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The Effects of the Look-Ask-Pick (LAP) Strategy on Struggling Grade 6 Learners' Ability to Add Fractions

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Fractions are an integral part of the mathematics curriculum. Most students acquire proficiency with these concepts during the course of their elementary education and are usually able to perform basic fractions operations when reaching middle-school age. However, a considerable number of students require extra help to not fall further and further behind in the curriculum. In this study, we extended the use of a simple strategy (Look, Ask, Pick; Test & Ellis, 2005) that holds the potential to help students with problems understanding and working with fractions catch up with their classmates. We applied a multiple-baseline design across four struggling sixth graders. After receiving the instruction, all participants' performance on fractions improved significantly; moreover, they viewed the strategy as highly useful. Limitations of the study, future directions of research, and implications for teachers regarding the instructional utility of the intervention are discussed.

Keywords: **fractions, strategy instruction, learning problems, single-case research**

Introduction

The Importance of Proficiency in Fractions

Conceptual knowledge of and computational fluency with fractions serve as the foundation for mastering more elaborate mathematical skills. As a prerequisite for more advanced study related to data analysis, probability, measurement, geometry, ratios, and algebra, mastery of fractions marks a milestone in the development of mathematically literate citizens (Brown & Quinn, 2007; Butler et al., 2003). For example, Siegler et al. (2013) found a strong relationship between older students' understanding of fractions and their general math achievements. Similarly, in an international study, Torbeyns et al. (2015) noted that sixth- and eighth-grade students' understanding of fraction magnitude correlated with their overall mathematics

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achievement. Moreover, Siegler et al. (2012) found that the fraction knowledge of fifth graders predicted mastery of algebra a couple of years later.

Being proficient in fractions is not only relevant for one's further mathematical development in particular and one's school career in general, however. It is also necessary for successfully managing many work-related activities and everyday tasks (e.g., adjusting recipe sizes, calculating the amount of wallpaper needed to renovate your room, or negotiating the contract rates when buying a car) (Booth & Newton, 2012).

Growing Numbers of Students Struggling With Math in General and Fractions in Particular

Unfortunately, all too many students struggle with math, especially those with an identified disability. The most recent results from the National Assessment of Educational Progress (NAEP) showed that in 2022, only 47% of fourth-grade students identified with disabilities (especially with nonverbal learning disabilities; LD) attained mathematics scores at the basic achievement level or higher, compared to 80% of their fourth-grade peers not identified as having a disability (U.S. Department of Education, 2022). Further, the number has been declining as students advance through the grades. In 2022, for example, only 28% of eighth graders with disabilities attained mathematics scores at the basic level or higher, compared to 67% of their eighth-grade peers not identified as having a disability. These scores were significantly lower than those found in the 2019 administration of the NAEP.

Further, to examine student achievement during the COVID-19 pandemic, the National Center for Education Statistics (NCES) conducted a special administration of the NAEP long-term trend in mathematics assessments for all 9-year-old students (both students with and without a disability). Average mathematics scores for those children in 2022 had declined by seven points compared to the 2020 administration, resulting in the first overall average score decline in mathematics recorded by the NAEP (U.S. Department of Education, 2022).

Looking at the specific component of fractions within math performance shows that understanding this crucial concept is especially challenging for many learners (Bailey et al., 2012). According to *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*, more than 40% of middle school students demonstrate severe difficulties in calculating fractions. Moreover, nearly half of these learners struggle with even the most fundamental level (U.S. Department of Education, 2008). These deficits are alarming, not only because basic proficiency in fractions is essential for the meaningful learning of subsequent school mathematics but also because it is important for overall daily functioning.

Effective Ways to Foster Fraction and Other Basic Math Skills

Given the above findings, a majority of students identified as having a disability, as well as a considerable number of learners not identified, are at risk of not obtaining the mathematical competencies necessary for pre- and postsecondary coursework. Thus, identification of effective, efficient, and practical instructional methods to improve proficiency in math in general and fractions in particular is critical.

Which aspects of instruction best facilitate the development of math skills? According to *Adding It Up: Helping Children Learn Mathematics* (Kilpatrick et al., 2001) and *Foundations for Success: The Final Report of the National Mathematics Advisory Panel* (U.S. Department of Education, 2008), mathematics instruction should foster proficiency. This includes conceptual understanding and procedural fluency (e.g., fluency with basic computational skills), strategic and adaptive mathematical thinking (problem solving and logical reasoning), and a productive disposition (the beliefs and confidence necessary to use mathematics effectively in everyday life) (Baroody, 2011; Schoenfeld, 1985, 1992). Identification and development of effective instructional methods in these areas should provide *all* students the opportunity to become mathematically proficient. For example, such strategies can augment and form the basis of a plan for furnishing instructional materials and activities that support student learning during regular class time, in before- or after-school programs, and for use by providers of supplemental services.

Overall, the current state of research in the area of math interventions for struggling students is robust. Thus, a number of well-conducted meta-analyses give reliable guidance on the efficacy of instruction (e.g., Küçükalkan et al., 2019; Schnepel & Aunio, 2022; Stevens et al., 2018). However, to date only few studies have specifically addressed fraction instruction for struggling students (Everett et al., 2014; Maccini et al., 2007). Among the existing studies, two are particularly relevant for our investigation: Ennis and Losinski (2019a) and Roesslein and Codding (2019).

Ennis and Losinski's (2019a) review of fraction interventions included studies that examined the effects of anchored instruction, explicit instruction, graduated instruction, strategy instruction, and video modeling. Overall, significant effects were found for each of these approaches. Specifically, in terms of effect size, video modeling was the most effective, although with only one study involving four students. The next highest effect was found for graduated conceptual instruction (e.g., concrete, representational, abstract) followed closely by strategy instruction and explicit instruction, which had the widest research base in Ennis and Losinski's (2019a) analysis.

Roesslein and Codding's (2019) review of fraction interventions for struggling elementary learners included studies based upon their (a) instructional focus (b) instructional components, and (c) effectiveness. Overall, the authors found that fraction interventions that employed multiple evidence-based instructional components (e.g., explicit, systematic instruction, visual representation) were likely to enhance student outcomes. Additionally, it was recommended that practitioners emphasize both conceptual *and* procedural learning.

Many of the studies in these authors' review demonstrated effective ways to develop conceptually focused fractions skills (e.g., fraction magnitude using the number line), while fewer devoted attention to procedural skills. As a result, Roesslein and Codding suggested that more research is needed to address an explicit linking of conceptual and procedural components within intervention settings, stressing that both are necessary for overall fraction skills development. This is consistent with the recommendations of the Mathematics Advisory Panel, which pointed out that proficiency not only includes this pairing of conceptual understanding and procedural fluency (e.g., fluency with basic computational skills) but also strategic and adaptive mathematical thinking (problem solving and logical reasoning), as well as a productive disposition (the beliefs and confidence necessary to use mathematics effectively in everyday life).

Students struggling to master mathematics tend to hold negative attitudes toward math, viewing math tasks as being too difficult or downright impossible (Mielicki et al., 2022). Teacher-level attitudes may perpetuate this behavior through a belief that students with LD possess an innate deficit or inability rather than simply understanding mathematical representations in atypical ways. This "difference versus deficit" paradigm (Lewis, 2014) challenges us to examine instruction not from a learning-deficit but from a teaching-deficit perspective, and brings us back to the question of how to implement effective instruction. Since some teachers have an uneven understanding of fractions, use of structured intervention programs may be an effective way to guide teachers to improve their own and their students' comprehension of this concept (Copur-Gencturk, 2022).

Most recent studies of methods to promote fraction proficiency have included a wide array of interventions, including virtual manipulatives and a graduated instructional sequence (Bouck et al., 2020), model drawing strategies (Dennis et al., 2016), point-of-view video modeling (Hughes, 2019), a game-based universal design for learning curriculum (Hunt et al., 2022), use of number lines (Morano et al., 2019), and interactive computer applications (McKevett et al., 2020; Shin & Bryant, 2017).

Acquisition of fraction skills is demanding, and many students find it difficult to understand the core concepts behind fraction proficiency (Brown & Quinn, 2007; Siegler et al., 2020). Comprehension of the underlying principles may be facilitated through group discourse and discussion of solution strategies, a concrete-representational-abstract instruction sequence (Zhang et al., 2022), and blending of conceptual and procedural knowledge (Moloto & Machaba, 2021). Newton and colleagues (2022) found that elements of instruction that included a focus on increased conceptual understanding of fractions, exposure to strategies for sense-making, and additional time and assistance yielded the most benefit to students. In addition, Crawford et al. (2019) noted that students benefited most, and interventions were implemented with greater fidelity, when the core intent of instruction or a strategy was made explicit to students, along with a conceptual understanding of fractions as numbers. Finally, Fuchs et al. (2013) observed that instruction that focused on fraction magnitude understanding was also effective for fraction arithmetic proficiency among students with math difficulties.

This complex array of interventions inevitably poses challenges to the time and resources typically available to teachers; therefore, we wanted to focus on structured interventions that have been found to be efficacious and that could feasibly be implemented in academically diverse classrooms. As mentioned, because teachers often have a limited understanding of fractions, the use of a structured intervention programs may be an effective way to guide them to improve both their own and their students' comprehension of the concept (Namkung & Fuchs, 2019). Indeed, Namkung and Fuchs (2019) demonstrated that a strategic self-regulation strategy can help students overcome any weaknesses related to motivation, self-regulation, and executive functioning, resulting in better fraction-solving outcomes.

Further, the results of a randomized controlled trial conducted by Fuchs and colleagues (2021) suggested that strong, deliberate, strategic intervention can address challenging mathematics standards for at-risk learners, providing more robust effects than growth mindset instruction alone. Thus, strategic intervention was found to be more effective than standard instruction in fraction problem-solving domains (Hwang et al., 2019). Ennis and Losinski (2019b) used self-regulated strategy development (SRSD) to assist students to add and subtract unlike fractions with unlike denominators, to simplify fractions, and to convert fractions to mixed numbers using mnemonics. Lastly, examining fifth graders' use of strategies, Erol (2021) found that students mostly preferred "applying the rules" of existing strategies to solve fractions where they used procedural information. Fewer students preferred "strategy development and discovery," which required conceptual comprehension.

The LAP Strategy as a Promising Approach to Fostering Fraction Skills

Our study focused on a specific "applying-the-rules" strategic method to promote younger secondary students' procedural fluency in fraction addition. Specifically, a strategy called LAP (Test & Ellis, 2005) was implemented. Using LAP, fractions with the same or a different denominator can be solved. Specifically, applying the strategy instruction approach to promoting numeracy skills, LAP utilizes a fraction-specific mnemonic technique for remembering three consecutive instructions for solving addition and subtraction problems of fractions:

1. "**L**ook at the sign and denominator."
2. "**A**sk yourself the question, 'Will the smallest denominator divide into the largest denominator an even number of times?'"
3. "**P**ick your fraction type" (Test & Ellis, 2005, p. 14).

For each type of task, there are corresponding sequences of actions that students use to instruct themselves. The LAP mnemonic has been shown to be effective. Test and Ellis (2005) demonstrated improvement in the fraction addition problem-solving for five of six students in their research. A subsequent study by Everett et al. (2014) replicated these positive results.

Research Question

Further studies using LAP are needed in order to replicate results with new participant groups; specifically, to expand current knowledge about the efficacy and social validity of the method both for students identified with disabilities and those at risk of being identified. Therefore, the purpose of this study was to determine the effects of this strategy on the ability to add fractions in four sixth graders with persistent math difficulties.

Method

Setting

The study was conducted in an urban secondary school in a large city in western Germany, comprising Grades 5–9. A total of 320 students were enrolled at the school at the time of this project. Almost 70% of the students or their families have migrated to Germany, with Arabic, Polish, Russian, and Turkish being the most common primary languages spoken at home. The socioeconomic status of the school as estimated by the mean occupational status of the families of its student population is considered below average.

Participants

The classroom teacher of a sixth-grade classroom contacted the first author because she needed help teaching some of her students how to add fractions. She had tried different approaches for a couple of weeks, but apparently to no avail. We selected our participants based on the following criteria: (a) basic arithmetic skills above the 50th percentile, (b) fraction skills below the 10th percentile, (c) perfect attendance in the last six weeks, and (d) willingness to participate in the study.

Attendance was determined by consulting the classroom teacher's attendance log. Math ability was measured using a standardized inventory (RZD 2–8; Jacobs & Petermann, 2020), which was administered to the whole class. Seven students scored above the 50th percentile on the basic arithmetic operations subtest and below the 10th percentile in the fractions subtest. Five of them had been present every school day within the previous six weeks and, therefore, met the eligibility criteria. However, one student stated that he was not interested in being part of the project, resulting in a sample size of four. The classroom teacher informed us after participant selection that these four students were among the ones she had in mind when she contacted us to request help.

The following descriptions are based upon the results of the standardized math test as well as on the information and the assessments provided by the classroom teacher. Indicators of cognitive functioning depend on school records of current results from standardized cognitive assessments. While the instruments that had been used varied, they provided us with a broad perspective of the participants' level of cognitive functioning. (All names have been changed to pseudonyms to protect privacy.)

Ayla

Ayla was born in Germany to parents of Turkish descent. The language predominantly spoken at her home was Turkish. Her German skills and general level of cognitive abilities were within the average range of functioning. Ayla reported that ever since she started school, she had struggled with math. She had previously been diagnosed with LD by a multi-professional team.

Bella

Both Bella and her family are natives of Germany, and her German skills and general level of cognitive abilities were within the average range of functioning.

Cemil

Cemil was born in Germany to parents of Turkish descent. The language predominantly spoken at his home was Turkish. Cemil's German language skills were emergent, and his general level of cognitive skills could be considered normal.

Daisy

Both Daisy and her family are native to Germany. Her German skills and general level of cognitive skills were within the average range.

All four students struggled severely with math in general, and with fractions in particular. However, Ayla was the only participant who had received a diagnosis of LD. The reason why the remaining three had not been diagnosed with a disability was not that they did not meet the respective criteria but that, in an attempt to avoid labeling, the school administration had not initiated an official procedure to test them for LD.

Research Team

The interventionist was a 25-year-old female graduate student in special education with three years of experience working as a tutor for children with LD. Before the study, she received four one-hour sessions of instruction by the first author on how to conduct the training. In addition, a 24-year-old female student research assistant recorded the data to establish procedural fidelity and interrater agreement.

Experimental Design and Measurement

Our study utilized a multiple-baseline design (AB) to evaluate the effectiveness of the LAP strategy (Kazdin, 2020). The dependent variable was the number of correctly solved fraction addition problems in a daily worksheet containing 15 tasks created using an online generator (https://fractions-worksheets.basic-mathematics.com). All summands were one-digit fractions. Problems were randomly drawn and arranged from a pool containing all possible options. The time limit for finishing the daily worksheet assignments was 3 minutes.

At the end of the study, students were interviewed by the interventionist about their attitudes toward using the LAP strategy. The conversations followed a script and lasted between 10 and 15 minutes. Students were asked (a) whether they liked the intervention, (b) whether they believed the strategy helped them learn how to add fractions, and (c) whether they would recommend the strategy to their classmates. Students' answers were documented in handwritten notes.

Procedures

Every day of the study, the interventionist brought the four sixth graders individually into a resource room of the school while the remainder of the class stayed with their teacher. Even though there were usually other students in the room, it was always possible to find a quiet place away from learners not receiving the intervention. The order in which Ayla, Bella, Cemil, and Daisy were asked to work with the interventionist varied daily.

Baseline

During baseline conditions, the interventionist played *Slap Jack,* a simple card game for two, with the participants. Players deal their cards from an evenly dealt deck one at a time into a center pile and slap the pile when a jack shows up, keeping all the cards in the pile if they are the first to slap the deck. After 20 minutes, students were given the worksheets with the math problems and told to complete as many of them as possible. After 3 minutes, they were asked to stop.

Intervention Phase

The baseline phase was followed by the intervention phase. The instruction was based on the self-regulated strategy development (SRSD) model by Harris and Graham (1992), which involves six stages: (a) activate background knowledge, (b) discuss the strategy, (c) model the strategy, (d) enable memorization of the strategy, (e) support the strategy, and (f) provide opportunities for independent strategy use.

In the first session, students were told that the strategy they were being taught had the potential to improve their fraction skills. The interventionist worked to build enthusiasm for the topic by explaining the importance of fractions and by reminding everyone of what they already knew (i.e., how to perform additions) (Stage a). She then introduced the LAP strategy by presenting students with a 11.7 x 16.5 poster that was placed on the wall next to them, outlining the respective steps. She also handed them an index card with the mnemonic and told them to do the following: (L) Look at the denominator and sign, (A) Ask the question, "Will the smallest denominator divide into the largest denominator an even number of times?," and (P) Pick a fraction type. She went through each step several times, asking the students guided questions to make sure they could follow along (Stage b).

The second and each subsequent lesson started with the students individually reviewing their index cards while supervised by the interventionist. Over the next couple of minutes during lesson two, the interventionist modeled the strategy three more times by performing three simple fraction addition problems while thinking aloud and referring to the steps on the poster (Stage c). Subsequently, the students tried to memorize the steps of the procedure by picking up cards with the letters L, A, or P printed on them. They then had to verbally explain what to do when performing each step. The interventionist provided scaffolded support and corrective feedback as the students tried to give an accurate description of each step (Stage d).

In the third session, after going through their index cards, each participant was asked to perform a couple of fraction addition problems by themselves while thinking aloud. In the process, the interventionist closely monitored every step. Whenever a student made a mistake, she intervened and completed the respective problem while explaining the correct application of the LAP procedure (Stage e).

During the remaining lessons, the students first went through their index cards and performed three to four fraction addition problems while the interventionist watched and scaffolded the process, if needed. They spent the remainder of each session completing practice worksheets. If a student made an error, the interventionist stepped in to help (Stage f).

Over the course of the training, the participants were continuously encouraged and praised for their achievements and persistence. Every correctly performed step and every correctly solved math problem was followed by the interventionist highlighting that this partial success was possible because the students tried hard and focused their attention on accomplishing their tasks.

Treatment Fidelity and Interrater Agreement

The extent to which the LAP strategy was delivered as intended was captured using a 10-item checklist that included every critical procedural feature of the intervention (available from the first author upon request). The research assistant observed two training sessions of each participant and determined that the intervention was carried out 100% as planned. Interrater agreement between the interventionist and the research assistant on the number of correctly solved fraction addition problems also reached a perfect score of 100%.

Results

The use of the LAP intervention led to remarkable treatment effects. Even though all four participants had received instruction on how to add fractions as part of the general classroom curriculum, none of them were able to solve a problem correctly during baseline. However, all of them responded well to the LAP intervention. As seen in Table 1, they reached an average score of between 9.60 and 14.22 during the intervention with a percentage of non-overlapping data (PND) of 100%, supporting the treatment's effectiveness.

We used a simple analysis tool, Visual Aid Implying an Objective Rule (VAIOR; Manolov & Vannest, 2019), to assess the changes in trend and level between two adjacent phases. This tool provides researchers with dichotomous decisions concerning the absence or presence of immediate, progressive, and overall effects. According to the VAIOR benchmarks, an immediate effect is present if the first three scores of the intervention phase are all above the median of absolute deviations (MAD) from the predicted baseline values. If the last three values during treatment fall above the MAD, we can assume a progressive effect. On overall effect exists if less than 40% of the treatment scores are located below the MAD. All of our participants benefitted from the LAP strategy in all three aspects.

Table 1. *Descriptive Statistics and Appraisals of Treatment Success for the Participants*

Student	***M/SD*** **(A)**	***M/SD*** **(B)**	**Immediate**	**Progressive**	**Overall**	**PND**
Ayla	0.00/0.00	14.22/1.50	Yes	Yes	Yes	100
Bella	0.00/0.00	10.00/3.14	Yes	Yes	Yes	100
Cemil	0.00/0.00	11.71/1.56	Yes	Yes	Yes	100
Daisy	0.00/0.00	9.60/2.92	Yes	Yes	Yes	100

Figure 1 shows line diagrams of the data sets for the four students. All graphs depict an immediate boost, as well as a subsequent increasing performance level after the intervention was initiated. Upon the start of the treatment, Ayla showed remarkable improvement, from 0 to 10 correctly solved fraction problems. Starting three days later, she achieved a perfect score every day of the remaining intervention sessions. Bella's immediate progress was not quite as impressive. However, she solved six fraction problems on the first day of Phase b and achieved maximum scores on Days 11 and 12. Cemil went from 0 to 8 points as soon as he received instruction on the LAP strategy. By the end of the treatment, he was able to answer 13 problems correctly. Finally, Daisy's scores also jumped from 0 to 8 after she received the first LAP lesson. At the last day of the intervention, she solved 14 fraction problems correctly.

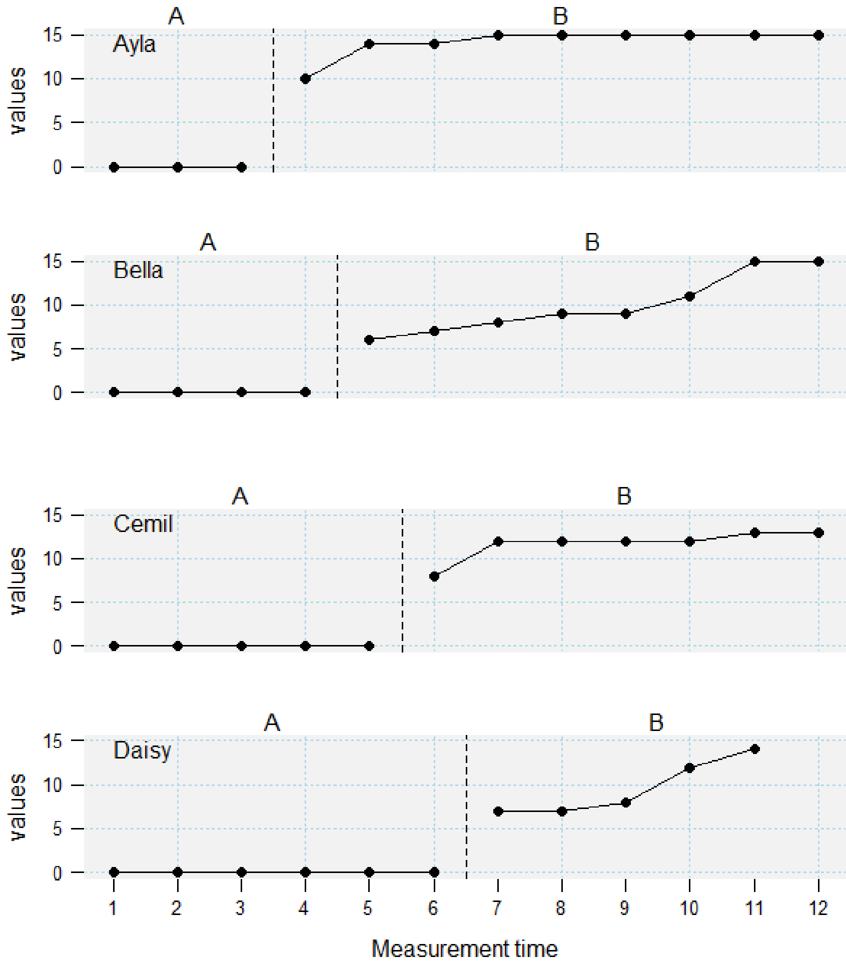

Figure 1. *Number of correctly solved fraction addition problems for the participants.*

Social Validity

Social validity was measured using the survey previously discussed. All sixth graders expressed satisfaction with the treatment, indicating that they enjoyed the lessons very much. All of them were very grateful for the support that they had received and voiced their conviction that the strategy helped them understand the procedure for adding fractions. Every student recommended the intervention to their classmates.

Discussion

Main Findings

The aim of this study was to evaluate the effectiveness of an innovative strategy to teach fraction addition skills to sixth graders struggling with math. Our findings show that the intervention was effective. All participants demonstrated an increase in performance. Whereas they were unable to correctly solve any of the problems prior to receiving instruction, by the end of the treatment, one student could answer 13, one could answer 14, and two could answer all 15 problems correctly. Each student reported that they liked the intervention and would encourage their classmates to use it as well.

These findings square well with the results of Everett et al. (2014) and Test and Ellis (2005). In all cases, the participants benefited from the instruction. Thus, the data affirm the effectiveness of the LAP strategy to support struggling math students in the acquisition of essential fraction addition skills.

Limitations and Further Research

This research is subject to several limitations. First, only four students participated and all of them attending the same classroom, thus limiting the generalizability of the findings. Future research should replicate the study with different student populations. Second, due to time restrictions, we did not collect any follow-up data. Therefore, it is not possible to determine if any gains were maintained. Prospective studies should plan for long-term evaluation to assess any long-term benefits of the treatment. A third limitation pertains to the intervention being delivered during one-to-one sessions, whereas group instruction is much more frequently used in classrooms. Thus, the ecological validity of the experiment is relatively low. Therefore, it is recommended that future studies test the strategy in whole-classroom situations in which the teacher instructs a group of 20 to 30 student.

A further limitation relates to the fact that the social validity interviews were conducted by the person who also delivered the treatment. This might have increased the likelihood of students trying to give answers telling the interventionist what she presumably wanted to hear. That is, if the participants wanted to criticize the training, voicing it to the very person who conducted it would probably have been harder than talking about it with an uninvolved individual. In future studies, “neutral” third parties, therefore, should conduct the interviews. Finally, all sessions were provided by a university graduate student. Although this was reasonable for an experiment like this, in the future, it would be beneficial to train educators in the LAP strategy and have them carry out the intervention.

Practical Implications and Conclusion

Despite these limitations, the results of this study demonstrate the value of strategy instruction to help struggling students to successfully perform addition of fractions. It took only a few sessions until our participants acquired the process and were able to correctly solve a majority of the problems presented.

Classroom educators need access to socially valid and effective targeted interventions for young learners who do not obtain math proficiency through general instruction alongside their classroom peers. If struggling students do not receive targeted and intense strategic support like that shown in this study, they risk falling further behind. Educators face an increased challenge to improve the achievement of both students with and without disabilities at a time when many learners are still struggling with the aftermath of remote or hybrid learning, quarantines, and the loss of face-to-face contact with their friends and teachers during the Covid-19 pandemic. The LAP strategy as part of classroom instruction (e.g., through peer-tutoring, through the help classroom aides, or through small-group work) offers teachers an effective means of meeting this challenge.

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