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Spatial Biases in Approximate Arithmetic Are Subject to Sequential Dependency Effects and Dissociate From Attentional Biases

Maria Glaser [1], André Knops [1,2]

[1] *Department of Psychology, Humboldt-Universität zu Berlin, Berlin, Germany.* [2] *CNRS UMR 8240, Laboratory for the Psychology of Child Development and Education, Université Paris Cité, Paris, France.*

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Corresponding Author: Maria Glaser, Department of Psychology, Humboldt-Universität zu Berlin, Rudower Chaussee 18, 12489 Berlin, Germany. E-mail: maria.glaser@hu-berlin.de

Supplementary Materials: Data, Materials [see Index of Supplementary Materials]

Abstract

The notion that mental arithmetic is associated with shifts of spatial attention along a spatially organised mental number representation has received empirical support from three lines of research. First, participants tend to overestimate results of addition and underestimate those of subtraction problems in both exact and approximate formats. This has been termed the operational momentum (OM) effect. Second, participants are faster in detecting right-sided targets presented in the course of addition problems and left-sided targets in subtraction problems (attentional bias). Third, participants are biased toward choosing right-sided response alternatives to indicate the results of addition problems and left-sided response alternatives for subtraction problems (Spatial Association Of Responses [SOAR] effect). These effects potentially have their origin in operation-specific shifts of attention along a spatially organised mental number representation: rightward for addition and leftward for subtraction. Using a lateralised target detection task during the calculation phase of non-symbolic additions and subtractions, the current study measured the attentional focus, the OM and SOAR effects. In two experiments, we replicated the OM and SOAR effects but did not observe operation-specific biases in the lateralised target-detection task. We describe two new characteristics of the OM effect: First, a time-resolved, block-wise analysis of both experiments revealed sequential dependency effects in that the OM effect builds up over the course of the experiment, driven by the increasing underestimation of subtraction over time. Second, the OM effect was enhanced after arithmetic operation repetition compared to trials where arithmetic operation switched from one trial to the next. These results call into question the operation-specific attentional biases as the sole generator of the observed effects and point to the involvement of additional, potentially decisional processes that operate across trials.

Keywords

spatial biases, MNL, approximate arithmetic, numerical cognition, operational momentum

Previous research has hinted at a functional association between the concepts of numbers and space (Knops, 2018). Several studies reported that attentional shifts to the left are elicited by small and shifts to the right are elicited by large number processing (attentional SNARC, Casarotti et al., 2007; Myachykov et al., 2016; Nicholls et al., 2008). A metaphor for how numbers are mentally represented that was proposed in that context is the *mental number line* (MNL): Numerical magnitude is supposedly represented in ascending order on an analogue number line with smaller numbers left from larger numbers.

Extending spatial-numerical associations to the realm of mental arithmetic, McCrink et al. (2007) and Knops et al. (2009) demonstrated that the estimated outcome of addition problems was overestimated while the outcome of subtraction problems was underestimated. This finding was termed *Operational Momentum* (OM) effect and was observed in non-symbolic (Knops et al., 2009; McCrink et al., 2007) and symbolic (Knops et al., 2009; Pinhas & Fischer, 2008) numerical formats. One hypothesis to explain that effect assumes attentional shifts on the internal mental representation (MNL) from the first operand by the amount of the second operand into the direction of the result. According to this *attentional shift hypothesis* (Hubbard et al., 2005), the OM is caused by an overshoot (momentum) on that internal representation to the right during addition and left during subtraction processing, resulting in relative over- and underestimation.

Other accounts were put forward to explain the OM effect. According to the *heuristics account*, the OM is the result of the simple heuristic to accept a result as long as it is more than the initial operand during addition and less during subtraction (McCrink et al., 2007; McCrink & Wynn, 2009). McCrink et al. (2007) and Chen and Verguts (2012) proposed that the OM is caused by a faulty decompression of the presumably logarithmically compressed numerical representation (*compression account*): Calculation with these faulty magnitudes will produce the typical overestimation and underestimation pattern of the OM. Finally, the *arithmetic heuristics and biases* model (AHAB, Shaki et al., 2018) assumes that three competing biases and heuristics (more-or-less heuristic, sign-space association, anchoring bias) are the cause of the OM effect. These biases contribute to arithmetic processing and depending on their relative weight produce the differential performance patterns. In summary, several theories exist that do not necessarily rely on spatial components to explain the OM effect.

Further evidence for the involvement of spatial representations during mental arithmetic comes from the *Space-Operation Association of Responses* (SOAR effect; Knops et al., 2009). When presented with an array of response options that are located equidistant from a central fixation point, participants preferentially selected (upper) right-hand options for addition problems, but more (upper) left-hand options for subtraction problems. Attentional deflections to the left during subtraction and right during addition have been reported by independent groups since then (e.g. Glaser & Knops, 2020; Liu et al., 2017; Masson & Pesenti, 2014). Similarly, modulating the locus of spatial attention affected addition and subtraction performance (e.g. Masson & Pesenti, 2016; Mathieu et al., 2016).

These attentional biases in mental arithmetic cannot be explained by all above theories. The heuristics account and the compression account alone do not predict spatial biases as they contain no spatial components. The AHAB model, on the other side, stipulates an association between addition signs and the right side of space and subtraction signs and the left side of space. This component which takes action in situations "when stimuli or responses are spatially distributed" can explain these spatial biases (Shaki et al., 2018, p. 141).

When it comes to the processes underlying mental arithmetic, a global and approximate evaluation is distinguished from an exact retrieval process. This dual-process assumption was based on the reaction time advantage for incorrect response alternatives with a large numerical distance from the correct outcome over correct response alternatives (Ashcraft & Stazyk, 1981). The extent to which approximate and exact processing strategies are concurrently applied depends on the stimuli and the task at hand (e.g. Klein et al., 2009).

The question arises what process gives rise to the hypothesised attentional shifts during mental arithmetic. The OM effect which has originally been interpreted as a consequence of the movement on the spatial representation of magnitude (attentional shift hypothesis) has been observed both during exact and approximate arithmetic but was stronger in a non-symbolic (i.e. purely approximate) calculation context (Katz & Knops, 2014; Knops et al., 2009). The SOAR effect emerged in non-symbolic (i.e. approximate) calculation but not in symbolic (i.e. exact) formats (Knops et al., 2009). Yet, the studies that tested the involvement of spatial attention in mental arithmetic by probing attentional modulations in target detection or temporal order judgment tasks as a consequence of the arithmetic operations, used only symbolic stimuli (e.g. Glaser & Knops, 2020; Liu et al., 2017; Masson & Pesenti, 2014). Such designs allowed for exact numerical processing that presumably involves both exact and approximate processing mechanisms (e.g. Li et al., 2018; Liu et al., 2017). This makes it impossible to trace back (spatial attention) effects to only one of either mechanism. Hence, no study has so far systematically investigated spatial biases in the context of (solely) approximate arithmetic.

The present study intended to fill this gap by investigating whether the approximate solution process to non-symbolic addition and subtraction tasks induces attentional shifts to the right and left, respectively. The arithmetic task made use of dot arrays that contained a minimum of 8 dots to avoid exact processing and rapid counting strategies. We sequentially presented an operation cue, the first operand dot array followed by a second presentation of the operation cue, the second operand dot array, and the concurrent presentation of four response option (RO) arrays. The participant's task was to indicate verbally which of these four ROs was the correct result to the arithmetic problem. Spatial attention during the approximate calculation phase was measured with a target detection task between the arithmetic problem presentation and the RO presentation. To investigate the time-course of potential attentional shifts, we varied the delay between the offset of the second operand and the onset of the target detection task (150, 300, 500 ms). Delays were chosen in accordance with previous studies (e.g. Liu et al., 2017). We predicted a rightward shift of attention for approximate addition processing (RTs right-sided targets < RTs left-sided targets) and a leftward shift of attention during subtraction processing (RTs left-sided targets < RTs right-sided targets).

We found no operation-dependent shifts in the target detection task but a spatial bias in the choice of RO locations of the arithmetic task (SOAR effect). To increase overall accuracy and overcome the overall underestimation bias that might have masked spatial effects in the target detection task, we conducted a second experiment that involved feedback on the arithmetic response during the practise phase. Again, no operation-dependent shifts were observed in the target detection task, but we replicated the SOAR effect in the arithmetic task. Exploratory analyses revealed that the OM effect is subject to sequence effects. The OM effect in trial *n* is more pronounced if the arithmetic operation is identical in trials *n* and *n*-1 compared to trials where the operation changes between *n* and *n*-1.

Experiment 1

Method

Participants

Eighteen German-speaking students from the Humboldt-Universität zu Berlin (M_{age} = 22.67 years, SD_{age} = 4.68 years, range: 18-32 years, 13 female, 18 right-handed) took part in the experiment in exchange for course credit. All participants had normal or corrected-to-normal vision and hearing. The experiment was non-invasive, and all procedures were carried out in accordance with the ethical standards established by the Declaration of Helsinki. Informed consent was obtained in written format from all individual participants.

Stimuli

9 addition and 9 subtraction problems that were matched with regard to their operands from Knops et al. (2009) were used (see the Supplementary Materials for the stimulus list). For all problems, 5 deviants (including the correct result) were created via the formula: $C \times 2^{(r+i/2)}$ (*C* is the correct result, *r* is drawn from a uniform distribution between -0.25 and 0.25, and *i* ranges from -2 to +2). Hence, without the jitter that was created with the *r* parameter, deviants ranged from *C*/2 to *C*×2. In the experiment, only four ROs out of five deviants were presented to avoid the strategy of always selecting the middle option. These ROs were later presented randomly in the four quadrants of the screen. To arrive at 162 tasks per operation, we created 18 variants per task by a) jittering the operands (O1+1 ± O2-1; O1 ± O2; O1-1 ± O2+1), and b) by varying the range of the ROs (upper vs lower four ROs of five deviants), so that in 50% of the trials the 2^{nd} RO (upper range)/ 3^{rd} RO (lower range) was correct.

The non-symbolic dot stimuli for the arithmetic task were created using MATLAB (R2016a) and the Psychtoolbox library (Brainard, 1997; Kleiner et al., 2007; Pelli, 1997) via a method that was adapted from Katz and Knops (2014) which in turn was based upon the method described by Gebuis and Reynvoet (2011). We first generated dot arrays for the operands. The dot arrays for the four ROs were controlled for intensive and extensive parameters to avoid selection strategies based upon these parameters instead of quantity. Two versions for each RO were created. Then, correlations between quantity and area subtended, between quantity and mean dot size, as well as between area subtended and mean dot size for all possible combinations of the four RO dot arrays were checked. We selected those stimuli with individual correlations $r < .2$. The mean correlation between quantity and area subtended (extensive visual parameter) was $r = .063$ ($SD = .116$) and between quantity and mean dot size (intensive visual parameter) was $r = -.018$ ($SD = .099$).

Apparatus

The experiment was presented via MATLAB (R2016a) and the Psychtoolbox package (Brainard, 1997; Kleiner et al., 2007; Pelli, 1997) on an LCD monitor (resolution 1080 x 1920; refresh rate: 100 Hz; distance to screen: ~60 cm). We attached a black cardboard with a central quadratic opening to the screen to reduce the possibility of priming towards the horizontal axis. For the target detection task, participants were instructed to press the space bar on a keyboard with their preferred hand once a target was detected. Verbal responses in the arithmetic task were recorded via microphone. The voice onset reflects the time the sound signal exceeded a threshold level and launched the next trial. Data was analysed via R (R Core Team, 2022).

Procedure

The visual stimuli were presented on a light grey background. For fixation we used a white asterisk. All dot stimuli consisted of white dots on a grey circular background (radius 9.3°). Targets consisted of dark grey squares (.75° × .75°) that were presented with a distance of 5.12° from fixation (measured from the centre of the target).

The 396 trials were presented in 11 blocks of 36 trials each. Practise blocks contained 30 trials drawn randomly from the set of experimental trials. One practise block was mandatory. Another block was optional. Addition and subtraction trials were presented in interleaved, randomised order.

Every trial started with the fixation asterisk at the centre of the screen for 300 ms (Figure 1). The operation cue (letter 'A'/'S') was presented for 750 ms and indicated the upcoming arithmetic operation. Then, the first operand stimulus was presented for 750 ms, followed by the operation cue (500 ms), and the second operand (750 ms). The fixation asterisk was presented again to variably (150, 300, 500 ms plus jitter -80, -40, +40, +80 ms) delay the presentation of a target on the right or left side of fixation in 82% of the trials. In 18% no target appeared. Participants were instructed to press the space bar with their preferred hand upon target detection. Once decided upon the hand to use, participants were instructed to not change the hand during the experiment (in 2 cases the hand used did not match the self-reported handedness). Participants were told to respond as fast and accurately as possible. The target stayed on screen until a button press or a maximum of 2 seconds. After the button press or 2 seconds after the onset of the fixation a 500 ms blank screen was presented, followed by four ROs in the quadrants of the screen. Participants had to indicate verbally which was the correct result to the arithmetic problem presented beforehand. The letters 'C', 'D', 'E', 'F' were presented next to the respective ROs. The letter 'A' was avoided because it was already serving as an operation cue, and the letter 'B' to avoid misunderstanding it for letter 'D'. 500 ms after voice onset, the ROs disappeared for an inter trial interval (ITI) of 500 ms. If 6 s after the onset of the ROs no verbal response was detected, participants were prompted to respond faster or louder by a message on the screen for 1500 ms before the ITI and the next trial were launched. The complete experimental session lasted 1.5 hours.

Design

The factors operation (addition, subtraction), delay (bins around 150, 300, 500 ms) and target side (left, right) were varied within subjects. All condition combinations were repeated 27 times (i.e. 162 trials per operation). Additionally, 72 catch trials (no target) were included to ensure participants' attention. Hence, the experiment consisted of a total of 396 trials.

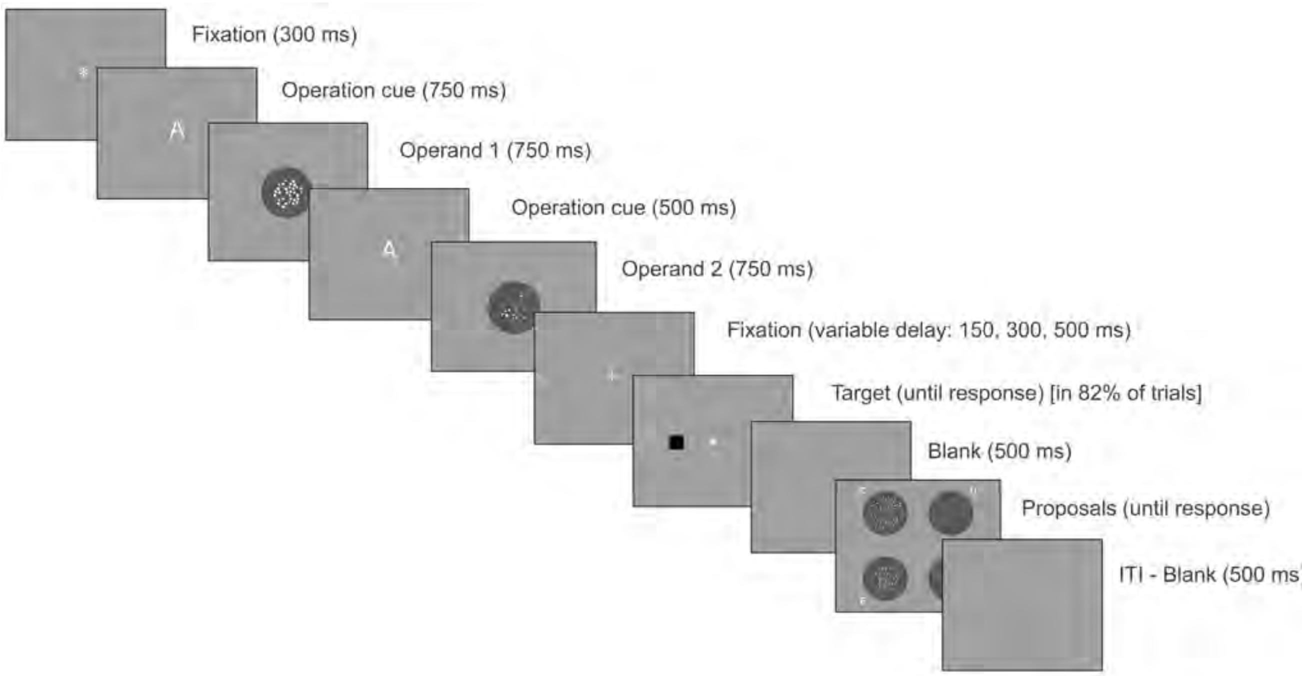
Data Analysis

Reaction time (RTs) measurement in the target detection task started with the presentation of the target. We eliminated RTs that deviated more than two standard deviations from the subject's mean (4.25%) or that were shorter than 200 ms (0.1%). For inferential analysis, RTs were $\log_{10}$-transformed because the distribution of non-transformed RTs is not symmetrical.

For the analysis of the arithmetic task, catch trials were included as they contained an arithmetic task response worth investigating. As the ROs were jittered, the correct RO did not contain the same amount of dots for all repetitions of that task. Therefore, we calculated the mean of these amounts over all repetitions in the experiment, so that the "correct value" in the result section constitutes the mean of the correct amounts. The correct value and the chosen value by the participant were $\log_{10}$-transformed. Whenever Mauchley's test of sphericity indicated a violation of the sphericity assumption, the Greenhouse-Geisser correction was used. Raw data are available as Supplementary Materials.

Figure 1

Trial Structure of Experiment 1 and Experiment 2



Results

Target Detection Task

In no-target (catch) trials the error rate (i.e. false alarm rate) was around 0.28%. For the further analysis of the target detection task, only left- and right-sided target trials (i.e. no catch trials) were considered. We predicted an interaction between target side and operation in the form of faster RTs for left-sided targets in subtraction trials and faster RTs for right-sided targets in addition trials. Mean RTs (and *SD*s) are shown in Table 1. Figure 2 depicts the difference values between RTs to left- and right-sided targets (deltaRT = RTleft - RTright). Negative values indicate that the left-sided targets were detected faster (than the right-sided targets) and vice versa for positive values. The figure indicates that in most of the conditions right-sided targets were detected faster, independent of operation. A repeated measures ANOVA with the factors operation, delay and side of target on the log-transformed mean-aggregated RT variable revealed main effects of operation, $F(1, 17) = 6.795$, $p < .05$, $\eta^2_G = 0.002$, and delay, $F(2, 34) = 32.096$, $p < .001$, $\eta^2_G = 0.026$. Crucially, the ANOVA did not support our hypothesis of a significant interaction between target side and operation, $F(1, 17) = 1.77$, $p = 0.2$, $\eta^2_G = 0.0004$.

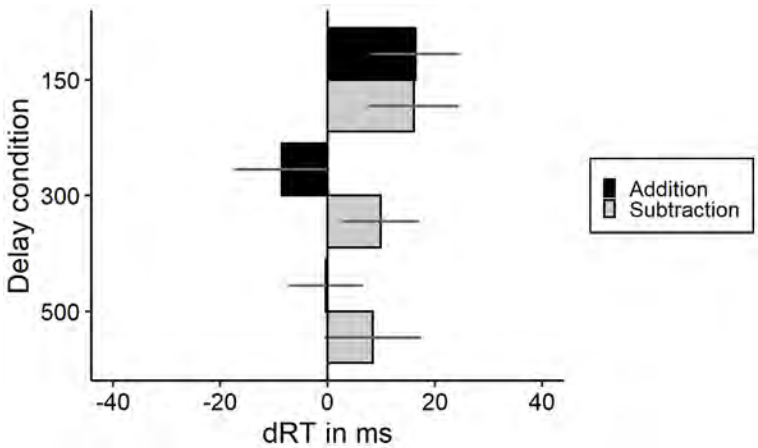
Table 1

Mean RT (and SD) in ms of the Target Detection Task by Operation, Delay and Target Side of Experiment 1

	Addition			Subtraction		
Target side	**150**	**300**	**500**	**150**	**300**	**500**
Left	527 (170)	477 (151)	475 (160)	508 (174)	481 (153)	471 (152)
Right	511 (165)	488 (168)	475 (165)	495 (161)	470 (150)	462 (160)

Figure 2

Difference in Reaction Times Between Left-Sided and Right-Sided Targets by Delay and Operation (deltaRT = RTleft - RTright) of Experiment 1



Note. Positive values indicate that the right-sided targets were detected faster, negative values indicate that the left-sided targets were detected faster.

Arithmetic Task: Non-Random Distribution of Responses

We needed to examine whether the participants chose among the four ROs randomly. A non-random distribution (centred around the correct value) would indicate that they did indeed base their judgments on the arithmetic problem at hand. Figure 3A shows the response percentages for the deviants of the present experiment. The correct outcome is labelled D3. Depending on the RO range, either the upper (D2-D5) or lower (D1-D4) four ROs were presented. The plot indicates that in case of addition, the distribution was centred around the correct value (D3) in the upper range condition, and around the second RO (D2) in the lower range condition. For subtraction, subjects predominantly selected the smallest available option (D1 in lower range, D2 in upper range condition). This suggests an underestimation bias and generally reflects a pattern that has already been observed before (Knops et al., 2009). An ANOVA on the response percentages by operation, RO rank (P1 - P4) and RO range (upper, lower) showed no main effect of operation, $F(1, 17) < .001$, $p > .99$, $\eta^2_G < 0.0001$, but a main effect of RO rank, $F(2.22, 37.75) = 24.609$, $p < .001$, $\eta^2_G = 0.36$, and significant interactions between operation and RO rank, $F(2, 34) = 32.336$, $p < .001$, $\eta^2_G = 0.443$, as well as between RO rank and RO range, $F(3, 51) = 15.901$, $p < .001$, $\eta^2_G = 0.089$. Finally, the ANOVA showed a significant three-way interaction between operation, RO rank and RO range, $F(3, 51) = 7.626$, $p < .001$, $\eta^2_G = 0.038$. Overall, the ANOVA indicated that in both operations the four ROs were not chosen at random. Instead, depending on the range condition, different patterns of RO choices emerged.

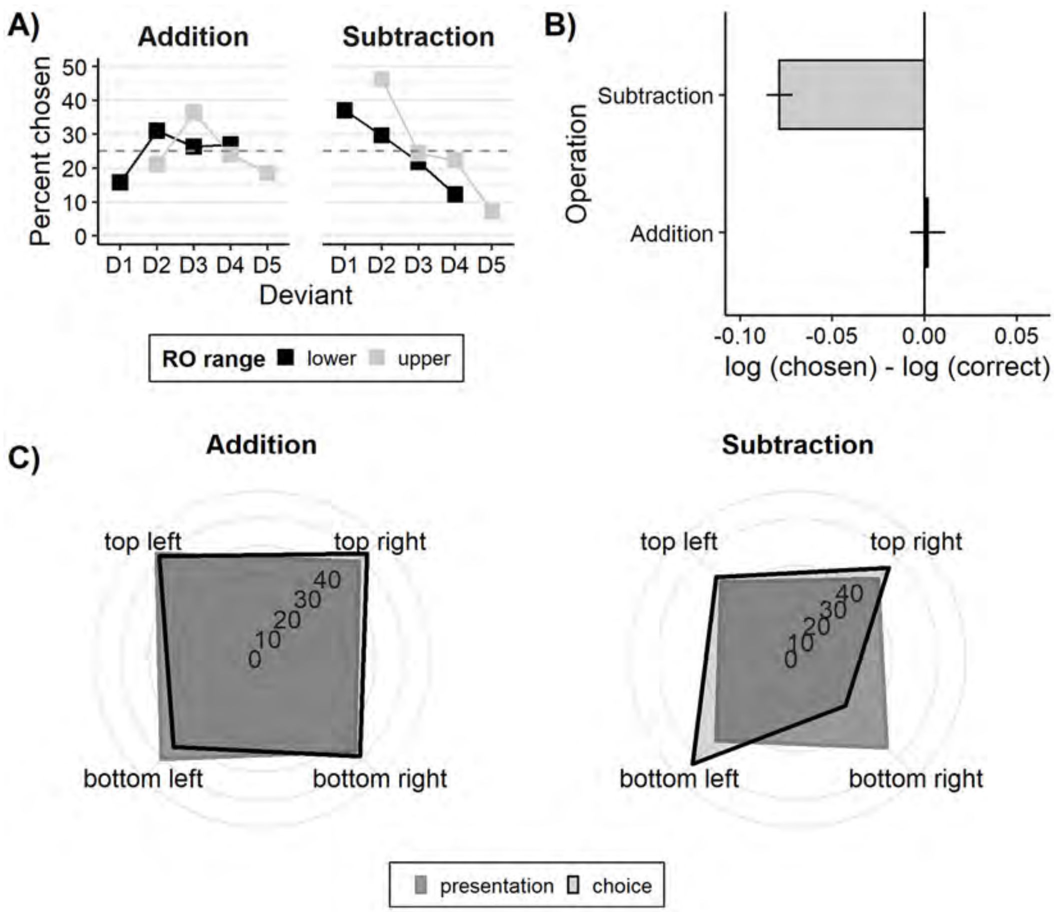
In line with previous results, we observed that responses for addition problems were on average more accurate compared to subtraction. Two observations underline this. First, on average the correct response alternative was chosen significantly more often in addition ($M = 0.316$, $SD = 0.05$) compared to subtraction, $M = 0.232$, $SD = 0.055$; $t(17) = 5.206$, $p < .001$. Second, while response distributions peaked at or close to the correct outcome for additions (see Figure 3A), in subtraction the most often chosen response alternatives were the smallest values on screen.

Arithmetic Task: Operational Momentum Effect

For the analysis of the Operational Momentum (OM) effect, we calculated the response bias as the difference between the logarithm of the chosen value and the logarithm of the correct value. Figure 3B shows the mean response bias for both operations. Participants tended to select smaller outcomes (compared to correct values) in subtraction tasks ($M = -0.079$, $SD = 0.029$) compared to addition tasks, $M = 0.002$, $SD = 0.041$, $t(17) = 6.356$, $p < .001$. Two one-sample t-tests against zero for both operations indicated that participants significantly underestimated subtraction problems, $t(17) = -11.38$, $p < .001$, but that their performance was fairly accurate for addition problems, $t(17) = .173$, $p = .865$.

Figure 3

Arithmetic Task of Experiment 1



Note. A) Distribution of the participants' choices across the five deviants. D3 was always the correct outcome. Depending on the RO range condition only the upper (D2-D5) or lower (D1-D4) four ROs were drawn from the set of five deviants. B) Mean response bias defined as the difference between the logarithm of the chosen value and the logarithm of the correct values. A negative value indicates an underestimation, and a positive value indicates an overestimation. C) The response frequencies of the four RO locations by arithmetic operation (black). Grey lines indicate the presentation frequencies of the correct RO at the given location.

Arithmetic Task: Influence of the Arithmetic Operation on the Spatial Distribution of Responses

Even though, we found no operation-dependent spatial bias in the target detection task, it is conceivable that the arithmetic operation had an impact on the locations of the ROs chosen (SOAR effect; cf. Knops et al., 2009). Figure 3C depicts the frequencies of how often the four RO locations were chosen, as well as the frequencies of how often the correct RO was presented at that location. The figure illustrates that especially for subtraction participants preferred left-sided response locations. A repeated measures ANOVA with the factors operation and side on the response frequencies confirmed this with a significant effect of side, $F(1, 17) = 5.799$, $p < .05$, $\eta^2_G = 0.193$, and, crucially, a significant interaction between operation and side, $F(1, 17) = 31.95$, $p < .001$, $\eta^2_G = 0.358$. To examine the operational origin of the significant interaction between operation and side, we conducted four separate, directional t-tests against zero for the difference values between the choice frequencies (i.e. how often that location was chosen) and the presentation frequencies (i.e. how often the correct RO was presented at that location) – pooled over the vertical axis. This analysis revealed that in the addition condition, participants chose significantly less options from the left-hand side, $M = -7.5$, $SD = 13.967$, $t(17) = -2.278$, $p < .05$, and significantly more options from the right-hand side, $M = 6.556$, $SD = 14.106$, $t(17) = 1.971$, $p < .05$. In the subtraction condition, participants chose significantly less options from the right-hand side, $M = -18.389$, $SD =$

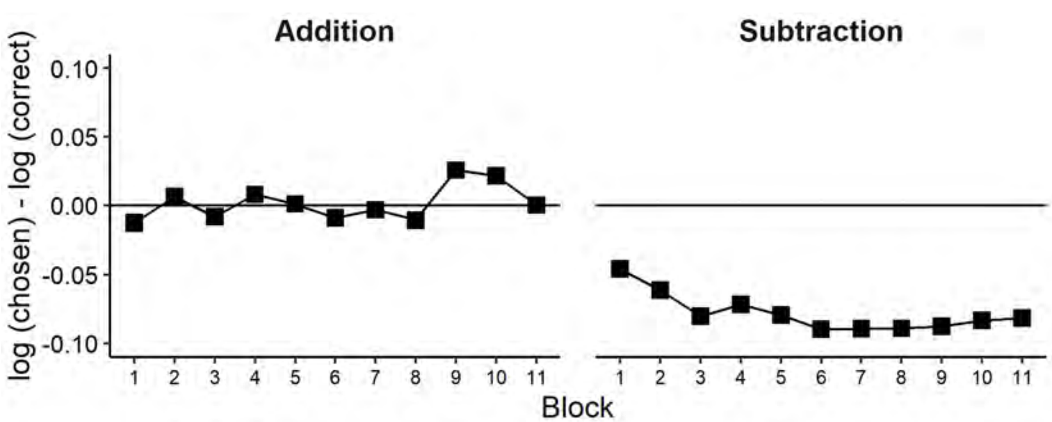
15.244, $t(17) = -5.118$, $p < .0001$, and significantly more options from the left-hand side, $M = 17.389$, $SD = 15.47$, $t(17) = 4.769$, $p < .0001$.

Arithmetic Task: Block-Wise Analysis

In this exploratory analysis we analysed the response bias (defined as the difference between the logarithm of the chosen value and the logarithm of the correct value) and a CV_block variable (CV = *SD/M*) over experimental blocks to investigate how performance developed over time. Figure 4 depicts the response bias variable separately for addition and subtraction tasks over the course of the experiment. In addition blocks, performance was constantly close to perfect. In subtraction blocks, however, performance deteriorated over time. More precisely, subjects performed fairly accurate at the beginning and then tended to underestimate results (OM effect). This pattern then remained constant over the rest of the experiment. A repeated measures ANOVA with the factors operation and block on the response bias variable revealed no main effect of block, $F(10, 170) = 1.361$, $p = .202$, $\eta_G^2 = 0.024$, but a main effect of operation, $F(1, 17) = 40.249$, $p < .001$, $\eta_G^2 = 0.379$, and, importantly, a significant interaction between block and operation, $F(10, 170) = 2.69$, $p < .01$, $\eta_G^2 = 0.035$.

Figure 4

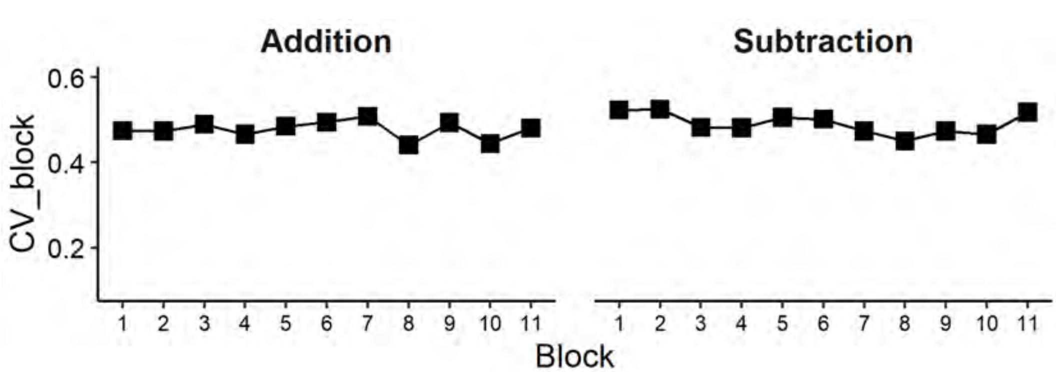
Mean Response Bias by Block



Note. The response bias is defined as the difference between the logarithm of the chosen value and the logarithm of the correct value.

Figure 5

Coefficient of Variation (CV) by Block



Note. CV is defined as the ratio between the standard deviation and mean of the subjects' response (CV = *SD/M*).

Figure 5 illustrates the CV variable by block. It shows that the CV stayed almost constant over the experimental blocks. This was confirmed by testing the regression slopes of the individual subject's CV over block against zero, $t(17) = .011$, $p = .991$.

Discussion

This study set out to investigate spatial biases and attentional shifts in the context of approximate addition and subtraction processing. We sequentially presented participants with arithmetic task components in the form of dot arrays. In the calculation phase, i.e. before the presentation of four ROs of which the correct solution had to be chosen, a target detection task was used to measure the locus of spatial attention.

The analysis of the target detection task showed no operation-dependent differences in target detection times for left-sided and right-sided targets. Hence, this experiment did not reveal spatial biases in approximate arithmetic in the target detection task. An increase in chosen numerosities and in response variability as a function of the correct value indicates that the ROs were not chosen randomly. We observed a response bias, defined as the difference between the chosen option and the correct result, that differed significantly between the operations (OM effect). Participants tended to select smaller ROs than the correct result in subtraction trials (underestimation). A block-wise analysis further revealed that this pattern of underestimation gradually develops over the course of the first blocks of the experiment and then remains nearly constant.

We also found that the arithmetic operation influenced the location of the ROs chosen: Participants preferentially selected left-sided ROs in subtraction trials, and right-sided ROs in addition trials. This SOAR effect implies some form of operation-dependent spatial bias. These findings are evocative of the observations made by Knops et al. (2009), despite the presentation of only four ROs in the current study compared to seven in Knops et al. (2009).

Although some spatial bias was observed in form of a SOAR effect, we hypothesised that providing feedback might increase overall accuracy and calibrate the responses on the correct outcome which would potentially help overcoming the overall underestimation bias. It is known from Izard and Dehaene (2008) that feedback can be used to improve performance in a dot numerosity estimation task. In their study, participants were provided with a comparison dot array together with a symbolic number that they were told to be the amount of the dots presented. Performance in the estimation task then improved, because the dot array numerosity that had to be estimated could then be compared with this comparison stimulus presented, i.e. it served as a form of calibration. Hence, we hypothesised that performance in an approximate arithmetic task would also benefit from feedback. The second experiment, therefore comprised an initial feedback during practise trials. After providing their verbal response to the arithmetic task (i.e. saying “C”, “D”, “E” or “F” for the respective RO/quadrant), participants saw a green frame appear around the RO that was the correct solution to the problem. Additionally, we increased the amount of mandatory practise blocks from one in Experiment 1 to three. Another two practise blocks with feedback were optional. The experimental blocks did not contain feedback and were identical to the ones used in Experiment 1.

Experiment 2

Method

Participants

Twenty-one participants took part in the experiment (M_{age} = 25.67 years, SD_{age} = 5.38 years, range: 18-33 years, 14 female, 18 right-handed). All subjects were German-speaking students from the Humboldt-Universität zu Berlin and had normal or corrected-to-normal vision and hearing. The experiment was non-invasive and all procedures were carried out in accordance with the ethical standards established by the Declaration of Helsinki. Informed consent was obtained in written format from all individual participants.

Stimuli, Apparatus, Procedure and Design

Experiment 2 was completely identical to Experiment 1 except for the fact that practise trials involved feedback and that the number of practise trials was increased. The feedback involved the presentation of a green frame for 2 seconds around the correct option once the participant had given a verbal response. Each practise block consisted of 25 trials drawn randomly from the set of experimental trials. Three practise blocks were mandatory and two further blocks were optional. Again, participants were instructed to press the space bar with their preferred hand once a target was detected in the target detection task (in one case the hand used did not match the self-reported handedness). Before the practise blocks, they were informed that after each response to the arithmetic task, a green frame around one of the four ROs would indicate the correct solution.

Data Analysis

For the analysis of the target detection task, again, we eliminated RTs that deviated more than two standard deviations from the subject's mean (4.26%) or that were shorter than 200 ms (0.12%). In Experiment 2 (in contrast to Experiment 1) we also analysed the practise data because it contained feedback and was, therefore, deemed worth investigating (see "Arithmetic Task: Effect of feedback"). We failed to collect the practise trial data of one subject, so that for the analysis of the arithmetic performance within the practise (feedback) blocks, we could only use the data of 20 subjects. The experiment offered five practise blocks, but only three blocks were mandatory. For that reason, data for the 4th and 5th practise block was not available from all subjects. In fact, only three subjects made use of the 4th block and no subject used the 5th practise block. Therefore, we only analysed the arithmetic performance data of the mandatory three practise blocks. Raw data are available as Supplementary Materials.

Results

Target Detection Task

In no-target (catch) trials the error rate (i.e. false alarm rate) was around 0.37%. For the further analysis of the target detection task, only left- and right-sided target trials (i.e. no catch trials) were considered. As in Experiment 1, we predicted an interaction between target side and operation (see Table 2 for RT means and *SD*s and Figure 6 for deltaRTs). A repeated measures ANOVA with the factors operation, delay and side of target on the log-transformed mean-aggregated RT variable showed main effects of side, $F(1, 20) = 4.434$, $p < .05$, $\eta^2_G = 0.002$, and delay, $F(2, 40) = 43.482$, $p < .001$, $\eta^2_G = 0.023$, but no interaction between target side and operation, $F(1, 20) = 3.373$, $p = .081$, $\eta^2_G = 0.0003$.

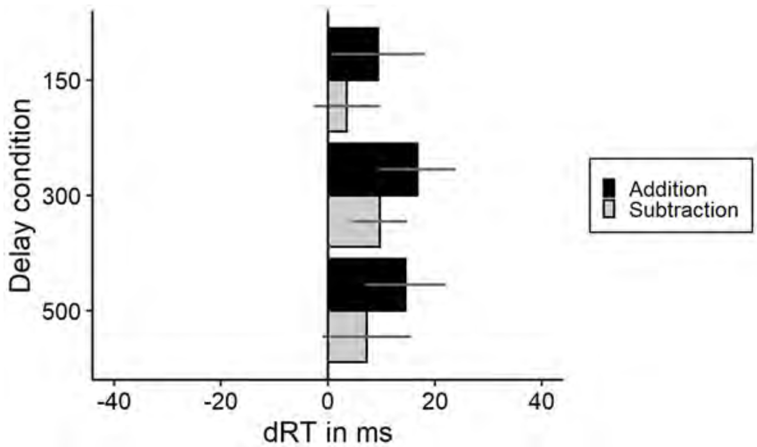
Table 2

Mean RT (and SD) in ms of the Target Detection Task by Operation, Delay and Side of Target of Experiment 2

	Addition			Subtraction		
Target side	**150**	**300**	**500**	**150**	**300**	**500**
Left	468 (151)	455 (151)	436 (141)	469 (144)	448 (157)	434 (154)
Right	459 (140)	440 (137)	423 (133)	467 (144)	437 (143)	426 (136)

Figure 6

Difference in Reaction Times Between Left-Sided and Right-Sided Targets by Delay and Operation (deltaRT = RTleft - RTright) for Experiment 2



Note. Positive values indicate that the right-sided targets were detected faster, negative values indicate that the left-sided targets were detected faster.

Arithmetic Task: Non-Random Distribution of Responses

Figure 7A shows the response percentages for the deviants of the second experiment. Note, that D3 is always the correct value. Again, the ANOVA with operation, RO rank (P1 – P4) and RO range (upper, lower) as factors showed no main effect of operation, $F(1, 20) < .001$, $p > .99$, $\eta^2_G < 0.0001$, a main effect of RO rank, $F(1.98, 39.63) = 22.342$, $p < .001$, $\eta^2_G = 0.331$, and significant interactions between operation and RO rank, $F(1.67, 33.33) = 22.839$, $p < .001$, $\eta^2_G = 0.338$, as well as between RO rank and RO range, $F(3, 60) = 41.168$, $p < .001$, $\eta^2_G = 0.082$. The three-way interaction between operation, RO rank and RO range was significant, $F(3, 60) = 10.579$, $p < .001$, $\eta^2_G = 0.033$. This is the same pattern of results as in Experiment 1 and indicates that the four ROs were not chosen at random.

Similar to Experiment 1, in addition tasks the correct RO was chosen significantly more often ($M = 0.322$, $SD = 0.072$) than in subtraction, $M = 0.239$, $SD = 0.031$, $t(20) = 5.548$, $p < .001$. Furthermore, for additions the distributions peaked around the correct outcome (see Figure 7A), while in subtraction the smallest value on the screen was chosen most often. These findings indicate that addition responses were on average more accurate than subtraction responses.

Arithmetic Task: Operational Momentum Effect

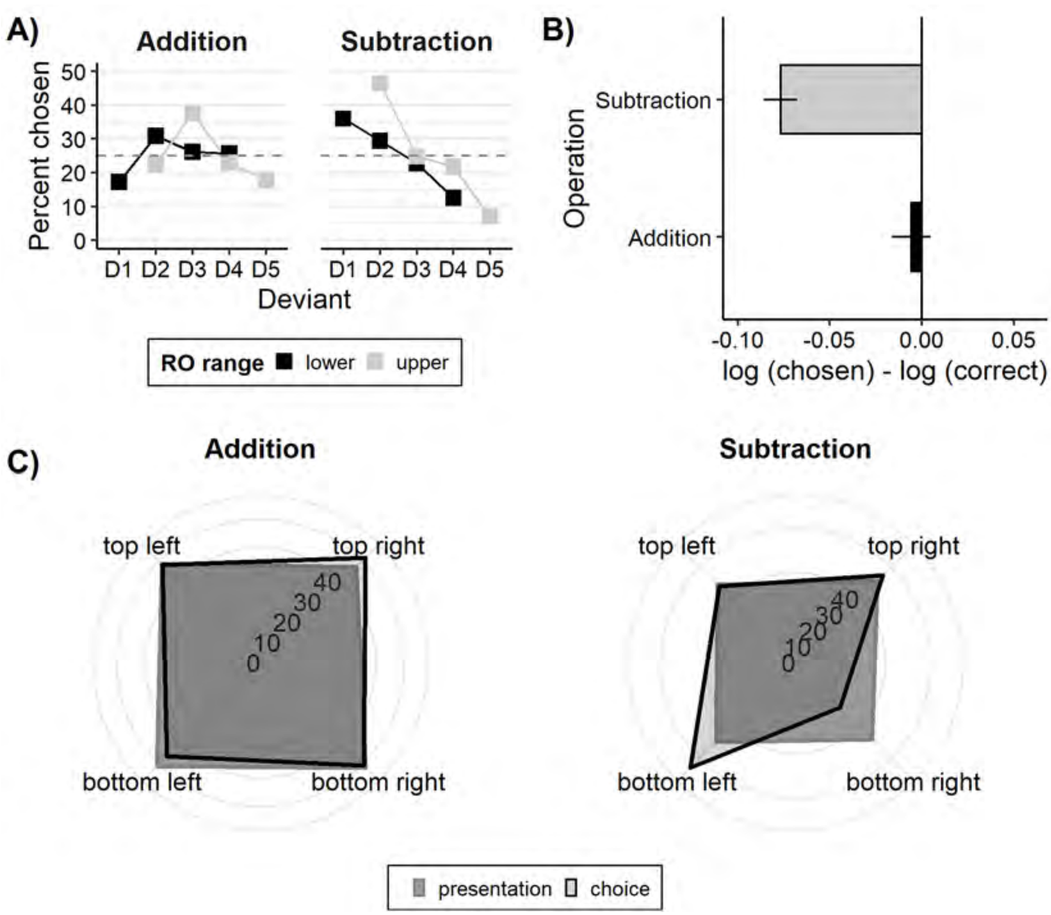
Figure 7B depicts the response bias variable of Experiment 2 for additions and subtractions. Similar to Experiment 1, participants tended to select smaller outcomes (compared to correct values) in subtraction tasks ($M = -0.077$, $SD = 0.042$) compared to addition tasks, $M = -0.006$, $SD = 0.048$, $t(20) = 4.705$, $p < .001$. Two one-sample t-tests against zero indicated that participants significantly underestimated subtraction problems, $t(20) = -8.42$, $p < .001$, but that performance was rather precise for addition problems, $t(20) = -.549$, $p = .589$.

Arithmetic Task: Influence of the Arithmetic Operation on the Spatial Distribution of Responses

Figure 7C depicts the frequencies of how often the four RO locations were chosen, as well as the frequencies of how often the correct RO was presented at that location of Experiment 2. Similar to Experiment 1, participants appeared to prefer left-sided response locations in subtractions tasks. A repeated measures ANOVA on the collapsed response frequency data of the left and right locations (collapsed top and bottom data) revealed a significant effect of side, $F(1, 20) = 8.105$, $p < .05$, $\eta^2_G = 0.226$, and a significant interaction between operation and side, $F(1, 20) = 63.448$, $p < .001$, $\eta^2_G = 0.467$. To examine the operational origin of the significant interaction between operation and side, we conducted four separate, directional t-tests against zero for the difference values between the choice frequencies (i.e. how often that location was chosen) and the presentation frequencies (i.e. how often the correct RO was presented at that location) – pooled over the vertical axis. This analysis revealed that in the addition condition, participants chose significantly less options from the left-hand side, $M = -5.143$, $SD = 10.561$, $t(20) = -2.231$, $p < .05$, but not significantly more options from the right-hand side, M = 3.762, SD = 10.667, t(20) = 1.616, p = .061. In the subtraction condition, participants chose significantly less options from the right-hand side, M = -16.381, SD = 11.465, t(20) = -6.548, p < .0001, and significantly more options from the left-hand side, M = 15.19, SD = 11.21, t(20) = 6.21, p < .0001.

Figure 7

Arithmetic Task of Experiment 2



Note. A) Distribution of the participants' choices across the five deviants. D3 was always the correct outcome. Depending on the RO range condition only the upper (D2-D5) or lower (D1-D4) four ROs were drawn from the set of five deviants. B) Mean response bias defined as the difference between the logarithm of the chosen value and the logarithm of the correct values. A negative value indicates an underestimation, and a positive value indicates an overestimation. C) The response frequencies of the four RO locations by arithmetic operation (black). Grey lines indicate the presentation frequencies of the correct RO at the given location.

Arithmetic Task: Block-Wise Analysis and Effect of Feedback

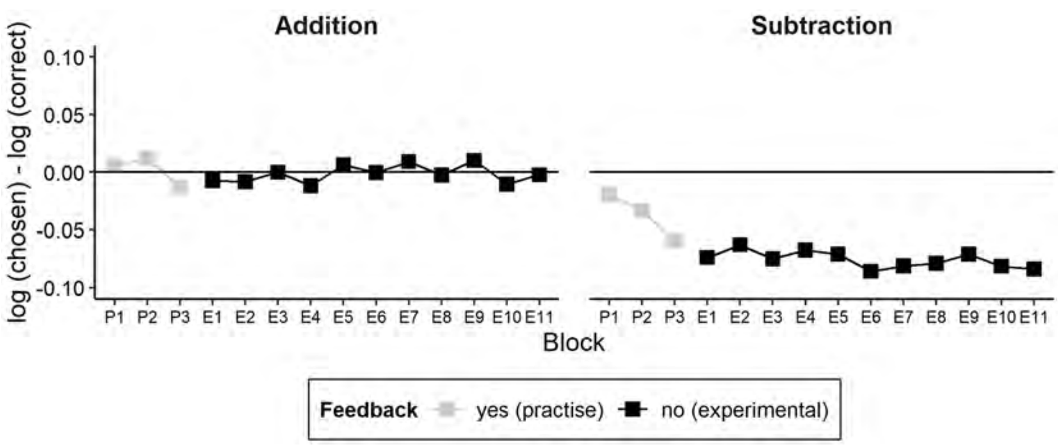
To investigate whether feedback during practise blocks of Experiment 2[1] had an impact on arithmetic performance we focussed on the response bias (defined as the difference between the logarithm of the chosen value and the logarithm of the correct value) to quantify the OM effect and a CV_block variable (CV = SD/M) to illustrate the constant response variability.

1) Note, that contrary to Experiment 2, in Experiment 1 no practice trial data was collected. Furthermore, it is important to point out that trial amounts slightly differed between practice and experimental blocks in both experiments: Experiment 1 included one mandatory and one optional practice block á 30 trials. Experiment 2 included three mandatory and two optional practice blocks á 25 trials. Both experiments consisted of 11 experimental blocks á 36 trials. Consequently, practice and experimental blocks are not fully comparable.

Figure 8 shows the response bias variable separately for addition and subtraction tasks over the course of the experiment. Performance was close to perfect in addition tasks and did not change over blocks. In subtraction tasks, subjects tended to underestimate the results. Accuracy decreased over the course of the practice trials and remained constant over the rest of the experiment. In other words, the OM effect in subtraction was increased with practise. Inferentially, a repeated measures ANOVA with the factors operation and block on the response bias variable revealed a main effect of block, $F(5.82, 104.75) = 2.401$, $p < .01$, $\eta^2_G = 0.039$, and a main effect of operation, $F(1, 18) = 31.787$, $p < .001$, $\eta^2_G = 0.284$. However, the interaction between block and operation was not statistically significant, $F(5.59, 100.71) = 1.761$, $p = .05$, $\eta^2_G = 0.028$. These results suggest that the feedback during the practise trials did not improve arithmetic performance. More specifically it did not reduce the response bias (underestimation) that was observed especially in subtraction tasks.

Figure 8

Mean Response Bias by Block

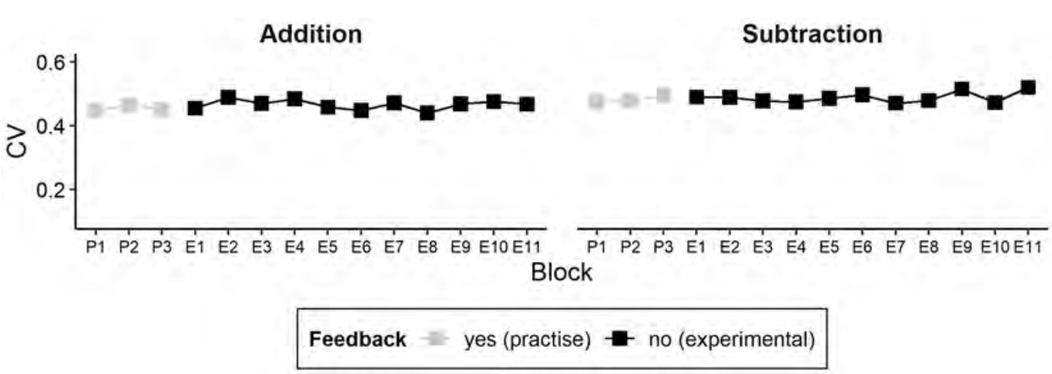


Note. P1-P3 indicate the practise blocks; E1-E11 indicate the experimental blocks. The response bias is defined as the difference between the logarithm of the chosen value and the logarithm of the correct value.

Figure 9 depicts the CV separately for each block and illustrates that the CV variable stayed nearly constant over the course of the experiment, as confirmed by testing the regression slopes of the individual subject's CV over block against zero, $t(19) = .586$, $p = .565$. This suggests that feedback during practise trials had no impact on the response variability over the course of the experiment.

Figure 9

Coefficient of Variation (CV) by Block



Note. CV is defined as the ratio between the standard deviation and mean of the subjects' response (CV = *SD/M*).

Joint Analysis of Experiment 1 and 2: Replication Check

To examine whether the results differed between experiments, we performed two mixed ANOVAs on the response bias variable (OM effect) and the response location variable (SOAR effect) data of both experiments. With regard to the OM effect, the ANOVA on the response bias variable revealed a significant effect of operation, $F(1, 37) = 56.619$, $p < .001$, $\eta^2_G = 0.471$, but, crucially, no significant interaction between operation and experiment, $F(1, 37) = .216$, $p = .645$, $\eta^2_G = 0.003$, indicating that the response bias effect (OM effect) did not differ between experiments. As regards the SOAR effect, the ANOVA on the response frequencies indicated a main effect of side, $F(1, 37) = 13.75$, $p < .01$, $\eta^2_G = 0.209$, and a significant interaction between operation and side (SOAR effect), $F(1, 37) = 90.999$, $p < .001$, $\eta^2_G = 0.414$, but crucially, no interactions with the factor experiment (all *p*s > .05) indicating that the SOAR effect did not differ between experiments.

Joint Analysis of Experiment 1 and 2: Analysis of the Operation-Side Interaction for the Location Analysis of the Arithmetic Task

In order to examine the spatial distribution of responses and their operational origin, we pooled the data of both experiments and performed four separate, directional t-tests against zero. In additions, participants chose significantly less options from the left-hand side, $M = -6.231$, $SD = 12.141$, $t(38) = -3.205$, $p < .01$, and significantly more options from the right-hand side, $M = 5.051$, $SD = 12.284$, $t(38) = 2.568$, $p < .01$. In subtractions, participants chose significantly less options from the right-hand side, $M = -17.308$, $SD = 13.197$, $t(38) = -8.19$, $p < .0001$, and significantly more options from the left-hand side, $M = 16.205$, $SD = 13.207$, $t(38) = 7.662$, $p < .0001$.

Joint Analysis of Experiment 1 and 2: Exploratory Analysis of Sequential Dependencies

In an exploratory analysis we analysed the response bias (OM) depending on whether the previous trial (n-1) involved the same or different operation as the current trial (repeat trial: Add ⟶ Add, Sub ⟶ Sub; switch trial: Add ⟶ Sub, Sub ⟶ Add)[2]. We observed a significant interaction between the operation and the switch trial property, $F(1, 38) = 21.59$, $p < .001$, $\eta^2_G = 0.362$. Post-hocs tests revealed that in subtraction trials, the response bias became more negative when the previous trial was a subtraction trial (repeat) compared to switch trials, $t(38) = -3.91$, $p < .001$. In addition trials, the response bias became more positive in repeat trials compared to switch trials, $t(38) = 2.61$, $p < .05$. These results suggest that the OM effect increases when an operation is repeated over trials.

Discussion

In Experiment 2, we provided feedback in the practise blocks to assure that participants performed the approximate calculation adequately. No operation-dependent effects were observed in the target detection task (as in Experiment 1). Additionally, the feedback did not lead to an improvement in the arithmetic performance. The remaining performance pattern matched the results from Experiment 1 extremely closely. Participants selected smaller numerosities as correct results in subtraction tasks compared to addition tasks (OM effect) and preferentially selected left-sided ROs in subtraction trials and right-sided ROs in addition trials (SOAR effect). A pooled analysis over both experiments revealed increased OM effects when the operation is repeated over trials.

Similar to Experiment 1, we found that performance in subtraction trials was initially centred on the mean outcome and only successively became biased towards underestimation.

General Discussion

This study set out to investigate spatial biases and attentional shifts in the context of approximate addition and subtraction. In two experiments, participants were presented with non-symbolic addition and subtraction problems and had to choose the correct solution amongst four ROs. Spatial attention was measured via a target detection task that was

2) The switch trial property was not experimentally varied, but the data involved a nearly equal amount of switch (.52%) and repeat trials (.48%).

presented after the arithmetic task and before the RO presentation. In the second experiment we introduced a feedback during initial training trials to increase the reliance on approximate calculation and its accuracy.

We replicated the OM effect in both experiments (Knops et al., 2009; McCrink et al., 2007). Participants selected smaller ROs than the correct result in subtraction trials (underestimation). We also replicated the SOAR effect in both experiments. Participants preferentially selected left-sided ROs in subtraction tasks and right-sided ROs in addition tasks. The results of the target detection task that we used to directly probe the presence of attentional shifts showed no operation-dependent bias, i.e. no reaction time difference between left- and right-sided targets for addition or subtraction tasks. Hence, while we failed in detecting horizontal shifts of spatial attention in the context of approximate addition and subtraction via the target detection task, we reliably observed spatial and arithmetic biases (SOAR effect, OM effect) in the context of an approximate calculation task. Latter observations (SOAR, OM) are in line with previous studies (e.g. Knops et al., 2009). Even more so, the observations of a SOAR effect are in accord with more general observations linking visuospatial and numerical processing (Grasso, Anobile, & Arrighi, 2021; Grasso, Anobile, Caponi, et al., 2021).

How can this pattern of results be linked to the models explaining the OM effect? The models differ regarding their predictions of spatial biases: The compression account and the heuristics account do not predict spatial biases, while the AHAB model contains a spatial component (sign-space association). Note however, that the present study did not involve operation signs but the letters “A” and “S”, respectively. Nevertheless, if this component would be generalised to an “operation-space association” it would still predict a spatial bias. The heuristics and compression accounts could make spatial predictions under the additional assumption of magnitude-space associations. Hence, all models could potentially predict operation-dependent spatial biases, i.e. effects in the target detection task and the arithmetic choice locations (SOAR). But this is not what we found.

This prompts the question of why spatial biases have been observed in one task (arithmetic task: locations) but not the other (target detection task). We would like to discuss two possible explanations. First, the way we realised the target detection task might not have been sensitive enough to measure spatial biases. Previous studies using this paradigm differ largely with respect to the timings (target onset and durations), use of no-target trials and response (to target or target side). In the present study, the choice to leave the target on screen until the response might have impeded the detection of RT effects: Participants had no incentive to react as fast as possible because even if they didn’t detect the target at first, they would at some point. However, previous experiments (Liu et al., 2017) with a comparable design observed such effects, suggesting that attentional biases are observable with such a target detection set-up and that other factors are more likely to have caused the absence of effects in the target detection task of the present study.

Another possible explanation might be that the time window of the target detection task did not capture the approximate calculation process. Attentional modulations of activity in posterior parietal cortex and of reaction times have been observed in response to the presentation of an arithmetic operator. For example, Mathieu and colleagues (Mathieu et al., 2016) serially presented participants with arithmetic problems. Crucially, the second operand was presented either to the left or to the right side of the operator, essentially acting as a cued target. They found that addition problems were solved faster when the second operand appeared to the right of the operator compared to its appearance on the left. An equivalent association between subtraction and left targets was observed (cf. Campbell et al., 2021, however, who did not find the latter effect). This effect was maximal when the second operand appeared 300 ms after the operator. More recently, the time-resolved analysis of gaze position during arithmetic problem solving revealed differences in horizontal eye positions between addition and subtraction in two time windows – immediately after the presentation of the arithmetic operator and during response preparation (Salvaggio et al., 2022). These results suggest that early attentional effects emerge very rapidly upon the presentation of the arithmetic operator. It should be noted, however, that these biases emerged in the context of exact arithmetic calculation tasks where problems were visually presented in symbolic format (Arabic digits). In contrast, we presented participants with non-symbolic stimuli that triggered approximate rather than exact arithmetic processing. The exact strategies underlying exact and approximate arithmetic are still poorly understood but both might involve a first approximation of the result involving the activation of the numerical magnitude representation. The early attentional effects associated with the operator might reflect a first activation of the numerical range of potentially eligible outcomes (i.e. results larger than the first operand in addition trials and results smaller than the first operand in subtraction trials) against which the final outcome of the operation is compared, leading to the late attentional modulations. The temporal structure of our task did not allow to capture the fast attentional modulation observed in previous studies. However, we observed late attentional modulations in the form of the SOAR effect. This is even more surprising when we consider that the processing of the four ROs in the quadrants of the screen might be associated with additional biaxial attentional deflections (into to the four positions) that would precede any response (bias).

The time window might also differ as a function of the type of response, depending on whether a free response (e.g. verbal input or dot production) must be given or a forced-choice decision has to be made on one or multiple ROs. The present study involved an approximate arithmetic task in combination with a forced-choice response (choice among four ROs). In that case, it is plausible that actual processing - i.e. operation upon the numerical representation - occurred only when the ROs were presented and the correct solution had to be selected. Consequently, spatial biases should have been observed in the RO presentation part of the arithmetic task but not before. This is exactly what we observed: Spatial biases were not observed before the ROs via the target detection task but during the RO presentation phase via the arithmetic task response. Future studies need to systematically measure spatial attention during the arithmetic response, i.e. during the RO selection phase in non-symbolic approximate arithmetic.

Another major finding of the present study is that both experiments did not differ in their result patterns even though the second experiment involved feedback during the practise blocks. In contrast with the hypotheses, however, participants' performance (accuracy and CV) did not improve via feedback, and the OM and SOAR effect remained the same. This means that the observed effects are relatively invariant to feedback. Two potential explanations for the lack of improvement in arithmetic performance after feedback can be discussed.

Firstly, it is plausible that due to its shortness, the feedback used in the present study was not efficient enough to improve arithmetic performance. Other studies involving some form of training often used multiple training sessions (e.g. Park & Brannon, 2013). In the present experiment, however, participants practised on average for 3 blocks á 25 trials, which might not have been enough to improve participants' arithmetic performance. Alternatively, the lack of an incentive in our study may have left participants unmotivated to improve performance by taking the feedback into account. In line with a recent study that shows how an incentive motivation improves the precision of the ANS, future studies may want to induce this type of reward-related enhancements (Dix & Li, 2020).

Secondly, simply relying on a feedback might not have been sufficient to induce improvements of performance. Several studies investigated the effects of correct/incorrect feedback on approximate numerical processing or approximate arithmetic - with mixed results: Two studies described improvement in symbolic arithmetic after approximate arithmetic training (Park & Brannon, 2013, 2014). However, Lindskog and Winman (2016) pointed out that the improvement during approximate arithmetic training was not indicative of learning. Similarly, Szkudlarek et al. (2021) were not able to replicate their own findings (Park & Brannon, 2013, 2014). These studies indicate that training via correct/incorrect feedback might only be suitable to improve approximate arithmetic performance under certain constraints. Of note, the present study did not involve a correct/incorrect feedback but a feedback on the correct RO so that participants did not only receive information about the correctness but also about their deviation from the correct result. Surprisingly, the additional information provided by the feedback did not have a strong impact on performance. This may reflect the overall robustness of the OM effect. Alternatively, the initial feedback period may not have been long enough or might require periodic topping-up.

Furthermore, earlier studies investigating the processing of non-symbolic stimuli were in fact able to observe reduced variability after providing feedback of the real numerosity of stimuli (Minturn & Reese, 1951) or after providing inducer arrays (Izard & Dehaene, 2008; Krueger, 1984). Compared to the feedback of the present study, this feedback involved a transcoding process from a non-symbolic stimulus into symbolic information which allowed for an exact numerical representation. Perhaps a transcoding process might be what is needed to sufficiently improve approximate performance (see Lindskog et al., 2013 for a similar argument).

Nevertheless, it is important to point out that studies by Park and Brannon (2013, 2014) involved training of non-symbolic arithmetic while in Izard and Dehaene (2008) mprovement of non-symbolic processing performance was only shown in an estimation task. Single numerosity processing and arithmetic processing do not overlap in all of their underlying mechanisms making it difficult to fully be able to generalise findings of estimation tasks onto arithmetic tasks. Therefore, further research is needed to investigate how exact/symbolic information can improve non-symbolic arithmetic.

The two experiments of the present study further revealed that the underestimation in subtraction trials gradually emerged over the first blocks of the experiment. In Experiment 1, where no practise trial data was collected, participants performed fairly accurate in the first experimental block and then tended to progressively underestimate subtraction results. In Experiment 2, this pattern was already detectable in the practise trials – despite them involving feedback. We did not expect this surprising finding. The current study is the first to describe this type of time-resolved performance in the context of a non-symbolic arithmetic task. Hence, we cannot know whether this is something seen frequently or not. We can only speculate what might have caused participants in subtraction trials to choose values that become smaller over time. The OM effect might require a contextual build-up phase in which participants create an internal distribution of the stimuli that are used in the experiment and elaborate their strategy. In our study, the operands were identical for addition and subtraction. Hence, participants may have learned over time that the average outcome for subtraction is smaller compared to addition which may have biased their decision. Note, that the CV remains constant throughout the entire experiment for both addition and subtraction, meaning that participants did not become increasingly inaccurate. Rather, the overall mean of the preferred outcome with respect to the correct outcome stabilizes only after ~75 trials during which the OM effect increases. This suggests that the OM effect builds up over time by biasing the decisional processes without affecting the precision of the perceptual basis. This is a new and exciting discovery that requires a systematic investigation in future experiments. It is unclear, for example, why this was observed only for subtraction. One potential reason might be that the approximate addition of visual stimuli is easier than approximate subtraction because such stimuli might be visually superimposed in visual working memory. Approximate subtraction, on the other hand, involves a more complex mechanism of mentally erasing stimuli from the mental representation of the minuend. What is more, real-life subtraction usually involves the "disappearance" of a certain amount of objects – whereas the operationalisation of approximate subtraction in the current study involved the additional presentation of a second visual stimulus as a subtrahend. As this mechanism deviates from the real-life visual subtraction, it might be more difficult for participants to achieve. Consequently, subjects might fall back to a simple heuristic of choosing the smallest option leading to the underestimation bias. Of course, this remains highly speculative, and it does not explain why participants start off fairly accurate in approximate subtraction. Future studies are needed to systematically investigate the differences between approximate addition and subtraction.

Finally, the joint analysis of both experiments revealed sequential dependencies in the form of increased OM biases in cases where the operation was the same as in the previous trial compared to when it was different (switch). This finding might indicate that more mechanisms than only attentional shifts are involved in the formation of the OM. This is an exploratory finding which needs to be treated with caution as the switch property was not experimentally manipulated. Nevertheless, it provides an interesting new field of research. In a variety of tasks, the perception of a current stimulus is modulated by previous perceptual history. Bayesian theories of perception propose that the perceptual history serves to predict current perception via changes of perceptual priors. One possible perceptual mechanism that could explain the observed increase in the OM bias on repeated operation trials and that does not require additional theoretical assumptions for explaining the OM effect operates on the perceived numerosity (Cicchini et al., 2014): Since we matched the operands between addition and subtraction, the average of the presented outcomes of subtraction problems was smaller compared to addition problems on any given trial. This might have biased the perceived numerosity of the operands of the successive trial. In line with previously described attractive serial dependency effects (Fornaciai & Park, 2020), the larger average values in addition trials may have increased the perceived quantity of the subsequent operand in subtraction trials when operation switched. This means that the outcome of the subtraction trial is larger compared to situations where operation did not switch and the operands of the subsequent subtraction trials was not biased in this direction. Hence, this serial dependency effect induces a bias towards smaller numerosities after subtractions and larger numerosities after additions, enhancing or diluting the OM effect if subsequent operation was repeated or switched, respectively. We tested this assumption by computing a linear regression of the mean outcome of the previous trial on the OM effect in switch and in repeat trials. We found that the OM effect was positively correlated with the average outcome of the previous trial in repeat trials (mean slope: 0.00094) while this relation was negative in switch trials (mean slope: -0.00075). The difference between these slopes was significant, $t(38) = 8.02$, $p < .001$. Alternatively, serial dependence effects may arise at post-perceptual information processing instances (Ceylan et al., 2021). According to this idea, the mean chosen outcome on a given trial might exert an influence on the chosen outcome in the next trial. To test this hypothesis, we computed the difference between the chosen outcome in the current trial and the previous trial (delta_CO), separately for switch and repeat trials and entered it in a liner regression on the OM effect. In switch trials, a larger absolute difference between current choice and past choice (i.e. more extreme values) lead to a larger OM effect in repeat trials compared to switch trials (steeper regression slope; $t(38) = 21.338$, $p < .001$).

We corroborated these results by comparing the relative importance of both variables (by partitioning of the total explained variance (R^2) of the model into individual R^2 contributions; see Grömping, 2007) when entered as predictors in a multiple regression on the strength of the OM effect. These two variables were able to explain (R^2) for 29% and 34% of the variance in switch and repeat trials, respectively. Of these, the choice in the previous trial explained more variance (i.e., was a better predictor) compared to the mean numerical size of the response alternatives (81% vs. 79% for repeat trials and 80% vs. 20% for switch trials).

Based on these (exploratory) results, we tentatively conclude that the locus of the amplification of the OM bias is at a central cognitive level, rather than on the perceptual side of the process. It may be interpreted as the consequence of an inhibition of the irrelevant task set (here: the alternative arithmetic operation; Koch et al., 2010) that impedes the execution of the previously inhibited operation in switch trials but not in trials where the operation is repeated. A cognitive interpretation also links nicely with the observed emergence of the OM effect in subtraction over the course of the first ~75 trials that also points to a successive built-up process that is independent of the core parameters of the perceptual performance (i.e. CV remained constant). These effects do not easily integrate into the attentional bias hypothesis of the OM effect and point to additional processes that influence performance in approximate arithmetic.

The present study investigated spatial attention in the context of approximate addition and subtraction. While no shifts could be observed via the target detection task, participants preferentially selected right ROs after addition and left ROs after subtraction processing (SOAR effect) implying a spatial bias in the context of approximate calculation during the RO selection stage. The feedback introduced in the second experiment in the form of highlighting the correct arithmetic answer after the participant had given their response during the practise trials, did not improve arithmetic performance. Consequently, the pattern of results was identical to the first experiment. Put positively, the observed effects and biases are robust and not easily malleable by feedback. The newly described serial dependency effect and the built-up of the OM effect over trials point to additional cognitive factors that require a more systematic exploration in future studies.

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Data Availability: For this article, a data set is freely available (Glaser & Knops, 2022).

Supplementary Materials

The Supplementary Materials contain the following items (for access see Index of Supplementary Materials below):

- Raw data and codebook
- Arithmetic stimuli used in both experiments. The stimuli were based on the stimuli of Knops et al. (2009).

Index of Supplementary Materials

Glaser, M., & Knops, A. (2022). *Supplementary materials to "Spatial biases in approximate arithmetic are subject to sequential dependency effects and dissociate from attentional biases"* [Research data and codebook]. PsychOpen GOLD. https://doi.org/10.23668/psycharchives.8268

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