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Preservice teachers’ knowledge mobilized in solving area tasks

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Abstract

Studies that address preservice teachers’ knowledge of area measurement emphasize their lack of knowledge and their tendency towards the use of formulas, without offering a body of knowledge that helps to address such difficulties. This study offers an approximation of the mathematical knowledge necessary for preservice teachers to solve area tasks. For this, the Mathematics Teacher's Specialized Knowledge model is used with emphasis on the subdomain of Knowledge of Topics and Knowledge of the Structure of Mathematics. Preservice teachers' resolutions and written justifications are analyzed using qualitative and quantitative tools. The results indicate that those resolutions that manage to mobilize mathematical knowledge are associated with the joint mobilization of different procedures, properties, and geometric principles. Results also indicate that the strategic coordination between different registers of representation allows Preservice Teachers to mobilize categories of specialized knowledge and establishing connections with other mathematical contents.

Keywords: Area of Flat Figures, Knowledge of the Structure of Mathematics, Knowledge of Topics, Mathematics Teacher's Specialized Knowledge, Mixed Methods

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Content knowledge in teachers is fundamental to direct and improve student learning (Hill et al., 2005), since it allows teachers, as well as preservice teachers (PST), to better understand and justify why they solve mathematical tasks in a certain way (Shulman, 1986). In the particular case of area, content knowledge is determinant for teachers to be able to propose productive questions, explain geometric principles and respond effectively to students' concerns (Baturo & Nason, 1996; Murphy, 2012). However, several researchers evidence that teachers and PST do not have the necessary knowledge about area measurement processes (i.e., Chamberlin & Candelaria, 2018; Livy et al., 2012). Teachers' lack of such knowledge strongly affects their use of different pedagogical tools and therefore their teaching methods (Carpenter et al., 1988; Hill et al., 2005), which could negatively impact student learning (Hill et al., 2005). To develop an understanding of area measurement processes in students, teachers must provide meaningful and sustained teaching-learning experiences over time (Sarama & Clements, 2009). This requires that teachers have a robust knowledge about area and its measurement processes. For example, it is necessary that teachers understand different procedures to solve a certain task, properties and principals involved in area measurement processes (Sarama & Clements, 2009), and different representations that can be used to solve a given task (Caviedes et al., 2021). The development of the aforementioned procedures and concepts is interactive in nature, this means that they are mutually reinforcing (Clements et al., 2018; Sarama & Clements, 2009), so it is necessary to understand how they are mobilized or put into practice (Caviedes et al., 2022b) in a resolution process. In this sense, this study offers a first approximation to those components of specialized knowledge necessary to solve area tasks. For this purpose, we rely on the model of Mathematics Teacher Specialized Knowledge (Carrillo-Yañez et al., 2013; Carrillo-Yañez et al., 2018).

Mathematics Teacher's Specialized Knowledge (MTSK)

The MTSK model is mainly used to study the practice of mathematics teachers (Carrillo-Yañez et al., 2013; Carrillo-Yañez et al., 2018), however it is possible to assume it as a referent of the desirable components that PSTs need for their future practice (Liñan et al., 2014). The model allows characterizing the mathematical knowledge needed to solve area tasks, through the domain of Mathematical Knowledge (MK) and its three associated subdomains: Knowledge of Topics (KoT), Knowledge of the Structure of Mathematics (KSM) and Knowledge of Mathematical Practice (KPM). Specifically, we are interested in KoT subdomain and complementarily in KSM subdomain. The KoT describes what and how mathematics teachers know the content they teach. In this way, it describes the conceptual knowledge that teachers have about area definitions at different school levels (e.g., what is area mathematically speaking?); the properties and principals involved in area measurement processes (e.g., area conservation and the geometric principles that support it); the phenomenology or contexts of use of the concept of area (e.g., equal sharing); the procedures that can be used to solve area tasks (e.g., unit iteration); and the systems of representation that are made explicit by means of statements, procedures or justifications. Intraconceptual connections are also part of this subdomain and correspond to the relationships that are established between the different elements mentioned above. For its part, KSM refers to interconceptual connections, for example, relationships between different mathematical topics (Montes et al., 2013). In this study we focus on auxiliary connections, which are related to the participation of an item in a larger process (Carrillo- Yañez et al., 2018), in other words, with "*the need to consider a notion -procedure or construct- as a support for the students to understand a given concept -procedure or notion*" (Policastro et al., 2019, p.3). For example, when a teacher uses Pick's theorem, since this knowledge helps students to differentiate between perimeter and area and to overcome difficulties related with both magnitudes (Palmas, Rojano, & Sutherland, 2021).

Some of the studies that have used MTSK model have shown its advantages for the characterization of teachers' knowledge in different aspects (e.g., Ribeiro & Amaral, 2015; Policastro et al., 2019). For example, Ribeiro and Amaral (2015) point out that the model allows identifying aspects of conceptual knowledge that are necessary to divide and to pose division problems. They also suggest the need for further research to gain a better understanding of MTSK model (and its subdomains) as an analytical tool. In this sense, this study uses the model to further explore its potential as an analytical tool and, in turn, to characterize the mathematical knowledge mobilized by PSTs when solving area tasks.

Preservice Teachers' Knowledge on Area Measurement

The diversity of situations in which area measurement can appear and the different representations used in area measurement processes allow us to infer an underlying complexity (Caviedes et al., 2021). This complexity is associated with specific PSTs difficulties, which are related to a scarce variety of procedures for solving area tasks and a tendency to the use of formulas (Baturo & Nason, 1996; Caviedes et al., 2022a; Runnalls & Hong, 2020; Simon & Blume, 1994); to the lack of acquisition of geometric properties and principles (Hong & Runnalls, 2020); to the lack of understanding of units of measurement

(Chamberlin & Candelaria, 2018); and to the relationship that exists between perimeter and area (Livy et al., 2012). For example, Hong and Runnalls (2020) report that PSTs present difficulties in accepting area conservation in non-prototypical figures, since lacking numerical values to compare areas of triangles equivalent in area, but different in shape, they justify their answers based on visual estimation. The same authors emphasize that the understanding of the concepts underlying area conservation would allow PSTs to develop an understanding of the formulas and procedural fluency. In Caviedes et al. (2022b) it is evidenced that area conservation can be accepted and justified by PSTs when tasks restrict the use of calculations, which could be an indicator of how to design tasks to work on this property with PSTs. The same study shows that the acquisition of certain geometric principles is fundamental to justify the area conservation property. Similarly, Baturo and Nason (1996) point out that PSTs have a scarce knowledge of mathematical concepts and the relationship between them, as they prioritize procedural knowledge associated with the memorization of formulas, which causes great problems in connecting concrete and abstract representations. The same authors show that PSTs make mistakes when they must operate with decimal numbers and do not reason about whether the value of the area may be correct or not.

The previous studies show that PSTs need to improve their mathematical knowledge and therefore their understanding of the mathematical elements underlying area measurement processes. For this reason, it is relevant to identify the mathematical knowledge that allows PSTs to solve area tasks and justify their procedures in a reasoned manner. We understand that the latter implies a broad knowledge of procedures that a task may admit, in addition to the study and analysis of the different concepts and properties involved in the measurement processes (Sarama & Clements, 2009). These elements are explicitly manifested using different representations (Caviedes et al., 2021). Therefore, we believe it is necessary to broaden and deepen the understanding of the relationships that PSTs can establish between the different representations involved in area measurement processes.

Registers of Semiotic Representations

Duval (2006; 2017) points out that the development of the understanding about geometric figures requires the coordination of two registers of semiotic representations: the discursive register (oral or written, in natural or symbolic language) and the non-discursive register (drawings, sketches, graphs, figures and geometric configurations). With the aim to acquire such coordination, it is necessary to carry out a visual deconstruction of the figural units that are imposed at first sight, to obtain a reconfiguration. Thus, the auxiliary line tracing presents itself as one of the main problems, since how to divide the figure is not obvious. There are two ways to decompose a figure into figural units, mereological division and dimensional deconstruction (Duval, 2017). In this study we focus on mereological division, which consists in the division of a whole into parts that can be juxtaposed or superimposed, and it is always done to reconstruct, with the parts obtained, a figure often very different visually but in the same dimension. This division constitutes one of the major heuristics in the transformations of geometric figures (Duval, 2017) and admits three types of decompositions: (1) strictly homogeneous, decompositions into figural units of the same shape as the starting figure; (2) homogeneous, decompositions into figural units different from the starting figure, but all the same shape; and (3) heterogeneous, decompositions into figural units of different shapes from each other and from the starting figure. All these decompositions allow a purely visual exploration of a starting figure, to detect the geometric properties to be used to solve a task and to obtain a reconfiguration that makes new shapes, not recognizable in the starting figure (Duval, 2017). However, PSTs present difficulties in performing this type of decompositions (Caviedes et al., 2022b; Hong & Runnalls, 2020), indicating a limited range of strategies related to mereological division.

Research Questions

The studies that address PSTs knowledge of area measurement emphasize their lack of knowledge and their tendency towards the use of formulas, without offering a body of knowledge that helps to address such difficulties. In this study, it is understood that PSTs difficulties are due to the complexity underlying area, as area measurement processes require specific mathematical knowledge to be executed accurately. Thus, the present study seeks to answer: what mathematical knowledge do PSTs bring into play when solving area tasks? Specifically, *the objectives are twofold: (1) to characterize the components of mathematical knowledge mobilized by PSTs when solving a set of area tasks; and (2) to identify the relationships between different components of mathematical knowledge mobilized by PSTs in the resolution of area tasks*. For this purpose, we use the Mathematics Teacher's Specialized Knowledge -MTSK- model (Carrillo-Yañez et al., 2018), mainly for two reasons. First, the model considers specialization as the core of the mathematics teacher's knowledge, in all its domains, subdomains and categories (Aguilar-González et al., 2018). Second, the model is a theoretical proposal that models the core of the teacher's professional knowledge and is, in turn, a methodological tool that allows us to analyze different practices of PSTs through its categories (Carrillo-Yañez et al., 2018). Hence, the MTSK model allows to characterize the components of mathematical knowledge that enables PSTs to solve area tasks, and which are not reported in previous studies. We focus on Knowledge of Topics (KoT) and Knowledge of the Structure of Mathematics (KSM).

METHODS

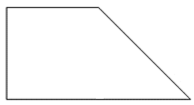
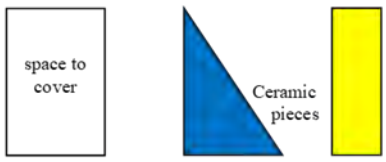
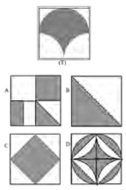
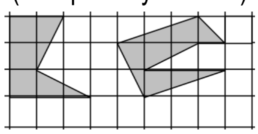
The study is situated in an interpretative paradigm with a mixed and exploratory approach (Rocco et al., 2003). A qualitative approach (Cohen et al., 2000) and a content analysis (Krippendorff, 2004) are used to identify the mathematical knowledge mobilized in PSTs resolutions. The categories of analysis proposed by MTSK model for KoT and KSM subdomains are used (described in section 2.1). For each of these categories we construct indicators of knowledge (see Table 2) and to facilitate the process of assigning these indicators to the PSTs responses, the MAXQDA plus software is used. In addition, a quantitative approach, and a statistical implicative analysis -SIA- (Gras & Kuntz, 2008) is performed to identify the relationships between different indicators of knowledge mobilized by PSTs. In this study the term mobilization indicates "putting into practice" a certain knowledge (Aguilar-González et al., 2018; Caviedes et al., 2022a, 2022b). The variables considered for this analysis are those indicators that emerged from qualitative analysis.

The SIA quantifies how likely it is that variable B will occur if variable A has been observed in the population (Gras & Kuntz, 2008). Thus, it makes it possible to identify and organize the quasi-implication relationships (implicative relationships between variables with a given probability) by means of a graph with arrows that relates the variables with the strongest implications, at different levels and intensities. In the implicative graph, we use the arrow → to indicate a quasi-implication relationship according to the meaning described above. The double arrow ↔ indicates a reciprocal quasi-implication relationship. The SIA is presented as an effective tool to establish possible relationships between concepts from empirical data (Trigueros & Escandón, 2008). Thus, SIA would be useful to find those indicators of knowledge that would allow PSTs to respond the demand of the tasks and, therefore, provide information on key components of mathematical knowledge for solving area tasks. In order to carry out the analysis a value of 1 was assigned to each indicator mobilized in the PSTs responses and a value of 0 to each indicator that was not mobilized in the responses. The package C.H.I.C -Classification Hierarchies, Implicative et Cohesitive- version 0.27 (free version) in the R console version 3.5.2 was used.

Instrument and Procedure

Data collection was conducted in the first quarter of the 2020-2021 school year. The participants were 147 PSTs, who were attending the course Management and Innovation in the Mathematics Classroom, in the third year of the Primary Education Degree at University X. PSTs as part of their study program had completed three courses related to magnitudes and their measurement. A semi-structured open-ended questionnaire (Bailey, 1994) was designed to be solved individually and PSTs were asked to justify each procedure in writing. The questionnaire consisted of 8 tasks, of which the first 5 sough to explore mathematical knowledge using procedures for their resolution. Tasks 1, 2 and 3 respond to contexts of equal sharing and comparison and reproduction of shapes (Freudenthal, 1983), with privilege of procedures of a geometric nature (Douady & Perrin-Glorian, 1989). In these tasks, the use of formulas and measuring instruments was prohibited. Tasks 4 and 5 respond to a measurement context (Freudenthal, 1983) and involve the joint use of geometric and numerical procedures (Douady & Perrin-Glorian, 1989). Here, the use of formulas and measuring instruments was allowed. The questionnaire was applied by the professor in charge of the course, in online format due to COVID-19 and PSTs had one week to answer it and send it in word or pdf format. Due to the objective of our study, evidence of the resolutions to Tasks 1, 2, 3, 4 and 5 are presented (Table 1).

Table 1. Tasks proposed to the group of PSTs

Statement of tasks	Graphic representations of tasks
TASK 1: A father wants to divide a piece of land, in the shape shown in the figure, among his six children. If each child should receive the same portion of land, how could the father divide the land equally? **Justify your answer.**	 (Compiled by authors)
TASK 2: Camila is making a ceramic mosaic with different colors. To finish her mosaic, she needs to cover a rectangular space, but she doesn't have any pieces of ceramic with that shape. What can Camila do to cover the missing space in her mosaic? **Justify your answer.**	 (Adapted from Puig & Guillén, 1983)
TASK 3: Taking the shaded surface of Figure (T) as a reference: Which of the following figures has a shaded surface equivalent to Figure (T)? **Why? Justify your answer.**	 (Compiled by authors)
TASK 4: Look at the triangles constructed on the geoboard. What is the area of each triangle? Which one has the largest area? **Justify your answers using two or three different procedures.**	 (Compiled by authors)
TASK 5: Look at the two figures drawn on the grid. Which one has the larger area and why? **Justify your answers using two or three different procedures.**	 (Adapted from Aguilera & Flores, 1998)

The categories constituting KoT subdomain are identified in advance and arise from an *epistemic configuration* on area measurement (Caviedes et al., 2021). The elements of such configuration are adapted to KoT categories and allow for deductive coding to be performed. The categories corresponding to KSM subdomain emerge from the analysis of PSTs responses through inductive coding. Table 2 shows categories of knowledge and their respective indicators according to MTSK model conceptualization.

Table 2. Categories of specialized knowledge

Categories of KoT	Indicators
Representations (R)	**(R1)** Written: use of adjectives such as "minor", "major", "double", "half", etc., related to surfaces. **(R2)** Manipulative: use of physical objects or dynamic geometry software. **(R3)** Geometric: use of convenient decompositions or partitions of known figures to calculate the area of unknown figures or to compare and-or estimate surfaces quantities. **(R4)** Symbolic: use of the R^+ set to compare two or more surfaces, for counting units or adding up areas and-or for the indirect calculation of the area.
Procedures (P) and justifications (J)	**(P1)** Compare two or more surfaces directly by total and-or partial overlapping. **(P2)** Compare two or more surfaces indirectly by cutting and pasting. **(P3)** Decompose in a convenient way, graphically or mentally, two or more surfaces. **(P4)** Carry out movements of rotation, translation, and superimposition of figures. **(P5)** Decompose surfaces into congruent units and/or sub-units to facilitate the process of measuring areas. **(P6)** Measure areas as an additive process by counting units or sub-units that cover the surface. **(P7)** Measure linear dimensions and use formulas. **(P8)** Calculate areas of known figures to obtain areas of unknown figures by decomposition and by adding up areas. **(J1)** The overlapping method to compare two or more surfaces is useful for establishing equivalence or to include relationships. **(J2)** The mental act of cutting the two-dimensional space into parts of equal area serve as a basis to compare areas. **(J3)** The change in the shape of a surface does not change the area of the surface, as the figures can be decomposed and reorganized while keeping the same "parts". **(J4)** The square is presented as the best choice for measuring areas of polygonal surfaces, due to the ease of iterating it and for covering rectangles and squares without overlaps or gaps. **(J5)** The area of a square and rectangular surface is determined by the product of their two linear dimensions. Thus, the base times height formula allows to find the area of squares and rectangles. **(J6)** The area of the triangle is half of a square or a rectangle with the same base and height that contains it. Thus, the formula of the triangle is base times height divided by two.
Properties (Pp) and principles (Pr)	**(Pp1)** Conservation. **(Pp2)** Accumulation and additivity. **(Pp3)** Transitivity. **(Pr1)** A parallelogram with the same base as a triangle, both placed between the

	same parallels, is twice the triangle. **(Pr2)** Triangles placed on equal bases and between the same parallels are equal. **(Pr3)** Two polygons are congruent if they have their sides and angles respectively equal or congruent. **(Pr4)** Every polygon can be decomposed into triangles. **(Pr5)** Every triangle is equidescomposable to a parallelogram. **(Pr6)** The unit of measurement can be divided into parts (fractionated) to ease the process of measuring areas. **(Pr7)** To calculate the area of a figure, the figure can be broken down into a finite number of parts so that these parts can be put back together to form a simpler figure.
Categories of KSM	**Indicators**
Auxiliary connections (Cau)	**(Cau1)** Use of a procedure to evoke a different procedure or topic.

Figures 1 to 4 show examples of resolutions that mobilize indicators of specialized knowledge from KoT and KSM. These resolutions are considered representative of the group of PSTs that evidence mobilization of specialized knowledge.

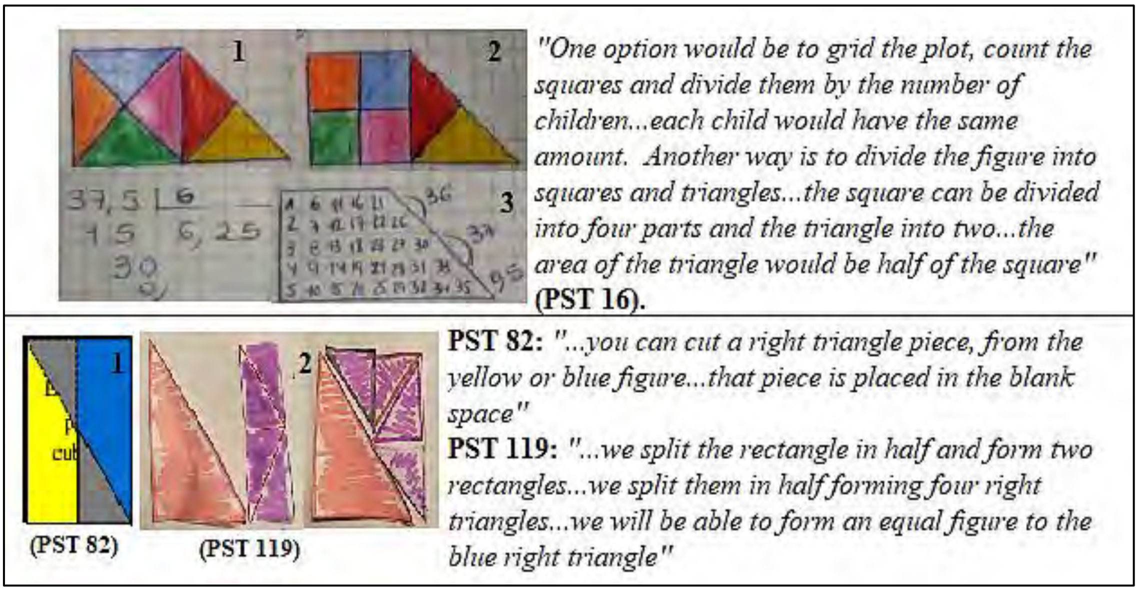
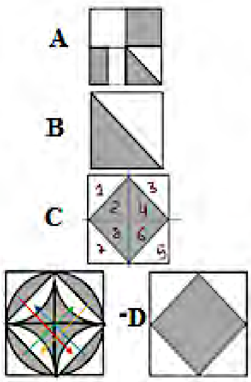


Figure 1. Examples of resolutions for Task 1 and Task 2

Figure 1 shows examples of resolutions that mobilize specialized knowledge in Tasks 1 and 2. PST 16 (Task 1) uses representations in its discursive registers, written (R1) and symbolic (R4); and non-discursive registers, geometric (R3). It is observed that PST 16 mobilizes surface decomposition procedures (P3 and P5) to distribute the total area of the figure equally. PST 16 uses two types of decompositions. In Figure 1(1) PST 16 performs homogeneous decompositions since she decomposes the trapezium into equivalent right triangles. Such a decomposition allows inferring the use that polygons can be decomposed into triangles (Pr4). In Figure 1(2) PST 16 uses heterogeneous decompositions since she decomposes the trapezium into squares and right triangles. The decompositions used, as well as the comparisons between them, allow PST 16 to mobilize the transitivity property (Pp2) and at the same time, to infer that triangles and squares have equal area (Pr2 and Pr3). The comparison between the square and the triangle that make up the area to be divided allows PST 16 to infer that the first is twice the second (Pr1). Finally, PST 16 uses a numerical procedure (P6) that allows her to obtain the total area of the figure and then divide it into six. This procedure allows PST 16 to use the square as a two-dimensional unit to measure areas (J4) and to establish an auxiliary connection (Cau1) with partitive division (an object is partitioned according to a known number of groups) and its algorithm.

The resolution of PST 82 to Task 2 shows representations in its discursive and non-discursive registers. The former, of written type (R1); the latter, of geometric type (R4). It is observed that PST 82 decomposes (heterogeneously) and superimposes surfaces in order to compare them (P1 and P3). In addition, PST 82 performs rotation and translation movements (P4) of the parts into which she divides the figures, in order to check that the triangle and the rectangle represent half of the piece that needs to be covered. PST 82 shows an implicit use of the transitivity property (Pp2) since she makes a comparison between the surface to be covered and those representing the chunks. Moreover, she shows an implicit use of the properties of accumulation and additivity (Pp2) and conservation (Pp1), since she recognizes that figures can be decomposed and recomposed into other figures conserving the same parts (J3). Finally, it is possible to infer that PST 82 recognizes that a triangle is equidecomposable to a parallelogram (Pr5) and that a parallelogram of equal base and height to a triangle, both placed between the same parallels, is twice the triangle (Pr1). The previous indicators are also mobilized in the resolution of PST 119. However, it is possible to infer that PST 119 also makes use of manipulative representations (R2), since she cuts and reorganizes the parts of the triangle (P2 and P4). This indicates that PST 119 makes use of homogeneous decompositions, since she decomposes the rectangle into small rectangular triangles. This procedure allows inferring that PST 119 mobilizes a geometric principle that indicates that every polygon can be decomposed into triangles (Pr4).



"If we look at figure A we see that there are four squares... we know that each square represents 1/4 of the total figure... so we have: $0+\frac{1}{4}+(\frac{1}{2}*\frac{1}{4})+(\frac{1}{2}*\frac{1}{4})=\frac{1}{4}+2(\frac{1}{8})=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

"... figure A is equivalent to figure T... figure B is 1/2 of the total figure... is equivalent to figure T... in figure C we can draw lines to visualize the total... there are 8 triangles that form the total figure... we have 4 triangles that represent the shaded area... 4/8 is the fraction that represents it... 1/2 of the total, therefore, it is equivalent to figure T... in figure D, if we put the external parts towards the interior, we obtain figure C, therefore, it also has a shaded area equivalent to figure T. ***(PST 6)***

Figure 2. Example of resolution for Task 3

Figure 2 shows the resolution of PST 6 to Task 3. It is observed that PST 6 makes use of representations in its discursive registers, written (R1) and symbolic (R4); and in a non-discursive register of geometric type (R4). It is inferred that PST 6 makes use of (J2) as she recognizes that the act of cutting a surface in equal parts is useful to make comparisons; and of (J3), as PST 6 recognizes that changing the shape of a surface does not produce changes around the surface (e.g., in Figure 2(A)). PST 6 identifies in Figure 2(A) that the shaded parts correspond to 1/8 or 1/4 of the square. Thus, she recognizes that the shaded square is twice the shaded triangle and rectangle (Pr1). Therefore, PST 6 establishes an auxiliary connection with procedures for adding and multiplying fractions and obtains that the shaded area is 1/2 of the total. Similarly, in Figure 2(B), PST 6 identifies that the shaded part equals 1/2 of the total. In the case of Figure 2(C), PST 6 decomposes the shaded area into triangles (P5) and obtains the total area by an additive process (P6). The homogeneous decomposition (into triangles) allows PST 6 to identify that the shaded area corresponds to 1/2 of the total area and to mobilize (Pr2). Figure 2(D) shows that PST 6 performs rotation and translation movements of the shaded parts (P4), in order to verify that the shaded area corresponds to 1/2 of the total area. The comparison made between the shaded and unshaded parts of all the figures allows PST 6 to mobilize the transitivity property (Pp2). It also facilitates the mobilization of the property of area conservation (Pp1) and accumulation and additivity (Pp3), since, although the figures have different shaded parts from the model, it is possible to reorganize them and verify that their area is equivalent in all cases.

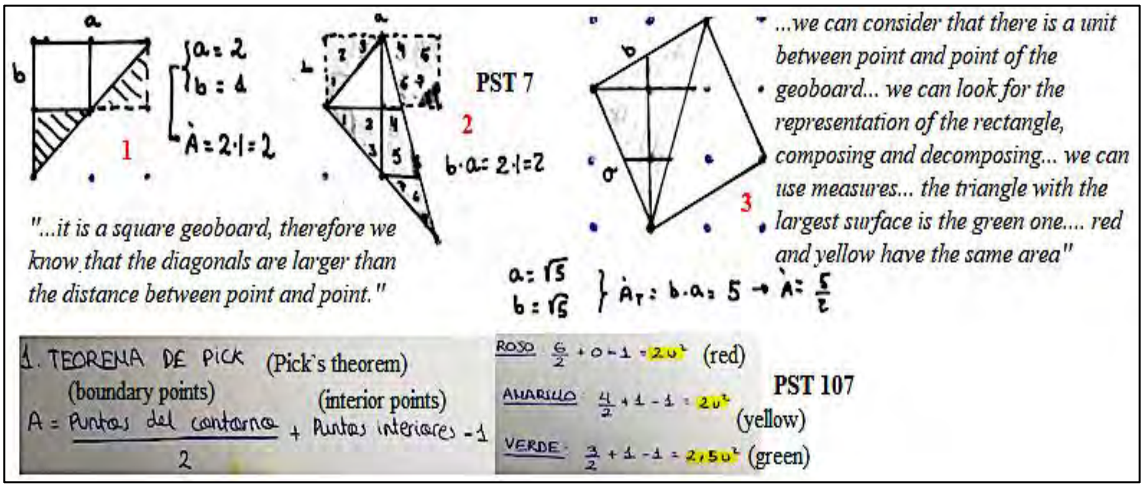
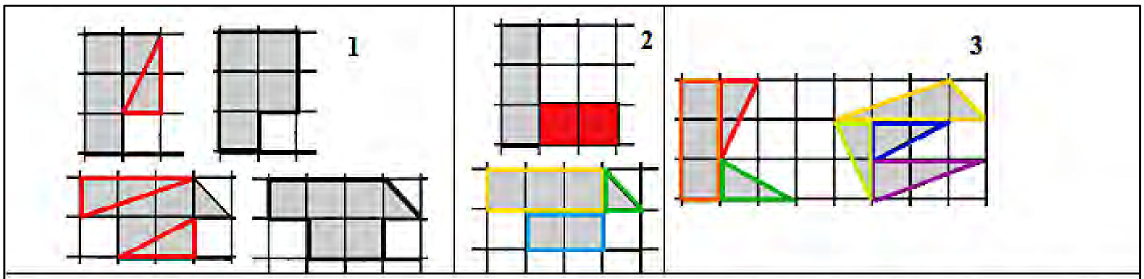


Figure 3. Examples of resolutions for Task 4

Figure 3 shows excerpts from the resolution of PST 7 and PST 107 to Task 4. Both PSTs use representations in their discursive registers, written (R1) and symbolic (R4); and non-discursive register, geometric (R3). It is observed that PST 7 decomposes triangles 1 and 2 and rearranges them into rectangles to apply the area formula (P7). The application of the formula for the area of triangles is supported by the use that the triangle is half of a square with the same base and height that contains it (J6). Similarly, PST 7 recognizes that cutting two-dimensional space is useful for comparing areas, and that figures can retain their parts when rearranged into a new figure (J2 and J3). The latter allows inferring that PST 7 mobilizes the property of conservation (Pp1) and accumulation and additivity (Pp3). We infer that the comparison between the parts that make up the triangles allows PST 7 to mobilize the property of transitivity (Pp2). Geometric type representations (R3) allow PST 7 to decompose the triangular surfaces (P3) in a heterogeneous way, and to reorganize the parts in a rectangular figure (P4 and Pr7). Such reorganization allows inferring that PST 7 recognizes that a triangle is equidecomposable to a parallelogram (Pr5) and that the unit of measurement can be fractionated to facilitate measuring areas (Pr6). In triangle 3, we observe that PST 7 establishes an auxiliary connection (Cau1) with the Pythagorean theorem, which allows her to subsequently use the area formula (P7). On the other hand, PST 107 uses Pick's Theorem, showing an auxiliary connection (Cau1) with another topic and procedure (although PST 107 makes an error when counting the interior points of the third (verde/green) triangle).

Figure 4 shows the resolution of PST 15 to Task 5. It is observed that PST 15 uses representations in its discursive registers, written (R1) and symbolic (R4); and non-discursive register of geometric type

(R3). Figure 4(1) shows that PST 15 decomposes both irregular figures (homogeneously and heterogeneously) to reorganize them into simpler figures (P3 and P4), which facilitate the counting of square units (P6). The procedures used allow us to infer that PST 16 recognizes that the calculation of area is facilitated by decomposing and reorganizing an unknown figure into another known figure (Ppr7). The reorganization and comparison of the parts that compose both surfaces, allows inferring that PST 15 mobilizes the property of accumulation and additivity (Pp3), transitivity (Pp2) and area conservation (Pp1). The latter is supported using (J3), which indicates that the figures can be decomposed and reorganized while conserving the same parts. Figure 4(2) shows mobilization of the indicators mentioned above, and in addition the application of the formula for the area of rectangles and triangles (P7 and P8). These operations are supported by justifications (J5 and J6) that indicate that the base times height formula allows finding the area of square and rectangular surfaces, and that the area of the triangle is half of the rectangle of equal base and height that contains it. Figure 4(3) shows that PST 15 decomposes both figures heterogeneously and homogeneously (P3) in order to calculate the area of each of the parts that make up the surface (P7) and then add them up together. Likewise, this procedure is supported by justifications mentioned above (J5 and J6).



*"**Procedure 1:** we assume that each square measures 1x1... group the figures into familiar shapes... the first figure is made up of 5 squares, each square is 1 cm2, the area is 5 cm2 ... the second figure has 5 squares and a triangle.... 5 and a half squares... 5.5 cm2... **Procedure 2:** calculate the area of the figures by rotating their parts... in the first figure, the gray rectangle has 1x3 = 3 cm2... the red rectangle 2x1 = 2 cm2... the figure has 3+2 = 5 cm2... in the second figure, the yellow rectangle has 3 cm2; the blue rectangle has 2 cm2... the green triangle has 0.5 cm2... **Procedure 3:** divide the figure into triangles and rectangles and calculate the area, the first figure has 3+1+1+1 = 5 cm2; the second figure has 2+1+1+1+1.5 = 5.5 cm2" **(PST 15).***

Figure 4. Examples of resolutions for Task 5

RESULTS AND DISCUSSION

The above examples (Figures 1, 2, 3 and 4) show resolutions from PSTs that succeed in mobilizing mathematical knowledge in each of the tasks. Table 3 shows that in Tasks 1, 2 and 3 the written and geometric type representations; the procedures of decomposition and reorganization of surfaces; as well as the properties, are mobilized by more than half of PSTs, which may be due to the explicit request to justify the procedures used and not to use rulers to measure the lengths of the figures. On the other hand, geometric principles, as well as some of the justifications that support the procedures used, are mobilized by a minority of PST.

Table 3. Percentage (%) of PSTs that mobilized mathematical knowledge in tasks 1,2 y 3

Code	%	Code	%	Code	%	Code	%	Code	%
R1	93	**P4**	61	**Pr4**	30	**P6**	2	**J6**	1
Pp2	92	**Pp3**	61	**Pr5**	29	**(N.R)**	2	**J5**	1
P3	91	**J2**	55	**R4**	15	**J1**	2	**Cau1 (Dv)**	0,5
R3	82	**Pr5**	33	**Cau1 (Fr)**	11	**P1**	1	(Fr) Fractions	
Pp1	61	**Pr1**	32	**R2**	7	**P5**	1	(Dv) Division	
J3	61	**Pr2**	30	**P2**	7	**J4**	1	(N.R) No response	

Table 4 shows that in Tasks 4 and 5 PSTs mostly resort to the use of symbolic type representations and therefore to procedures involving formulas. The property of transitivity is mobilized by more than half of PSTs, which is because both tasks required comparing surfaces. It is possible to observe that 59% of PSTs use geometric type representations, which is associated with surface decomposition procedures. Thus, more than half of PSTs use some alternative procedure to the use of formulas. The conservation and accumulation and additivity properties, as well as geometric principles and justifications that can support the procedures used, are mobilized by less than half of PSTs. Finally, it is observed that a reduced number of PSTs establish auxiliary and complexity connections to solve the tasks.

Table 4. Percentage (%) of PSTs that mobilized mathematical knowledge in tasks 4 y 5

Code	%	Code	%	Code	%	Code	%	Code	%
R4	85	**P3**	53	**P4**	23	**Cau 1 (Tp)**	14	**P2**	1
R1	84	**P5**	44	**Pp3**	21	**Cau 1 (Fr)**	5	**Ccp 1 (Fsp)**	1
Pp2	85	**Pr6**	42	**J3**	21	**R2**	4	**Cau 1 (Tpk)**	0,3
P7	69	**J2**	39	**Pp1**	21	**P1**	3	(Tp) Pythagorean theorem	
R3	59	**P8**	31	**Pr7**	18	**Pr5**	2		
P6	54	**J4**	30	**J6**	15	**J1**	2	(Fsp) Semi-perimeter's formula	
								(Tpck) Pick's theorem	

Tables 3 and 4 allow us to identify those indicators of mathematical knowledge that PSTs mobilize in the resolution of the tasks; however, they do not report on the relationships between them. To identify these relationships, a SIA (Gras & Kuntz, 2008) is carried out. We performed the analysis of tasks 1, 2 and 3 together, as these tasks involved similar procedures by restricting the use of measuring instruments and formulas. Tasks 4 and 5 are analyzed separately, because the complexity of procedures and connections that could emerge from PSTs resolutions were varied. The implicative graphs in Figure 5 shows relationships between different indicators of mathematical knowledge mobilized in Tasks 1, 2 and 3 (with 98 % significance indicated by the red arrows and 95 %, indicated by the green arrow). To distinguish each indicator, we use colors. Thus, justifications are shown in green, procedures in purple, properties in black, geometric principles in orange, connections in brown, and representations in blue.

Figure 5(1) shows that those PSTs that use geometric type representations (R3), also use written type representations (R1). This might be because PSTs also justify in a written type of register procedures related to the decomposition and reorganization of figures (J2 and J3). Likewise, those PSTs that establish auxiliary connections with procedures linked to fractions do so by means of symbolic type register (R4) and written type registers (R1), but with lower significance. Figure 5(2) shows that PSTs mobilize geometric principles jointly since PSTs that mobilize (Pr3) also mobilize (Pr2) and (Pr4). The mobilization of these principles implies the use of a geometric type register (R3) which indicates that auxiliary line tracing that allows the comparison, decomposition and reorganization of the figures, is presented as a key element to illustrate that triangles with the same base, and between the same parallels, are equal (Pr2); that figures are congruent if their sides and angles are equal (Pr3), and that every polygon can be decomposed into triangles (Pr4). Figure 5(3) shows that procedures related to indirect comparison of surfaces (P2) involve the use of a geometric type of register (R3). Procedures that require decomposing surfaces (P3) and executing rotation and translation movements of their parts (P4) involve the use of a written type of register (R1), indicating that PSTs justify the use of such procedures in a written form. Finally, Figure 5(4) shows a reciprocal implication relationship between conservation property (Pp1) and accumulation and additivity (Pp3). Both properties are also linked with the use of transitivity property (Pp2), indicating a joint mobilization by PSTs. Furthermore, it is observed that PSTs justify the use of such properties through a written type of register (R1) and that there is a reciprocal relationship between transitivity property (Pp2) and a written type of register (R1). This means that those PSTs that mobilize transitivity property do so through their justifications and vice versa. It is also observed that manipulative type register (R2) implies the use of a geometric type of register (R3), since the auxiliary line tracing serves as a basis for obtaining the pieces that make up a figure. Finally, the geometric type of register (R3) implies the use of a written type register (R1) and transitivity (Pp2), since PSTs justify in writing the auxiliary line tracing and the comparisons that allow mobilizing the property of transitivity (Pp2).

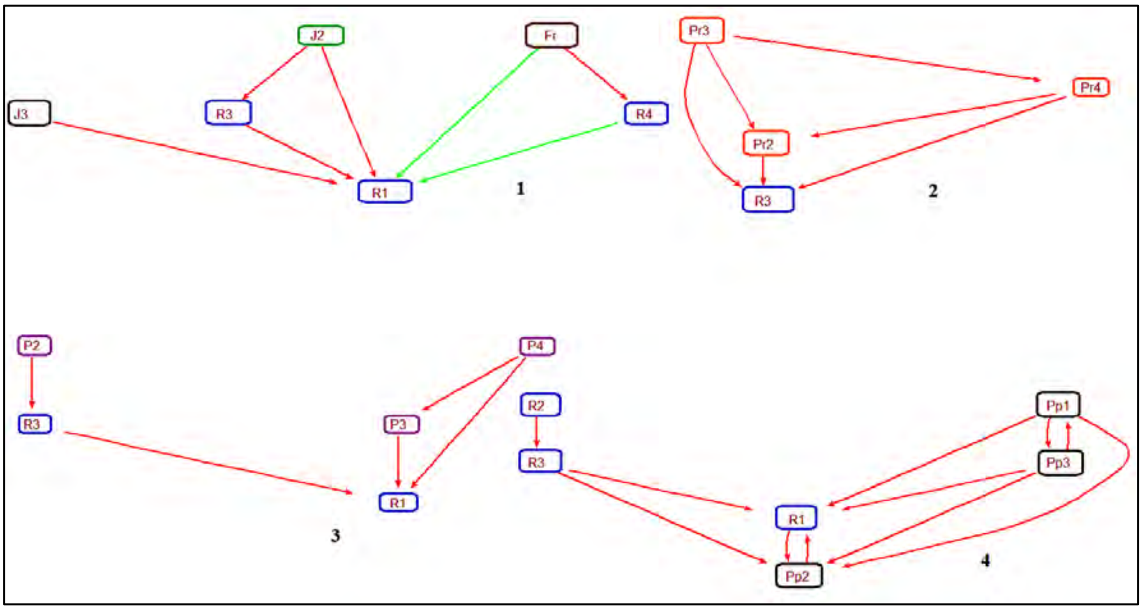


Figure 5. Implicative relationships between KoT and KSM indicators in Tasks 1, 2 and 3

The implicative graphs in Figure 6 shows relationships between different indicators of mathematical knowledge mobilized in Task 4 (with 98% significance indicated by the red arrows). Figure 6(1) shows that those PSTs that mobilize (J2) do so through the use of a geometric type of register (R3), since the auxiliary line tracing serves as a basis for justifying procedures that require decomposing or cutting surfaces. On the other hand, the geometric type (R3) implies the use of a written and symbolic type register (R1 and R4), which indicates that PSTs justify the auxiliary line tracing in writing and in addition mobilize the geometric (R3) and symbolic (R4) type registers together. This, because Task 4 had to be solved using two or more procedures (this relationship is also observed in Figures 6(3), 6(4) and 6(5)). It is observed that the joint use of (R1) and (R4) is necessary for PSTs to justify the use of the triangle area formula (J6). Figure 6(2) shows a reciprocal relationship between the properties of accumulation and additivity (Pp3) and conservation (Pp1), which indicates that PSTs mobilize them jointly in their resolutions. Both properties imply the use of the geometric type of register (R3), that is, the auxiliary line tracing that allow the decomposition and reorganization of the triangles, serves as a basis for the mobilization of properties. The use of geometric type register (R3) implies the use of the transitivity property (Pp2), which indicates that the auxiliary line tracing that allow comparisons between triangles serves as a basis for mobilizing this property.

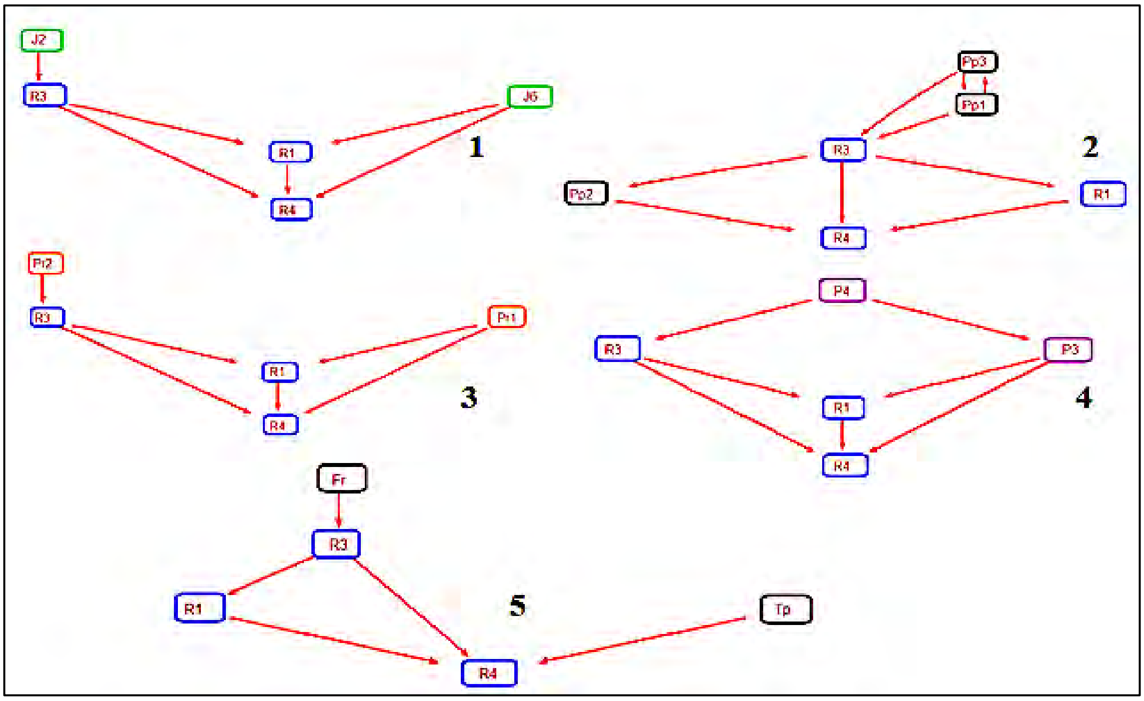


Figure 6. Implicative relationships between KoT and KSM indicators in Task 4

It is also observed that (Pp2) and (R3) imply the use of a written register (R1), which indicates that PSTs justify the auxiliary line tracing and comparisons between triangles in a written form. Figure 6(3) shows that the geometric principle (Pr2) involves the use of a geometric type of register (R3), meaning, PSTs use auxiliary line tracing to evidence that triangles that have the same height and are between the same parallels, are equivalents. The geometric principle (Pr1) implies the use of a written (R1) and symbolic (R4) type register. Therefore, PSTs justify in writing and by calculations, that a parallelogram having the same base as a triangle, both placed between the same parallels, is twice the triangle. Figure 6(4) shows that auxiliary line tracing (R3) and figure decompositions (P3) are a necessary condition for PSTs to mobilize procedures associated with isometric transformations (P4). At the same time, figure decompositions (P3) involve the use of a written (R1) and symbolic (R4) type registers, indicating that PSTs justify in writing and by means of calculations their procedures. Finally, Figure 6(5) shows that PSTs that establish auxiliary connections with procedures linked to fractions (Fr) use auxiliary line tracing to visualize relationships between the parts that make up the triangles. On the other hand, those PSTs that establish auxiliary connections with the Pythagorean theorem, do so using calculations (R4), which in turn is associated with written justifications (R1).

The implicative graphs in Figure 7 shows relationships between different indicators of mathematical knowledge mobilized in Task 5 (with 98% significance indicated by the red arrows). Figure 7(1) shows that PSTs who justify that changes in the shape of a figure do not alter its area (J3) use a geometric type of register (R3) and, at the same time, recognize that the act of cutting a surface is useful to compare areas (J2). The latter, they do so by using a written (R1) and symbolic (R4) type register since they justify the usefulness of cutting a surface in writing and through calculations. It is observed that (R1) and (R4) have a reciprocal relationship, indicating that those PSTs that use written justifications also use calculations and vice versa. In turn, those PSTs that use geometric type registers (R3) also use written and symbolic type registers (R1 and R4), which is due to the demand of Task 5 (use two or more procedures).

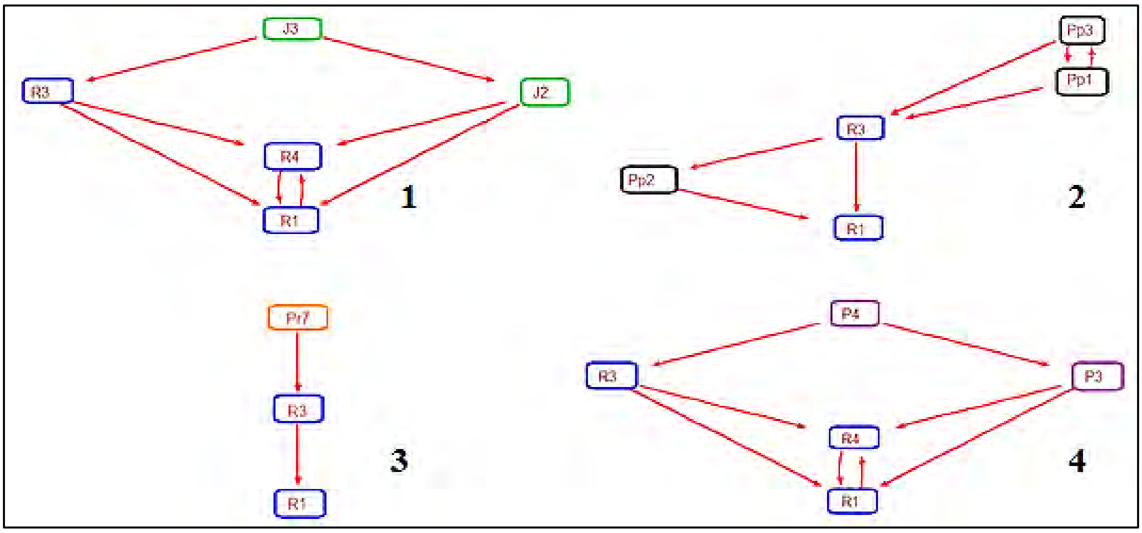


Figure 7. Implicative relationships between KoT indicators in Task 5

Figure 7(2) shows that PSTs mobilize the properties of accumulation and additivity (Pp3) and conservation (Pp1) jointly. Both properties imply the use of a geometric type of register (R3) which indicates that the auxiliary line tracing to compare, decompose and reorganize a figure, serves as a basis for the mobilization of properties. For its part, the transitivity property (Pp2) implies the use of a written type of register (R1). Thus, PSTs justify the comparison between figures in writing. The same occurs with the geometric type of register (R3). Figure 7(3) shows that PSTs that show that the area can be calculated by rearranging a figure into a simpler one (Pr7) also use geometric type register (R3), justifying the auxiliary line tracing in writing (R1). Figure 7(4) shows the same relationships as Figure 6(4). It is observed that geometric type register (R3) and figure decompositions (P3) are a necessary condition for PSTs to mobilize procedures associated with isometric transformations (P4). In addition, figure decompositions (P3) involve the use of written (R1) and symbolic (R4) type register, indicating that PSTs justify their procedures in writing and through calculations.

CONCLUSIONS

The present study sought to answer what mathematical knowledge do PSTs bring into play when solving area tasks. In order to answer this question, we aimed to characterize the components of mathematical knowledge mobilized by PSTs when solving a set of area tasks, and to identify the relationships between different components of mathematical knowledge mobilized by PSTs in the resolution of area tasks.

The results of the qualitative analysis for Tasks 1, 2 and 3 suggest that PSTs that support their procedures in some of the geometric principles (e.g., a triangle is equidecomposable to a rectangle) mobilize the properties of conservation, transitivity, and accumulation and additivity jointly. Furthermore, it is possible to see that PSTs use geometric properties and principles to support procedures related to the decomposition and reorganization of surfaces and in turn, these procedures are supported using diverse justifications (Caviedes et al., 2022a). Despite this, the mobilization of geometric principles was not present in most of the written resolutions and justifications provided by PSTs, indicating that their mathematical knowledge is limited (Chamberlin & Candelaria, 2018; Livy et al., 2012; Murphy, 2012) and that there is a gap in the specialized knowledge of PSTs.

In Task 4 PSTs have difficulties in accepting area conservation which is mainly due to the tendency towards the use of calculations and formulas, and to the unfamiliarity with non-prototypical triangles (Baturo & Nason, 1996; Caviedes et al., 2022b; Hong & Runnalls, 2020; Runnalls & Hong, 2020; Simon & Blume, 1994). However, in Tasks 1, 2, 3 and 5 the use of such property is implicit in the justifications and procedures used and PSTs do not present major difficulties. This may be due to the nature of the tasks, since by restricting the use of ruler to measure lengths (Tasks 1, 2 and 3) PST must look for alternative procedures to the use of formulas. In Task 5 it is possible that the grid facilitates the decomposition of the figures and the use of alternative procedures, that allow the mobilization of the conservation property. The strictly homogeneous, homogeneous, and heterogeneous decompositions, present in the resolutions of Tasks 1, 2, 3, 4 and 5 allow PSTs to perform a visual exploration of the figures presented in each of the tasks, an aspect that facilitates the identification of geometric properties that can be used (such as the conservation of area) and the obtainment of reconfigurations that allow responding to the demand of the tasks (Duval, 2017).

The results of the quantitative analysis indicate that the properties of conservation, transitivity, and accumulation and additivity are jointly mobilized in PSTs resolutions, which shows a certain dependence among them. The mobilization of these properties may allow PSTs to broaden their repertoire of strategies to solve tasks, since they support the procedures of mereological division and work with two-dimensional units of measurement. Thus, properties would be presented as a category of knowledge that allows PSTs to simplify a resolution process in each task. Moreover, the coordination between the discursive and non-discursive register allows PSTs to justify their resolutions in terms of what they do and why they do it. Hence, such coordination would be a necessary condition for PSTs to be able to solve area tasks mobilizing different categories and indicators of KoT and KSM subdomain. The use of both registers implies the coordination of procedures based on calculations, decompositions, and reconfigurations of figures, in addition to the justifications, properties and geometric principles mentioned above. In this context, the different registers of representation have an instrumental and organizing value within the KoT subdomain which is not yet recognized by MTSK model. That is, certain representations allow the use of certain procedures that would not be possible with the use of other representations. For example, the use of a geometric register allows PSTs to use procedures related to the decomposition and reorganization of surfaces (as well as the properties, principles and justifications that support such procedures), which would not be possible using a symbolic register.

The examples shown in the analysis and the indicators used allowed us to characterize the specialized knowledge mobilized by PSTs. The mixed approach allowed us to evidence the way in which the different indicators of mathematical knowledge are coordinated, in a context that requires comparing and calculating areas of diverse figures. Thus, the joint mobilization of different knowledge indicators shows evidence of PSTs´ specialized knowledge on area measurement and, on the contrary, the lack of mobilization of such indicators, a gap in specialized knowledge about area and its measurement processes. This joint mobilization implies a mutual reinforcing (Clements et al., 2018; Sarama & Clements, 2009) of different representation, procedures, justifications, properties, and principles, which are made explicit through intraconceptual connections within area measurement topic, and interconceptual connections between area measurement and other topics (i.e., fraction as a part whole, Pythagorean theorem). Therefore, both kind of connections would show evidence of specialized knowledge on area measurement.

The contribution of this study has a dual character. On the one hand, it presents a set of indicators that constitute the mathematical specialized knowledge needed by PSTs to solve area tasks; and on the other hand, it highlights the instrumental and organizing value of representations within the KoT subdomain. These findings could have implications in the design and strategic sequencing of tasks for PSTs trainers, who could gradually increase the mathematical elements to be developed. However, given that the study has been conducted in a single context, it is not possible to assume that the use of the indicators provided could serve as a basis for developing or enhancing specialized knowledge of PSTs from a different context. In this sense, the need for further research seems evident in order to refine or enhance the indicators proposed to characterize PSTs specialized knowledge on area measurement. Further research considering and exploring intraconceptual and interconceptual connections with different tasks would be also needed, to clarify the importance of such connections on specialized knowledge.

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Declarations

Author Contribution	:	SC: Conceptualization, Writing - Original Draft, Editing and Visualization, Formal analysis, and Methodology. GG: Writing - Review & Editing, Formal analysis, and Methodology. EB: Validation and Supervision.
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Additional Information	:	Additional information is available for this paper.

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