conflating energy and charge, where energized particles do not "want" to occupy the same space. Despite productively describing the wave behavior of light earlier, Arthur generated an explanation rooted in the particle behavior of light to explain the outward radiation. While his prediction was correct, it did not build upon his previous knowledge about wave features. Rather, Arthur's explanation of energy repelling served as a p-prim. The use of the word "obviously" is a cue for identifying p-prims (diSessa, 2018) because to students, p-prims are generated from seemingly obvious observations of the world.

After his prediction, Arthur watched the Barrier with Single Slit simulation (Figure 1b). He remarked that the simulation somewhat showed what he expected but he was not expecting to see the "blurred region" near the top and bottom. He interpreted the blurred regions as "chaos" with particles spreading out. He then goes on to explain that it seems like lines are being reformed which he inferred as particles reforming as waves.

> That's kind of what I expected but there's this blurred region on the top and bottom which I was not expecting… You have these clean bars of light that are moving through [the slit] and then when they come out on the other side, it's kind of just mayhem and chaos as they're spreading out…It looks like it's starting to form lines again with the green-black alternation. So I'm guessing somehow it goes from being a mess of particles into somehow forming waves again. I suppose it has something to do with the attraction between the energized particles.

Arthur's interpretation of the simulation showed that he was thinking about particles of light and how that relates to light waves. He postulated that the light particles are spreading out and go on to reform into waves because there is some sort of attraction between light particles. In Arthur's explanation, he continued to build on the idea of energized particles. While he now considered energized particles both attracting and repelling, he still aligned this explanation to previous explanations by conflating energy and charge. We also can see that Arthur used particles of light to think about some big picture ideas regarding light behavior, in this case, diffraction.

Next in the interview, Arthur watched the Single Slit with Screen (Figure 1c). He explained that the brighter region on the screen is due to "clear waves" that have a certain amount of energy to make it to the screen, and the edges of the screen are not illuminated because the light particles have spread out and lost energy.

> I think that hazy portion is like a mayhem of particles and because it's not really organized into those individual unique distinguishable waves, it's not producing a lot of light [on the screen] because those energized particles are just spreading out, dissipating, and losing the energy. Whereas in the middle, because they're very clear waves, they have the energy and the endurance to make it all the way to the [screen].

Arthur connected his observations of the simulation to both particle and wave behavior. For instance, he attributed the hazy regions of Figure 1c to light behaving like a particle and described them as being disorganized. He later explained that the middle area of Figure 1c depicted light waves. Arthur's explanation of the Single Slit with Screen simulation showed that he tried to organize the ideas he has about particle and wave behavior by assigning particle behavior to certain aspects of the simulation and wave behavior to others.

After observing the single slit simulations, Arthur watched the Barrier with Two Slits (Figure 1d). He explained that the light exiting the slits collides, and when the light collides, they begin to move forward.

> So as the light exits both of those openings, it collides. And I suppose when it collides it, they kind of push off each other and start going in that forward direction instead of going straight outward.

Arthur explained that rather than light radiating outward, it has now collided with another light source which caused the light to travel forward.

After observing the Double-slit with Screen simulation (Figure 1e), Arthur explained that it reminded him of something he had seen in high school. Upon observing Figure 1e, he remembered previous observations of the double-slit experiment.

> I'm starting to think back to like high school when we did some experiment like this where we had those two slits in a sheet of paper. It makes sense that of course we get two light sources on that screen from each of the openings. But then those waves are colliding in the middle and kind of combining to organize themselves into a third [region].

Arthur described an experiment he observed in high school where he had a piece of paper with two slits. He went on to explain that Regions 2 and 4 are illuminated from the two slits, building off his earlier prediction. He also explained that Region 3 is a result of waves colliding and organizing into an illuminated region. Arthur recognized that some regions are a result of light sources interacting with each other. However, he did not extend that interaction to all illuminated regions.

When asked to further explain how light was combining to organize into a third illuminated region, Arthur brought up two analogies, sound waves and pool balls to help him describe his thinking. At this point in the interview, Arthur introduced a particulate explanation of light behavior that he relied on for the duration of the interview. He explained:

> If you have two sound sources, they kind of collide and then merge into one. Um, and I guess it's kind of the same thought process as if you had two pool balls and you were to push them toward each other. They're going to collide and then start moving forward because when they collide, they cancel out the side-to-side motion, if you will. And the only thing that's left is that forward motion.

Arthur's explanation of how light collides highlights his attempt to connect and fit particle behavior into wave behavior in the context of the double-slit experiment. This explanation also shows how Arthur used particle behavior, in this case, the analogy of pool balls, to think about light waves colliding. Arthur built upon his previous explanations grounded in particle behavior to explain how the collision of light results in the illuminated middle region (Region 3 on Figure 1e).

Arthur was then asked to explain why some regions on the screen were dark on the Double-slit with Screen simulation (Figure 1e). He built upon his ideas about light organizing into waves and how it collides and ultimately moves forward. He goes on to further explain that the remaining light that is not organized into waves is scattered light. The scattered light resulted in dark regions on the screen and Arthur related that to intensity:

> Some of [the light] is organized and moving forward and then some of it is thrown off from the collision and scattered. And so it's much lower in intensity than the [light] that was organized in a single wave.

This explanation provided by Arthur shows that he continued to fit particle-like behavior, specifically collisions and scattering, into his explanation of light waves. Further, he connected the ideas of scattering and colliding to intensity. This explanation highlights that Arthur was considering the observations from the simulation and aligned to his prior knowledge of wave-like properties such as intensity. It also shows that Arthur relied on particle behavior to generate a mechanism of light interactions. However, Arthur's knowledge of wave interference was limited, which resulted in a reliance on particle behavior to generate explanations for his observations.

Towards the end of the interview, Arthur was asked to draw conclusions about light behavior based on his observations of the simulations. He concluded that these simulations are evidence of light acting as particles and explained that light particles organize themselves into waves.

> This supports the theory that light acts as particles. Somehow those particles come together and organize themselves into light waves. And that's why we see [the diffraction pattern]. Because even though they're acting in that weird manner when they first exit the two openings, they still are able to come together and form very distinct [lines].

Arthur conclusively stated that the observations from the double-slit experiment show that light acts as particles. Throughout the interview, Arthur continued to build upon the idea that light acts as a particle and frequently tried to fit his ideas about light particles into big picture ideas about light waves. Here, Arthur discussed how the particles exit the slits and eventually organize into distinct lines.

When asked how the light particles organize themselves into lines or light waves, Arthur postulated that the organization was related to the frequency of the light particles. He explained that because the light particles are all traveling with the same frequency, they can organize themselves resulting in an increase in intensity:

> I suppose it has something to do with the frequency of the light particles. If they're traveling at that frequency before they hit the opening, and then if they're still maintaining that frequency as they're moving through the opening, then they should align with the other particles because they're moving the same frequency. Even though they're spreading out and dissipating, you still have all these particles traveling at the same frequency. So some of them that are going in the same direction are going to line up and their intensity will be increased as a group.

Arthur explained that upon exiting the slit, the particles of light are spreading out, but all of the particles of light are traveling with the same frequency. Because of this, the particles that either move forward directly out of the slit, *or* collide and move forward, will line up and increase the overall intensity. In this explanation, Arthur combined ideas productively to explain his observations of the double-slit experiment. He considered the role of intensity and how that was related to the amount of light on the screen. He further connected that to how particles collide as they exit the slits to move in a forward direction. He also correctly incorporated the relationship between the number of light particles and intensity.

During Arthur's interview, he initially described the simulations using wave-like properties and then transitioned into explaining light as particles, indicating context dependent explanations and fragmentation. Arthur transitioned from wave to particle behavior once the slits were introduced in the simulation and light was now interacting with another object. He began to generate explanations heavily focused on collisions and energized particles. While Arthur's ideas of energized light particles are scientifically non-normative, he consistently returned to particle behavior to explain his observations of light behavior. When explaining the double-slit experiment, Arthur recalled conducting a similar experiment in high school. This was productive for Arthur, indicating he connected his observations during the interview to prior observations of the experiment. Following this connection, Arthur continued to organize his ideas about light particles into his observations of wave behavior. For instance, he considered wave-like properties such as frequency and intensity to explain how light particles organized themselves into waves resulting in the pattern on the screen.

While Arthur's explanation of wave interference is not correct, he made productive attempts to generate a mechanism for the simulations he observed. He aligned his explanations of the simulations with his prior knowledge of particle behavior and wave features such as frequency and intensity. He used particle behavior to help explain the wave behavior he observed in the double-slit experiment and to generate a mechanism of the resulting diffraction pattern. Arthur is evidence of the developmental category because his ideas are still partially fragmented with his use of p-prims. However, Arthur coordinated knowledge elements together by continuing to use energized particles as an explanatory tool and coordinated this with extractions from the simulations. Despite his inferences about particle behavior being incorrect, they are temporally stable and served as a tool for generating a mechanism of his observations of the double-slit experiment. Like Arthur, other students in the developmental category relied on particles to explain the merging or canceling of waves and often connected that to the dual nature of light. Other students in the developmental category also displayed a less sophisticated understanding of interference, which surfaced when they incorporated ideas of interference when considering both the single and double-slit experiment.

A key distinction between developmental students and fragmented students was their ability to engage in the generation of a mechanism of interference. With their mechanisms, we observed students making connections between the simulations they observed and including elements of prior knowledge of light behavior. However, the developmental students' explanations are less sophisticated and often included incorrect knowledge elements, which set them apart from the most sophisticated group of students.

Coordinated – Destiny

The third category, coordinated, describes students who easily accessed and applied useful prior knowledge and generated a mechanistic explanation of the double-slit experiment. Students in this category explained both constructive and destructive interference and the effect of changing the wavelength of light. Students were categorized as coordinated when they provided explanations that included relevant prior knowledge and built on that prior knowledge throughout the course of the interview. Destiny, a student in the general chemistry course for chemistry majors, fell into the coordinated category. Throughout the interview, Destiny incorporated relevant pieces of knowledge and displaced pieces that were not relevant. Additionally, she exhibited span across simulations by recognizing the role of interference in each simulation.

After observing the Single Continuous Light (Figure 1a) and the Barrier with Single Slit (Figure 1b), Destiny was asked to predict what would appear on the screen after the Barrier with a Single slit (Figure 1c). In her prediction, she described light passing through the center of the slit where the light hitting the edge of the barrier would deflect or diffract. Here, Destiny was unsure of which vocabulary word best describes what she was predicting. However, she was able to describe what she was thinking:

> I guess just that if it passes through the center of the slit, then it has minimal impact or deflection or diffraction or whatever the proper word is to use. And so essentially, you know, the very center will keep on a straight path, but beyond that it will kind of be turned outwards.

Despite the vocabulary barrier Destiny experienced, she explained that she would expect to see the light interacting with the edges of the barrier and turning outwards which indicated her understanding of light diffraction.

Destiny then watched the Single Slit with Screen simulation (Figure 1c) where she explained why she observed light radiate outwards:

> [With] only one slit, there isn't actually, there's no other wave to interfere with, just kind of photons amidst their own wave, there isn't really another wave for them to interfere.

Destiny explained that with only one slit, there was only one light source, or one wave, exiting the barrier. Because of this, she recognized there was no reason for interference with another wave to occur. Destiny's explanation highlighted her displacement of interference in this context. Specifically, she recognized that interference was not a relevant resource to employ.

Later in the interview, Destiny was asked to predict and explain the Double-slit Experiment with the Screen (Figure 1e). She described that she expected to see an interference pattern with alternating bars of light and dark regions. She further explained that the pattern is a result of constructive and destructive interference because there are now two waves exiting the barrier:

> An interference pattern of bright and dark little bars not continuing on since there's only the two slits, but like kind of getting dimmer and like less distinct as you move out (motions with both pointer fingers spreading out to represent the spreading of light). [There will be] bright spots and dark spots intermittently as the lights interact and to have constructive or destructive interference with the two different waves that come from the individual slits.

Here Destiny appropriately incorporated her prior knowledge of constructive and destructive interference. She recognized and explained that because the barrier with two slits resulted in two different waves exiting the barrier, interference occurred. This aligned with her explanation and understanding of interference earlier in the interview, where she determined interference was not occurring in the simulation depicting Barrier with Single Slit (Figure 1b) and Single Slit with Screen (Figure 1c).

After Destiny predicted and explained the double-slit experiment, she was then asked to predict what she would expect to happen if the frequency of light changed. Up until this point in the interview, each simulation depicted a green light. When asked to consider the effect of frequency, students compared a red and violet light (Figure 1f). Destiny explained that with a change in frequency, she would not expect to see any changes in intensity. Instead, she predicted that changing the frequency would impact the angle of interference by considering Equation 1.

$$d \sin\theta = m\lambda \qquad (1)$$

> So, I'm trying to think here. Frequency. If you raise the frequency, obviously you would lower the wavelength. I guess you would see a change in the angle at which the bright spots appear depending on if we raised or lowered the frequency … But you wouldn't see a change in like brightness or anything like that…Constructive and destructive interference is dependent on the wavelength. If you think of it like a wave, then you want the crest and the trough to cancel

> out. And when you change the frequency, then you change the distance between those crests and troughs. And so when you change the frequency, you in turn change the wavelength, which impacts that distance. And that distance is related to the angle (of interference) at which the bright spots and dark spots appear.

In Destiny's prediction, she explained that the instances of interference were related to the wavelength or the distance between the peaks and troughs. She exhibited an understanding of the relationship between wavelength, frequency, and intensity by explaining that the brightness will remain the same. Destiny incorporated relevant prior knowledge of how interference occurs and aligned that with her understanding of changes in wavelength. This resulted in a prediction accompanied by a thorough explanation of constructive and destructive interference.

Following her prediction, Destiny observed the Double-slit at Two Different Wavelengths simulation (Figure 1f). She saw that the simulation matched her prediction, and she further explained why she observed more violet bars than red bars:

> I see more bars [with violet] because the angle at which those bright spots and dark spots appear is smaller. You know, it's the distance between, um, it's like there's a relationship, it's not random where they appear at what angle because they maybe are closer together. [For the red] there's more out here [pointing off screen], you just can't see them because the screen is too small in a sense that the angle (of interference) is larger and so it kind of goes off of your visible area.

Destiny explained that if the screen had been larger, she would have observed more red bars. Because Destiny was able to recognize that the wavelength impacts the spacing between the instances of interference, she has shown that she has a mechanistic understanding of interference. Further, all of Destiny's explanations were grounded in the wave behavior of light, which indicated her understanding that the simulations are evidence of wave behavior rather than particle behavior. Additionally, Destiny exhibited span throughout her explanations of each simulation by applying the resources of interference across the simulations through both displacement—recognizing the inappropriateness of the resource—and alignment—recognizing the relevance of the resource.

Over the course of Destiny's interview, she displayed a robust understanding of how light waves travel and how they interfere with one another. Destiny exhibited a coordinated framework of ideas by incorporating and displacing relevant knowledge elements and aligning her prior knowledge with extractions from the simulation. She accurately determined the relevancy of interference across contexts by displacing interference with one source of light and incorporating interference with two sources of light. Her ability to recognize the relevancy across contexts also corresponded to a detailed mechanism of how interference occurs. Her understanding of the mechanism of interference allowed her to explain that interference requires two light sources. Additionally, Destiny's framework is considered coordinated because she exhibited span across the multiple simulations because her understanding of interference is consistent across contexts (i.e., the multiple simulations over the course of the interview). This indicates that she was able to access relevant prior knowledge and coordinate with extractions across simulations, which ultimately led her to generate scientifically normative explanations of the double-slit experiment.

Conclusions

Based on our qualitative investigation of students' understanding of the wave nature of light, we found a variation in student reasoning ranging from fragmented to coordinated. Informed by our theoretical framework, we categorized students' knowledge structure as fragmented, developmental, or coordinated.

We observed students using their daily observations of light to explain wave behavior with some observations being more relevant and productive than others. One productive experience that was frequently used by students to explain the single slit was light coming into a dark room with the door ajar. The observations from this experience were transferred to the single slit often resulting in an accurate prediction. Other experiences were less productive. For example, some students relied on shadows to explain the dark regions on the screen due to destructive interference. Rather than developing an explanation grounded in light waves interfering, students relied on their experiences and intuition. Similar to other studies with physics students, the absence of a basic wave model or a limited understanding of wave phenomena prompted intuitive reasoning (Henriksen et al., 2018; Singh, 2020).

In the developmental knowledge structure category, many students showed productive reasoning and attempts at aligning relevant prior knowledge. However, their limited understanding of wave behavior often resulted in partially correct explanations. Many students within this category were comfortable with the concept of constructive interference, but only applied it to the middle-illuminated region, rather than all illuminated regions. In addition, destructive interference proved to be particularly challenging with some students introducing particle-like behavior to explain the dark regions on the screen which could be influenced by a chemistry context requiring students to consider the behavior of small particles.

Students who exhibited more expert-like knowledge structures were able to accurately displace and incorporate relevant prior knowledge to explain the mechanism of interference. Students in this category provided detailed explanations of how interference was occurring, and which regions on the screen corresponded to constructive or destructive interference. Students verified a mechanistic understanding by generating accurate predictions when considering two different frequencies of light and further explaining the relationship between wavelength and instances of interference. Students who demonstrated a more robust understanding of wave phenomena are likely to be more comfortable extending these ideas when considering the dual nature of matter.

After grouping the students into three categories, we examined the distribution of students by course (see Table 2). The majority of general chemistry students were categorized as fragmented, which could be because this is the first time many of these students have been introduced to light behavior. In the general chemistry for majors course, some of the students we interviewed were either double majoring in physics (e.g., Destiny) or enrolled in other physics courses, which accounts for a large number of GCM students falling in the developmental or coordinated groups. We saw that most organic chemistry students were categorized in fragmented or developments. This may be because OC students have likely not received additional instruction on the conceptual basis of light behaviors, but rather, received more instruction regarding the light-based instrumentation techniques such as spectroscopy. Finally, our sample of physical chemistry students was limited to one student and therefore limits the drawing of any conclusions about which group physical chemistry students at large may belong to.

Based on our observations of student reasoning and the structures of their knowledge frameworks, certain areas can be specifically addressed in instruction. We observed many students struggle with the features of waves, particularly nodes and antinodes, which limited their understanding of interference. This resulted in students generating explanations for constructive interference rooted in intuition because it is relatively intuitive that the region on the screen directly across from the barrier is a result of waves joining together. This was further evidenced by the fact that many students did not assign constructive interference to all regions on the screen and provided non-normative explanations for destructive interference regions. It is important to note that across all levels, students incorporated instances of intuition. However, there were differences in the use of intuition. Students who exhibited more novice-like knowledge structures relied on intuition to explain observations where they had limited prior knowledge. In contrast, students exhibiting more expert-like thinking incorporated intuitive reasoning with their prior knowledge and those students could provide more detailed explanations following their intuitive explanations.

Understanding light waves has implications for learning the dual nature of matter. In first-year chemistry, the double-slit experiment is one of the first steps to introducing the dual nature of light and matter. We expect students to transfer their ideas of light waves to a new context, specifically electrons. When introducing the complex idea of duality, it might be assumed that students already understand basic wave phenomena. However, it is important to first make sure students possess a detailed understanding of the wave model in the context of light before expecting them to apply that model to matter. Students in this qualitative investigation, regardless of their knowledge structure, used the PhET simulations to generate informed predictions and explanations. Presenting wave behavior in this sequential manner could be beneficial in promoting a coherent understanding of wave behavior. Finally, it would be beneficial for the chemistry and physics community to discuss light-matter interactions, as this is a central idea across science disciplines. Together both communities can better support students by having conversations about how the double-slit experiment is introduced across introductory courses and the features each community attends to when considering the wave behavior of light and matter.

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