INFUSION OF POLYA AND DIGITAL BAR MODEL: AN ALGEBRAIC THINKING MODULE FOR SEVENTH GRADERS

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Abstract

Algebraic Thinking Skills (ATS) are one of the skills that students need to master in order to solve non-routine problems. These skills are also necessary as a foundation for students preparing to enter university studies and fields of work that require logical and analytical thinking. However, Malaysian students' performance in solving algebraic problems still needs to be satisfactory, according to the TIMSS 2019 and PISA 2018 reports. Therefore, the Algebraic Thinking Skills Module (ATSM) was developed to cultivate ATS through three constructs, namely i) arithmetic generalization, ii) functions, and iii) modelling. The ATSM was developed using the heuristic method by infusing the Polya and digital bars model. The bar model illustrates a rectangle representing known and unknown quantities and the relationship between quantities. The digital bar model refers to the free application of the bar model at https://mathsbot.com/manipulatives/bar. An ATS test was developed and administered as a pre-and post-test on 120 seventh graders from rural schools in Sabah. The paired sample t-test results showed a significant difference in the mean scores between the pre-test and post-test after the intervention using the ATSM. This shows that the ATSM can improve ATS through the infusion of the Polya and digital bars model. The ATSM is able to help rural schools to shape algebraic thinkers and digitally savvy students.
Keywords: *algebraic thinking skills, digital bar model, non-routine problem solving, Polya*

Introduction

Algebraic thinking skills are one of the skills that students need to master to solve non-routine problems. Algebraic thinking skills are also necessary as a foundation for students to enter university studies and fields of work that require logical and analytical thinking.

To foster algebraic thinking skills among students, especially in early algebra, it is crucial for teachers to proactively diversify their teaching strategies across various algebraic topics (Blanton & Kaput, 2003:10). This is necessary because teacher-centred approaches are no longer sufficient to meet the demands of 21st-century education, which emphasizes student-centred learning. According to Ramsden (1992), quality teaching involves utilizing teaching methods and tasks that require active student participation, collaboration, and the cultivation of competency among peers. An effective teaching method that promotes student-centred learning and simultaneously fosters algebraic thinking skills is problem-solving.

By employing problem-solving approaches, students are encouraged to actively engage in mathematical tasks that involve analysing problems, devising strategies, and applying algebraic concepts to find solutions. This approach facilitates the development of critical thinking, logical reasoning, and creativity in students' problem-solving processes. Additionally, it provides opportunities for collaborative learning, enabling students to discuss and exchange ideas as they work together to solve problems. This collaborative aspect fosters a deeper understanding of algebraic concepts.

Moreover, incorporating real-world and contextualized problems helps bridge the gap

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between abstract algebraic concepts and their practical applications. By connecting algebra to real-life situations, students can recognize the relevance and significance of algebra in solving everyday problems. This enhances the meaningfulness and engagement of the learning experience. By embracing problem-solving approaches and promoting student-centred learning, teachers can create an environment that nurtures algebraic thinking skills among students. This approach encourages active participation, collaboration, and critical thinking, enabling students to develop a profound understanding of algebraic concepts and their practical applications.

Research Problem

According to the results of the Trends in International Mathematics and Science Study (TIMSS) conducted in 2017 (Mullis, 2017), students in Malaysia are still at a deficient level in algebraic thinking skills when it comes to problem-solving. Malaysia only managed to rank 29th out of 42 countries participating in TIMSS, and findings from the TIMSS' achievement also show that only 5% of students could apply, make inferences, and generalize in questions involving algebraic problem-solving in non-routine problems.

According to Stacey (2005), the main challenge in acquiring non-routine problem-solving skills is the requirement for students to possess a diverse set of abilities to effectively solve a problem. Therefore, teaching problem-solving is considered a complex topic to teach (Dendane, 2009). Reiss and Renkl (2002) and Novotná (2014) stated that heuristic methods can improve non-routine problem-solving skills. Heuristic methods are mathematical problem-solving strategies formulated independently of context and carried out systematically (Koichu et al., 2004).

Koichu et al. (2004) also agreed that math classes need to emphasise heuristic methods. Additionally, according to Hoon et al. (2013), bar modelling is one of the heuristic methods that help students solve non-routine problems. Similarly, Polya's problem-solving model relies on heuristic methods in problem-solving (Polya, 1945). Therefore, the Algebraic Thinking Skills Module is developed using heuristic methods as an effort to cultivate algebraic thinking skills, especially in solving non-routine problems.

Theoretical Framework

Theories and models are essential references in developing a module or approach. In developing the ATS module using the digital bar model approach, among the theories and models used are a) Cognitive Constructivism Theory by Piaget; b) Social Constructivism Theory by Vygotsky; c) Behaviourism Theory; d) Polya's Problem-Solving Model; e) Bar Model, and f) Swartz and Parks' Infusion Model (1994). Figure 1 depicts the theoretical framework used for designing and developing the ATS teaching and learning module.

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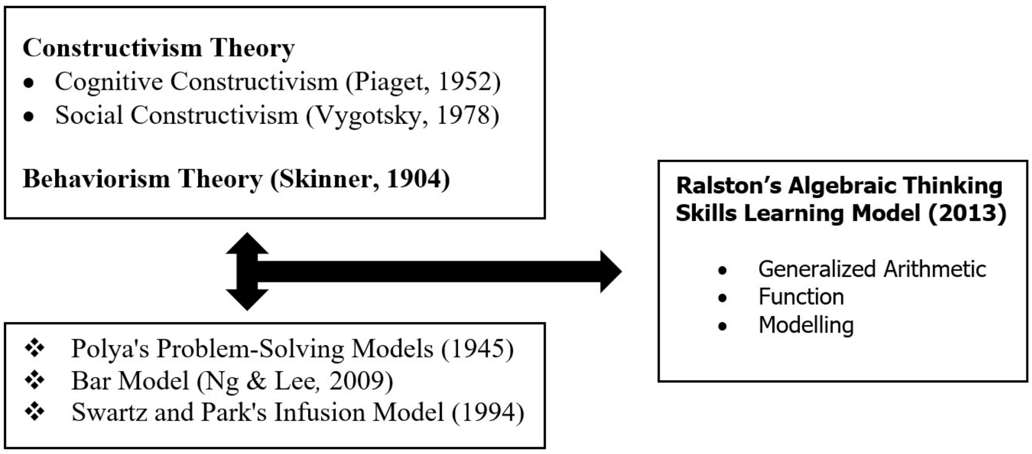
Figure 1
Theoretical Framework Used for the Design and Development of the ATS Teaching and Learning Module

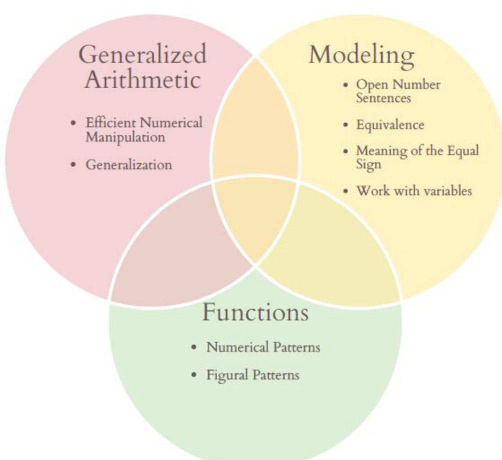


The models used for the development of the Algebraic Thinking Skills Module are based on i) Ralston's Algebraic Thinking Skills Learning Model (2013), ii) Polya's Problem-Solving Model (Polya, 1945), and iii) the bar model.

Ralston's Algebraic Thinking Skills Learning Model (2013)

The algebraic thinking skills (ATS) developed in the Algebraic Thinking Skills Module are based on Ralston's Algebraic Thinking Skills Model (2013). Ralston's model focuses on three constructs: i) Arithmetic Generalization - involving efficient number manipulation and generalization; ii) Function - involving the relationship between quantities including the process to identify, articulate, and form patterns from generalization skills; and iii) Modelling - involving equality and equivalence, open number systems, and computation involving variables (Figure 2).

Figure 2
Ralston's Algebraic Thinking Skills Learning Model (2013)



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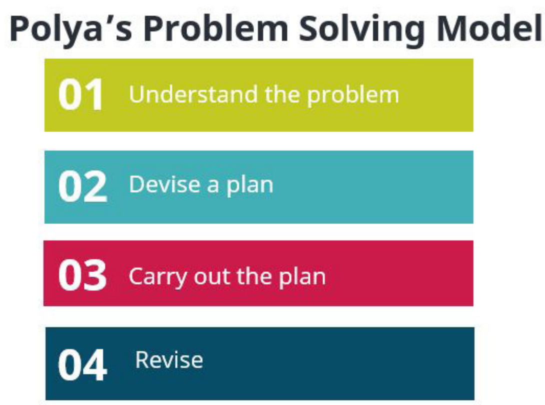
Polya's Problem-Solving Model (1945)

The Polya's Problem-Solving Model (1945) is the core activity in the Algebraic Thinking Skills Module, which includes four stages: (i) understanding the problem, (ii) devising a plan, (iii) carrying out the plan, and (iv) reviewing. Figure 3 shows the phases of problem-solving through the Polya Model.

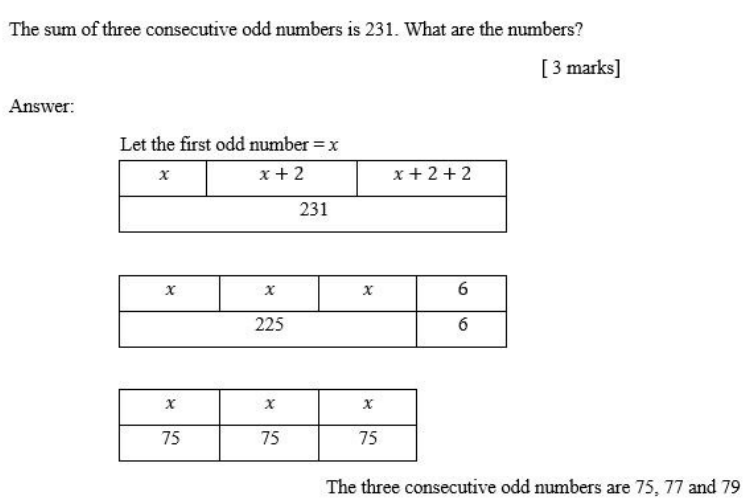
Figure 3
Polya's Problem-Solving Model (1945)



Bar Model

The Bar Model is a diagram in the form of a rectangular bar that is constructed to assist in solving word problems adapted from Ng and Lee (2009). The bar model is a visual representation of a rectangle that depicts both known and unknown quantities, as well as the relationships between these quantities, as indicated in the problem. An example of the Bar Model applied to solve an algebraic problem is illustrated in Figure 4.

Figure 4
Illustration of the Application of Bar Model in Algebraic Problem Solving



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Through the implementation of the Bar Model in this question, students can see the relationship between three consecutive odd numbers and then convert that relationship into an equation. The top and bottom bars are constructed to be of equal length to represent the equality of values between the two bars. This helps students understand the meaning of equality as an effort to solve the given problem.

The digital bar model is the heart of the ATS Module. The digital bar model refers to the free Bar Model application at https://mathsbot.com/manipulatives/bar. Students can manipulate this application by developing their own designed bar model solutions. The digital bar model (DMB) serves as a method to uncover and sharpen students' algebraic thinking skills through digital technology. Below is an example of a problem and suggested solution utilizing the digital bar model. Figure 5 shows the main page of the digital bar model application, while Figure 6 shows an example of a solution using the digital bar model application.

Example of problem-solving question

An association has sold 150 tickets for a charity concert held in conjunction with Independence Month. Adult tickets cost RM15 each, while children's tickets cost RM5 each. The total sales amount is RM1360. What is the number of adult tickets and children tickets sold?

Based on the given information, the linear equation that can be formed is as follows.

$$d + k = 150$$
$$15d + 5k = 1360$$

$*d$ = adult, k = children

Figure 5
The Main Page of the Digital Bar Model Application

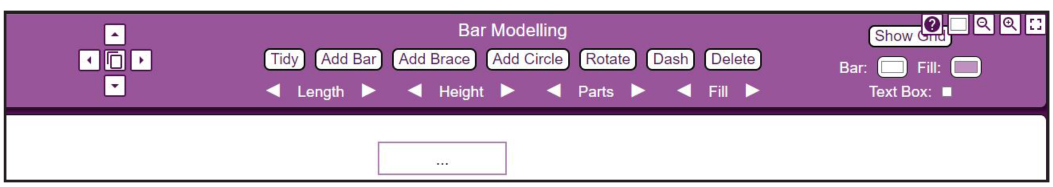
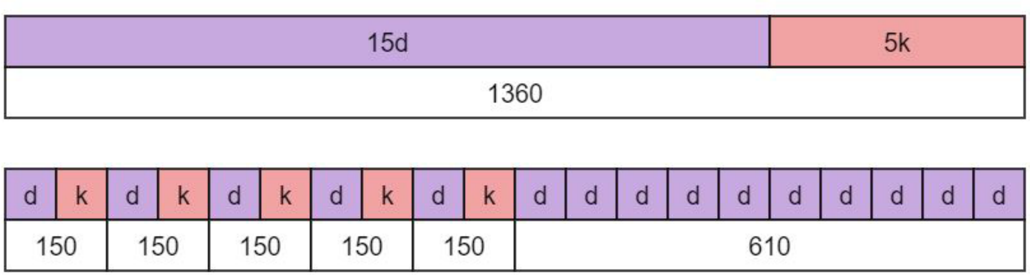


Figure 6
Example of Solution Using the Digital Bar Model Application

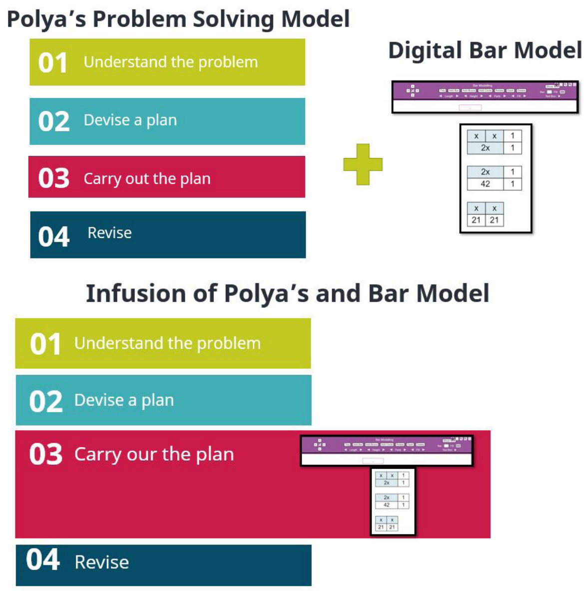


Through the Bar Model in Figure 6, students can find that 10*d* equals 610. This finding helps students determine the value of *d* and, subsequently, the value of *k*.

Infusion of Polya and Digital Bar Model

The digital bar model serves as an additional strategy and is integrated into the third step of Polya's problem-solving model, which is the implementation of strategies. The infusion of the Polya and Digital Bar was used as unit activities in the ATS Module, designed as a teacher's guide to implementing the process of teaching and learning (Figure 7).

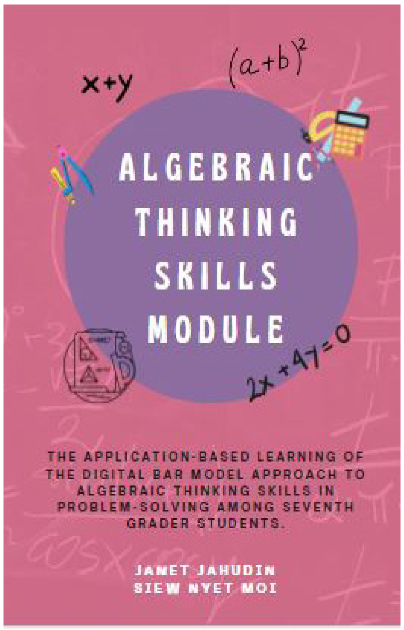
Figure 7
Infusion of Digital Bar Model into the Polya's Problem-Solving Model



Algebraic Thinking Skills Module

The Algebraic Thinking Skills Module (Figure 8) is specifically designed for seventh graders. The objective of the Algebraic Thinking Skills Module is to cultivate algebraic thinking skills in solving algebraic problems involving three constructs: i) Arithmetic Generalization (GA), ii) Function (F), and iii) Modelling (M). Students are required to form, label, and present problem-solving steps using the digital bar model method.

Figure 8
Front Cover of the Algebraic Thinking Skills Module



Swartz and Parks Model (1994)

In Swartz and Park's (1994) infusion model, the focus is on simultaneously identifying and teaching thinking skills alongside the content of the lessons. Two types of infusion are discussed: comprehensive infusion and partial infusion. Comprehensive infusion involves teaching thinking skills in five instructional steps: introduction, active thinking, thinking about thinking, reinforcement practice, and applying thinking. Both content and thinking skills are integrated throughout these steps. On the other hand, partial infusion refers to teaching specific thinking skills in one or two steps, offering an additional strategy that can be implemented at any suitable point. In this study, the partial infusion is considered appropriate as the aims are to assess the effects of thinking-based teaching by applying thinking skills in the procedural process and integrating the bar model method into Polya's Problem-Solving Model. This model is incorporated into the Daily Lesson Plan (RPH) within the Algebraic Thinking Skills (ATS) module, serving as a guide for teachers during the teaching and learning process.

ADDIE Model

The ATS teaching and learning module was designed and developed using the ADDIE Model (Branch, 2010), comprising five phases: analysis, design, development, implementation, and evaluation. The selection of the ADDIE Model was based on its focus on student-centred learning and goal-oriented instructional design to facilitate practical problem-solving and meaningful actions.

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Research Aim and Questions

The purpose of this study was to develop and assess a teaching and learning module that combined the problem-solving approach and digital bar model to foster algebraic thinking skills among seventh-grade students. The guiding framework for this study was established by three research questions presented below.

Q1: Were the results of the evaluation process indicative of the ATS Module's validity, reliability, and feasibility for seventh-grade students?

Q2: Did the implementation of the ATS module lead to a statistically significant change in the mean scores of algebraic thinking skills for seventh-grade students, as indicated by the comparison of pre-test and post-test data?

Q3: Was there a significant difference between i) pre-test mean scores and ii) post-test mean scores for the three constructs of algebraic thinking skills among seventh graders taught using the ATS module?

Research Methodology

Research Design

This study utilized both descriptive and quasi-experimental research designs. The descriptive design focused on assessing the validity, reliability, and feasibility of the ATS teaching and learning module. On the other hand, the quasi-experimental research design aimed to assess the effects of the ATS module on the algebraic thinking skills of seventh graders. The effects of the module were assessed through the performance of seventh graders in the algebraic thinking test, as specified in the second and third research questions. The research was conducted over 12 weeks, from October to December 2022.

Sample

The student selection process was based on the total number of rural secondary schools in Tuaran district of Sabah, Malaysia which amounted to 10 schools. Among these schools, two were randomly chosen based on the similar level of mathematics achievement among their seventh-grade students. Subsequently, two classes, each consisting of 30 students, were randomly selected from two of the chosen schools to form the study sample. The study sample consisted of 64 females (53.3%) and 56 males (46.7%). Chua (2011) recommended a sample size of 30 as sufficient for assessing internal consistency or reliability using Cronbach's alpha. This suggestion aligns with the recommendations of Yurdugül (2008), Bujang et al. (2018), and Conroy (2021) for pilot testing and determining Cronbach's alpha.

Ethical Considerations

Prior to initiating the study, consent was obtained from the parents of the participants and approval was obtained from the school principals. The consent letter included comprehensive information regarding the research's purpose and the extent of student involvement. Parents indicated their comprehension of the research objectives and granted permission for their child's participation. Participants were provided assurance that their responses would remain confidential and anonymous. Furthermore, they were informed of their right to withdraw from the study at any time without experiencing any adverse consequences.

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Descriptive Research

Analyse

The primary aim of designing the Algebraic Thinking Skills (ATS) module was to foster algebraic thinking skills among seventh-grade students, specifically in the context of problem-solving within the mathematics subject. An analysis was conducted to assess the student's needs and the study's context to accomplish the research objectives. During the needs analysis phase, three mathematics teachers from secondary schools in Tuaran, Sabah, were interviewed. The interviews took place from 18th to 24th June 2022 in the teachers' room, aiming to gain insights into the infusion of the bar model and problem-solving techniques for inculcating algebraic thinking skills in the teaching and learning process. The teachers reported that they had limited exposure to the bar model concept and its application in algebraic thinking skills due to the absence of teaching guides or learning modules specifically addressing algebraic thinking skills in secondary school. Additionally, the interviews revealed that the teachers had yet to receive any prior training or courses related to algebraic thinking skills.

The analysis of students and the context involved the utilization of criteria derived from Carlton et al. (2000), which aimed to evaluate students' low performance levels and their existing understanding of Linear Equations. The analysis indicates that students have a relatively low level of achievement in the topic of Linear Equations. The students' knowledge was also assessed on the topic of Linear Equations. For the subtopic of linear equations with one variable, the researcher found that 14 students were able to answer the given questions correctly, indicating their understanding of this subtopic. However, 16 students were unable to answer correctly. Similarly, for the subtopic of linear equations with two variables, 11 students demonstrated knowledge of this unit, while 19 students were unable to answer correctly. There is a need to assist students in improving both their algebraic thinking skills and problem-solving skills. Regarding computer literacy skills, all students demonstrated proficiency in using computers and showed enthusiasm in engaging in group activities. Therefore, teaching will be tailored to the level and needs of the seventh graders as best as possible.

Design and Development of ATS Module

The design phase is the stage where decisions are made on how the information will be presented and learned by the students. In this phase, the information gathered from the analysis phase, including needs analysis, student analysis, and context analysis, is used to plan the strategies for developing the Algebraic Thinking Skills (ATS) module. In other words, during this phase, the teaching methods, learning activities, and assessment processes become clearer (Kemp et al., 1998). The focus of the design phase is to create a draft of the module before developing the actual module materials. This process consists of four parts: (a) stating the learning objectives; (b) identifying relevant theories and models; (c) designing assessment instruments; and (d) planning the teaching strategies within the ATS module.

Stating the Learning Objectives

The aim of the conducted study is to enhance students' algebraic thinking skills. Based on the conducted analysis phase, the researcher has formulated four learning outcomes students will achieve after going through the ATS module. In accordance with the recommendations put forth by Morrison et al. (2012), these learning objectives guide the selection and organization of content, teaching materials, learning resources, learning strategies, assessment methods, and the skills and knowledge that need to be acquired. After completing each unit within the ATS

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module, students are expected to master algebraic thinking skills through the following steps: i. understanding the problem situation through brainstorming activities; ii. planning strategies by identifying variables and mathematical operations involved through brainstorming activities; iii. implementing strategies using the digital bar model approach; and iv. reviewing and revising answers.

Specific Objectives

The specific objectives of this ATS module were as follows:

a. To foster algebraic thinking skills in problem-solving among seventh graders using the Polya and digital bar model infusion approach in the constructs of i) generalized arithmetic, ii) function, iii) modelling.
b. To find out the interrelationship between generalized arithmetic thinking skills, function thinking skills, and modelling thinking skills through the infusion approach of Polya and the digital bar model.

Designing Assessment Instruments

At the initial stage of the planning, the researcher has set the objective of developing this module to cultivate algebraic thinking skills through problem-solving among seventh graders. Therefore, the researcher planned to develop an assessment instrument on algebraic thinking skills that involve problem-solving in the topic of Linear Equations, namely the Algebraic Thinking Skills Test (ATT).

Designing Teaching Strategies

The final step in the design phase is to plan the teaching strategies, which include instructional and learning strategies, lesson content writing for each objective, and lesson activity planning. The chosen learning strategy in developing this module is Polya's Problem-Solving Model approach with integration of the Digital Bar Model method. Next, the researcher proceeds to write the necessary lesson content to achieve the predetermined learning objectives. Then, the researcher carries out the planning of learning activities. In designing the activities, the researcher searches for problem-solving questions on the topic of Linear Equations that serve as a stimulus for each step based on Polya's Problem-Solving Model, ultimately leading to the development of students' algebraic thinking skills. The problem-solving questions are designed based on real-life situations that are relatable to students and presented in a graphical format to capture students' attention. All these problem-solving questions fall under the domain of Relations and Algebra.

Evaluation Phase

According to Taherdoost (2016), it is essential to verify the content of a module before implementing it. The second phase of evaluation aimed to assess the module's reliability, feasibility, and acceptance among the participants. This evaluation phase took place after the ATS Module was introduced to the participants through the teaching and learning process. As suggested by Cohen and Swedlik (2018), a high-quality module typically exhibits a strong level of validity and reliability.

To ensure the module's content validity, a panel of five experts was involved in the verification process. The panel included university lecturers, lecturers from a teaching institute, and secondary school mathematics teachers. It consisted of two professors from a public university, two lecturers from a teaching institute, and an experienced teacher with a wealth of knowledge in secondary school mathematics education. The experts were provided with an evaluation form to assess the pedagogical content of the module. This included aspects such as daily lesson plans, standards and learning outcomes, activity implementation, integration of the digital bar model method with algebraic thinking skills, algebraic thinking skills, overall content, and the opportunity to provide written comments for module improvement. The ATS module was developed in Malay language to ensure the comprehension of secondary school teachers regarding the module's content.

The researchers distributed a 5-point scale questionnaire adapted from Ambo (2019) to assess students' perceptions of each unit in the ATS Module. The evaluation of students' perceptions of each unit in the ATS Module was measured using Cronbach's alpha coefficient.

Quasi-Experimental Research

During the implementation phase of the experimental research, the ATS module was introduced and its effects on students' algebraic thinking skills were assessed. The study employed a pre-test-post-test research design and a total of 120 students from four classes were randomly selected from two secondary schools in the Tuaran district. The participants were given access to the ATS module, which consisted of six learning units, for a duration of 12 weeks between October and December 2022. Each unit required 60 minutes for implementation and included the completion of three missions; Mission 1 involved students engaging in discussions and arguments regarding the given problem-solving questions. The mission focused on discussing and planning solution strategies; Mission 2: Students implement the planned strategies and carry out the problem-solving process; and Mission 3: Students are asked to review and reflect on their findings through an activity involving presenting their results. These three missions can be divided into three stages based on the compatibility with the school's teaching and learning schedule.

To evaluate the module's effects, the Algebraic Thinking Test (ATT) (Jahudin & Siew, 2023) that the researchers designed was used. This instrument was proven valid, reliable, and suitable for evaluating seventh graders' algebraic thinking skills. The Algebraic Thinking Test (ATT) included nine open-ended questions that prompted students to apply, analyse, and evaluate their understanding. These questions were designed to align with the three constructs of algebraic thinking as outlined by Ralston et al. (2018) and were developed based on the content of the Curriculum Standard of Secondary School (CSSS) as well as the Standard Document of Curriculum and Assessment (SDCA) of Form One Mathematics under the field of Relations and Algebra (Curriculum Development Division, 2017: 53 – 56). Each ATT item has a scoring system ranging from 0 to 6, with a minimum and maximum score of 0 and 6, respectively. The student's response determines the score awarded, ranging from 0 to 6 marks for each item.

Data Analysis

Descriptive research data were analysed by calculating the percentage, mean, and standard deviation. Inferential research data were analysed using IBM SPSS (version 26), and the significance level was set at .05.

Research Results

Content Validity

Following the experts' advice, several amendments were made towards the content and activities in the ATS module; i) the format of writing a lesson plan (RPH) needs to be summarized for clear understanding and divided into three steps; ii) the learning outcomes should be specified according to the content standards of each activity, and the success criteria must be stated; iii) the problem-solving using the DMB method needs to be improved in terms of ensuring the consistent length of bars.; iv) the steps for using the digital bar model application should be presented in a concise and clear manner to avoid any confusion; and v) brief explanations for each important concept can be added to enhance the reader's understanding of the module. In general, the expert evaluators reached a consensus that the Algebraic Thinking Skills (ATS) module was well-suited for fostering algebraic thinking abilities among seventh-grade students.

Reliability

The reliability of the module activities can be ascertained when the study participants are able to achieve the objectives and successfully follow the steps for each activity in the module. Therefore, the reliability of the module activities is determined based on the internal consistency coefficient of Cronbach's alpha to assess the measurement level of activities within the module and the same construct to obtain consistent results (Cohen et al., 2018). Hair et al. (2006) suggested that a Cronbach's alpha ≥ .70 is acceptable, while Sekaran and Bougie (2010) stated that a Cronbach's alpha ≥ .80 usually indicates high reliability. The preliminary study results showed that Cronbach's alpha values for all activity units in the module ranged from .81 to .86, with an overall Cronbach's alpha value of .83 for the ATS Module. This indicates a substantial level of reliability for each activity unit within the module and the overall ATS Module. Table 1 shows the Cronbach's Alpha values for the learning activities in the ATS module that were gained from the 30-item questionnaire on the 120 respondents.

Table 1
Cronbach's Alpha Value for Activity Units in the ATS Module

Activity	Units in the module	Cronbach Alpha Value
1.	I am Mr. Integer	.86
2.	Love the Environment	.83
3.	Let's go camping	.83
4.	Small Entrepreneur	.82
5.	Bicycle Factory	.81
6.	Spirit of Patriotism	.82
	Total	.83

Students' Perceptions of the Feasibility of the ATS Module

Hu and Adey (2002) stated that any newly developed tool should have a level of module acceptance by students. To obtain the level of module acceptance by students, a questionnaire

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measuring overall student acceptance level, adapted from Al-Brahmi et al. (2015) and Supasorn et al. (2014), was distributed to 92 study participants involved in the pilot study. The determination of student acceptance level was adapted from Sanger et al. (2007), which stated that a minimum score between 1.00 and 2.33 indicates low acceptance, 2.34 to 3.66 as moderate acceptance, and a score of 3.67 to 5.00 as high acceptance. The summary of criteria for determining student acceptance level towards the ATS Module is presented in Table 2 below, and the finding of a minimum overall score of 4.07 indicates that the level of student acceptance towards the module is high.

Teachers' Perceptions about the Feasibility of ATS Module

In ensuring the implementation of the ATS Module in the teaching and learning process for seventh graders, the researcher conducted a feasibility questionnaire adapted from Ambo (2019). The questionnaire assessing the feasibility of the ATS Module was distributed to 20 lower secondary mathematics teachers. The rationale for selecting 20 teachers to assess the module's feasibility is based on statistical research design, which suggests that an experimental design requires a minimum sample size of 15 for an evaluation (Cohen et al., 2011; Rovai et al., 2013). The teachers were requested to evaluate the module's feasibility by indicating their level of agreement on a scale of one to five for various aspects. These aspects included the implementation of teaching and learning activities, integration of the digital bar model method with constructs of algebraic thinking, algebraic thinking skills, and the overall module. Table 3 displays the teachers' assessment of the module's feasibility.

Table 2
Percentage and Mean for Students' Perceptions of the Feasibility of ATS Module

		Percentage and mean according to Likert Scale					
No	**Pedagogical feasibility criteria**	**1 Strongly Disagree**	**2 Disagree**	**3 Neutral**	**4 Agree**	**5 Strongly Agree**	**Mean ($\bar{x}$)**
1	I feel excited throughout the learning process of Mathematics using the ATS Module	0 (0)	4 (4)	30 (33)	32 (35)	26 (28)	3.87
2	I will become more interested in participating in learning activities using the ATS Module in the future.	2 (2)	1 (1)	25 (27)	27 (29)	37 (41)	4.09
3	The activities in the ATS Module have stimulated my interest in algebra problem-solving questions.	0 (0)	0 (0)	20 (22)	32 (35)	40 (43)	4.22
4	My skills in manipulating numbers efficiently have improved after participating in Mathematics learning using the ATS Module.	0 (0)	0 (0)	11 (12)	40 (43)	41 (45)	4.33
5	My skills in making arithmetic generalizations have developed further after participating in Mathematics learning using the ATS Module.	0 (0)	15 (16)	27 (29)	43 (47)	7 (8)	3.46

6	I understand generalization through the diagram, where two bars of equal length have the same value after participating in Mathematics learning using the ATS Module.	0 (0)	0 (0)	34 (37)	37 (40)	21 (23)	3.42
7	I can express the relationship between two or more variables in the form of quantities after participating in Mathematics learning using the ATS Module.	0 (0)	6 (7)	39 (42)	17 (18)	30 (33)	3.77
8	I can express the relationship between two or more variables in the form of number patterns after participating in Mathematics learning using the ATS Module.	0 (0)	0 (0)	15 (16)	25 (27)	52 (57)	4.40
9	I have become more proficient in using variables, especially in problem-solving, after participating in Mathematics learning using the ATS Module.	0 (0)	0 (0)	8 (9)	39 (42)	45 (49)	4.45
10	I can understand that the symbol "equals to" signifies equality after participating in Mathematics learning using the ATS Module.	0 (0)	6 (7)	39 (42)	17 (18)	30 (33)	3.77
11	I can represent equations in the form of a digital bar model after participating in Mathematics learning using the ATS Module.	0 (0)	0 (0)	15 (16)	25 (27)	52 (57)	4.40
12	The ATS Module allows me to discuss with my peers while solving non-routine problems.	0 (0)	0 (0)	8 (9)	39 (42)	45 (49)	4.40
13	The ATS Module enables me to communicate with my peers and teachers while solving non-routine problems.	0 (0)	0 (0)	11 (12)	40 (43)	41 (45)	4.33
14	The ATS Module encourages me to actively engage during teaching and learning sessions.	0 (0)	13 (14)	15 (16)	53 (58)	11 (12)	3.67
	Total	2 (0.16)	45 (3.5)	297 (23.1)	466 (36.2)	478 (37.10)	4.07

Table 3
Teachers' Assessment of the Feasibility of the ATS Module

Activity	Units in the module	Mean ($\bar{x}$)
1.	Implementation of Teaching and Learning Activities	4.89
2.	Integration of Bar Model Digital Method with Algebraic Thinking Skills	4.78
3.	Algebraic Thinking Skills	4.83
4.	Overall Module	4.93
	Overall Mean	4.87

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Effects of the ATS Module

The Algebraic Thinking Skills module was tested on 120 seventh graders in two secondary schools in Tuaran District, Sabah. The students were given an Algebraic Thinking Skills Test instrument to assess their attainment of algebraic thinking skills before and after the intervention using the Algebraic Thinking Skills module. Inferential data analysis was conducted using paired samples *t*-test to determine if there was a significant difference between the pre-test and post-test scores of the Algebraic Thinking Skills. Table 4 shows the results of the paired samples *t*-test analysis for algebraic thinking skills (t (119) = -17.553, $p < .05$).

Table 4
Results of Paired Sample t-test Analysis

Paired Differences								
Pair	$\bar{x}$	s	σ_M	95% Confidence Interval of the Difference		t	df	p
				Lower	Upper			
Pre-Test – Post Test	-21.542	13.444	1.227	-23.972	-19.112	-17.553	119	$p < .05$

Table 5 shows the results of the Paired Samples *t*-test analysis for three constructs of algebraic thinking skills. The analysis results are significant for the differentiation of generalized arithmetic skills (t (119) = -17.964, $p < .05$), function skills (t (119) = -18.713, $p < .05$), and modelling skills (t (119) = -15.368, $p < .05$). This indicates that there are significant differences in the mean scores between the pre-test and post-test for all three constructs of algebraic thinking skills.

Table 5
Results of Paired Sample t-test Analysis Based on Constructs

Paired Differences								
Pair	$\bar{x}$	s	σ_M	95% Confidence Interval of the Difference		t	df	p
				Lower	Upper			
Pre GA – Post GA	-4.733	2.886	.264	-5.255	-4.212	-17.964	119	$p < .05$
Pre F – Post F	-5.950	3.483	.318	-6.580	-5.320	-18.713	119	$p < .05$
Pre M – Post M	-10.858	7.740	.707	-12.257	-9.459	-15.368	119	$p < .05$

Discussion

The purpose of this study was to evaluate a module for developing Algebraic Thinking Skills that specifically addresses the demands faced by students in the 21st century. The researcher evaluated the module's validity, reliability, feasibility, and effects to ensure its practicality in secondary schools. The development of the module was guided by a comprehensive theoretical framework that incorporated Polya's problem-solving approach and digital bar model based

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on Swartz and Park's infusion model. The researchers measured the module's effectiveness in fostering three key aspects of algebraic thinking skills: generalized arithmetic, function, and modelling.

Overall, the findings of the study indicate that the Algebraic Thinking Skills (ATS) module demonstrated good content validity and reliability. The content validity assessment involved the evaluation of the module by a panel of five expert evaluators, who positively received it. The module underwent revisions based on their feedback to enhance certain aspects. The reliability analysis, measured using Cronbach's Alpha, indicated that the ATS module's reliability fell within an acceptable range. Furthermore, the study confirmed that the module is feasible for improving seventh graders' algebraic thinking skills.

The results of the paired sample *t*-test demonstrated a significant improvement in the three aspects of algebraic thinking skills among seventh graders after they received intervention using the ATS module. This indicates that the ATS module's teaching and learning approach had a positive impact on the students' algebraic thinking abilities. The utilization of the heuristic method, incorporating Polya's problem-solving approach and digital bar model, proved to be an effective tool for students when solving non-routine algebraic problems. Additionally, as stated by Morin et al. (2017), the bar model method not only aids in visualizing problems but also enhances students' cognitive development during the problem-solving process.

The utilization of Polya's problem-solving approach has demonstrated its benefits for seventh graders, as it offers a broader context for developing successful problem-solving skills in algebra. The ATS module's focus on problem solving allowed students to make connections between mathematical problem-solving and the development of algebraic thinking. It became evident that teaching problem-solving strategies within a mathematical context contributes to the enhancement of complex problem-solving skills, thereby strengthening algebraic thinking skills. This aligns with the findings of Pehkonen (2019), who emphasized the primary role of mathematics education in fostering students' understanding of mathematical structures and promoting mathematical thinking across various age groups. The teaching process should support learners' mathematical thinking, provide a solid grasp of mathematical concepts, and equip them with the necessary skills to handle information and solve problems effectively.

Engaging students in problem-solving activities through the ATS module has led to a shift in learning theory from behaviourism to constructivism, which has significantly influenced the teaching and learning of algebra (Rosli et al., 2013). When students are involved in complex problem-solving tasks, it means that they are not merely solving problems to apply prior knowledge of algebra but rather using problem solving to learn new algebraic concepts. This approach facilitates the development of relational understanding among students (Walle et al., 2014). This emphasizes the belief that problem solving is fundamental in learning algebra, as it emphasizes reasoning over memorization. Through problem solving in the ATS module, students are encouraged to develop a deeper understanding of algebraic concepts and can explain the processes they use to arrive at solutions, rather than relying on rote memorization and application of procedures (Klerlein et al., 2019).

Furthermore, exposure to the problem-solving approach in the ATS module creates an opportunity for students to collaboratively construct diagrams, schemes, or maps of the problems at hand, fostering dynamic and collaborative discussions (Szabo et al., 2020). This approach emphasizes the inclusion of problem-solving tasks in each activity unit, and by integrating the digital bar model, it enables students to develop solutions following Polya's approach. The incorporation of the digital bar model in the ATS module serves as a tool that guides students in generating solutions and aids their understanding of the problem-solving process.

The test results also provide evidence that the Algebraic Thinking Skills Module has a positive impact on fostering generalized arithmetic, function, and modelling skills among seventh graders. In mastering generalized arithmetic skills, students need to master the skills of determining operations and variables and transforming text problems into mathematical statements. This is supported by Maputol (2019), who stated that basic addition, subtraction, multiplication, and division operations can be gradually applied to algebra by emphasizing patterns, generalization, and the relationship between numbers.

The Algebraic Thinking Skills Module also supports improvement in function skills, which emphasize pattern recognition, number patterns, and the relationship between variables. Kaput (1999) suggests the use of various visual representations such as diagrams, value tables, equations, and graphs to represent function relationships as it supports conceptual learning. The module also positively affects mastering modelling skills, which focus on understanding the "equals" symbol in linear equations and computations involving variables. This is supported by the study of Jan and Rodrigues (2012), which found that bar model representation and Polya's problem-solving model enable students to formulate mathematical equations easily and confidently. Through the generated bar models, students can visualize the relationship between the known and unknown in solving non-routine problems. Similarly, according to Warren and Cooper (2009) and Vlassis (2002), the bar model method assists students in understanding the meaning of the "equals" symbol, representing equivalence between both sides of linear equations rather than as a symbol for the final answer.

The above discussion shows that the infusion model of Polya and digital bar ensure an improvement in the mastery of algebraic thinking skills among seventh graders.

Conclusions and Implications

The Algebraic Thinking Skills Module has been proven to be effective in improving the mastery of algebraic thinking skills among seventh graders through the three constructs of Ralston's Algebraic Thinking Skills: generalized arithmetic, function, and modelling. This demonstrates that the problem-solving strategies applied in the Algebraic Thinking Skills Module through the Infusion Model of Polya and Digital Bar can assist students in solving non-routine algebraic problems. Both Polya's Problem-Solving Model and the Bar Model approach are heuristic methods, which are systematic problem-solving strategies. This means that the heuristic method through Polya's Infusion Model and Digital Bar can have an impact on the mastery of algebraic thinking skills among seventh graders.

The Algebraic Thinking Skills Module provides added value to classroom practices and curriculum innovation by presenting alternative problem-solving strategies through the Infusion Model of Polya and Digital Bar for solving non-routine algebraic problems. Moreover, these alternative problem-solving strategies can be highlighted as strategies for teaching students to solve word problems at the elementary school level. Additionally, the Algebraic Thinking Skills Module can be used to promote digital and active learning in Mathematics classrooms. Furthermore, technology application experts can transform the bar model approach into digital games to engage students in non-routine problem-solving during the teaching and learning of Mathematics.

Declaration of Interest

The authors declare no competing interest.

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