How substantial and efficacious is the learning of linear algebra at undergraduate level?

Lekë Pepkolaj[1], Siditë Duraj[2]

[1]Albanian University, Tirana, Albania, [2]University of Shkodra "Luigj Gurakuqi", Shkodër, Albania

l.pepkolaj@albanianuniversity.edu.al, sidite.duraj@unishk.edu.al

Abstract: Linear algebra is a crucial part of mathematical training for engineering students and there are many problems in the application of this subject in engineering, physical, and even mathematical disciplines. In this scope, a study on the sustainability of learning this subject and how it can be improved is considered necessary. This article aims to delve into how linear algebra learning is effective and at examining three critical parameters: a) Comparison of summative assessment test results on specific topics with the ones produced in the previous year to assess student learning sustainability. b) Testing the topics based on the approach taken, i.e., based on other mathematical topics and real problems. c) Comparing the effectiveness of learning of problems solved in an analytical fashion and the learning of problems solved with mathematical software's help. The methodology used is based on constructionism that depends on the type of activity given in both the design phase and management phase. From the obtained results of the study, the state of learning linear algebra was measured after one year. From the mathematical retest, it turns out that learning algebra was not effective and the exam grades showed temporary learning. There is a skewed relationship, in the next test those with better grades perform worse than those with lower grades. While in the applied aspect the learning was more sustainable. The scrutiny accentuates the noteworthiness of coordinating the importance of coordinating the semiotic systems for them to produce meaningful and durable learning.

INTRODUCTION

The effectiveness of student learning depends on the individual's willingness and interest in identifying the student's knowledge and the new knowledge that he has acquired. The social environment, which includes parents, teachers, and stakeholders, can significantly encourage students to pay attention to durable learning. Durable learning alludes to what happens when knowledge and skills remain in the memory bank for a long time (Brown, 2014). Our aspiration as teachers is to remember and apply the knowledge and skills that they have acquired even after the test in their daily practices.

For the most part, student assessment tests serve to indicate the students' appreciation of the course they have attended. The assessment of student learning is considered an integral and fundamental part of the educational activity. In this article, we examine how practical student assessment is in producing good learning. Learning is durable when students can perform in subsequent situations in which the new material is relevant to prior knowledge: student assessment in the subject matter after a year or, is even more significant, in a job in which the mathematics that is supposed to have been learned is to be used. Assessment of learning occurs when teachers use student learning evidence to make judgments about student success against goals and standards. Therefore, the assessment process is connected to two conceptions of learning: learning concerning goals and learning to construct knowledge.

In addition, the article elucidates the question: Is the mnemonic study only useful for exam purposes, or has learning happened? Finally, this article and reclaiming linear algebra learning will also help review the curriculum, the subject matter's initial goals, and the long-term planning.

THEORETICAL FRAMEWORK

Learning is an individual engagement, and as such it is an individual responsibility. The meanings given to reality, on the other hand, can be pooled, compared, and agreed upon (Novak, 2012). Our Guiding Principle is simple: Learning is the consequence of thinking. To comprehend, is to conjecture. To surmise aptly, our students have to be immersed in. Being knuckle down to the knowledge, increases the comprehension and engagement.

Everything we are going to design should help our students stay engaged to pay attention long enough to think effectively and deposit the learning in long-term memory. According to Hermann Ebbinghaus's theory, every time a new concept enters our head, it undergoes a deterioration over time. According to the graph in the figure, we have already lost 40% of what we have learned after twenty minutes. After one day, only 30% of what we have learned is left, and over time the graph flattens out more slowly until only 20/25% is left in our head (Ebbinghaus, 1885).

Consequently, what can incite the more enduring learning to result less scattered but more assidious?

The retrieval practice, spaced practice, and interleaving elicit the knowledge from long-term memory (Brown et al., 2014; Agarwal & Bain, 2019). Ebbinghaus comprehended that teachers often obliviate that covering only the content does not significantly connote that students have attained it. Covering content does not equate necessarily to mastery of learning, mostly when our students are disengaged and apathetic in our classes (Ebbinghaus, 1885).

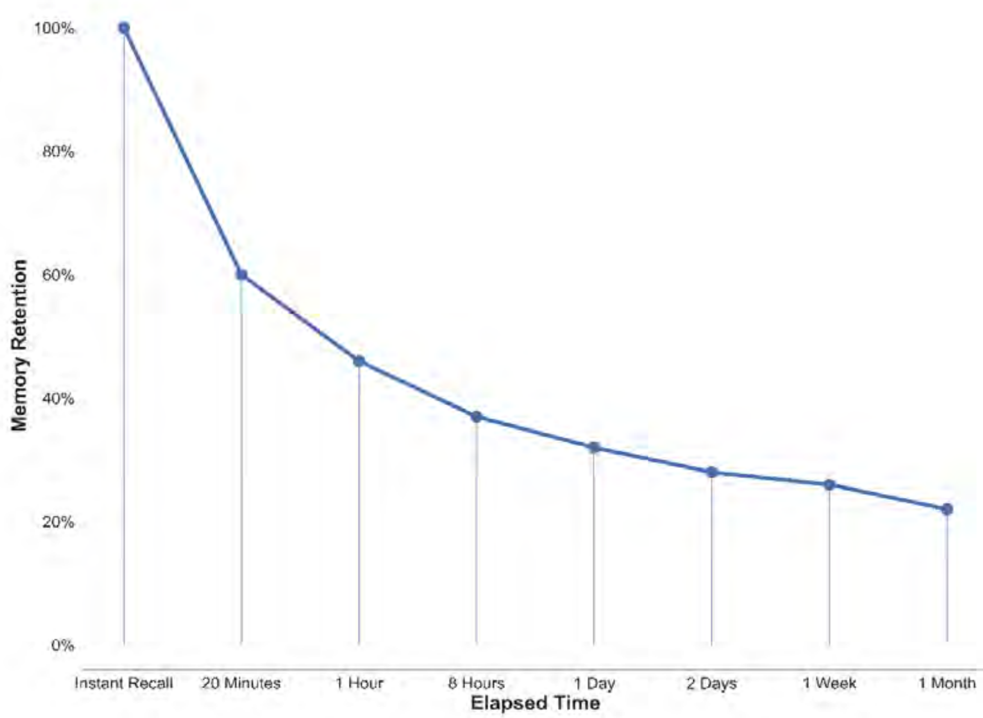

Figure 1: Ebbinghaus curve

At this level of inquiry, psychology, neuroscience, and molecular biology closely interact to invoke the students' learning engagement to be tenacious but not fallacious. The more engaged students are, the more likely they are to recall what they have learned. By definition, student engagement (Ashwin & McVitty, 2015) is that dynamic intersection between being challenged by and loving what one is learning. Think about the times you have been engaged in what you are learning. What does that engagement encompass? Your head and heart are both engrossed...you are thinking hard and enjoying what you are learning.

Many people believe that their intellectual abilities are ingrained from birth and that failure to meet a learning challenge is an indictment of their birth capacity. However, every time you learn something new, you change your brain - the residue of your experiences is stored. It is true that we begin life with the gift of our genes, but it is also true that we become capable through learning and developing mental models that allow us to reason, solve, and create. In other words, the elements that shape your intellectual abilities lie to a surprising degree within your control. Understanding that this is so allows you to see failure as a sign of effort and a source of useful information - the need to dig deeper or try a different strategy. The need to understand that when learning is hard, you are doing important work.

Neuroscience tells us that the brain believes the hands are the most important part of the body. Making things with our hands stimulates the part of our brain that forms neural pathways and causes synapsis to snap into place. We physically alter our brain by making with our hands. This is why the guiding principle here is that making things visible is one of the best ways for students to learn. The classroom application then is to look for opportunities to have your students "make things" that make visible their understanding of an idea, concept, or term (McQuinn, 2018; Kelleher & Whitman, 2016).

Traditionally, when speaking about evaluation, we refer to particular operations that enhance students' progress. The term "assessment of progress" implies all the operations traditionally carried out by the instructor or an education authority in charge of attending to student activities (Calvani, 2004). Assessment of learning is the process of collecting and interpreting evidence and ultimately summarizing learning at a given time. It helps make judgments about the quality of student learning based on established experience and assigning a value that represents quality (Ontario, 2010).

A search of Brown (2014) has pointed out that there are several unchangeable aspects of learning that all researchers probably need to agree upon. They include:

- Learning to be useful requires memory, which means that what we have learned is still in our memory and can be used later when needed.

Another research (Karpicke et al., 2009) argues that we are poor judges; hence it is hard to judge whether learning is satisfactory or not when we learn well and when we do not. When the going is more challenging and slower and does not seem productive, we are drawn to strategies that seem more fruitful, unaware that the gains from these strategies are often temporary.

Meaningful learning means being able to solve everyday life problems. Problem solving gives purpose to learning, which can only become "meaningful" to the person if they understand its usefulness and if it serves e certain purposes (Jonassen & Rohrer, 1999). Meaningful learning relies on constructivist pedagogy which places special emphasis on the learners, their prior knowledge, and their motivation to learn.

In order to assess how effective a particular course has been or if real meaningful learning has happened, you need to see how well students perform when they are given challenging tasks for which they have been taught in the course, or, at least they are aware of the importance of the task, and its difficulty and but that is no easy task.

Research studies have confirmed that student evaluations are very helpful in predicting the quality of learning (Scott & West 2008). However, other studies have proved the opposite. The correlation between student evaluations and the quality of learning is negative. The higher the instructor's score on student evaluations, the worse the learning. Conversely, when the assessment score is lower, the better the learning (Keith, 2019).

Instructors may increase grades or reduce instructional clarifications to elevate student evaluations. In this case, it is hard to realize how each of these measures correlates with the desired outcome of actual student learning. Standardized tests are not administered at the postsecondary level, and the grades generally cannot assess student academic achievement due to the heterogeneity of assignments/exams and the mapping of these assessment tools applied by individual professors' final grades (Scott et al., 2010). Research has revealed that student evaluations positively predict student outcomes in contemporary courses but are low indicators of student outcomes in subsequent courses (Keith, 2019).

It could be argued that learning occurs when mistakes get corrected by students, so an especially effective way to teach something in a way that sticks. There is no doubt that students will make mistakes, but when at a later stage, they correct the mistakes on their own while taking into account the instructor's suggestion, means that the learning goal has been achieved. Nevertheless, getting everything right is a counterproductive goal in education. Quality learning occurs when error correction happens after some time the error was made (Bjork, 1994).

The practice of retrieval (Roediger et al., 2011; Smith et al., 2016) by recalling facts, concepts, or events from memory - is a more effective learning strategy than revising through rereading. Retrieval strengthens memory and interrupts forgetfulness. A single simple quiz after reading a text or listening to a lecture produces better learning, and better recall, than rereading the text or reviewing lecture notes. Periodic practice arrests forgetting, reinforces pathways to recovery, and is essential for students to focus on the knowledge they want to acquire.

Agarwal (2019) explains "Rather than asking students to retrieve similar types of information in one continuous session, apply the principle of interleaving, which mixes up content from different areas. Students retain and learn more information when they mix it up" (p.22). When information is processed in different ways, establishing more connections across the neutral branding network and encoding learning more deeply (Kelleher & Whitman, 2016). In addition to doing regular retrieval practice on material that was covered recently, it's also important to revisit old information, asking students to retrieve information a few days, weeks, or even months after they learned it. Because that information is harder to recall, it actually makes the learning that comes from it that much more durable (Agarwal & Bain 2019).

The quality of learning assessment depends on "what you observe" so that you can verify it: it must refer to a meaningful, non-scholastic assimilation of knowledge. When assessing learning assessment tools that can predict the outcomes of the assessment must be in place, in other words what is the student able to do with the knowledge he has acquired real life situation (Sergiovanni & Starratt, 2003). A research warns that when students carry out a task and spend some time between sessions, or the task is more complex and involves two tasks or subjects simultaneously, error correction is more challenging, while learning is sustainable and the implementation of forthcoming tasks is less challenging (Sternberg & Grigorenko, 2002).

According to Nickerson (1979) when the student is able to establish the principles or the "rules" which differentiate the types of problems, he is more successful in choosing the right solutions in unfamiliar situations. This skill is better acquired when interrelated and varied practices are applied than through the application of group practice. Any new learning requires a foundation of prior knowledge (Callender & McDaniel, 2009). In order for the student to learn numerical analysis, one must remember algebra and mathematical analysis. If the student is only engaged in mechanical repetition, the ability to store all the information in his brain is compromised.

However, elaboration of information is the process of giving new meaning to the new material in which the student uses your own words and connects prior knowledge with new knowledge. The more one can explain how new learning relates to previous knowledge, the stronger the understanding of the new learning will be, and the easier it is to remember and use it in the future (Brown et al., 2014; Callender & McDaniel, 2009; McCabe, 2010).

Assessment of learning is often based on the themes or projects and the judgments are made on the basis of student performance on tasks, and evaluation involves various areas (Comoglio, 2005). So, this process involves:

- Information-gathering that occurs at the end of a teaching/learning process for the purpose of verifying whether the learning goals have been met.

METHODOLOGY

In this section, we present the methodology used in designing the activity learning path. The methodology used is based on constructionism that depends on the type of activity given in both the design phase and the management phase. The type of activities in the scheme of the created methodology is based on the given theoretical framework. A descriptive statistic was used to present the data. The transition from one phase of the scheme to another is based on qualitative analysis of data in both phases, management, and design.

The topics that are being tested are part of linear algebra and were taught to students a year ago. The students are sophomores, majoring in engineering in total 30. The students who participated in the study are from the same tutorial group and with the same teacher. 82 % of them are male, 18 % female, and 3 % of participants are of Egyptian ethnicity. The class chosen for the study contained 34 students. The level of the students is average.

Students have been notified about the test and the study a week in advance. As it was a non-examination test the students did well, especially with the application problems and in the use of Matlab.

The activities to be implemented were accomplished according to the following outline presented on Figure 2:

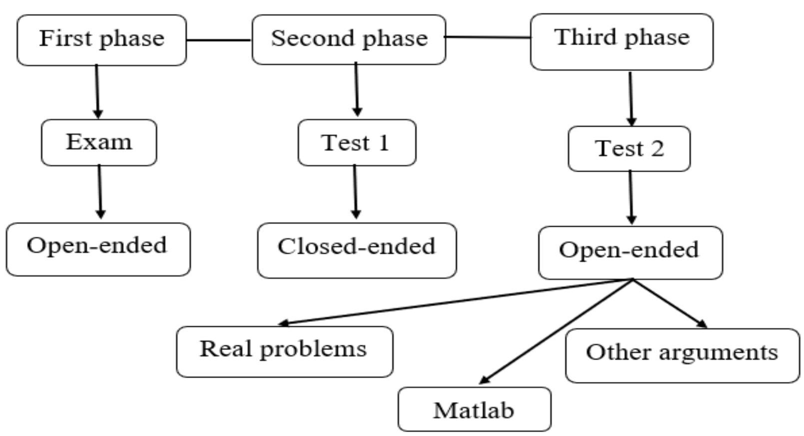

Figure 2: The schematic configuration of the scrutiny approach

In the first [Examples 1 and 2 in appendices] and third phase [Examples 5, 6 and 7 in appendices], the examination and test 2 include open-ended questions. As a result, at this stage, a summative assessment is conducted. In the second phase, test 1 [Examples 3 and 4 in appendices], includes activities in the form of a quiz with feedback for each answer. Here we have a diagnostic assessment. In the first and second phase, these are cognitive activities. In the third phase, test 2, we render activities that are not only cognitive but for the most part metacognitive. They fall into three parts: real problems from other disciplines, mathematical topics which require students to apply previously learned topics, and in the end, topics deciphered in Matlab software.

AN APPLICATION TO A CASE STUDY

The methodology described in the previous section has been applied to the case of the module of Linear Algebra for engineering undergraduates. Question:

What evidence did students learning demonstrate?

Students were asked to produce but not to reproduce knowledge, the case in test 1 and test 2. The tests are connected to the real world and to parts of the mathematics curriculum. Tests take account of student differences; tests that require of the students to apply the analytical skills, and tests in which the mathematical software can be applied. Tests allow for assessment of complex skills and allows students to demonstrate student success in a variety of ways.

What tools can be used to gather information?

Summative assessment information should be sought based on tasks/performances that demonstrate the mastery of objectives, essential skills, and competencies required in teaching. To see how effective the learning is, the first parameter in question should be discerned, that is, the comparison of the summative evaluation, the exam, [Example 1 and 2 in appendices] with topics tested after one year [Example 3 and 4 in appendices]. The tests are in a closed version, i.e. different from the tests in the first phase (examination), they are quizzes.

In the exam that was carried in the first phase, the summative assessment of linear algebra is given by the following figure:

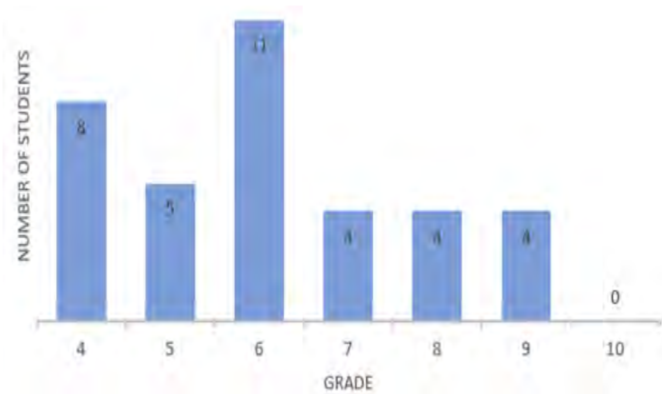

Figure 3: The summative assessment of linear algebra

Student results are for the most part below average 7, 23. In the event of getting grade 4 the student does not pass the exam. While 10 is the highest grade. From the results of the first test, we identify two issues:

- The report between the exam grade versus the grade in test 1, in the second stage.
- The qualitative results of students in the version of the quiz-type questions, test 1.

The comparison of the exam grade with the test grade 1 in the second phase is given in the figure below:

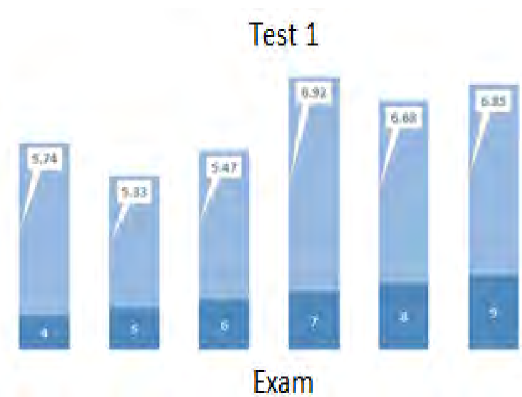

Figure 4: Comparing the exam grade with the test grade 1

Grade 4 in the exam becomes 5.74 in test 1, grade 5 becomes 5.33 continuing in the same way. The most stable were the students with grade 7. Then, as it increases, the grade of test 1 (of later) decreases, while with the decrease of grade 7 the grade of test 1 rises. Finally, after one year the number of students with low grades than with high grades. This is indicative of the quality of learning.

Straighaway, we should analyze the students' results in the second phase (test 1) by comparing them with the results of the first phase, in other words with the test results of the previous year. The most common errors in the second phase, test 1, are:

- Lack of concepts, algorithms, theorems that are increased compared to the exam test.
- Many students with lower grades do not give feedback to the answers; this might indicate that they are not used to questions in which the answers are closed.
- There is no increase in linguistic errors which might be attributed to the types of the question.
- Very often we observe Harlow's error factor (they do well but not what they are asked).
- There are contradictions between the feedback and the answers given.
- Reduced number of errors when reading the questions and answers.
- They demonstrate less than in the exam test.
- There are more mistakes than errors (Pepkolaj, 2015).

In the third phase, test 2, the topics that are addressed are the same with the ones in the second phase (test 1) but require students to apply the knowledge in a variety of situations. The topics address problems that relate to:

- Everyday life.
- Disciplines such as engineering, physics, chemistry.
- Other mathematical topics in which the topics of the second phase serve us.
- Problems that can be solved with the help of the Matlab software.

The table below indicated the average grades of the students grouped into the exam tests, test 1 and test 2:

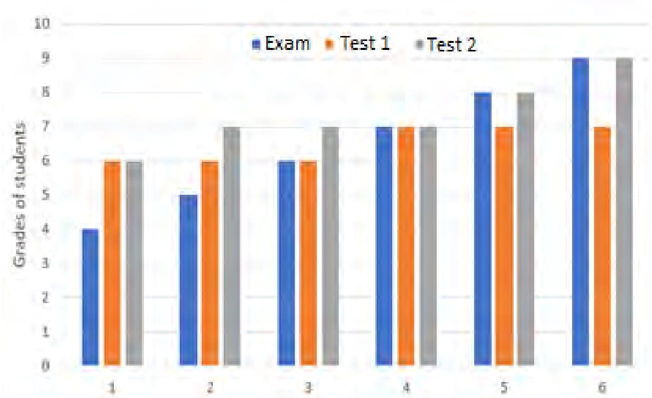

Figure 5: The exam tests, test 1 and test 2

Group 1 includes students who have received grade 4 in the exam test. In test 1 the grade becomes 6 and in test 2 the grade is 6. Group 2 includes students who have received grade 5 in the exam test. In test 1 the grade becomes 6 and in test 2 it becomes 8. If we are analyze the data of the third phase, consequently test 2, in quantitative terms, we observe that students with initial grade 7 (group 4) are more stable. On the other hand, the results of students with grade below 7 in the previous two phases increase but the results of students with grade above 7 do not arrive. It should be noted that the students with the best grade in the exam test, also do better in the third phase test.

However, students with the worst grades in the exam test get better results in test 2 but do not add students with better grades. If we compare test 1 with test 2 the results get better and the results of students with better grades are also maintained. Based on this analysis, we could conclude that more learning occurs in the application stage because previous activities focus more on theory.

We should do a qualitative analysis of the students' results from the third phase. The problems that were given to students to solve focused mainly on the application version. Students were asked to solve them by applying their analytical skills and employing graphical and numerical ways. The aim of this approach was to observe whether the student can easily pass from one solution way to another, from one semiotic representation to another. It ran out in all types of problems the analytical method produces more errors, usually the same as the other phases, but the numerical method and particularly the Matlab software helps students to increase students' performance.

They use Matlab commands and scripts well, but do not write comments. They only solve the problem in the numerical version which was also asked for the analytical one. The most important thing is that the student switches easily from one semiotic representation into another thus making learning sustainable.

One of the crucial objectives of the subject of algebra is the application of this knowledge in solving engineering and practical problems. From the above analysis it was seen that the objectives of the course are partially achieved. Even the program with its objectives needs changes. In the subject of algebra there is no Matlab program but it is done separately and not in the same semester. The connection between assessment and learning is a delicate issue, and when we discuss about effective learning the problem is even more obvious. Generally, we are poor evaluators to when we learn well and when we don't.

CONCLUSIONS AND RECOMMENDATIONS

The parameters brought into play in this article sought to demonstrate when learning linear algebra is stable and meaningful. Comparing the summative assessment by testing topics but in a different way after one year indicates the correlation between them is negative. After one year, more learning happens, but with the lower grades when contrasted to the higher grades in the previous test. The quality of learning decreases when previous grades measure a momentary learning. However, the approaches/ lessons, which they have learned, should remain and be implemented throughout life.

Furthermore, the topics that have been tested in the application version i.e. in the other mathematical topics and real life problems, show that students do better, and that there is no negative correlation between the earlier grades with the later grades. There is more learning in the application version and the best results are achieved when using the mathematical software, which allows the student to move easily from one semiotic representation to another.

However, the transition from the open-ended test of the exam to the close-ended test in the second phase enabled us to see the mnemonic study applied only in exam situations (for exam purposes). Therefore, there is a need to create balanced, well-designed assessment items that accurately measure what students know and are available to do. In conclusion, the learning of linear algebra was not sustainable and was not carried out properly and we suggest a review of the program and objectives of the algebra course. The following suggestions can be given:

- A text that includes theories, exercises, engineering problems, Matlab codes of their solution because it is very important to be all together. The student must move freely from one presentation to another through which guarantees us a stable education.
- This study suggests that course programs or amelioration should be done based on the difficulties of students (recovery of previous knowledge) and not just based on the labor market. We see this point as essential.

APPENDICES

In this addenda, we encompassed a series of sample questions, illustrating the three phases of testing. Some questions from the first phase (exam) are:

Example 1 Consider the following linear system

$$\begin{cases} x + y + hz = h + 1 \\ x + hy + z = 2 \\ tx + y + z = 2h \end{cases}$$

Discuss compatibility and calculate possible solutions for $\forall h \in R$

Example 2 Let $f : R^3 \rightarrow R^2$ be the endomorphism such that:

f(x, y, z) = (x + hz, x + hy +2z, x + y +hz)

a) Say for which values of h, f is not injective and calculate the dimension and a base B of ker f

b) Say for which values of h, f is not surjective and calculate the dimension and a base B' of Imf

c) Calculate $f^{-1}(3, -1, 1)$, varying $h \in R$

The examination track in the first phase contained six questions in the open version like the examples above, for a total of maximum 100 marks, and there were four tracks. The grade were according to the evaluation of the Bologna system.

Some sample questions from the second phase (test 1) are:

Example 3 In trying to find out if the lines in 3D space

$x = \begin{pmatrix}1\\3\\4\end{pmatrix} + l\begin{pmatrix}2\\1\\2\end{pmatrix}$ and $x = \begin{pmatrix}2\\5\\1\end{pmatrix} + m\begin{pmatrix}1\\1\\3\end{pmatrix}$ have a common point of intersection, we solve a system of 3 equations in the two unknowns l and m. After applying a number of elementary row operation there is yielded $\left(\begin{array}{cc|c}1 & 0 & -1\\0 & 1 & -3\\0 & 0 & 1\end{array}\right)$. What conclusion can be drawn about the point of intersection?

- The point of intersection is (-1, -3, 0)
- The point of intersection is (1, 3, -1)
- The point of intersection is (-1, -3, 1)
- The two lines do not intersect
- There is not enough information given to answer the question

Example 4 Let $A \in M_{88}(R)$ be the matrix in which the elements a_{ij} with $i \geq j$ are equal to 88 and the others are null, then A:

- has exactly 88 eigenvalues where 44 are real numbers and 44 are complex numbers with non-zero imaginary part
- μ=88 is an eigenvalue of A with multiplicity 88
- has exactly one complex eigenvalue with zero imaginary part
- it has no real eigenvalues
- it has exactly two distinct real eigenvalues

Test track 1 contained 18 questions in the closed version quizzes like the examples above for a total of 100 points and there were three tracks.

Some sample questions from the third phase (test 2) are:

Example 5 Let us find coefficients of a cubic polynomial $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ that satisfies $p(1.1) = 2.3, p(-0.3) = 3.3, p(0.4) = -4.2, p(0.7) = 6.1$

Example 6 For example Alka Seltzer makes fizzy soothing bubbles through a chemical reaction of the following type: $NaHCO_3 + H_3C_6H_5O_7 \rightarrow Na_3C_6H_5O_7 + H_2O + CO_2$

The reaction above is unbalanced because it lacks weight to describe the relative numbers of the various molecules involved in a particular reaction. In a chemical reaction the atoms that enter the reactions must also yield reactions. How do you write and balance chemical equations?

Example 7 Consider the electrical network shown in figure 6

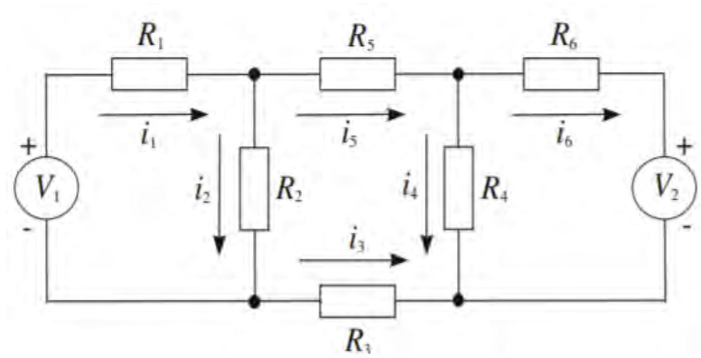

Figure 6: The electrical network

Consequently, the R_ks are positive numbers denoting resistances (unit: ohm (Ω)), the i_k are currents (unit: ampere (A)), and V_1 and V_2 are the voltages (unit: volt (V)) of two batteries represented by the circles. Using Kirchhof's law, assume that $R_1 = 6\,\Omega$, $R_2 = 16\,\Omega$, $R_3 = 2\,\Omega$, $R_4 = 5\,\Omega$, $R_5 = 3\,\Omega$, $R_6 = 6\,\Omega$, $V_1 = 70\,V$ and $V_2 = 61\,V$. Find the six unknown currents. Test track 2 containing 4 questions in the open version like the examples above for a total of 100 marks, and there were eight tracks.

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