

## Mathematical Proficiency as the Basis for Assessment: A Literature Review and its Potentialities

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**Abstract:** *Mathematics teaching and learning goes beyond computations and procedures; it rather includes complex problem-solving and critical thinking. Kilpatrick, Swafford, and Findell (2001) identify five mathematical competencies that are present in mathematics learning: conceptual understanding, procedural fluency, adaptive reasoning, strategic competence, and productive disposition. Although these competencies are named and discussed throughout mathematics literature, there is little to be said about the assessment practices that could be used to evaluate these five competencies. This paper offers a literature review that portrays the scarce ways in which mathematical proficiency is partially being used as the basis to assess mathematics. Most of the work that has been researched shows the use of the mathematical proficiency competencies in mathematics instruction and not in mathematics assessment. Using an action research approach, this study intends to have teacher participants and researchers working collectively in a classroom-based assessment methodology, which applies and evaluates assessment practices grounded on all five components of mathematical proficiency. These practices have the potential to inform teachers' practices towards the further development of students' mathematical proficiency.*

### Introduction

*“It goes without saying that ‘knowing’ mathematics, in the sense of being able to produce facts and definitions, and execute procedures on command, is not enough.” (Schoenfeld, 2007B, p.64)*

Mathematics learning can bring up frustrating and anxiety-driven memories of step-by-step procedures and formulas. There is a common belief that mathematics is only about memorization and procedures. Kilpatrick, Swafford, and Findell (2001) argue that, although mathematics

involves a lot of deductive reasoning, school mathematics does not necessarily reflect that. The authors state that, for a long time, the school system involved sophisticated knowledge, but superficially, without a concern about understanding it; “mathematics learning has often been more a matter of memorizing than of understanding” (Kilpatrick et al., 2001, p.16). It is undeniable that mathematics involves procedures and formulas. However, this is far from the essence of mathematics. Mathematics involves thinking, reasoning, analyzing and conjecturing; that is why it involves also frustration. Processes of doing mathematics are not usually straightforward and exempted from errors or misleading paths.

In 2016, The World Economic Forum anticipated that by 2020 the 10 most desirable skills to thrive would be in this order: complex problem solving, critical thinking, creativity, people management, coordinating with others, emotional intelligence, judgement and decision making, service orientation, negotiation, and cognitive flexibility (<https://www.weforum.org/agenda/2016/01/the-10-skills-you-need-to-thrive-in-the-fourth-industrial-revolution/>). These skills contemplate different nuances of the doing of mathematics and they differ in nature from what was required from school graduates in the past. For example, computational skills did not make it to this recent list, as it made it back in 1970 in 2nd place, and in 1999 in 12th, in the Fortune 500 list (Boaler, 2016). As a recent report from the National Research Council put it, “To be employable in the modern economy, high school graduates need to be more than merely literate. They must be able to read challenging material, to perform sophisticated computations, and to solve problems independently” (Kilpatrick et al., 2001, p.17). Indeed, Kilpatrick et al. (2001) state that “[t]he mathematics students need to learn today is not the same mathematics that their parents and grandparents needed to learn” (p.1). Simmt (2017) complements this thought highlighting that “Canada will need people with (...) stronger mathematical reasoning, stronger computational thinking skills, and the capacity to work on hard problems” (p.129). Therefore, why narrow students' experiences within mathematics to procedural experiences? Why focus on computational skills in 2020 if what is expected from school graduates is complex problem-solving? Mathematics classes needed and still need to adapt according to these trends.

Mathematics classes might be allowing for a broader focus, involving conceptual discussions and explorations. However, when it comes to assessment, it is not uncommon to see the broader focus of mathematics focused on procedures. When assessing students' progress in mathematics, there is often a limited focus on the students' solutions rather than the students' learning and working processes (Burkhardt, 2007). As Kilpatrick and Swafford (2002) mention “[m]ost current math tests, whether standardized achievement tests or classroom quizzes, address only a fraction of math proficiency — usually just the computing strand and simple parts of the understanding and applying strands” (p.32); mathematical proficiency being defined as a cohesive blend of conceptual understanding, strategic competence, procedural fluency, adaptive reasoning, and productive disposition. When mathematics assessments overvalue the procedural aspect of doing

mathematics, it favours the perception that mathematics education is only about procedures and formulas. This is reflected, for instance, in the “back to the basics” movement, which depicts the opinion of a group of people that “think of mathematical proficiency mainly in terms of procedure skill” (Groth, 2017, p.104). This practice comes with drawbacks, and a significant one is an emphasis on the idea that students should only worry about the “how” to do mathematics, disregarding the “when”, the “what,” and the “why.” Students tend to concentrate their learning efforts on what they perceive the teacher values or expects. Even when students’ experiences in mathematics classes are not solely focused on procedures, if the assessment expectation is only about knowing how to do the math, chances are that students will focus on procedures. In accordance with that, Schoenfeld (2007B) states that

teachers feel pressured to teach to the test — and if the test focuses on skills, other aspects of mathematical proficiency tend to be given short shrift. (...) Similarly, students take tests as models of what they are to know. Thus, assessment shapes what students attend to, and what they learn (p.72).

As Burkhardt’s acronym WYTIWYG says, “What You Test Is What You Get” (Schoenfeld, 2007B, p.72), which is a relevant reason why teachers should pay close attention to mathematics assessments. If assessments promote a poor engagement with mathematics learning, teachers and students might have to deal with a superficial development of mathematical skills.

Kilpatrick et al. (2001) emphasize the fundamental need for coordination between instruction and assessment (among other aspects) to foster the development of mathematical proficiency. The authors also indicate the need for more research about mathematics proficiency, both concerning its development and its assessment. In fact, the assessment of mathematical proficiency is a potential tool for the development of mathematical proficiency, instead of just a tool to report students’ mathematical proficiency at a certain point; “[m]athematics assessments need to enable and not just gauge the development of proficiency” (Kilpatrick et al., 2001, p.13). In this sense, this research is of relevance, given that it uses the notion of mathematical proficiency proposed by Kilpatrick et al. to analyze assessment possibilities that will not only holistically assess students’ mathematical learning, but that will also inform teachers’ practices to further develop students’ mathematical proficiency. As such, this study research question is posed as: *In what ways does an assessment based on mathematical proficiency result in a holistic understanding of students’ mathematical learning processes, and ultimately lead to the development of students’ mathematical proficiency?*

To answer this research question, this investigation will explore a classroom-based assessment methodology that investigates students’ mathematical proficiency, providing teachers with information to plan their classes accordingly. The implementation of the assessment methodology will yield data to be analyzed, aiming to better understand assessment tools that support students’ development of mathematical proficiency. This research is in its early stages and this paper focuses

on a literature review of what has been done so far in terms of mathematical proficiency in mathematics classrooms, assessments, and instruction. The next section of this paper speaks to mathematical assessment practices in general. Then, mathematical proficiency is defined, and a discussion on mathematical assessments based on the principles of mathematical proficiency is presented. Finally, an overview of the research done to date is offered, followed by final comments and research next stages.

### **Assessment Practices in Mathematics**

Assessment practices can have a traditional and procedural approach when it comes to mathematics. If mathematics courses assess only mathematics procedural skills, one might get the impression that a procedure-based course is enough to build on students' mathematical knowledge, given that students will probably succeed in procedure-based assessments. However, this could be a false impression given by limited assessments. Schoenfeld (2007B) explains that “[a]spects of strategy, metacognition, and beliefs are much more subtle and difficult to assess. Yet, doing so is essential” (p.72). It is interesting to notice that students who attend skills-based mathematics courses do not present achievement results that significantly differ from the achievement results of students that attend mathematics courses with a broader approach to the curriculum (Schoenfeld, 2007B). On the other hand, the former group does not tend to present good results in problem solving and conceptual tests, while the latter group tends to present good results when tested in these same skills (Schoenfeld, 2007B). It is of relevance to investigate mathematical assessments in parallel with all the different aspects involved in the development of mathematical knowledge and proficiency.

In accordance with these thoughts, Schoenfeld's work (2007A) highlights that mathematics education researchers have a thorough grasp of what thinking mathematically and understanding mathematics encompass, and, as a result, they tend to advocate for assessment practices that are comprehensive in terms of content and processes. In contrast, assessment in mathematics classes is usually concentrated on products instead of processes. The repetition and reproduction of procedures are not sufficient to develop the mathematical skills that are expected from the students. When students mindlessly practice mathematical procedures, they are becoming proficient at utilizing a procedure without understanding the ideas that underline the procedure (Schoenfeld, 2007A). Schoenfeld calls that an “Illusion of Competence” (p.10) because although students may think they are competent at that specific skill, they might fail if they have to deal with a slightly different problem or procedure. Because this focus on mindless practice is a long-dated modus operandi, it is extremely assimilated in students' experiences, teachers' habits, and people's common sense. Yet, if teachers from elementary to university level are asked about tasks that go in the opposite direction, requiring students to mathematically model or prove something, teachers will agree that this sort of task is successful to evaluate students' understanding (Schoenfeld, 2007A). So why wouldn't teachers try different methods of assessment that holistically assess students' mathematical work? Teachers can face a lot of resistance when trying new approaches



in their mathematics classes, as it can be difficult to change these deep-rooted ways of thinking about assessment in mathematics. Schoenfeld claims that in “some cases, curricular innovators have faced the problem that without ‘proof of concept’ (evidence that a non-standard approach will guarantee high enough test scores) school districts are reluctant to let people try new ideas” (p.13).

Changing the focus of assessments requires a focus on the types of assessment tasks chosen for students to complete. Therefore, when building assessments, teachers need to be mindful of the intentions of their tasks. Ramaley (2007) indicates that, according to a National Research Council’s report on assessment, it is recommended that classroom assessments should: “(a) share a common model of student learning, (b) focus on what is most highly valued rather than what is easy to measure, (c) signal to teachers and students what is important for them to teach and learn” (p.18). This understanding emphasizes assessment not only as a tool to report on students’ state of learning, but as a resource to inform students’ processes of learning. Burkhardt (2007) unpacks task intentions by discussing what teachers should care about regarding assessment. He argues that we should strive to teach to “societal goals,” and this starts with the intention of developing “thinkers” (p.84). Burkhardt offers a mathematical model for solving a math problem and argues that we often focus on the “solve phase” instead of on the “formulate,” “evaluate” or “interpret” phases (p.86). When working with students and assessing their mathematical proficiency, teachers often focus on the final product of the students’ work rather than the complex mathematical thinking that leads to that product (the process). When considering how teachers can assess a student’s performance, Burkhardt highlights one “holistic dimension, task type” (p.89). The author offers two assessment considerations: “[m]easure what is important, not just what is easy to measure” and “[a]ssess valued aspects of mathematical proficiency, not just its separate components” (p.78 and 79). In the first principle, Burkhardt argues that mathematical proficiencies cannot be assessed through traditional testing measures, like multiple-choice tests, as these methods are often riddled with bias. The reliance on such testing measures is because of their ease and readability. In the second principle, Burkhardt argues that teachers should consider the student’s entire performance instead of isolated aspects of students’ work. Suurtamm (2018) adds to the discussion, claiming that four features will better mathematics assessment: “ongoing and embedded in instruction, uses a variety of assessment strategies, reflects meaningful mathematics, and includes students in the process” (p.475).

Mathematics assessment intentions need to be visible in all aspects of teaching (planning, instruction, and assessment). The practices that teachers engage in while teaching and assessing provide students with the grounds needed to deepen their mathematics understanding. Mathematics assessment practices must have a relationship with the tasks used to evaluate student learning (de Lange, 2007). de Lange (2007) states that there is a relationship between “‘expected performances’ (or ‘competencies’ or ‘learning goals’) and assessment items” (p.100). When teachers use problem-solving methods in mathematics, they are usually focused on the question

and the solution, instead of focusing on the competencies that are present in the problem-solving process. de Lange identifies 3 “clusters of competencies” that are used in classroom assessment: “reproduction,” “connections,” and “reflection” (p.102). In the first cluster, students complete standard equations and computations to solve for a solution. In the connections cluster, students move toward problem-solving by making connections to other topics in mathematics or the context of the problem. While in the reflection cluster, students are engaging in abstract math thinking that involves “mathematical thinking, generalization, [and] abstraction and reflection” (p.102). The current study intends to go beyond the reproduction cluster and explore the connections and the reflection clusters, allowing students and teachers to further examine teaching and learning processes. As Schoenfeld (2007A) states, “[f]rom the teacher’s perspective, assessment should help both student and teacher to understand what the student knows, and to identify areas in which the student needs improvement” (p.9).

Although assessment tasks and practices have been discussed at length (e.g. Pai, 2018; Straumberger, 2018; Swan & Foster, 2018), there is very little research that discusses mathematical assessment practices, in which teachers assess all five strands of mathematical proficiency: conceptual understanding, procedural fluency, adaptive reasoning, strategic competence, and productive disposition. For this research study, the goal is to work with elementary students and teachers to provide enriched mathematical tasks combined with a holistic mathematical assessment that evaluates the five mathematical proficiencies. Indeed, mathematics education research indicates that teachers and students can benefit from holistic assessment practices that go beyond the reproduction of procedures and is not solely product-based (e.g. Foster, Noyce, & Spiegel, 2007). Besides, de Lange (2007) argues that rich, holistic assessments should begin in the early years of education through to secondary education. Unfortunately, this is not a common practice when it comes to mathematics education. As Suh (2007) states, in “many traditional classrooms, procedural fluency plays a dominant role in defining mathematical proficiency” (p.164). As a result, instead of encountering holistic assessments, students encounter procedural assessments. This research aims to explore holistic assessments that identify students’ areas of improvement and inform teachers’ practice towards students’ development of mathematical proficiency.

### **Mathematical Proficiency-Based Assessments – An Option**

But how does one begin to create assessments that are framed by mathematical proficiency? How mathematical proficiencies should be tested? What types of assessments should teachers be using? To conjecture about these questions, it is necessary to first understand what exactly defines mathematical proficiency.

According to the website dictionary.com (accessed on October 19, 2020), the word proficiency can be defined as “the state of being proficient”; proficient can be defined as “well-advanced or competent in any art, science, or subject”; and competent can be defined as “having suitable or sufficient skill, knowledge, experience, etc., for some purpose.” Based on these definitions, being

mathematical proficiency could be defined as *having suitable or sufficient skill, knowledge, or experience in mathematics*. Kilpatrick et al. (2001) created a framework to explain what mathematical proficiency encompasses. The authors argue that to be mathematically proficient, students need to be skilled in five different strands that contemplate concepts, procedures, strategies, reasoning and attitudes. The strands of mathematical proficiency are named as conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Each strand is briefly defined in Table 1.

<i>Conceptual Understanding</i>	[C]omprehension of mathematical concepts, operations, and relations.
<i>Procedural Fluency</i>	[S]kill in carrying out procedures flexibly, accurately, efficiently, and appropriately.
<i>Strategic Competence</i>	[A]bility to formulate, represent, and solve mathematical problems.
<i>Adaptive Reasoning</i>	[C]apacity for logical thought, reflection, explanation, and justification.
<i>Productive Disposition</i>	[H]abitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Table 1. The strands of mathematical proficiency (Kilpatrick et al., 2001, p.116)

The five strands of mathematical proficiency, according to Kilpatrick et al. (2001), are intrinsically related to each other, to the point that one depends on the others. The correlation between these strands is organic in such ways, that it can be difficult to describe which strand(s) is(are) in place in a certain piece of students' work. Kilpatrick and Swafford (2002) explain that

[t]he most important feature of mathematical proficiency is that these five strands are interwoven and interdependent. Other views of mathematics learning have tended to emphasize only one aspect of proficiency, with the expectation that other aspects will develop as a consequence. For example, some people who have emphasized the need for students to master computations have assumed that understanding would follow. Others, focusing on students' understanding of concepts, have assumed that skill would develop naturally. By using these five strands, we have attempted to give a more rounded portrayal of successful mathematics learning (p.9).

This interdependency between the strands of mathematical proficiency implies that teachers need to consider all five strands in the teaching and learning processes of their students, which includes assessments. The way students are taught, learn, and develop proficiency, as well as the way

teachers are educated to promote students' mathematical proficiency need to mirror this inherent relation between the strands (Kilpatrick et al., 2001). The interrelation between the five strands can also give some insight into the learning difficulties students might have with mathematics in the long run. Given that mathematics proficiency encompasses five different strands and school mathematics frequently focuses on only one of them, namely procedural fluency, it would make sense that students encounter challenges or do not properly learn mathematics due to the absence of the other four strands. Kilpatrick et al. mention that, in the United States, there is a tendency "to concentrate on one strand of proficiency to the exclusion of the rest" (p.11); this tendency is not an exclusive behaviour of the United States mathematics classrooms.

Aiming to authentically assess students' entire mathematical performance and knowing that Kilpatrick et al.'s (2001) model of mathematical proficiency considers five comprehensive nuances of students' work, Kilpatrick et al. framework was the chosen one to support this research approaches on mathematical assessment. Kilpatrick and Swafford (2002) confirm that if students are to become mathematically proficient, different aspects of their education need to undergo significant changes, assessments being one of them.

Does improving students' math proficiency require new types of tests?

Yes. New tests may be needed, and old tests may need to be changed. (...) Teachers need tests and other assessment procedures that let them gauge how far students have come along in all five proficiency strands. Furthermore, instead of taking time away from learning, these instruments should allow students simultaneously to build and exhibit their proficiency. (Kilpatrick and Swafford, 2002, p.32)

Because of the challenges in changing assessment practices and in accepting new assessment possibilities, this research intends to portray how assessment approaches that consider the five strands of mathematical proficiency can be beneficial in mathematics elementary classes to effectively inform teachers' practice and foster students' development of mathematical proficiency. In other words, the goal is to analyze assessment options that coherently and comprehensively examine students' strands of mathematical proficiency to inform teachers' practice, allowing for the further development or improvement of these strands. Kilpatrick and Swafford (2002) acknowledge that some changes might have already been reflected in instructional materials and even in assessments, nevertheless, they state that "progress has been uneven and poorly documented" (p.4), reinforcing the importance of research studies in this realm. In addition, Kilpatrick et al. (2001) state that they "find real progress toward mathematical proficiency to be woefully inadequate" (p.11). This research intends to contribute to the mathematics education research field by offering practical guidance and options that can contribute to the progress toward mathematical proficiency.



## Mathematical Proficiency-Based Assessment and Instruction – An Overview

To the extent of this study, research surrounding the five strands of mathematics proficiency in the mathematics classroom is limited. Much of the relevant research for this study only partially considers the five strands. Besides, most of the research found relates to instructional practices. This section discusses the ways in which the five strands have been used and the findings of these studies as it relates to assessment and instruction.

Khairani and Nordin (2011) created a mathematical proficiency test to assess the development and relationship between conceptual understanding, procedural fluency, and strategic competence of 588 14-year-old students. These three strands were chosen apart from the remaining two (productive disposition and adaptive reasoning) as Khairani and Nordin argue that the excluded strands are not yet mature enough to assess and that conceptual understanding, procedural fluency, and strategic competence are easier to assess using a “standardized achievement test” (p.35). The test was structured in the following manner: 50% of the test focused on conceptual understanding, 32% of the test focused on procedural fluency, and 18% focused on strategic competence. The topics covered on the test were “Linear Equation, Algebraic Expressions II, Ratios, Rates, and Proportions I, and Coordinates and Circles I” (Khairani & Nordin, 2011, p.37), and there was no specific training or special classes to prepare students for the test. The questions on the instrument concerning conceptual understanding were deemed the easiest for students to complete. Next was strategic competence, and then procedural fluency was considered the most difficult strand for the students. Students did well with questions that stated the information “explicitly” but struggled when they had to “use their prior knowledge” to solve for solutions (Khairani & Nordin, 2011, p.41). Although this is one of the very few studies that claims to assess mathematical proficiency (based on this research scope), one could question the employed assessment tool. This research study used a set of 50 multiple choice questions despite the often-cited notion that multiple-choice questions are not an effective assessment model (Burkhardt, 2007; Boaler, 2016). Multiple-choice exams can provide an inaccurate, and superficial assessment of students’ mathematical knowledge.

Suh (2007) reinforces the idea that there is still a prevailing focus on procedural fluency in mathematics classrooms. Willing to change this scenario, she proposed five different classroom activities to elementary students focused on building on the five strands of mathematical proficiency. The first activity is called “Modeling Math Meaningfully” and intends to have students working with different representations when solving problems or doing exercises, as a way of working on conceptual knowledge. The second activity is called “Math Curse” and pretends that students are under a curse that makes them see mathematics in everything. Students are then supposed to find mathematics in their daily lives, share and discuss with their classmates. The third activity is called “Math Happenings” and consists of the exploration of likely-to-happen problems proposed by the class teacher. Finally, the last two activities are called “Convince Me” and “Poster Proofs”; these activities are meant to work on deductive reasoning, argumentation, justification, and collaboration. Suh indicates that because of the nature of the activities she was able to

differentiate her instruction according to students' needs. She also observed a considerable change in terms of students' productive disposition.

Groth (2017) argues that the qualitative data that can be gathered during mathematics classes is relevant for improving instruction; his research investigates classroom data through a mathematical proficiency lens. Groth's research presents an iterative cycle: 1) qualitative data is gathered from students' interactions; 2) this data is used to portray students' mathematical proficiency; 3) lesson plan is designed to develop students' mathematical proficiency; and 4) feedback about the lesson is obtained from peers and mentor. Pieces of evidence informed instructional strategies to build on students' mathematical proficiency, and results were observed. Groth underlines situations, in which correct responses were not matching correct explanations, and vice-versa, as examples of when the final answer may not be sufficient to portray students' mathematical proficiency. The author indicates productive disposition as the most challenging strand to observe and suggests that teaching approaches might have an important role when fostering productive disposition. Groth also mentions the challenges involved in creating tasks that promote mathematical proficiency. One of the author's final thoughts about mathematical proficiency is that teachers "can develop the fundamental habit of mind of basing daily instructional decisions on observations of their students' strengths and needs along the five strands of mathematical proficiency" (Groth, 2017, p.107). Groth's study gives insight into the current study in the sense that it suggests that the strands of mathematical proficiency should permeate teaching practices in the classroom. The current study intends to extend Groth's proposition by offering an analysis of mathematics assessment practices, instead of mathematics instructional practices.

In a research study that discusses problem-based teaching models, Ozdemir and Pape (2012) describe the instructional approaches that supported students' development of strategic competence. The teacher and the respective class were chosen for this study because the teacher had attended a Self-Regulated Learning (SRL) workshop. There is a connection between SRL and strategic competence in that students are meant to reflect on the strategies to use when given a problem and use their autonomy to carry out their plan. In their observations of the lessons and students' learning, Ozdemir and Pape identified four categories that support the development of students' strategic competence: "(a) the nature of tasks and activities, (b) practices supporting understanding, (c) practices supporting strategic knowledge and skills, and (d) practices supporting motivation" (p.160). Regarding the first category, the tasks that the students engaged in were intentionally planned to support student understanding through collaborative work in small groups during lessons that introduced topics. In order to support student understanding (second category), the teacher gave students detailed explanations as well as provided multiple methods of showing understanding. The concepts always had a real-life connection for students. For the third category, the teacher would invite the students to engage in group problem solving where "students exercised strategic competence by analyzing the task (e.g., rereading, under-lining, and using context clues),

selecting, adapting, and implementing strategies, as well as comparing and contrasting each other's strategies" (Ozdemir & Pape, 2012, p.161). Finally, to support students' motivation (fourth category) to persist in problem-solving, the teacher would acknowledge students' understanding and highlight their strategies and ideas used to solve the problems.

Freund (2011) sought to understand teachers' approaches when teaching for mathematical proficiency in an urban school context. The author discussed how teachers engaged in problem-based teaching to develop mathematical proficiency. The teachers in the study were filmed teaching lessons about algebraic thinking after attending a professional development seminar. Productive disposition was excluded from this study as it was deemed too difficult to measure. Based on the lessons taught and on whole class and small group discussions, the researcher observed each student's mathematical proficiency on a scale from 1-5 to determine if the student was "proficient strong, proficient-limited, non-proficient-strong, non-proficient-limited, and no-rating-none" (Freund, 2011, p.49). According to students' performances, the teachers were organized into three groupings: majority proficient, mixed proficiency group, and low proficiency group. In the majority proficient group, the lesson structure was formatted as follows: "problem introduction, small group work, and whole class discussion" (Freund, 2011, p.70). The teachers focused their discussions on explanations, multiple solutions, and student-to-student conversations. The mixed proficiency group shared some similarities with the majority proficiency group, while the low proficiency group had some more traditional instructional approaches, for instance, independent work and teacher-led problem-solving (Freund, 2011). It is clear to conclude that the open-ended, student-led, problem-based mathematics tasks serve students much more positively regarding mathematical proficiency development as opposed to the traditional teacher-centered model.

Student's self-assessment of their productive disposition has been evaluated by Graven (2012). The author sought to understand how the use of mathematics clubs could influence the productive disposition of students. The students were to reflect on productive dispositions and rate themselves on a self-assessment. In this self-assessment, the students were asked to rate their proficiency in mathematics based on a 9-point scale. The students were asked to relate their performance to a hypothetical low performing student named Mpho (at the low-end of the scale), and a hypothetical high student named Sam (at the high-end of the scale). They were asked to reflect on both types of learners in terms of classroom behaviour and overall performance. The students identified that the proficiency low student, Mpho, was distracting and off task, while the proficiency strong student, Sam, was focused and completed the homework without any struggle. In the interviews, Graven found that when students were struggling with a question "only two learners suggested asking the teacher and the remaining eight learners referred to ways of solving that did not involve the teacher, e.g. 'I thought it in my mind', 'I work it out', 'I take scrap paper or counters or my brain', 'I stretch my brain a bit and don't copy'" (p.57). The students shared strategies and

struggles that they encountered in their mathematics work. These comments indicated students' high productive disposition.

In a different perspective, Siegfried (2012) conducted a two-part study that examined productive dispositions as well, but of pre-service and in-service teachers while assessing student work. In the first study, Siegfried designed a rubric that evaluated the productive disposition of teachers that were assessing a student's math task. In the second study, Siegfried took 10 teachers that were found to have strong productive dispositions to engage in interviews. In the first study, pre-service teachers had lower productive dispositions than in-service teachers, and Siegfried credited this to a difference in experience. In the second study, the participants that were deemed to have strong productive dispositions exhibited the following characteristics while engaging in the math tasks: determination, unstoppable effort, multi-modal approach, honesty, passionate about the excitement of mathematics, and the belief that success could be achieved by anyone (Siegfried, 2012). These qualities are arguably quite important when teaching and assessing a proficiency-based math program. These characteristics of the teacher participants in Siegfried's study mirror students' anecdotes in Graven's (2012) work.

Although research does not directly address the five strands of mathematical proficiency in terms of assessment, there has been some evidence that highlights important teaching and assessing strategies that address the holistic development of mathematical knowledge. Foster, Noyce, and Spiegel (2007) discuss the ways that the Mathematics Assessment Collaborative (MAC) – a program that replaces standardized tests with a “coordinated program of support and learning for teachers based on a common set of assessments given to students” (p.138) – impacted student success. This assessment considered five main ideas about mathematics per grade level, and, in place of tests, students had five tasks to complete. These tasks “require students to evaluate, optimize, design, plan, model, transform, generalize, justify, interpret, represent, estimate, and calculate their solutions” (Foster et al., 2007, p.139) through open, problem-based tasks. This exam was given to students in grades three, five, seven, and in algebra classes in 24 school districts. The teachers evaluated each exam after receiving training on the assessment rubric. The training involved in the assessment of the tasks was a professional development exercise as teachers were taught to consider all aspects of the math task that they were assessing, and then they were expected to find evidence of that in the students' work. The students' exams received a grade in 4 levels, where Level 1 showed limited success and Level 4 showed success at an increased level. The exams were returned to the schools for teachers to reflect on and use in future teaching scenarios. The test was also used as “valuable information for professional development, district policy, and instruction” (Foster, Noyce, & Spiegel, 2007, p.141). For instance, if a school has shown an unsuccessful trend in a strand of mathematics, the teachers would receive professional development sessions targeting the core ideas of the strand, and later, the students would be reassessed to evaluate any changes. This was the case with a proportional reasoning task for grade 7 students. Initially, only 37% of the grade seven students were able to “meet the standard” (p.142),

and 20% could complete all the questions. This meant that 63% of the students were unsuccessful. Four years later, the students were given another proportional reasoning task and their success had increased to 59%. The exam results of the students that participated in the MAC program were compared to students that were not in the program, and even though the students in the MAC program had a lower socioeconomic status, they still were able to meet the standards on the state exam compared to those not in the MAC program. The findings support Foster et al. (2007) argument that “when teachers teach to the big ideas (...) participate in ongoing content-based professional development, and use specific assessment information to inform instruction, their students will learn and achieve more” (p.152). Furthermore, teachers were empowered by the results of these assessments as teachers were able to collaborate with other teachers to discuss strategies and student progress with other knowledgeable teachers. The MAC shows teachers where their students share similar struggles, how the big ideas are interrelated across grade levels, the importance of collaboration with other grade levels, and how to reflect on their intentions related to teaching and assessment (Foster et al., 2007).

As illustrated in the above literature review, research around mathematical proficiency and assessment is scarce. Most of the literature review refers to mathematics proficiency and instruction, and even within instructional approaches, research is still limited. Indeed, mathematics assessment practices have proven to be more challenging to change than mathematics instructional practices. Given that fostering students’ mathematical proficiency involves both instruction and assessment, this research focuses on offering new possibilities in terms of mathematics proficiency and assessment.

### **Researching Mathematical Proficiency-based Assessments – A Proposal**

*“Not only is it the case that one assessment size does not fit all, it may well be the case that one assessment size does not fit anyone’s needs very well.” (Schoenfeld, 2007A, p.14)*

Challenging teaching and assessment practices can be demanding given teachers’ previous experiences as learners, beliefs, teacher education, and approaches to teaching. Besides, it is not uncommon to have teachers discouraged about new tendencies and understandings from mathematics education research, as teachers may perceive research as disconnected to teachers’ teaching practices and classroom realities. Considering this perspective and aiming to develop a study that has the potential to enlighten teachers’ approaches when it comes to mathematics assessment and proficiency, this study is planned to be an action research study that has in-service teachers fully engaged as co-researchers. This way, teachers can collaborate with researchers as they study their “own instructional methods, their own students, and their own assessments” (Mertler, 2009, p.4). As Greenwood and Levin (2007) state, action research “democratizes the relationship between the professional researcher and the local interested parties” (p.4). In the specific case of this study, the local interested parties refer to teachers concerned about assessment

practices that have the potential to assist in evaluating their students' mathematical proficiency and inform their teaching practice in meaningful ways. The goal is to develop a practice-based research methodology with a practical purpose (McAteer, 2013), which aims to apply the five strands of mathematical proficiency to assessments more seamlessly.

This research acknowledges the relevance of integrated work between teachers and researchers. Greenwood and Levin (2007) argue that when theorists and actors (teachers in this case) are not engaged in the same way in a research study, the disengagement might do a disservice in terms of meaningful, practical, methodological and theoretical results. Action research moves towards "closing the gap between the *roles* of theorist and practitioner" (Kemmis, 2009, p.468, original italics). When teachers take part also as researchers, practice and theory are investigated in parallel, yielding beneficial outcomes on both ends. This desired engagement between theorists and practitioners is in tune with McNiff and Whitehead's (2002) view of action research, in what they argue that in action research "[n]o distinction is made between who is a researcher and who is a practitioner. Practitioners are potential researchers, and researchers are practitioners" (p.15). As such, the term researchers, when used in this study, will refer to both researchers and practitioners.

Kemmis (2009) states that action research aims at transforming three things: practitioners' practices, their understandings of their practices, and the conditions in which they practice. In this study, researchers will reflect on classroom current assessment practices, how these practices inform the development of students' mathematical proficiency, and what are the circumstances in place, so that they can transform their practices. In other words, researchers investigate and make judgements about their own practices, in order to enhance them (Kemmis & McTaggart, 2000; McAteer, 2013). To figure out ways of improving assessment approaches, researchers can ask themselves: What strands of mathematical proficiency are or are not being considered in the current assessment practices? What changes or additions need to be made in assessment tools to ensure that the strands of mathematical proficiency are being evaluated in its entirety? How can these assessment tools help in promoting students' mathematical proficiency? According to the outcomes of this reflection process, researchers plan an action and implement it in class. The action taken needs to be documented and analyzed, so that the plan can be modified, improved, put into practice again, and reanalyzed until researchers' goals are achieved. This action research process has huge importance in the accomplishment of the research goals. As Greenwood and Levin (2007) argue, the "direction of an AR [action research] project is guided by the learning gained through the process, not by a set of a priori norms or expectations imposed on the situation and actors" (p.134).

McNiff and Whitehead (2002) explain that in an action research project:

[w]e review our current practice, identify an aspect we want to improve, imagine a way forward, try it out, and take stock of what happens. We modify our plan in the

light of what we have found and continue with the ‘action’, evaluate the modified action, and so on until we are satisfied with that aspect of our work (p.71).

This research study adheres to this format and follows the cycle depicted in Figure 1. The action research cycle has six distinct phases: Review, Identify, Plan, Act, Reflect, and Discuss. The initial Review stage encompasses a critical analysis of the assessment practices being used to date in the researched classes. Then, the Identify stage investigates these current assessment practices aiming to find aspects for improvement. In the Plan stage, researchers are engaged in rich planning as they modify or create teaching contexts and tools for the assessment of mathematical proficiency. This occurs both independently and in focus group discussions. The researchers then use the assessment model in the Act phase. The researchers meet again for reflection and discussion, offering adjustments to the assessment tool based on the practical applications done in the classroom. After the Reflection and Discussion phases, researchers may go back to the Plan stage to modify the initial plan as needed and reapply it. The cycle is repeated until the goals of the research are achieved. Researchers have a collaborative relationship in this project as they are actively involved in the creation and revisions of the assessment model used to evaluate the five strands of mathematical proficiency.

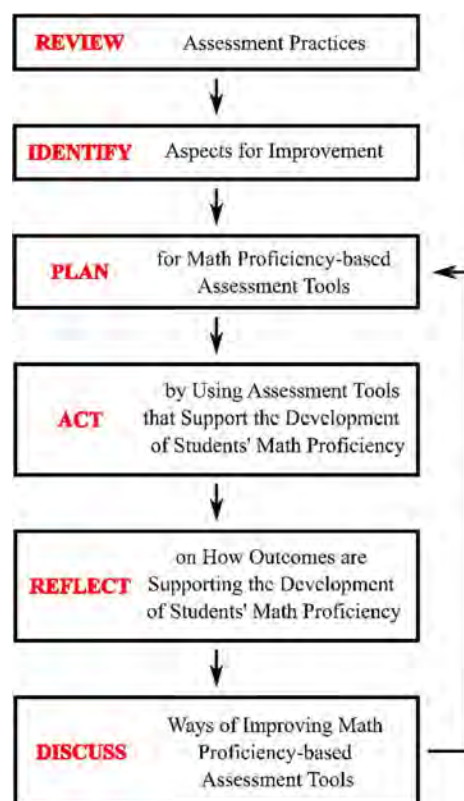


Figure 1. Action Research Cycle.

## Final Remarks

The main goal of school mathematics education could be described as the development of mathematical skills and knowledge that will promote mathematical literacy, allowing citizens to effectively and consciously participate in society and in all that it entails. Kilpatrick et al. (2001) argue for a teaching approach that is holistic and balanced in both teaching and assessing so that "[w]hen today's students become adults, they [can] face new demands for mathematical proficiency that school mathematics should attempt to anticipate." (p.1). School mathematics education is supposed to form students that have a practical, but also an analytical understanding of mathematics. This paper briefly explains what mathematical proficiency is about and argues that mathematics teachers and educators should have a close look into students' mathematical proficiency if they intend to develop long-lasting essential mathematical skills and knowledge. This research study claims that assessments focused on mathematical proficiency can be a strong ally in the development of these skills and knowledge. The understanding of students' learning processes, through assessments that investigate mathematical proficiency, may effectively inform teachers' practice towards the holistic development of mathematical proficiency. Grounded on a literature review about what has been done to date in terms of instruction, assessment and mathematical proficiency, this study proposes an action research classroom-based investigation that aims to fill in gaps related to the potential benefits of mathematical proficiency-based assessments.

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