Practicing Teachers' Perspectives on the Purposes of Mathematics Lesson Study

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Although lesson study has been shown to improve the teaching and learning of mathematics, little research has focused on reasons for engaging in it from a teacher's perspective, particularly after sustained engagement with lesson study. This qualitative study reports on thirty-three teachers from the United States who were involved in twelve lesson study cycles during a three-year partnership aimed at improving algebraic instruction for students aged 8–14 years during a period of educational reform. It used grounded theory techniques to analyse interview data to determine the purposes of lesson study from teachers' perspectives. Results highlight six purposes for engaging in a mathematics lesson study: understand reform standards, focus on student thinking, develop pedagogy, support collaboration, develop curricular materials, and learn mathematics. Particularly noticeable was its use for focusing on student mathematical thinking and understanding of reform initiatives and requirements. Research implications suggest positioning lesson study as a useful, teacher-vetted tool for unpacking educational reforms.

Keywords • mathematics teacher education research • lesson study • professional growth • professional development

Introduction

The collaborative form of teacher professional development, lesson study, has been used to improve mathematics instruction in the United States since the 1990s. Its implementation was instigated by the *Third International Mathematics and Science Study* (TIMMS) video study (Stigler & Hiebert 1999). Although much research has suggested its potential for improving instruction (e.g., Huang & Shimizu, 2016), it remains under-theorised, which can prevent its selection as a vehicle for teacher growth (Hart & Carriere, 2011). In their call for how lesson study research should contribute to instructional improvement, Lewis et al. (2006) described the challenge of "inadequately formulated or shared knowledge about what actually constitutes the innovation" (p. 5). Similarly, Lewis et al. (2009) noted foundational questions about key features of lesson study that still need answering.

Yoshida's (2012) found that an obstacle to engaging in high-quality and effective lesson study was a lack of understanding of the professional development strategy. A better shared understanding of the potential lesson study may support the mathematics education community to improve mathematics learning for students. It would also allow for theoretical research on lesson study to be better connected to mathematics teacher practice, a feat that would support addressing critical issues of mathematics teaching and learning identified by the National Council of Teachers of Mathematics (2012).

Recent research has focused on describing conceptualisations and theoretical perspectives of lesson study (e.g., Huang & Shimizu, 2016), and the mechanisms through which teachers' professional growth developed (Widjaja et al., 2017). Yet little research currently exists on what teachers believe to be the purposes of lesson study (e.g., Cajkler et al., 2015; Gero, 2015; Richit & Ponte, 2017), particularly those with sustained lesson study experience during a time of educational reform. The challenges from understanding and adapting lesson study to United States educational contexts, along with institutional structures that play a large role in determining time teachers have for growth (e.g., Takahashi &

McDougal, 2016; Yoshida, 2012), often prevent teachers, districts, and students from reaping full benefits of lesson study.

This study was part of a grant funded through the National Science Foundation's Math and Science Partnership Program, referred to hereon as ProjectX, which engaged over 80 teachers, principals, and district administrators in a university partnership to provide professional development (PD) and cycles of lesson studies during the school years on algebraic thinking for Grades 3 through to high school algebra (Grade 10). This article explores the purposes of mathematics lesson study as told by the participating teachers after the three-year PD, which occurred during a unique period of educational reform. Understanding the purposes of lesson study from the teacher's point of view can motivate new teachers to begin lesson study and improve how districts invest resources and support educators in participating in collaborative, teacher-led PD experiences (Fernandez, 2005).

Lesson Study

Lesson study is a practice-based, research-focused form of PD that has roots in the Asian educational practice of Japanese Lesson Study (Lewis & Tsuchida, 1999; Lewis et al., 2009; Stigler & Hiebert, 1999; Yoshida, 1999). The study reported in this paper was conceptualised using Lewis et al.'s (2006) conceptualisation of lesson study: (a) studying curriculum, including standards, curricula, and other resources to plan activities, teacher questioning, and anticipated student responses, and formulating goals about student learning and broader, school-based goals; (b) planning the research lesson by designing or modifying a lesson to answer a goal; (c) teaching, observing, and collecting student data; and (d) reflecting on the research lesson through sharing student data, analysing what went well, considering whether goals were met, discussing any changes they might make based on the collected data, and reflecting on the broad lessons learned from the cycle of lesson study that could extend to other areas of the teachers' professional work (see Figure 1). Participants may optionally reteach, observe, and debrief for a second time. In this project, each lesson study was planned in approximately six hours, where each teacher had a substitute take over their teaching responsibilities for the day while planning. Lessons were then taught two times in total, by two different teachers on the same team. Additionally, each lesson was observed by an additional team of teachers, along with at least one researcher, administrator, and/or graduate student involved in ProjectX.

Typically, a knowledgable other or commentator is invited to watch and debrief the lesson with LS participants (Takahashi, 2014). During the studying curriculum phase, this knowledgeable other supports the lesson study by suggesting relevant articles, curriculum, and standards and helps teachers incorporate their goal and inquiry into their students' mathematical learning. In the planning research lesson phase, the knowledgable other asks questions on how the selected and sequenced activities support teachers in achieving their learning goals and provide guidance on mathematical content questions that may arise. In the teaching, observing, and collecting student data phase, the knowledgeable other observes the research lesson. Lastly, during the reflecting phase, the knowledgeable other provides a summary after all the teachers debrief on how the lesson supported teachers in reaching their mathematical learning goals for their students given the enactment of the lesson and data collected. Other members of the lesson study team are next invited to comment, as well as those from the observing team. Finally, the knowledgeable other would comment, offering overall insights, summaries, and responses about the lesson goal and any broad educational goals (e.g., school-based goals).

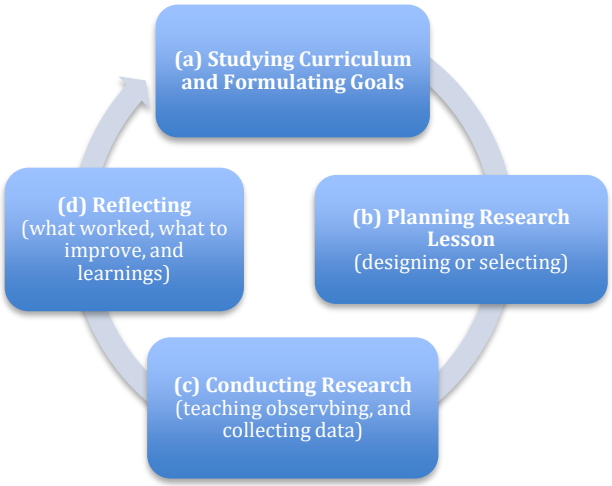

Figure 1. Components to one lesson study cycle, including studying, planning, conducting, and debriefing the research lesson. Adapted from Lewis et al. (2006).

Artefacts such as, research articles, subject concepts, curriculum documents, and learning frameworks, are often used during the initial planning stages to determine learning goals and guide lesson planning. Participants return to their research questions and goals during the final reflection stage to understand whether these were answered through their lesson study. Collaborative reflection occurs during planning and reflection conversations.

Recent research suggested that lesson study can be used to improve the teaching and learning of mathematics (e.g., Huang & Shimizu, 2016). It allows participants to deepen mathematics content knowledge (Alston et al., 2011; Fernandez, 2005; Lewis et al., 2009; Meyer & Wilkerson, 2011; Robinson & Leikin, 2012; Yoshida, 2012); improve overall teaching practice (Hart & Carriere, 2011; Murata et al., 2012; Olson et al., 2011; Widjaja et al., 2017); and develop communities of practice (Lieberman, 2009; Lewis et al., 2009; Saito et al., 2012). More recently, there has been work suggesting lesson study as a tool for understanding educational reforms (Lee & Lo, 2013; Lewis & Takahashi, 2013; Ni Shuilleabhain & Seery 2018; Takahashi et al., 2013). Yet, obstacles exist to facilitating lesson study in the United States, including awareness of the nature of lesson study, access to material resources, support from administrators, and structural opportunities to engage in meaningful PD (e.g., Yoshida, 2012). The study reported here, sought to contribute understanding of how teachers interpret the activity of lesson study so that the mathematics education community can better understand how to support teachers engaging in lesson study.

Purposes for Engaging in a Lesson Study

Engaging in lesson study requires resources and local support to coordinate the study, plan, teach, and debrief components of a research lesson (Druken, 2015). Teachers and districts may be more likely to support lesson study if they understand the affordances that can come from investing time and money into it. Thus, understanding through the eyes of teachers provides practice-based reasons to try a lesson study and can support the mathematics education community in using lesson study as a tool for improving students' mathematical thinking.

While limited, research investigating the purposes of lesson study from a teacher's perspective—including in-service, pre-service, and university faculty—has led to several findings. Fernández and Robinson (2006) described lesson study perspectives from pre-service secondary teachers who engaged in micro-lesson study. They found that participants viewed lesson study as particularly useful for connecting theory to practice, engaging in collaboration, and reflecting on practice. Similarly, Richit and Ponte (2017) studied elementary and middle school teachers three years after they participated in three lesson study cycles. They found that teachers reported developing their mathematical knowledge, increasing their knowledge of teaching strategies centred on student thinking, and changing

professional and personal relationships at school. In a study with university faculty teaching in an undergraduate mathematics content course and credential math methods course, lesson study was found to be a vehicle for creating community, enabling attention to detail, improving knowledge of issues related to teaching mathematics, improving curricular materials, and extending its effect beyond more than the research lesson (Druken et al., 2020).

Studies report changes in teachers' beliefs after engaging in lesson study (e.g., Gero, 2015; Samaranayake et al., 2018; Tachie, 2020). Gero (2015) explored teacher attitudes about lesson study, including teacher attitudes towards collaboration, reflective practice, continuous improvement, lesson observations, and levels of comfort with critiques. The study involved fifty-five teachers from two schools who participated in a two-day lesson study experience. Gero found that those comfortable with being observed, reflecting on teaching practice, and planning collaborative lessons viewed lesson study favourably, suggesting that a teacher's prior beliefs and comfort affect engagement with lesson study. Importantly, Gero (2015) highlighted the importance of maintaining lesson study as a teacher-driven reform, rather than reform within a "traditional hierarchical, top-down culture" (p. 24). Gero argued models that fail to give voice to teachers threaten to "alienate teachers from the process" and run the risk of being "swallowed up by the existing system, preventing any real changes to the status quo" (p. 24). Engaging in lesson study, consequently, involves redistributing authority in a way that supports teachers in engaging in teacher-driven professional growth experiences rather than mandated experiences. Tachie (2020) studied teachers' attitudes towards lesson study in South Africa and found positive teacher attitudes towards lesson study despite limited time to participate.

To better understand the purposes of lesson study as told by teachers in the United States, the investigative research reported in this paper was conducted after the ProjectX grant officially ended. The research question motivating this study is,

> *What do teachers who have engaged in lesson studies for three years describe as the purposes of lesson study?*

This study contributes to mathematics lesson study research by providing motivation and reasons for teachers new to lesson study to try it. The word "purposes" is used to capture broadly teachers' reports on uses for, perspectives on, and reasons to engage in a lesson study. Reports from teachers' interviews that reflect on three years of lesson study experiences are shared, along with implications for future lesson study experiences.

Methods

Out of five qualitative inquiry approaches to research—narrative research, phenomenology, grounded theory, ethnography, and case study (Creswell, 2007)—this study used grounded theory (Corbin & Strauss, 1990) to describe and work towards generating a theory on purposes of lesson study from the perspective of over thirty practicing teachers. Grounded theory research aims to make sense of and find meaning in data, especially over time or periods of change, through using methodological guidelines (Creswell et. al, 2007; Merriam, 2009). Doing so allows theory to be built from the data and methods used to analyse the data, a differentiating characteristic of grounded theory compared to other qualitative approaches (Merriam, 2009). Grounded theory processes include theoretical sampling, coding, constant comparison of data, development of concepts and categories, theoretical saturation, and theoretical integration (Corbin & Strauss, 1990). Since no frameworks of purposes of lesson study existed at the time of this study, grounded theory was deemed appropriate due to its focus on generating meaning from data.

Research Setting and Participants

This study was part of a larger study, referred to hereon as ProjectX[1] that sought to engage teachers, principals, and district administrators in a three-year university partnership to provide professional development and school-year cycles of lesson study on algebraic thinking for Grade 3 (8 years of age) through to Grade 10 algebra (16 years of age). ProjectX occurred during a unique educational time, as the state in which this study took place began implementing new Common Core State Standards of Mathematics (CCSSM) around 2010, which coincided with the start of ProjectX. The general goal of the standards was to teach mathematical concepts more deeply for understanding and reduce strategies that required memorisation. The standards also marked a shift towards making mathematical practices prominent in students' learning processes. Consequently, the teaching and learning implications related to the standards involved a significant shift in many teachers' practices. After ProjectX ended in May 2013, the 2013–2014 school year served as the first year of implementing standards without assessing students, which was the period of data collection for the study reported in this paper.

The ProjectX grant involved three yearly week-long institutes facilitated by district mathematics specialists and university faculty, and twice a year lesson studies focused on improving algebraic instruction in grades three through high school algebra. The focus of the institutes centred on problem solving tasks that elicited algebraic reasoning, understanding student thinking, and connecting multiple representations of mathematics. All lesson study teams involved 3–6 teachers in grade-level bands from multiple sites. All lesson study cycles included approximately six hours of planning, where teacher participants were supported by the provision of a substitute teacher who covered their class so participants could all meet in person in the same location. Also, four hours of teaching and debriefing time were allocated. The lesson studies were typically taught within a week or two of the planning meeting and the project manager along with one or more university researchers attended. Lesson studies were recorded and housed online where participants could review the video lesson, lesson plan, and any debriefing documents that were submitted to the research assistants. In addition to two annual lesson study cycles, each teacher participant also observed a live research lesson and subsequent debriefing of other lesson study groups twice per year. This totals to each participant experiencing approximately four lesson studies per year for three years.

All ProjectX participants were invited to participate in the study reported in this paper on the last day of the three-year project. ProjectX survey data collected showed 75% of the 80 teachers expressing interest in continuing lesson study after the project ended. This high response served as a rationale for following up about perspectives of continuing and sustaining lesson study. In total, 33 teacher participants of students aged 8–14 years were recruited for this study, and represented two school districts and 18 elementary, middle, and high school sites in southwestern United States.

Informed consent was collected, and all participants participated voluntarily. The participants included 19 elementary teachers, 10 middle school teachers, three resource specialists who focused on special education, and one high school teacher. Four of these participants also served as a mathematics coach during ProjectX in addition to their typical teaching duties—Ben, Carmen, Gem, and Hope. These participants engaged in leadership training, met regularly with the grant's leadership team, and led one of the 18 ProjectX lesson study teams. Although five principals involved in ProjectX were also interviewed, their data were not analysed for this paper.

Data Collection

Data collection began mid-2013, shortly after the conclusion of ProjectX. Three data collection instruments were used. They included an online survey, semi-structured interviews, and theoretical memos. In grounded theory, the investigator serves as the main instrument of data collection, and "analysis strives to derive meaning from the data" (Merriam, 2009, p. 29). These sources aligned with

[1] Pseudonyms were used for teacher names, districts, and the joint partnership grant between two districts and a university.

grounded theory methods due to the ability to capture change over time through coded survey responses, transcripts, and memos that captured teachers' perspectives on lesson study purposes.

Online survey

An online survey was administered to all ProjectX participants to collect information about whether teachers continued lesson study practices at their site, who they collaborated with, what supported or limited their ability to participate in a lesson study, whether they believed teachers in their school exhibited a commitment to student learning in mathematics and science, whether they felt encouraged to experiment with their teaching and continue to learn new ideas, levels of perceived administrative support, and information on their professional development climate. These data were used to contact participants for a follow-up interview.

Semi-structured interviews

Semi-structured interviews are an effective method of collecting in-depth data about participants' perspectives and ideas about a topic (Fontana & Frey, 2000). Semi-structured interview data, ranging from 30–60-minutes, were collected over a period of 6–12 months starting mid-2013. Interviews were video-recorded, except in two instances where the interviews were audiotaped, and were conducted by one researcher with the exception of two interviews where a mentoring researcher involved in ProjectX supported the interview process. Interview questions asked participants to describe: colleagues with whom they collaborate, the nature of teachers' collaborative activities, resources that supported their work, resources that might support lesson study, the structure and purposes or nature of lesson study, what they thought lesson study was useful for, and changes they would make if they were to engage in lesson study again (see Appendix A). For this paper, primary data sources included responses to a subset of questions focused on the purposes of lesson study. The data were taken from transcripts of the audio data from the video and audiotaped interviews.

Theoretical memos

Theoretical memos, or documentation of the researcher's ideas on evolving codes and relationships among them, were recorded in audio form by the author after each interview was collected. The purpose of this was to continually formulate and revise theory during the research process (Corbin & Strauss, 1990). These memos were used to locate potential themes and triangulate claims made by the researcher about emerging relationships gleaned from the interview data.

Data Analysis

Transcription using InqScribe[2] software and analysis using MaxQDA[3] of data began shortly after the first interviews were collected and continued until the last interview. Participant responses related to the definition of, purpose of, and examples of lesson study were coded by the author using open, axial, and selective coding (Corbin & Strauss, 1990). The remainder of interview data was then analysed to identify additional information related to purposes of lesson study.

An initial set of twelve interviews were open coded, or broken into discrete parts and labelled, to search for major themes. These initial themes were compared across the twelve interviews for similarities and differences and refined to identify themes related to the research question. Several documents containing codes, their definitions, and examples of each code, were created by the researcher over the span of analysis to track their development. Axial coding, which involves organising and drawing connections among codes created during open coding (Simmons, 2017) was used to refine themes that were then applied to the 21 additional interview transcripts. Table 1 illustrates some initial codes that were later collapsed into broader categories through this process. Selective coding was used to connect all relevant codes into a single theme related to teachers' perspectives on purposes of lesson study.

[2] https://www.inqscribe.com/
[3] https://www.maxqda.com/

Table 1.
Examples of Initial and Collapsed Codes

	Initial Codes	Collapsed Code
Nature of lesson study	Student learning Listen to students Anticipate student performance, Anticipate student prior knowledge	Focusing on student thinking
Nature of lesson study	Framework Activities Questioning Teaching/observing	Developing pedagogy
Lesson study goal	Lesson plan Process	
Nature of lesson study	Assessments Developing lesson	Developing curricular materials
Nature of lesson study	Mathematics content Real world context/application	Learning mathematics

Oftentimes, data had more than one code associated with it. For example, when a participant described rehearsing questions for students while planning a lesson study (e.g., "We would develop questions before we delved in … oh! Remember to ask them this! … If they're not understanding that it is ¾ of one half, why? They are not understanding that it's groups of."), the data were coded for both *focusing on student thinking* and *developing pedagogy*. A constant comparative method of data analysis between data and emerging categories promoted greater precision and consistency of coding and helped to guard against bias (Corbin & Strauss, 1990).

Once a robust coding scheme emerged from reviewing 12 interview datasets and collapsing and refining codes, the remaining 21 interviews were coded. Comparisons between new codes and initial codes were made to determine whether a new code was warranted given the new data. Once all participant interviews were coded, supporting and disconfirming data, such as interviews with grant leadership and research memos, were sought to triangulate claims made about purpose codes.

Six themes emerged to answer the research question around purposes of lesson study reported by participants and include: understanding reform standards, focusing on student thinking, developing pedagogy, supporting collaboration, developing curricular materials, and learning mathematics (see Table 2). Each perspective emerged "by repeatedly being present in interviews, documents, and observations in one form or another—or by being significantly absent" (Corbin & Strauss, 1990, p. 7). These perspectives were built from events and happenings described during the survey, interview, and analytical memo datasets.

Table 2.
Definitions of Purposes of Lesson Study, and Number of Participants

Emergent Themes	Definition	% Participants (*N* = 33)
Understanding reform	Unpacking and reflecting on reform standards and its implications for teaching through conversing with peers. focusing on student thinking, and teaching the lesson.	100% (*n* = 33)
Focusing on student thinking	Anticipating, observing, analysing, or improving student mathematical understanding.	84% (*n* = 27)
Developing pedagogy	Developing instructional strategies.	70% (*n* = 23)
Supporting collaboration	Working with colleagues to discuss, share, network, and improve teaching practices.	48% (*n* = 16)
Developing curricular materials	Refining lessons, activities, and assessments used to promote student learning.	30% (*n* = 9)
Learning mathematics	Better understanding mathematics content and ways it connected to other mathematical ideas	15% (*n* = 5)

Results

Findings are organised by six themes about teachers' reported usefulness of lesson study: understanding reform, focusing on student thinking, developing pedagogy, supporting collaboration, developing curricular material, and learning mathematics (see Table 2). These categories describe the purposes of lesson study from the teacher participants' point of view after sustained and repeated engagement in lesson study as a form of professional development. Table 3 provides the range of perspectives reported across grade level taught. Pseudonyms are used to maintain participant anonymity and to protect their privacy.

Table 3.
Count of Participants Reporting Each Perspective by Grade Level

	Elementary (*n* =19)	Middle (*n* = 10)	Resource (*n* = 3)	High (*n* = 1)	Total (*N* = 33)
Understanding reform	19	10	3	1	33
Focusing on student thinking	17	7	1	1	27
Developing pedagogy	13	7	3	0	23
Supporting collaboration	11	4	0	1	16
Developing curricular materials	6	3	1	0	10
Learning mathematics	3	2	0	0	5

Understanding Reform

All thirty-three participants (100%) reported lesson study as useful for unpacking and reflecting on the reform standard and its implications for teaching practice. Reasons centred on support for (a) conversing about reform standards and (b) teaching reform-aligned lessons. While all questions posed during the interview were analysed, one question in particular was analysed closely for this finding,

which was "Do you think lesson study would be useful for figuring out Common Core?" Over two-thirds of teachers' initial response to this question gave an emphatic positive reply (e.g., "definitely"; "obviously"; "invaluable"; "yes, yes, yes, and yes"; "oh, absolutely"). Elementary ProjectX coach Gem referred to lesson study as a "steppingstone to understanding Common Core" because lesson study bridged the work completed in ProjectX to understanding the new standards.

> A lot of the work that we've already done for lesson study, those of us that have been through the program, it's like we got a head start on Common Core. And it's not as scary an idea to think about jumping into because we've already been looking at student thinking.

Lesson study went "hand-in-hand" with getting to know the new standards and allowed teachers to develop an understanding of reform standards through their students' eyes. The following sub-themes help to further characterise the broad theme on how lesson study supports teachers in understanding reform implications.

Conversations about new standards

Teachers stated repeatedly that conversations during planning and debriefing sessions with colleagues were useful for unpacking the new standards' meaning.

> It's been really helpful to sit down and talk about the standards with other teachers, and say, "What does this standard mean to you? What does that mean that we're supposed to teach?" That's almost like the beginning of lesson study. That's really helpful. That's extremely helpful.

Being able to discuss instructional implications of new standards was important to many teachers and lesson study provided a space to do so. Similarly, middle school resource teacher Anita described: "You're talking about, Okay, we got to teach this standard. And you're discussing, 'What does this mean? What does it look like? What do the kids have to do?'" Lesson study requires participants to be specific about their teaching goals, which often were connected to learning standards. Elementary teacher Mia noted how lesson study provided an opportunity to actively discover pedagogical strategies that support teaching new standards, rather than passively learn about new standard implications.

> We're teachers. We either research what we don't know, or we want to put a solution idea out there right away and not admit that we don't know. And so, as teachers, we need to have programs like this that kind of inadvertently tell us what we don't know, and [allow us to] discover on our own, rather than someone say, 'Hey brush up on that a little bit.' … So just delve in. And to have other minds talk about it. To devote time to talk about it.

Mia's response suggests that lesson study allows teachers to feel ownership over directing their own learning of new pedagogies, rather than engage passively with reform implications through reading or watching a video. Some appreciated the focus of the lesson study being on understanding the reforms through lesson debrief conversations. Elementary teacher Jackie described discussing the "breaking point", which was where she identified during critical debriefs that the lesson failed to meet the intended goals. She said,

> And lesson study really lets you get in and say, 'Is that the section? Is that where [students] do it?' By watching [students] develop their answers in this real deep questions, which you can't do either, on a day-to-day basis.

Teaching reform-aligned lessons

Participants reported opportunities to rethink ways to represent mathematics and the types of problem situations posed to students by collaborating with teachers in a lesson study. Teachers learned new pedagogies by working with other teachers, coaches, and partnering university researchers. The following excerpt highlights the teacher putting students at the centre of activities to observe and explore student thinking: "By watching [students] develop their answers to these real deep questions, which you can't do either on a day-to-day basis. … And lesson study lets you have the chance to do a deeper lesson that lets [students] really explore." A deep focus on student thinking was typically not available to teachers during a non-lesson-study lesson.

Participants noted connections between reform standards and a focus on problem solving through participation rather than direct instruction after participating in the grant. Elementary ProjectX coach Ben explained,

> Whether it's Common Core or lesson study, my focus is on what are the students doing. That's the focus. It's not on my lecture. It's not on the kid sitting there listening to me speak. It's on them actually doing the work.

Other connections between teaching reform-aligned lessons and lesson study centred on investigating mathematics through using many representations of math and questioning students and revisiting mathematical topics that they had not previously or recently taught. Elementary teacher Layla said,

> The whole Common Core math is what we did with the lesson study. Questioning and investigation and multiple representations, and explaining your thinking, and showing all your work. And that's basically what we had [students] doing in all of the lessons we did with lesson study."

Providing students with multiple representations of mathematics allowed teachers to give students another tool for thinking mathematically.

One middle school teacher provided evidence that lesson study was most useful only after understanding the meaning of the standards. This perspective contrasted with the other 32 teachers' responses. Tonya reported that lesson study could provide the most help "once we know it [CCSS], then we could do lessons that are shaped about it." Tonya described an unfamiliarity with the meaning and implication of the new standards, which would be needed prior to engaging in a lesson study about those standards:

> When you read the standards, and it's sort of hard to decide what they're actually want us to teach them. "How do you show the number and its opposite have a sum of zero? Okay, what are you looking for on that? Are you looking for a model? Are you looking for a number line? Are you looking for all of them? How do you develop understanding?"

She continued on to say, "Once we know what's going on, lesson study might be helpful for meeting the requirements." Tonya's desire to understand standards prior to designing lessons contrasted using lesson study as a tool for making sense of new standards[4].

This result highlights how conversing about and teaching in a way that aligned with reform were supported through lesson study experiences.

Focusing on Student Thinking

Twenty-seven teachers (84%) reported lesson study useful for focusing on student thinking. Four subthemes related to focusing on student thinking included: (a) anticipating, (b) observing, (c) analysing, and (d) improving.

Anticipating student thinking

Some teachers described lesson study as useful for anticipating student thinking. This theme is not surprising, as one column of the lesson plan template used during the lesson studies was labelled, *Anticipate Student Misconceptions.* This structural design supported teachers in focusing on predicting student (mis)conceptions. Teachers described this process as "pre-thinking and pre-planning about student misconceptions for the first time you do the lesson." Anticipating strengths and weak areas during planning allowed teachers to reflect during the debrief to check in to see whether the misconceptions were present.

> What are the students going to get, and what are they not going to get? So, you plan that. And then it's most useful then after you do the lesson, and you go back and do the debriefing to really look at what were their misconceptions.

[4] Notably, Tonya reported only one purpose of lesson study, developing curricular materials, while all other participants noted more than one purpose of lesson study.

Elementary teacher Jordan contrasts her ability to anticipate student thinking before and after the grant changing to include an intentional focus about their prior knowledge.

> I think they called it misconceptions in ProjectX. Trying to figure out what those might be, trying to anticipate them. That was really good. Like, we'd just teach our lesson and we'd just assume that kids know, whatever-whatever. [For example,] "Oh, all the kids know numerators and denominators." And then there's always a few [students] that are like, "What's the numerator?" And you're like, "Why don't you know this?" So, to anticipate that, kind of attack it first, I really like that about lesson study.

Deciding to systematically address students' prior knowledge prior to teaching a topic helped teachers tailor their lessons to students. Prior to lesson study, Elementary teacher Jimmy shared, "You don't think about the pitfalls of where [students] are going to go wrong with it, you're just thinking about how you're going to teach it right." Anticipating student thinking helped a teacher focus on where students might struggle so that teachers can provide appropriate support. Some teachers even described planning a lesson around the (mis)conception. Overall, lesson study afforded teachers the chance to anticipate student thinking ahead of teaching so that they can prepare for (mis)conceptions arising and help steer students in a more productive direction.

Observing student thinking

Some teachers described being able to observe student thinking. Teachers focused on listening to student contributions and "getting to see the way students think." Elementary teacher Jackie stated,

> [lesson study] gives you the chance to stop and watch what kids are doing, instead of spending all your time preparing how to teach …. And it lets you get around to the kids and hear what they say.

When teachers stopped to notice students' mathematical thinking, it allowed them to appreciate multiple ways of solving a problem. Elementary teacher Jane stated:

> It was very invaluable. Because just really sitting back and watching and observing what students do naturally when posed with a different question. Rather than me just going around saying, "This is the formula, this is how we're going to apply it. Let's see how you all do." It's now, I might pose an open-ended question, give them a few manipulatives. And it's interesting to see how many people could come up with the different, you know, with the same answer but in a different way. And I thought that was a really neat thing to do because it just shows that there's more than one way to really approach a problem and answer it.

In this excerpt, Jane highlighted how asking an open-ended question and listening to multiple approaches to solving a problem provided access into student thinking and supported her in valuing multiple solutions. Similarly, elementary teacher Jimmy discussed the ability to listen actively to student explanations during lesson study:

> To me, the most important part [of lesson study] is to, is to be an observer and watch the way kids learn, from, you know, a role that's outside of the typical teacher role. To be able to, you know, even if a teacher's giving some sort of instruction, direct instruction or whatever, to be the one standing back and listening and, you know, asking the kids to explain themselves.

Jimmy also highlighted how engaging in lesson study allowed him to ask student to explain their thinking, which was not typically available to him during non-lesson study moments. Designing lessons that made student thinking visible was new for many participants. Middle school teacher Scott exemplifies this when he said,

> It's really useful for just learning how kids think about things. I mean, it's kinda cool just to see how they react. If you purposely try to set up the situation for them and just kind of see how they take it, I think it's cool just to see how they think about things.

Perhaps the most powerful result from observing student thinking is the ability for the teacher to move away from doing most of the talking and mathematical thinking towards allowing students to have more ownership over their learning. Middle school teacher and ProjectX coach Kai described being able to shift attention from the teacher towards students during lesson studies.

> I got to be quiet. And I got to listen to [the student]. And I got to show your work. I got to walk around. I got to look. I got to think. And it's hard, because teachers think, I'm talking and doing, not that, I'm watching, I'm listening, I'm thinking, I'm doing.

Analysing student thinking

Some teachers highlighted lesson study's usefulness for analysing student thinking, particularly during the debrief component of lesson study. By examining student data after the research lesson, teachers connected the impact of a part of the lesson to evidence of student thinking. The teachers reported that looking at the student work after the lesson put them in a different state of mind where they could identify parts of the lesson the students understood and the parts that needed further development. Lesson study served as a vehicle for systematically investigating students' thinking and for the teachers to establish a process for analysing student thinking. Doing so helped the teachers identify what students understood. Middle school teacher, Scott, noted that lesson study was useful for "really analysing how they [students] think," which in turn, "helps modify our teaching." High school teacher Markus described that lesson study helped teachers share an understanding of common errors by "seeing what kind of mistakes the kids make." He continued to say that his lesson study group would analyse common mistakes and address them collectively:

> All four of us would see, "Okay, this kid is having the same mistake." And then being able to address it later on. So doing something like the lesson study where everybody sees the kids doing the same thing is a valuable tool because it makes it a little bit more clear what they're struggling with and what not. And where some of the adjustments might come in.

Improving student learning

The last way teachers focused on student thinking was for improving student learning. Elementary teacher Jaime noted broadly that lesson study helped teachers focus on improving student learning.

> It helps you get back to what teaching is about. You just can kind of talk with people and realise that there's like-minded people, and that you are all here for the kids. That's what it's all about. You're here for the kids, you're here to help them learn.

Elementary teacher, Kerry, noted the usefulness in improving student learning due to tools, such as manipulatives for understanding volume of rectangular prisms, that the research lesson provided students. Kerry said she now gives students snap cubes along with various rectangular boxes, and asks students to predict the number of snap cubes it takes to fill each one. Kerry reflected on the volume activity that, "If I can give them the tools for them to be successful, but they develop the understanding through their own work and activity, then it's theirs. It's not something that they'll ever forget." Mia noted the usefulness of improving student learning since students "definitely [are] going to learn more from a well-designed lesson." Don discussed how lesson studies gave him a lens for framing instruction.

> It's like a self-check. Are you reaching the kids that you want to reach? Are you doing what you need to do to make those students successful at it? Are they going to be able to take what you're teaching them, and use that in their experiences in the future to make them better students and better people?

Middle teacher Kate noted that lesson study was useful for improving student thinking due to intentional designs to engage students "in that investigative experimental mode. They're learning the whole time. They're making their little synapses in their brain go, 'Oh yeah, I remember that.'"

Overall, this result highlights the ability of lesson study to spotlight student thinking, in particular to anticipate, observe, analyse, and improve it.

Developing Pedagogy

Twenty-three teachers (70%) reported lesson study to be useful for *developing instructional strategies*. Developing pedagogy has four subthemes: (a) a structure for improving, (b) try new techniques, (c) learn by doing, and (d) beyond one lesson.

A structure for improving

Several teachers described lesson study as useful *structure for improving*, or for connecting content to pedagogies for teaching the content. Mia explained it as "knowing what to do with what you're given." Fellow elementary teacher and coach Ben described lesson study as useful because it provided, "a structure, a matrix, so to speak, on how to do things in the classroom with your colleagues."

Try new techniques

Four teachers described lesson study allowed them to *try new techniques*. They can "go in and try new things," like using manipulatives, and in some cases, were pushed "outside of my box." Elementary school teacher Jaime described the importance of being able to work together for "that breathe of fresh air, or those new ideas that you think, 'Okay, I can try it out.' Or, 'I don't like it that way but I can modify it to look like this so it fits me.'" She elaborated on the importance of constantly challenging herself to improve as a teacher:

> The day that you said you've learned it all, like, you need to get out of teaching. Because that's not what it is. You need to change, you need to change with your students, you need to change with the time. So, I think [lesson study is] something that's good for everyone to do.

Middle school teacher Charlotte described how hearing feedback from other colleagues gave her new strategies to try with students. This allows for outside perspectives to be integrated with previous ways of teaching mathematics.

> You get stuck in your own head sometimes, and you can't really think outside of the box. And so, you get all these people together that talk about it; "What if we did this?" and "I actually saw a teacher once that did this. We can try that."

Middle school teacher Alice described discomfort that trying a new routine brought, and stated, "I was not happy or comfortable there, not at all. I'm still not today, but I'm getting better." High school teacher Markus summarised that lesson study allowed teachers to update unsuccessful lessons with the following mathematics teaching example:

> This is kind of a classic example. It was right before [my colleague] left. We were teaching quadratics. And we finished our unit on quadratics. And once again, the averages were like 60%, and the kids just didn't get it. And I told [my colleagues], that I just remember saying, this is that whole quote of Einstein—doing something over and over again the same exact way but expecting different results is stupid. And that's what we're doing. We teach it the same way every single year hoping that the kids are going to get it this time. There's got to be a better way to teach it, but none of us know it. Cause we all taught it the same way. So, we need somebody that's outside of our box to say, "Hey, why don't you try this?" So that aspect right there, when you have teachers from different areas coming together and talking, anything like that is beneficial to a teacher as far as I'm concerned.

Learn by doing

Some teachers reported that lesson study provided the opportunity to actively engage in planning, teaching, observing student thinking, and debriefing on instruction. This stood in contrast to developing teacher practices through reading or attending workshops disconnected from teaching practice. Both elementary teachers Ben and Mia explained their preference to meet with colleagues rather than reading a book for professional growth.

> You can give me a staff meeting and tell me how to be effective. You can give me a book that tells me how to be effective. I can read research and whatnot. But I won't know how until I've done it, or until I've seen the difference between a lesser lesson rather than, oh the kids really seemed to understand what you're saying.

Fellow elementary teacher Jimmy noted that he liked how lesson study allowed him to "dive in headfirst and figure it out." He continued by saying, "It's not about failing, you know. It's about figuring it out." This active way of learning centers the teacher in their own growth process. Resource teacher Pete noted that a valuable part to lesson study was its active engagement in improving mathematics instruction. "You want to sell a car? Go sell a car. We could talk about what it's like and what you might encounter, and what you might want to do. But you have to actually go sell the car."

Beyond one lesson

Some teachers reported lesson study as useful for changing instruction beyond the research lesson. Elementary teacher and coach Gem explained that planning one lesson affected how she planned other non-lesson study lessons.

> Obviously, you're not going to be doing it [lesson study] for every single lesson. But you will understand the process of doing it. So that as you are planning your lessons, in the back of your mind you've got, "Oh I need to think about how I'm going to respond to this." … Those ideas are still going to be with you. And they're going to make your everyday lessons stronger.

Elementary teacher Kerry explained ways her lesson studies extended to other lessons: use more manipulatives during lessons, introduce symbolic representations of a concept after using concrete manipulatives, and incorporate clothesline number lines into her classroom, an activity learned during lesson study. Middle grades teacher Kamille noted that lesson study provided her an example lesson that could be used for planning and improving other lessons, to "fix it and make it better so that student understanding will be better."

Overall, the structure of lesson study supported teachers in actively developing new pedagogy techniques that have carried beyond the single research lesson.

Supporting Collaboration

Sixteen teachers (48%) reported lesson study useful for working with colleagues. Teachers explained that discussing challenges and sharing different, often opposing, perspectives was helpful for their professional growth. Elementary teacher and ProjectX coach Ben described that meeting with colleagues was useful for improving teaching practice, even with differences in classroom approaches. He said,

> Going through the process is where you learn about your content that you're teaching. I can read it in a book, and I can you know, see it go on the internet and research it. But there's something about meeting with colleagues who all have a different opinion. And it's in that struggle that I think it broadens your thinking. Hearing Teacher A, Teacher B, Teacher C, "no I think it's this way," "I think it's that way," "no I think it's that way."

Claudia also described how lesson study allowed for sharing of teaching practices and "thinking about things in ways that you might not have thought about them" as a result of planning together. She noted that, "hearing just the diversity of opinions on that helps broaden your own thinking." Several teachers shared the view that they were stronger together compared to being alone. Elementary teacher and ProjectX coach Gem described the benefit of sharing different experiences as beneficial for planning due to pooled resources—"two heads, three heads, ten heads together are better than one." Middle school teacher and ProjectX coach Hope noted that "two brains are going to be better than one. Four brains are going to be better than two." Jaime noted how lesson study brought teachers with seven years' experience together with teachers with twenty years' experience to share ideas and "bounce things off one another" to get multiple perspectives. As Kerry stated, "you're seeing it not just through your eyes, you're seeing it through other peoples' eyes."

Networking and discussing teaching issues were reported by elementary teachers Alice, Gem, and Mia. Alice stated,

> For me the best part of ProjectX was talking with other teachers. Having that time to sit down, and listen to people go, I did this, I did that. … Cause every once in a while, you need to know someone else is feeling the same way you are.

In this way, lesson study created space for teachers to discuss important aspects of their teaching profession and be in community with those who are interested in improving the mathematics learning for students.

Another theme of collaboration centred on bringing teachers together simultaneously rather than stay within the confines of their single classroom. Elementary teacher Jackie noted how having teachers together to "see it at the same time and then be able to discuss it" during the enactment of the research lesson helped build teaching skills more quickly than "trying to do it in isolation." This collective

acknowledgement of specific occurrences in the classroom was present in elementary and high school classrooms alike. High school teacher Markus specified that collaborating through lesson study made it clearer to colleagues what students struggled with and how to address their struggles.

This result highlights that lesson study supported teachers in coming together, seeing strength in sharing differences, and engaging in discussions about teaching and learning mathematics.

Developing Curricular Materials

Ten teachers (30%) reported lesson study as useful for developing and refining lessons. While teachers noted that while "you're never always going to have that perfect lesson," lesson studies helped teachers "refin[e] lessons as you go." Elementary school teachers Jane, Becky, and Hannah reported that engaging in lesson studies helped to design lessons. This was important for Hannah since at the time her site had not selected a mathematics curriculum.

> Being able to have those times to design those lessons is huge because we don't know when we are going to have access to the next thing or whatever it is. Just knowing how to pull in all those different activities and learning how to design the lessons so it is more Common Core.

Hannah also suggested that since lesson studies occurred during a period of curriculum reform, it was a good time to "totally make up new lessons and figure out what we want [students] to learn, and how can we get them to where we need them to be, so that they'll be successful and get it."

Elementary teacher Kerry described how a research lesson is deeper than other typical lessons because of investigating standards while planning, rehearsing what teachers will say during the lesson, and providing quality learning experiences for students.

> It's a lesson that is deeper and more important really than the one you took 30 minutes to develop. Because you've actually had the time to investigate in depth what that standard said, what you need to do, what part of it you want to cover. And you've put quality experiences together for the students. And by having [the lesson] be exploratory, you have not been delivering information. You have been delivering the opportunity for them [students] to experience and wrestle with the concepts themselves. And that kids remember. They remember that. They retain that because it then becomes part of their thought process. And it's so much more valuable. I wish every lesson could be [a lesson study].

The lesson study process brought teachers together with different skill sets to create well-designed lessons for students. Elementary teacher Ceci described lesson study as helping to ease the challenge of creating or adapting previous lessons to address new standards, a daunting task for many:

> If you get together as a team and can at least create one or two lessons a week together, and then that lesson is pretty well done and pretty tight and pretty clean, it's just one more step in helping the kids to master content. And then it's one more piece of the job that's done, you know? Because there's just so many standards to learn, and so many new lessons to create now.

Elementary teacher and ProjectX coach, Gem, noted the helpful influence of multiple perspectives while creating lessons by stating, "probably some of the lessons you plan together are much more powerful than something you would plan on your own, just because of so many different peoples' experiences and resources being drawn into it." Along similar lines, middle school teacher Tonya stated that, "When those [exemplary] teachers can meet with good teachers, good teachers like me can use their lessons and be great teachers for the kids' sake." She continued on to say, "That's the good part about lesson study. I think it helps everybody, helps hone excellent lessons, and then disseminating them a little better." Overall, this result suggests a purpose of lesson study for developing curricular materials that "really challenge and get the kids hooked" on mathematics.

Learning Mathematics

Five teachers (15%) reported lesson study as useful for learning mathematics. Comments on deepening mathematical knowledge for teaching included: "going through the process is where you learn about

your content that you're teaching" and "make(s) you more knowledgeable about the concepts themselves." Elementary teacher Mia noted learning mathematics content from participating in lesson study conversations by "delving into the skeleton of math and what it means." Mia reflected that, "if I'm going to ask kids to represent [mathematics] in multiple ways, I have to know it multiple ways. If I want the kids to be able to critique work constructively, do I do that with them?" These reflections highlight her focus of representing mathematics and engaging in mathematical practices, which she reported were not present before lesson study. Elementary teacher Bertha described how collaborating with others helped her deepen her knowledge about mathematics. She provided an example about multiplication not always resulting in a product greater than the multiplier or multiplicand.

> If I say something, Oh what about this? Somebody who knows more than me says, 'Oh yeah, but did you think about this? Is it always true?' It's like, Oh my gosh. I didn't even think about that! So, it's really good in the sense that you learn content and you learn connections.

This result highlights the ability of lesson study to deepen participants' mathematical ways of knowing.

Discussion

Findings offer a detailed categorisation on purposes of lesson study from a teacher's perspective and provide evidence of six distinct areas of potential professional growth through its sustained use. The most occurring themes reported by teachers—understanding reform (100%), focusing on student thinking (87%), and developing pedagogy (70%)—suggest lesson study to be a collaborative, student-centred process for improving the teaching of mathematics especially during times of change. This coincides with research identifying features of high-quality professional development, specifically that which centres on student learning, provides an integrated learning experience for teachers that develops teachers' content knowledge and pedagogical knowledge, and sustains learning in an active way (Darling-Hammond & Richardson, 2009).

The six areas of professional growth indicate for researchers and practitioners alike, reasons that teachers wanted to continue with lesson study. It is hypothesised that these reasons could be used to spark curiosity and motivate new participation in lesson study. The strongest reported theme—understanding reform—suggests teachers' belief of the usefulness of collaborative PDs like lesson study to process change (Lee & Lo, 2013; Lewis & Takahashi, 2013; Takahashi et al., 2013). This result aligns with findings from Ni Shuilleabhain and Seery (2018), who found that engaging Irish secondary teachers in lesson study supported them in learning new pedagogical practices during curriculum reform. Akiba and Wilkinson (2016) documented Florida's state-wide efforts to use lesson study as a vehicle for implementing the Common Core State Standards, which adds further support for this outcome.

Lesson study, in its intentional attention to student thinking before, during, and after the research lesson (Guner & Akyuz, 2020; Huang & Shimizu, 2016; Richit & Ponte, 2017), was shown to help teachers focus on student thinking. Focusing on student thinking shares many similarities with professional noticing of children's mathematical thinking, or the attending to children's strategies, interpreting children's understandings, and deciding how to respond based on children's understandings (Jacobs et al., 2010). Recent studies suggest that lesson study may support both practicing and pre-service teachers in noticing students' mathematical thinking with appropriate scaffolds (Amador & Carter, 2018; Amador & Weiland, 2015; Choy, 2013; Guner & Akyuz, 2020; Lee, 2019). Using lesson study as a vehicle to support the professional noticing of children's mathematical thinking could combine a structure and focus for teachers interested in their own professional growth.

Another theme present in approximately half of the interviews was its usefulness for supporting collaboration (Cajkler et al., 2015; Gero, 2015; Huang & Shimizu, 2016). By bringing teachers into the same room, collectively acknowledging what occurred in the lesson, sharing diverse perspectives, discussing challenges, and offering suggestions for improving student learning, lesson study can be used as a vehicle to support professional development. Other findings, such as learning mathematics (Richit & Ponte, 2017), developing pedagogy (Richit & Ponte, 2017), and developing curricular materials (Lewis et al., 2009), echo research results and provide evidence of the practicality of many lesson study

findings. This study found that lesson study was seen as useful for developing pedagogy, important since some research questioned whether the process of lesson study can change teacher practice (Chokshi & Fernandez, 2004).

As others have noted, lesson study activities are research tasks in and of themselves that invite participants to apply a researcher lens to study student learning (Hart & Carriere, 2011). ProjectX had several foci across the lesson study phases that teachers and knowledgeable others prioritised. This is common in lesson studies since it helps teachers address their reasons for engaging in a lesson study. ProjectX included a purposeful focus on student thinking, research, and collaborative reflection on shared experiences in both the week-long institutes and school-year lesson studies. Addressing student thinking during lesson studies occurred while ProjectX participants planned (e.g., anticipate what students might do), taught (e.g., observing or responding to students in-the-moment), and debriefed (e.g., what students do/write/draw/say).

Whereas most teachers understood the multi-purpose foci of lesson study, Tonya seemed to believe that lesson study was mostly useful for disseminating well-designed lessons. She was the only teacher to report one purpose for engaging in lesson study. Future work may use the overall results to further examine how lesson study may contribute to improving instruction. Which lesson study purposes are more productive for achieving particular instructional goals? What do teachers who described a single purpose notice in their lesson study experiences, and how does this compare to others who described multiple purposes? Takahashi and McDougal's (2016) called for structural and social supports by districts for teachers leading their own lesson studies. They questioned how teachers become aware of the purposes of lesson study and decide to initiate a lesson study, and how may the process be supported by the district.

Although holding multiple perspectives and positive beliefs about lesson study is not sufficient to ensure high quality lessons are Implemented, it is important to listen to teachers' accounts of the purposes of lesson study to contribute to improving mathematics teachings. As Fernandez et al. (2003) caution, "Explaining to teachers that lesson study is meant to be a form of systematic research will not guarantee that teachers can immediately adopt and sustain the researcher lens when conducting lesson study" (p. 182). They continued to say, "… perhaps most importantly, in order for any of this to occur, teachers will need to develop a disposition towards their practice that is grounded in a vision of teaching as a site for learning and of themselves as actively in charge of their ongoing learning process" (p. 182).

Limitations

One limitation of this study was that it focused mainly on teachers' self-reported interview data and did not extensively include survey responses (e.g., Fernández & Robinson, 2006; Gero 2015) or classroom observations. Individual interviews were collected to learn about nuances of teachers' lesson study experiences in a one-on-one setting. Efforts to collect data during planning, teaching & observing, and debriefing sessions could support continued research on purposes for engaging in lesson study from the teacher's perspective. During the interviews, it could be that teachers responded with more favourable summaries of their experiences, either due to not wanting to cause friction among their colleagues or since several months that had passed since the grant ended. Efforts were made to mitigate these limitations by probing participants' thinking during interviews and triangulating claims with research memos.

Conclusion

Mathematics lesson studies were reported to be useful for understanding reform implications, focusing on students' mathematical thinking, and developing mathematics pedagogy. These results are important because they provide a method of professional learning can occur in local contexts while also strengthening partnerships with other teachers, district administrators, and university members. Additional purposes for engaging in lesson study from teachers' perspective included its ability to

support collaboration among teachers, learn mathematics more deeply, and develop curricular materials. This study captured and synthesised thirty-three teacher perspectives after a three-year PD effort during a unique period, the implementation of new standards. This study connects practice to research by attending to what teachers found meaningful after twelve lesson studies, particularly while unpacking meanings and implications of new standards. Teachers interested in beginning or continuing lesson study may use these study teachers' rationales for engaging in lesson study at their site. Principals, mathematics specialists, and district administrators may be better informed about ways mathematics lesson study could support their site and district goals and contribute to the improvement of teaching and learning mathematics in their context.

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Appendix A: Interview Protocol

Before Interview:

1. ***Explain the purpose of the interview:*** Thank you for taking the time to meet with me today. As you know, we are interested in learning about your experiences after the ProjectX grant ended. In this interview, I am going to ask you questions about the groups of teachers that you collaborate with, the kinds of activities that you do together, and resources that support your collaboration.

2. ***Consent Process****:* Please be sure to consent the participant if s/he has not consented before (see Interview Process document). In all cases, tell the participant: Before we begin the interview, I want to remind you that participating in this study is voluntary and your responses will only be shared with the researchers on our team. When your responses are shared, any identifying information will be removed. At any point during the interview, if you would like me to turn off the recorder, just tell me to do so. Do you have any questions about the study before we begin?

During Interview:

* [make sure MIC is plugged in to jack and turned on, and that all three recording devices are recording]

Thank you (participant's name) for taking the time to participate in this interview. We are interested in hearing your experiences after the grant has ended, who you are working with, and what sorts of activities you do with them.

GENERAL/BACKGROUND

Q1: First we have some questions about your experiences as a teacher after participating in the ProjectX grant.

- Are you still working at the same school where you were working during ProjectX?
- Are you still teaching at the same grade level?
- If not, were there district changes that affected your placement?

Q2: Is there a community or group of teachers that you collaborate with?

- Would you say this group was formally or informally arranged?
- How long have you been working together? How often do you meet?
- What sorts of activities do you do in this group? Would you say you do lesson study with this group?

Q3: Are there any other communities or groups that you collaborate with?

Ask the following questions for each community/group that they identify and use the language that they use to label the group [e.g., if they call their group their 'grade level' group, use that. Others: PLC group, informal group, grant teachers, or others]).

- Would you say this group was formally or informally arranged?
- How long has this group been going?
- What sorts of activities do you do in this group? Would you say you do lesson study with this group?

Q4: Are you working with some of the teachers from ProjectX?
If yes, are these teachers at the same site?

Q5: Have you completed a round of lesson study since the grant ended?

- How did you arrange time to plan? How often?
- How did you arrange time for the research lesson? How often?
- How did you arrange time to debrief? How often?

Q6: Did you teach the research lesson during lesson study? How did your group decide that? (Optional: What is it like to have teachers observe your practice?)

CONCEIVING OF LESSON STUDY

Q7: Can you briefly describe what goes on during a lesson study round?

Q8: If they have difficulty with XXX, ask if they would have liked to have done lesson study. If they say yes, ask, were there obstacles to observing? (e.g., were there things that prevented you from doing XXX?)

RESOURCES

Now I'll ask some questions about resources for your group.

Q9: What sorts of resources does your school give to support your group?

- In addition to _____ (summarize what they stated, such as a district leader, books, or PE teacher), were there any other resources that your school/district provided that supported your group's work at lesson study?

Q10: If they have done lesson study, what resources were for lesson study activities?

Q11: If they have not done lesson study, what resources would support your lesson study activities?

Q12: Do you feel like your group has connections to resources/people/groups outside of your group?

GOALS

I'm going to ask you some questions about group, school & district goals for each of your described groups.

Q13: Do you feel like your professional group has shared values about what constitutes effective teaching? (Integration)

Q14: Do you feel like the goals of your group are aligned? (Synergy)

Q15: Are your group's goals aligned with ProjectX PD goals?

Q16: Are your group's goals aligned with the goals of your district?

END OF SURVEY

Q17: If you were to do / if you did lesson study again, would you change anything / did you change anything from the way it was done in ProjectX?

Q18: What problem is your group trying to solve as you try to go forward with lesson study?

Q19: Do you think lesson study is or would be useful for figuring out (particular aspect of) the CCSS-M? Why or why not?