Journal of Pedagogical Research
Volume 7, Issue 2, 2023
https://doi.org/10.33902/JPR.202318531

Research Article

Integrating computational thinking to enhance students' mathematical understanding

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This study aimed to examine whether a computational thinking (CT) intervention related to a) number knowledge and arithmetic b) algebra, and c) geometry impacts students' learning performance in primary schools. To this end, a quasi-experimental, nonequivalent group design was employed, with 61 students assigned to the experimental group and 47 students to the control group (n = 108). The experimental group comprised students in primary school who were to be followed across the second and third grades. The experimental group underwent work with digital CT activities, while the control group did not receive such interventions. A one-way analysis of variance (ANOVA) was performed on the data gathered to assess the ability differences between students from the experimental and control groups. The pre-and post-test results revealed that the experimental group's performance was significantly better than the control group's performance for the content areas involving CT activities.

Keywords: Computational thinking; Mathematics; Technology; Robots; Primary level

Article History: Submitted 15 October 2022; Revised 6 March 2023; Published online 10 June 2023

1. Introduction

Computational thinking (CT) is central to the discussion on using technology's potential for education in many countries worldwide (e.g., Bocconi et al., 2016; Brown et al., 2014; Niemelä et al., 2017). Not surprisingly, CT has been implemented for example in the curricula of several Nordic countries (Bocconi et al., 2016). This reflects that many now acknowledge that CT is as fundamental as numeracy and literacy skills (e.g., Wing, 2006; Barr & Stephenson, 2011; Grover & Pea, 2018).

Therefore, CT has been increasingly linked to and discussed in the context of mathematics curricula in particular—that is, whether it should be a part of curricula (e.g., Weintrop et al. 2016). Integrating CT with mathematics in the classroom can create the opportunity to present authentic, real-world examples that enhance students' mathematical thinking (Weintrop et al., 2016; Pérez, 2018). However, this growing interest in integrating CT into a mathematics curriculum or classroom activities has proven to be challenging (Israel & Lash, 2020; Grover & Pea, 2018; Bocconi et al., 2016). Few studies have investigated using CT with mathematics in primary schools in Nordic countries, while multiple studies elsewhere have focused on middle, high, and college

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How to cite: Kaup, C. F., Pedersen, P. L., & Tvedebrink, T. (2023). Integrating computational thinking to enhance students' mathematical understanding. *Journal of Pedagogical Research, 7*(2), 127-142. https://doi.org/10.33902/JPR.202319187

students (Sung & Black, 2021). Further research is thus needed to understand how CT relates to mathematics in a practical classroom setting at the primary school level, including in the Nordic countries.

This study aims to examine whether a CT intervention related to geometry, algebra, and number knowledge may impact students' development in these three areas in primary schools in Denmark. The following sections present theories regarding CT and its connection to the three domains of mathematics: a) number knowledge and arithmetic, b) algebra, and c) geometry.

2. Computational Thinking and Mathematical Domains

The term *computational thinking* is attributed to Papert (1980), who argued that programming and CT support and develop students' mathematics through testing, evaluating, and correcting their ideas with coding (Papert, 1980; 1996). Papert's ideas about CT were seen to develop one's mathematical understanding, but he did not have much influence on the mathematical curriculum at the time. This may have been because digital technology had not yet developed and become a natural part of people's daily lives (Kotsopoulos et al., 2017).

Wing (2006) discussed CT about 30 years later, arguing that it should be taught on the same level as reading, writing, and math in schools. She defined CT as an approach used to solve problems, design systems, and understand human behavior founded on computer science concepts (p. 33). Wing (2006) argued that CT "represents a universally applicable attitude and skill set everyone, not just computer scientists, would be eager to learn and use" (p. 33). According to Wing's approach, CT could be perceived as part of all subjects across the school system and does not particularly relate to mathematics. Nevertheless, research has shown that participating in CT activities can benefit an individual's mathematical abilities (e.g., Clements, 2002). Various studies have been conducted on disparate mathematical topics, for example, geometry; these intervention studies have often been based on games, and programmed robots, for example, "turtle geometry" (e.g., Hickmott et al. 2018). Across countries, there is growing interest in integrating CT and mathematics; however, finding a way to integrate the two disciplines in a meaningful manner remains a challenge (e.g., Israel & Lash, 2020; Grover & Pea, 2018). The integration of CT with mathematical domains has often been considered concerning overall mathematical understanding. Studies have not investigated how CT influences individual domains in a single study. In the following sections, we briefly discuss the following three mathematical domains: a) arithmetic b) algebra and c) geometry, including how CT may be connected in these three domains.

2.1. Arithmetic in Relation to Computational Thinking

The distinctions and similarities between number knowledge, operations, and arithmetic within mathematics are complex and multifaceted (Van de Walle, 1998). When connected to CT, they become even more complex. In this section, we define *arithmetic* as the inclusion of both number knowledge and the four operations in arithmetic. Arithmetic is a domain that has a long tradition and is not merely connected to a mathematical topic in school. For example, Lakoff and Núñez (2000) argued that learning arithmetic concepts begins earlier than mathematical instruction in school and is based on everyday experiences, constructed representations, and metaphorical or symbolic thought. They emphasized that humans learn abstract concepts through concrete representations and situations, so learning and understanding numbers are based on conceptual metaphors. Lakoff and Núñez (2000) defined four arithmetic metaphors: object collection, object construction, the measuring stick, and the object along a path. We argue that all four metaphors are present in many activities associated with CT. For example, CT activities may be based on the definition of an object or construction of the objects to be worked on or may even involve a robot moving a defined distance. We note that Lakoff and Núñez's research is based on a theoretical approach to arithmetic.

Recently, research has been conducted on CT and arithmetic in empirical studies. Xu et al. (2022) found that arithmetic fluency mediates the connection between CT and reasoning ability, indicating that these three concepts might share conceptual commonalities. It has also been found that CT does not necessarily lead to an improved understanding of numbers. For example, in Chan et al. (2021), students did not significantly improve their ability to solve number patterns through CT activities. Overall, how CT and arithmetic can support or share common ground is unclear.

2.2. Algebraic Thinking in Relation to Computational Thinking

In the mathematical domain of algebra, there has been much attention to this domain within the mathematical research community during the last year. Over the last few decades, algebra and algebraic reasoning have moved down the school system from higher education to the primary school level (Kieran et al., 2016). There are multiple definitions of algebraic thinking, each stemming from discussions of the reasoning and approaches used in this domain. Studies involve a broad range of discussions, including the types of reasoning and approaches to the representation used when engaging in algebraic work, especially when defining algebra in the school context (e.g., Kaas, 2022; Kaput, 2008; Kieran et al., 2016). For example, Radford (2018) defined *algebraic thinking* as involving a) indeterminate or unknown quantities, b) culturally and historical modes of representing or symbolizing these unknown quantities, including operations, and c) analytically working with these unknown quantities. Radford emphasized the importance of the symbolic representations of unknown quantities. Another central definition targeting early algebra – that is, algebra in earlier grades– was given by Kaput (2008) in terms of three strands: a) the study of structures, b) the study of functions, and c) the application of a cluster of modeling languages. These can be found inside and outside mathematics or in the mathematics classroom. When considering these two central definitions of CT, the question is whether programming language can be perceived as a central element of CT, a culturally developed system of symbolic representations, with programming and algebra being two sides of the same coin regarding content. However, when defining CT as a broader concept (e.g., Wing, 2006), it is not narrowly connected to programming and the languages involved in this process. CT is a way of solving problems. More specifically, it can be argued that CT is a fundamental way of systematically solving problems (Wing, 2006).

This broader approach to understanding CT is supported by Bagley and Rabin (2015), who showed that CT could enhance algebra learning. Their study revealed that undergraduate students use computational modes of thinking in various creative and reflective ways when working with linear algebra, thus indicating that CT involves learning a symbolic representation of language and a broader problem-solving process.

2.3. Geometry in Relation to Computational Thinking

Studies have found and elaborated on the connection between computational stimuli and students' sense of geometry (Echeverría et al., 2019; Papert, 1996; Barcelos et al., 2018). For example, based on observations from Mindstorms (1996), which involved students playing with robot turtles, Papert (1996) found parallels between the turtle system of computational geometry and Euclid's axiomatic geometry. For example, Clements et al. (2001) found that elementary school students could learn geometric ideas by participating in CT activities.

A case study with fourth graders found that CT activities enhance students' motivation and performance in geometry (Echeverría et al. 2019). In Niemelä's (2018) study, geometry was the most popular subject when combining CT with mathematical topics: 54.7% (N = 206) of the participating teachers sketched out geometry-oriented lessons, whereas about 20% of the teachers chose topics from either algebra or arithmetic.

When considering CT, geometry is often linked to spatial reasoning (Città et al., 2019; Lee, 2019). For example, a positive correlation between CT skills and mental rotation has been found. There appears to be a logical link between spatial thinking/reasoning and geometry since spatial reasoning involves knowing the shape of one's environment, which is also essential to the mathematical understanding of geometry (Clements & Sarama, 2004). One aspect of spatial reasoning involves knowing the shape of one's environment, which is also crucial to a mathematic understanding of geometry (Clements & Sarama, 2004).

However, the definitions of the mathematical differences and similarities between geometry and spatial reasoning need further investigation. One could argue that geometry is based on a mathematical theoretical approach, whereas spatial thinking is based on a cognitive psychological framework. In the present study, spatial reasoning/thinking was considered part of developing students' understanding of geometry, emphasizing that the connection between CT and geometry is likely linked to spatial thinking and reasoning.

2.4. Present Study

Although past studies have examined CT in relation to different mathematical topics, there appear to be very few studies that have investigated the effects of CT interventions on students' progress in each of these topics in primary school. Therefore, the following question guided the present study:

> RQ) Do two groups of elementary school students, one with a CT intervention (experimental) and the other without intervention (control), differ in their learning progress in three mathematical topics: a) number knowledge and arithmetic, b) algebra, and c) geometry?

If so, we need to ask how this can be explained through students' work with digital computational thinking activities. This paper will not address this question directly, but will provide a short discussion of two interventions as a way to provide initial insights.

3. Materials and Methods

Our larger study consists of a mixed-methods sequential design. We conducted a quasi-experiment with non-equivalent groups at pre- and post-test to collect quantitative data. A control group was compared with the experimental groups, with 61 students in the experimental group and 47 in the control group (n = 108). The pre-test and post-test non-equivalent group design examined whether one group of students benefited from the intervention. Table 1 shows the experimental and control groups. By gathering information sequentially, we expand our quantitative results with observations between pre-and post-test (Creswell and Creswell, 2018). In this paper, we mainly report on the quantitative part of the study. We also provide preliminary insights into our qualitative analysis by describing two selected parts of the intervention with connections to the literature discussed above.

Table 1
Experimental design in this present study

Group		*Intervention*	
Experimental Group	Pretest	CT	Posttest
Control Group	Pretest		Posttest

The students in the two groups were pre-tested using a first-grade mathematical test before the intervention. Both groups were assessed with a third-grade post-test after the intervention. The mathematical test is explained in section 3.4.

In the experimental approach, looking at the relationship between cause and effect is fundamental. The concept of validity deals with the independent variable showing an effect in the dependent variable and that this effect is not due to other factors. The quantitative part of the study has a descriptive aim. The focus has been on whether working with CT may lead to better performance in mathematics. A causal question can be viewed here as the cause of an effect, where a cause (in this case, a CT intervention) leads to an effect (improved understanding and achievement). In this form of knowledge development, X leads to Y by viewing the cause-effect relationship (Shadish et al., 2002). From this research approach, it will be possible to draw conclusions that tell whether X leads to Y, but it may be challenging to say anything about why X has led to Y. As mentioned above, this article will only give preliminary insights into the qualitative analysis by presenting a brief description of the intervention, focusing on two examples centered on the use of robots and CT in mathematics. The article's primary focus will be to present the results from the quantitative part of the study, focusing on the students' pre- and post-tests.

3.1. Participants

The school administrations in the Northern Denmark municipalities were contacted to gain access to the schools. Teachers were recruited using purposive sampling based on the socio-economic factors of the schools, their experience with digital technology, and their commitment to mathematics education. Suter et al. (2006) found that purposive sampling could be helpful when working with a quasi-experimental approach to educational research, as there is a need for researchers and school staff to be able to work together. Table 2 sums up the students' data.

Table 2
Students participants

	Students Total	*Boys*	*Girls*	*Socio-economic status*
Intervention	61	30	31	Middle SES
Control	47	27	20	Middle SES

The intervention carried out in the 2019/2020 and 2020/2021 school years required close collaboration with teachers. Therefore, teachers need to know the expectations for participation and be willing to spend time developing their practice.

Schools that expressed an interest in participating in the intervention component all received a letter regarding the intervention. One school was accepted to participate in the intervention with three classes. Two schools then indicated they would be willing to participate as control schools; one participated with one class and the other with two. The selection itself was thus voluntary, which impacts our ability to generalize data (Creswell & Creswell, 2018). Before the study began, consent forms were obtained from the student's parents to collect observation data from the intervention classes and student test responses. This allowed subsequent analysis of the data at the student level. Consent forms were also obtained from the teachers to observe their teaching. Furthermore, all participants were provided anonymous ID numbers, and all data was labeled.

3.2. The Target Group of the Intervention

In this intervention, the 2nd-grade classes would be followed for two years. The purpose is to follow the classes over a more extended period and conduct observations and time with reflection to determine whether CT became an integral part of their teaching practice under the intervention. Additionally, the target group was chosen to study how CT could be used in mathematics in primary school since CT has been understudied at the primary level (Lee et al., 2022; Chongo et al., 2020).

3.3. Procedure

The procedure can be explained in Tables 3 and 4. The teachers in the experimental group participated in two workshops to learn about CT and technology. The first workshop focused on CT and how to relate it to mathematical content specific to 2nd grade. The second workshop had the same focus but was geared toward third-grade students. Tables 3 and 4 show how data was collected. The tables include the date of the workshop, the test, the reflection or observation, and the time in minutes. It also includes which technology was featured and what mathematical content the student is required to learn. The teachers in the experimental and control classes had the same information about the mathematical test and how to perform it.

As is shown in Tables 3 and 4, there was a change between reflection and observation in the classroom for the experimental group. The reflections work as a formative intervention, where the teachers and researcher (author number one) discuss the previous observation. This reflection helps the teachers across the three classes develop their teaching and make didactical choices on

Table 3
Data collection in the experimental group

	Workshop	*Pretest*	*Reflection*	*Observation*	*Reflection*	*Observation*	*Reflection*	*Interview*
Date	03.9.2019	September 2019 Experimental And control classes	25.9.2019	A: 29.10.2019 B: 31.10.2019 C: 01.11.2019	13.11.2019	C: 28.11.2019 A: 03.12.2019 B: 12.12.2019	29.01.2020	02.07.2020
Minutes	360		72	A: 105 B: 95 C: 105	80	C: 105 A: 125 B: 105	81	55
Technology				BeeBot		Beebot		
Mathematical content				*Numbers* The student can use multiple-digit natural numbers to describe number and sequence. The student has knowledge of the structure of natural numbers in the number system.		*Geometry* The student can categorize planar figures according to geometric properties. The student has knowledge of geometric properties of plane figures.		

Table 4
Second year of the intervention in the experimental group

	Workshop	*Reflection*	*Observation*	*Reflection*	*Reflection*	*Observation*	*Posttest*	*Reflection*
Date	01.09.2020	21.10.2020	B: 22.09.2020 C: 25.09.2020 A: 23.10.2020 30.10.2020	02.12.2020	07.04.2021	A: 04.05.2021 05.05.2021 06.05.2021 C. 07.05.2021	June 2021 Experimental And control classes	28.06.2021
Minutes	360	68	B: 60 C: 60 A: 90 + 90	53	79	A: 150 150 150 C: 60		116
Technology			Scratch Ozobot Micro:bits			Scratch Ozobot Micro:bits		
Mathematical content			*Geometry* The student can discover relationships between planes and simple spatial shapes. The student has knowledge of geometric properties of simple spatial figures.			*Numbers* The student can recognize simple decimals and fractions in everyday situations. The student has knowledge of simple decimals and fractions.		

integrating CT in their mathematical teaching. The students had five weekly mathematics lessons, and teachers integrated CT's work into the math lessons. All quotes included in the article are translated from Danish to English.

3.4. Measurements

A pre-and post-test was conducted to examine students' mathematical skills development. CT skills were not directly measured but discussed during the teacher reflection meetings. Measurements were thus made in both intervention and control schools. The pre-test was administered as a baseline during the first month of the 2019/2020 school year. The post-test was administered as an endline at the end of the 2020/2021 school year. Intervention teachers were informed that they could only begin the actual intervention part after the test.

Students' mathematical skills were measured using a standardized test designed to measure students' skills related to the curriculum and students' specific grade levels. The test measures students' skills, knowledge, and competencies in the following subject areas: number and algebra; geometry and measurement; statistics and probability (Gyldendal: Om Matematikprofilen, n.d.). The test is validated using psychometric analysis and is thus expected to provide a valid measure (Kreiner, in press). A Rasch test was conducted to assess the scale's psychometric properties and conducted various tests, including local independence, targeting, midpoint, location, item fit, and ordered response categories (Kreiner, in press).

The mathematics test provides an overall score that classifies students into five categories. The categories are used to tell the student's current level at the end of the grade. The categories are as follows: 1 started, 2 in progress, 3 well underway, 4 longer than expected, and 5 much longer than expected.

In the pre-test, students completed Matematikprofilen to grade 1, wherein in the post-test, they completed the test to grade 3. Students who perform well in the 1st-grade test will have progressed accordingly in the 3rd-grade test if they score similarly. Additionally, if the student improved his or her score from 1st to 3^{d} grade, this would indicate progress. The same student would experience a decline if, on the other hand, the student was placed at the "in progress" level. Student progress can be determined through test analysis by comparing 1st to 3rd-grade results. The tests were chosen to consider the students' natural development, as intervention and control classes are followed throughout the two grades (Creswell & Creswell, 2018). Assessments of students' mathematical proficiency are criterion-referenced and based on curriculum from the Danish ministry. The assessment consisted of 54 questions with three parts: A, B, and C. All three parts tested students' skills, knowledge, and competencies in mathematical subject areas and mathematical competencies. Tasks in part A are based on subject areas, tasks in part B are based on both subject areas and competencies, and tasks in part C are based on competencies.

3.5. Strategy for Statistical Analysis

A pre-and post-intervention test assessed each student. An overall score and topic-specific scores are calculated and standardized according to each topic's maximum obtainable overall score. Visual inspections of the standardized score indicated a sufficient fit to normality to apply ANOVA and t-tests to assess the study's hypotheses. The table below shows the kurtosis (a measure of tailedness of the distribution) and skewness computed and reported for the overall data (i.e., divided by intervened and non-intervened topics) and topic-specific values. We have tested against the theoretical values for a normal distribution. The deviations are not greater than what can be accepted for the t-test, as this test is very robust to deviations from normality. Their means are compared due to the central limit theorem.

To evaluate the hypotheses, the difference in standardized overall scores within the intervention and control groups was assessed by two paired t-tests. These differences include intervened topics (algebra, geometry, and numbers) and non-intervened topics (motion, measuring, probability, and statistics), where a significant difference would suggest an overall effect of the intervention.

Table 5
Kurtosis and skewness

Intervention	*Topic*	*Kurtosis*	*p*	*Skewness*	*p*
Overall					
Reference	Intervened topic	3.75	0.04	0.23	0.23
Reference	Non-intervened topic	2.93	0.83	0.40	0.02
Intervention	Intervened topic	2.71	0.39	0.18	0.32
Intervention	Non-intervened topic	4.15	0.01	−0.07	0.66
Topic specific					
Reference	Algebra	4.54	0.02	0.41	0.20
Reference	Geometry	2.57	0.49	0.01	0.97
Reference	Numbers	2.62	0.51	0.41	0.16
Reference	Motion	3.37	0.53	0.70	0.03
Reference	Measuring	3.26	0.65	0.22	0.44
Reference	Probability	3.27	0.66	−0.25	0.42
Reference	Statistics	2.98	0.97	0.54	0.09
Intervention	Algebra	3.43	0.42	0.29	0.30
Intervention	Geometry	2.36	0.22	−0.11	0.70
Intervention	Numbers	2.30	0.18	−0.13	0.63
Intervention	Motion	3.58	0.27	−0.02	0.93
Intervention	Measuring	2.63	0.51	0.10	0.74
Intervention	Probability	3.30	0.60	0.01	0.98
Intervention	Statistics	3.12	0.84	0.45	0.13

In addition, similar statistical comparisons were made for each topic-specific difference, where significant differences would indicate an effect of the intervention on the specific topic. This holds for both the intervened and non-intervened topics, i.e., would test for both intended intervention improvements and side effects (a general improvement from increased mathematical skills and understanding or deterioration due to e.g., increased focus on the intervened topics). A Benjamini-Hochberg adjustment was used to account for multiple testing.

Based on their overall scores, the students were categorized into five groups according to pre-specified bins on the scores. It is interesting to investigate if the intervention or other demographic variables (e.g., gender) improved the mobility probability of moving from a lower to a higher-ranking group. To assess this hypothesis, ordinal regression was used to test for the significant effects of such influences. The estimated ordinal regression can be used to predict the expected mobility probabilities, where increased probabilities for the intervention group in moving to higher categories is one way of reporting the effect of the intervention on a categorical level.

4. Results

Across the specific topics comparison of the two teaching approaches (intervention and control) showed a significant improvement in the scores for the intervention group (p-value < 0.001). The control group showed no significant improvement (p-value = 0.9).

Table 6
The overall comparison between groups

Approach	*Difference of differences*	*Reference difference*	*Intervention difference*	*p*
Intervention	13.10 (8.15; 18.04)	6.53	−6.57	0.00
Control	0.38 (−5.78; 6.53)	1.06	0.69	0.90

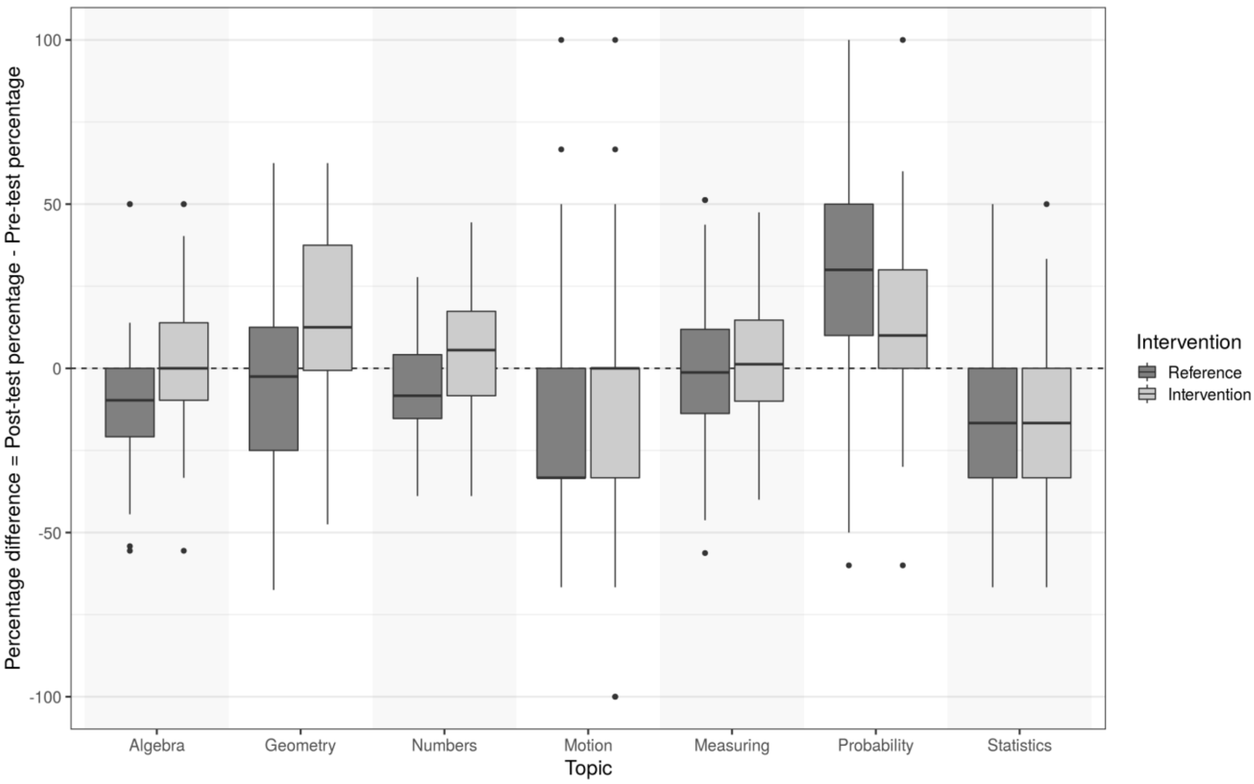
To account for multiple testing, the p-values for the individual topics were adjusted using the Benjamini-Hochberg approach (cf. Table 7). The adjusted p-values in the table show that the differences between the intervention and control groups are significant for algebra, geometry, and numbers (the intervened topics). In contrast, the differences are insignificant for the remaining topics. However, we see a close-to-significant effect (adjusted p-value = 0.08) for a worsening in the probability of the intervention.

Table 7
The p-values for each topic in the test

Approach	*Difference of differences*	*Reference difference*	*Intervention difference*	*p-adjust*
Algebra	11.85 (4.33; 19.38)	10.83	−1.02	0.01
Geometry	18.70 (7.93; 29.48)	3.95	−14.75	0.01
Numbers	8.73 (2.09; 15.37)	4.80	-3.94	0.02
Motion	4.22 (−10.05; 18.48)	13.94	9.72	0.57
Measuring	2.22 (−5.52; 9.95)	0.14	−2.08	0.57
Probability	−12.08 (−23.84; −0.31)	−28.91	−16.83	0.08
Statistics	7.15 (−2.17; 16.47)	19.09	11.94	0.18

In Figure 1, the boxplots show the differences in percentage (of the total obtainable score) between the post- and pre-tests for each student. We see that differences for the intervention group are positive for the three intervention topics. The topic probability shows a negative difference between the intervention and the reference group. However, both groups showed progress during the intervention time. In the topic statistics, both groups show a negative development with almost no difference between the two groups. In the topic, measuring, the results show almost no development in both groups during the intervention. In the last topic, motion, the results show a negative development for both groups; hence, the intervention groups' median is higher than that of the reference groups.

Figure 1
Boxplots with differences in percentage



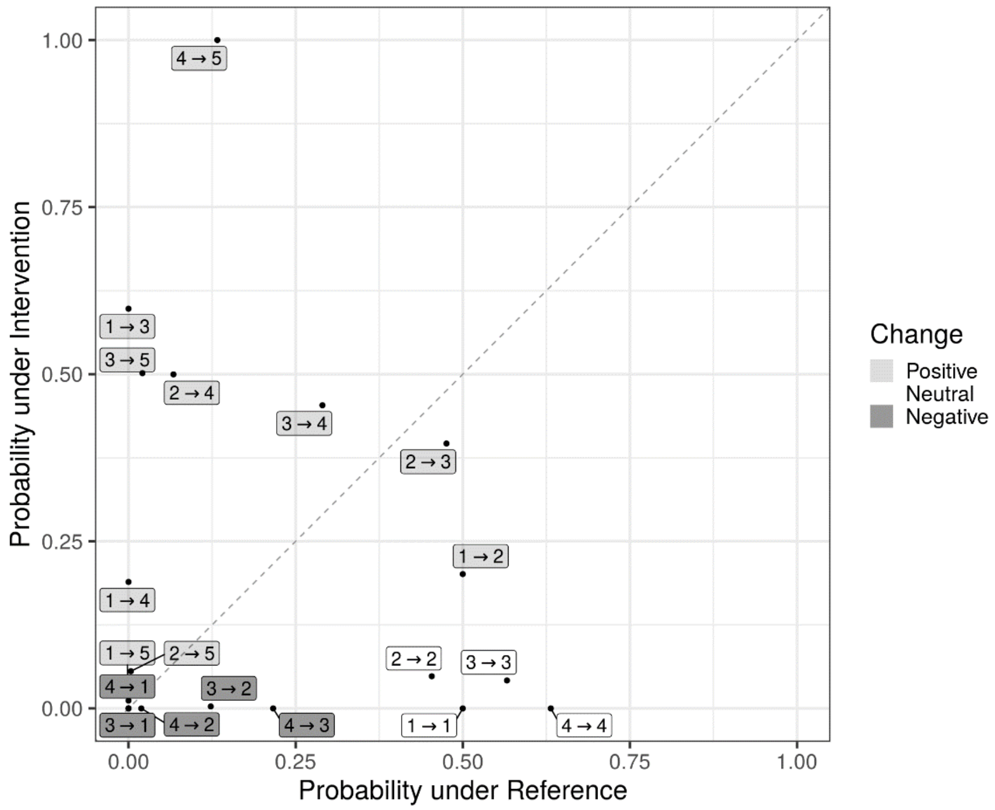
In Table 8, the empirical relative frequency of the mobility rates is given for the reference and intervention groups. The table shows that the chance of moving upwards in category (i.e., in terms of 'performance level') increases in the intervention group compared to the reference group.

Table 8
Empirical relative frequency of the mobility rates

			Post-category				
Intervention	*Pre-category*	*n*	*1*	*2*	*3*	*4*	*5*
Reference	1	4	50%	50.0%			
Reference	2	18		44.4%	50%	5.6%	
Reference	3	20		10.0%	60%	30.0%	
Reference	4	13		7.7%	23%	46.2%	23.1%
Intervention	1	1			100%		
Intervention	2	34		5.9%	38%	50.0%	5.9%
Intervention	3	23				52.2%	47.8%
Intervention	4	5					100.0%

These empirical findings are further supported by the estimated probabilities based on the ordinal regression. In the plot below, the probabilities are visualized, where light gray labels highlight the improvement probabilities, white the neutral, and in dark gray the worsening outcomes (i.e., decreasing the category). It is clear that the intervention group has a higher chance of ending up in the higher categories, and in particular, improving by more than a single category is notably higher for the intervention group.

Figure 2
The estimated probabilities based on the ordinal regression



5. An Illustration of the CT Interventions

This study examines whether a CT intervention can improve students' learning in primary schools. Results presented in the previous section suggest that it may have been the case for the Grade 2-3 classes studied. In this section, we will briefly describe selected parts of the intervention, with connections to the literature, as a way to provide preliminary insights into explaining why this may be the case.

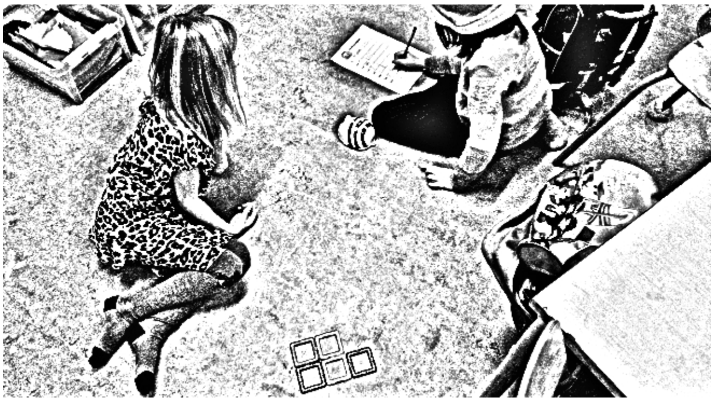
5.1. BeeBot and Geometry

The following example describes an activity from the first year of the intervention, where the student in 2nd grade worked with a robot called BeeBot. The task described in this example was the fourth task of a teaching sequence. Students worked in pairs with one robot per pair, and 10 groups were in the class.

When introducing the task, the teacher drew on students' preexisting knowledge of polygons' characteristics and previous work with the robot. The student's job was to determine, through problem-solving, whether the robot, which makes 90-degree turns, could make a series of polygons numbered 1–10. First, the students had to count the angles and sides of each polygon. Second, they had to collect square magnetic tiles from the teacher and program the robot to create the polygon. The purpose of the magnetic tiles was to increase students' ability to understand and visualize the robot's movements. We argue that the students were using pattern recognition as they decomposed the movement of the BeeBot. The tiles thus worked as a mediating artifact to help students visualize creating an algorithm for the robot to design a polygon, as seen in Figure 3. The magnetic tiles further helped the students grasp spatial thinking by working with the tiles and BeeBot (Clements & Sarama, 2004).

Within their groups, students discussed how to create the different polygons. Some students used gestures to communicate which way the robot should turn. For example, if the robot were supposed to turn to the right, the student would first move their body to the right before pushing the button to make the robot move to the right. Other students used the word "right" and gestured to the right with their bodies when the robot should turn to the right. This relates to what Sung et al. (2017) argued that an embodied approach may help make the thought process more explicit and visible. At the start of the task, most groups used the magnetic tiles. However, after the first few polygons were built, it appears that they could internalize the structure of the magnetic tiles and complete the remainder of the task without using them. The magnetic tiles, therefore, could be seen as serving as an auxiliary artifact until the students did not need them anymore. The activity became more student-centered when the students no longer needed the magnetic tiles and attempted different solutions to complete the task. To complete the assigned task, some students counted the sides of the polygons using language, whereas others used their fingers to count or draw on paper.

Figure 3
Students on the floor working with BeeBots and magnetic tiles



For this task, we argue that students used CT to relate to the robots and then created an algorithm to instruct each robot to make a polygon. In this case, the robots act as a mediating artifact through which students may learn to describe and make representations of different polygons. The robots help students shift between different semiotic representations, both polygons made using the robots and drawings by hand (Barcelos et al., 2018).

The instructional task allows students to think mathematically using CT practices such as pattern recognition and decomposition in connection to geometry (Pei et al., 2018; Clements &

Sarama, 2004). The end of the lesson included a shift back to a teacher-centered perspective as the teacher guided the students in identifying the task's pattern; this helped the students establish a relationship and identify patterns between the different polygons (Barcelos et al., 2018). This last step enabled the teacher to identify students' misconceptions and support them in developing a deeper understanding of the concepts. During a class discussion, each group demonstrated and drew the polygons from the task on the blackboard. One group was convinced that the robot could make a nine-sided polygon, and they were asked to make an argument for their solution. During their presentation on the blackboard, the students discovered that they had been drawing an eight-sided polygon instead of a nine-sided one. In this example, the students used evaluation and argumentation to determine whether their solution was correct. Argumentation was then used to help students identify a pattern in the creation of the polygons. With help from the teacher, the students observed that BeeBot could only make polygons with even sides and right angles. The students, therefore, learned that the robot could make polygons with four, six, eight, or 10 sides. Through the final discussion, the students learned how to describe and represent models of different polygons with the help of the robots by using mathematical and algorithmic language that, according to Barcelos et al. (2018), explains and reveals the student's understanding of the problem-solving process.

5.2. Micro:bit and Calculation

In the second year of the intervention, students worked on making a calculator with a micro:bit. Students worked together in pairs and followed an introduction through video. On one computer, the students watched the video; on the other, they created an algorithm through block programming in the program MakeCode. As the student watched the video, they could stop at any time, make the same block as shown, and then create the algorithm step-by-step. Figure 4 shows two students creating the calculator in MakeCode on one computer and watching a video on the other.

Figure 4
Two students creating the calculator in MakeCode

After making the algorithm, the students tested it in MakeCode before downloading it on the micro:bit. After completing the download to the micro:bit the student got an assignment from the teacher. As seen in Figure 5, the teacher presented the assignment on the smartboard and paper. One student had to calculate multiplication through micro:bit, and the other had to do it by hand. Some students found that the input and display of numbers were not optimal on the micro:bit. If the product exceeded two digits, entering and reading the display took too long. Likewise, some students found they were faster than the micro:bit at doing the calculations in this case. In this lesson, students were introduced to using variables to store data and as the results of mathematical operations. As Lakoff and Núñez (2000) note, students learn via concrete representations such as the micro:bit as a calculator. We argue that this task also illustrated students' CT thought process since students could recognize when it is more efficient to let the computer do the work or calculate it themselves.

Figure 5
The teacher presents the assignment

During the intervention, the teachers noticed that some students were growing in the classroom. "...there are certainly some children who come up with the oral because they think you are talking to a toy rather than doing something right or wrong. Some kids are beginning to use mathematical language" (Teacher, C class, CL-2). The teachers also reported that some students talked more about math than they usually do in their ordinary math lessons.

6. Discussion and Conclusion

This paper presents a quasi-experiment with pre- and post-test. Our quantitative results (e.g., those summarized in Table 8 and Figure 2) showed that the intervention group significantly differed, compared to the reference group, in their performance on the tests about the 'intervention topics', namely: numbers, algebra, and geometry. These results suggest that students benefitted from the CT intervention. Furthermore, in the intervention group, students were more likely to move up one or more categories (i.e., performance levels) from the pre-test to the post-test than in their control group. These results are consistent with those reported by Bagley and Rabin (2015), who found that CT could enhance algebra learning. Especially when using various creative and reflective approaches, thus implying that CT involves learning a symbolic representation of language and a wider problem-solving approach. Xu et al. (2022) also argue that arithmetic fluency mediates the link between CT and reasoning ability, indicating a conceptual similarity between these three concepts. As in geometry, Echeverría et al. (2019) found that CT activities may enhance motivation and performance. By contrast, Chan et al. (2021) found, in their research involving Secondary One students, that CT activities did not significantly improve the students' ability to solve number patterns. As one can see, studies related to CT and mathematics have produced divergent conclusions, which could potentially be explained by numerous factors, such as the type of CT task studied, the pedagogical context, etc. Nevertheless, several studies led to suggest that CT can help mediate and create representations that students must relate to when working with technologies. This can be accomplished through embodiment and explanations to other peers (Sung & Black, 2021). In the study by Sung and Black (2021), it was found that working with computational perspectives, including robots, could help make thought processes more explicit and transparent. We argue that this could explain why, in our study, all students in the intervention class moved one or more categories on the mathematical test.

A limitation of quasi-experimental studies is that randomization is not used, which limits the ability to conclude causal associations between interventions and outcomes (Creswell & Creswell, 2018). However, our study suggests a significant effect on the specific topics that the students in the study had worked on in their math lessons. The teachers in this project were self-selected both in the intervention and control classes. To avoid selection bias, the intervention and control teachers completed a survey to map their knowledge of using CT and robots in an educational context. This was to ensure they were at the same level of expertise when working with technology. Both groups of teachers had only limited or no previous knowledge of using CT or robots in their teaching. Preliminary insights into this intervention may suggest an increase in students' mathematical knowledge: teachers reported observing that students, when working with robots, create new relationships and a new division of labor in the classroom, and that they may use mathematical expressions more precisely when using robots. They also observed that students showed greater motivation when working with robots and collaborated more than usual during math lessons. These processes, however, should be studied in more depth in future studies.

Author contributions: All authors are agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Declaration of interest: No conflict of interest is declared by authors.

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