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Teaching Partial Differential Equations With Didactic Situations

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Abstract

This article shows results on the simulation of partial differential equations such as the heat equation, the Laplace equation and the wave equation in a course called mathematical methods at a university in Bogotá Colombia.. In this course, the concept of partial differential equations and different solution methods were explained to them, both in the analytical and numerical part, the importance they have in the physical sciences, the exact sciences and many of their different applications in engineering, since students They are in the fifth semester of the mechatronic engineering degree, then under the methodology of didactic situations they were presented with a particular situation where by developing code in the mathematics and electronic simulation software called Matlab, some of these could be simulated,giving different values to constants and describing their behaviorjust as it was possible to visualize some results of the main partial differential equations, with quite significant results.

Keywords: Partial differential equations, simulation, didactic situations.

1. Introduction

One of the approaches to teaching and learning mathematics is called didactic situations, which is part of the French school of mathematics education, which was created under the direction of the Mathematician Guy Brousseau in 1995, as an alternative of solution to the problems that at that time faced what was called modern mathematics (García 2021).

This paper aims to show how a didactic situation can be applied in a mathematics class at the mid-career university level that leads the student to understand a complex concept such as partial differential equations, particularly the partial equations of wave, heat and Laplace.

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For Brousseau, "a situation is called a model of interaction of a subject with a certain environment that determines a given knowledge, as the resource available to the subject to achieve or maintain a favorable state in this environment" (Brousseau 2007). Therefore, to reach these situations, some previous concepts are required, which for our research are the explanation by the teacher of the concepts of the partial equations that are intended to be modeled, but there are also others where it is intended that the student through from a given situation build your new knowledge.

2. METHODOLOGY

The methodology used for the development of this research was qualitative since it is recommended in the research work of mathematics education.

A didactic situation was applied in a university institution to fifth semester students of a subject called mathematical methods, groups of three students were made and they were asked to carry out the following application:For the frontier value problems of partial differential equations of the heat, wave and Laplace equations, a simulation of these must be carried out using the MATLAB package, the graphs of the surfaces must be visualized, for , giving different values to the constants and describe their behavior.$u(x, y)$

In the case of the Laplace function, the maximum values that can take must be shown.$u(x, y)$

In the case of the heat equation, you must show the graphs of some partial sums and describe their behavior.

In the case of the wave equation, approximations to the first eigenvalues must be found.

The students had 20 days to carry out and present the results and in this work the results of one of the groups are shown.

3. RESULTS AND DISCUSSION

Next, two procedures are presented to solve partial differential equations that frequently arise in problems involving vibrations, potentials, and temperature distributions. These problems are called boundary value problems and are described by second-order partial differential equations, which are relatively simple. What is done is to find the particular solutions of a partial differential equation by reducing it to two or more ordinary differential equations.

For the frontier value problems of partial differential equations of the heat, wave and Laplace equations, a simulation of these must be carried out using the MATLAB package, the graphs of the surfaces must be visualized, for , giving different values to the constants and describe their behavior.$u(x, y)$

Laplace's partial differential equation, in this case the maximum values that it can take must be shown.$u(x, y)$

Laplace's equation in two and three dimensions occurs in time-independent problems involving potentials such as electrostatic, gravitational, and velocity in fluid mechanics. Furthermore, a solution of Laplace's equation can also be interpreted as a steady-state temperature distribution. One solution could represent the temperature that varies from point to point, but not with time, of a rectangular plate. Laplace's equation in two dimensions and in three dimensions is abbreviated as$u(x, y)$. $\nabla^2 u = 0$, where and$\nabla^2 u = \frac{d^2u}{dx^2} + \frac{d^2u}{dy^2}$ $\nabla^2 u = \frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2}$

Next, the work carried out by a group of three students is shown, which, when using Matlab with the following code, which consists of a thermally insulated thin plate with a rectangular domain, for which initial conditions are needed, which are the different temperatures possessing the plate on each side. The objective of this is to observe the intersections inside their temperatures.

The Laplace equation in its general form (Zill 2008):

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < a, \quad 0 < y < b$$
$$\frac{\partial u}{\partial x}|_{x=0} = 0, \quad \frac{\partial u}{\partial x}|_{x=a} = 0, \quad 0 < y < b$$
$$u(x,0), \quad u(x,b) = f(x), \quad 0 < x < a$$

```
function y=Ecuacion_LaPlaces_Toledo_Melo_Pareja_Fonseca(ua,ub,uc,ud,n,m,maxiter,error)
clear u;
for i=1:n+2
        u(i,1)=uc;
        u(i,m+2)=ud;
end

for j=1:m+2
        u(1,j)=ua;
        u(n+2,j)=ub;
end p=(ua+ub+uc+ud)/4;
for i=2:n+1
        for j=2:m+1
              u(i,j)=p;
  end
  end

k=0;
conv =0;
while k<maxiter & conv==0
        k=k+1;
        t=u;
        for i=2:n+1
              for j=2:m+1
                    u(i,j)=0.25*(u(i-1,j)+u(i+1,j)+u(i,j+1)+u(i,j-1));
              end
        end
              if norm((u-t),inf)/norm(u,inf)<error
                    conv=1;
        end
end
if conv==1
        disp(u)
        disp(k)
        [x,y]=meshgrid(1:m+2,1:n+2);
        surf(x,y,u)
        shading flat
end
```

Figure 1, Laplace's PDE code

The following figure shows both its maximum and minimum points, likewise it also shows how the temperature is distributed and its variation.

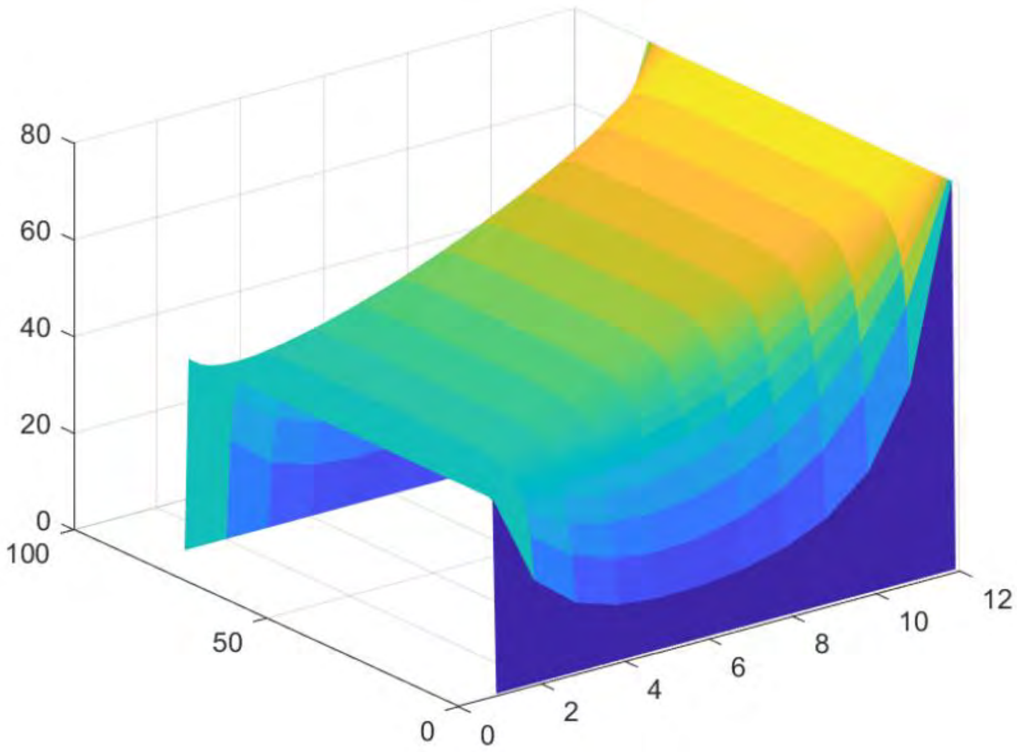

Figure 2, Extreme points (own source)

heat equation,is a partial differential equation in which students show their behavior by simulating some constants of their general equation:
For the analysis of the behavior of the heat equation, it was carried out based on a rod of length 0.5, with a coefficient of thermal diffusivity of 79733 corresponding to steel. Where for the extreme left it has a temperature of T1= 100°C and the extreme right a temperature of T2=100°C, in the same way the quantity of 95 terms to add in the sum is taken. It starts at the instant t=0, and the temperature of the rod at a point x is given by: $f(x) = 100 * x^2$
Then they present the visual representation of , which represents the temperature at an instant t. $u(x, t)$

$$u(x,t) = \frac{1}{L}(T_2 - T_1)x + T_1 + \sum_{n=1}^{\infty} B_n sin\left(\frac{n^*\Pi^*x}{L}\right)e^{-\left(\frac{n^*\pi^*c}{L}\right)^2 t}$$

$$B_n = \frac{2}{L}\int_0^L (f(x) - (T_2 - T_1)x + T_1)sin\left(\frac{n^*\Pi^*x}{L}\right)dx$$

The code elaborated by the students in Matlab is shown below:

```
%Ecuacion de calor - Santiago melo - Angie toledo - Daniela Pareja - Camilo Fonseca clear, clc
syms x n t
disp('Graficar la ecuacion de calor')
f = 100*x^2;
T1 = 100; %temperatura en el extremo izquierdo
T2 = 100; %temperatura en el extremo derecho
T = 0;
L = 0.5; %longitud de la varilla
c = 79733; %Coeficiente (ACERO)
z = 95;

disp(' ')
Bn = (2/L)*int((f-1/L*(T2-T1)*x-T1)*sin(n*pi*x/L),x, 0, L);
Bn = subs(Bn,{sin(2*n*pi),sin(n*pi),cos(n*pi),cos(2*n*pi),sin(n*pi/2)^2,cos(n*pi/2)^2},{0,0,(-1)^n,1,(1-(-1)^n)/2,(1+(-1)^n)/2});
disp(' ')
disp('Bn* = ')
pretty(simplify(Bn))
 disp(' ')
disp(' ')
disp(' u(x,t) = ')
pretty(Bn*sin(n*pi*x/L)*exp(-(n*pi*c/L)^2*t)+(x/L)*(T2-T1)+T1)
x = 0 :0.01: L;
s_f = (1/L)*(T2-T1)*x+T1+symsum( Bn*sin(n*pi*x/L)*exp(-(n*pi*c/L)^2*T), n, 1, z); h = plot(x, s_f);
set(h, 'color','red', 'LineWidth', 1.5);
grid on, xlabel('Posicion'), ylabel('Temperatura'), hold on
```

En la siguiente imagen se observa que la varilla cuenta con temperaturas en los extremos de 100° C y en un punto x está dado por = $100 * x^2$:

Figure 3. EDP heat code (own source)

The hypotheses that were considered were the following: a rod of length L with an initial temperature f(x) throughout the rod and at the ends of the rod the temperature is kept at zero all the time, t > 0. The partial differential equations with their boundary conditions are (Nagle 2001):

$$k\frac{\partial^2 x}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < L, \;\; t > 0$$
$$u(0,t) = 0, \qquad u(L,t) = 0, t > 0$$
$$u(x,0) = f(x), \;\; 0 < x < L$$

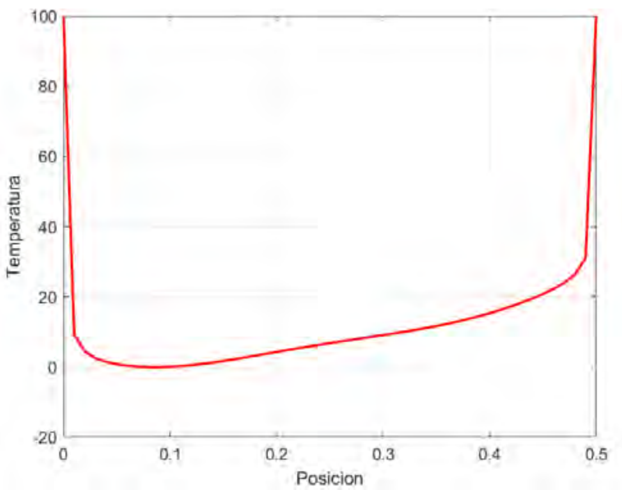

Figure 4. Position (own source)

Partial wave equation.

For this partial differential equation, the students foundapproximations to the first eigenvalues. The assumption is a string of length L, like a guitar string, stretched between two points on the axis, for example, at at L . When the string begins to vibrate, we assume that the motion is in the plane in such a way that each point on the string moves in a direction perpendicular to the axis (transverse vibrations). Since the string is said to be perfectly flexible, it is homogeneous, that is, its mass per unit length p is a constant. Their displacements are small compared to the length of the string, the tension T acts tangent to the string and its magnitude is the same at all points and no other external force acts on the string.$xx - 0 - y\ x - Lxux$

The equations are given by (Zill 2007):

$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad 0 < x < L, \quad t > 0$$

$$u(0, t) = 0, \quad u(L, t) = 0, \quad t > 0$$

$$u(x, 0) = f(x), \quad \frac{\partial u}{\partial t}\Big|_{t=0} = g(x), \quad 0 < x < L$$

Your code in Matlab is:

```
%Ecuacion de onda - Santiago melo - Angie toledo - Daniela Pareja - Camilo Fonseca

clear all
%Datos en x
a=0;  b=5;  %Longitud  del  cable  L=b-a.  h=0.1;
%Paso de la onda
x=a:h:b;      %Discretización      espacial      del      cable.
N=round((b-a)/h);

 %Definimos las matrices de la ecuación ME=x(2:N);
ME=ME';
 ua=0;ub=0;         %Condiciones          de          contorno.
 U0=zeros(size(ME)); %Preasignación de U0.

%Recorremos mediante un bucle U0, y añadimos los valores que correspondan.
 for j=1:length(ME);
          if ME(j)<b/3
            U0(j)=3*ME(j)/b;
        end
 else end

U0(j)=1.5-1.5*ME(j)/b;

V0=zeros(size(ME)); %Preasignación de V0.

%Matriz K
K=1/h^2*(2*diag(ones(1,N-1))-diag(ones(1,N-2),-1) -diag(ones(1,N-2),1));

%Término F y valor inicial
F=0*ME;
F(1)=F(1)+ua/h^2;
F(end)=F(end)+ub/h^2;

%Resolución del sistema de ecuaciones de EDO de orden 1.
t0=0;tM=40;
k=h; %Paso en t.
t=t0:k:tM; %Discretización del vector de tiempos.
M=length(t)-1; %Número de subintervalos.

%Añadimos en la primera columna las condiciones iniciales.
U(:,1)=U0;
V(:,1)=V0;
        for i=1:M
              %Resolución del sistema de ecuaciones por el método del trapecio
              U(:,i+1)=(eye(size(K))+0.25*(k^2)*K)\(U(:,i)+0.5*k*(2*V(:,i)+0.5*k*(-K*U(:,i)+2*F)));
             V(:,i+1)=(eye(size(K))+0.25*(k^2)*K)\(V(:,i)+0.5*k*(-K*U(:,i)+2*F)-0.5*k*K*(U (:,i)+0.5*k*V(:,i)));
        end
```

Graph 5. EDP wave code (own source)

In addition, the students included the Dirichlet conditions:

```
%Incluimos          condiciones          Dirichlet.
UA=ua*ones(1,length(t));
UB=ub*ones(1,length(t));
U=[UA;U;UB];

%Dibujamos      el      gráfico.
[Mt,Mx]=meshgrid(t,x);
mesh(Mx,Mt,U)
xlabel('Longitud [m]'); ylabel('Tiempo [s]'); zlabel('Posición [m]');
```

Figure 5. EDP code, Dirichlet conditions. (Own source)

```
%Dibujamos el gráfico.
[Mt,Mx]=meshgrid(t,x);
mesh(Mx,Mt,U)
xlabel('Longitud [m]'); ylabel('Tiempo [s]'); zlabel('Posición [m]');
```

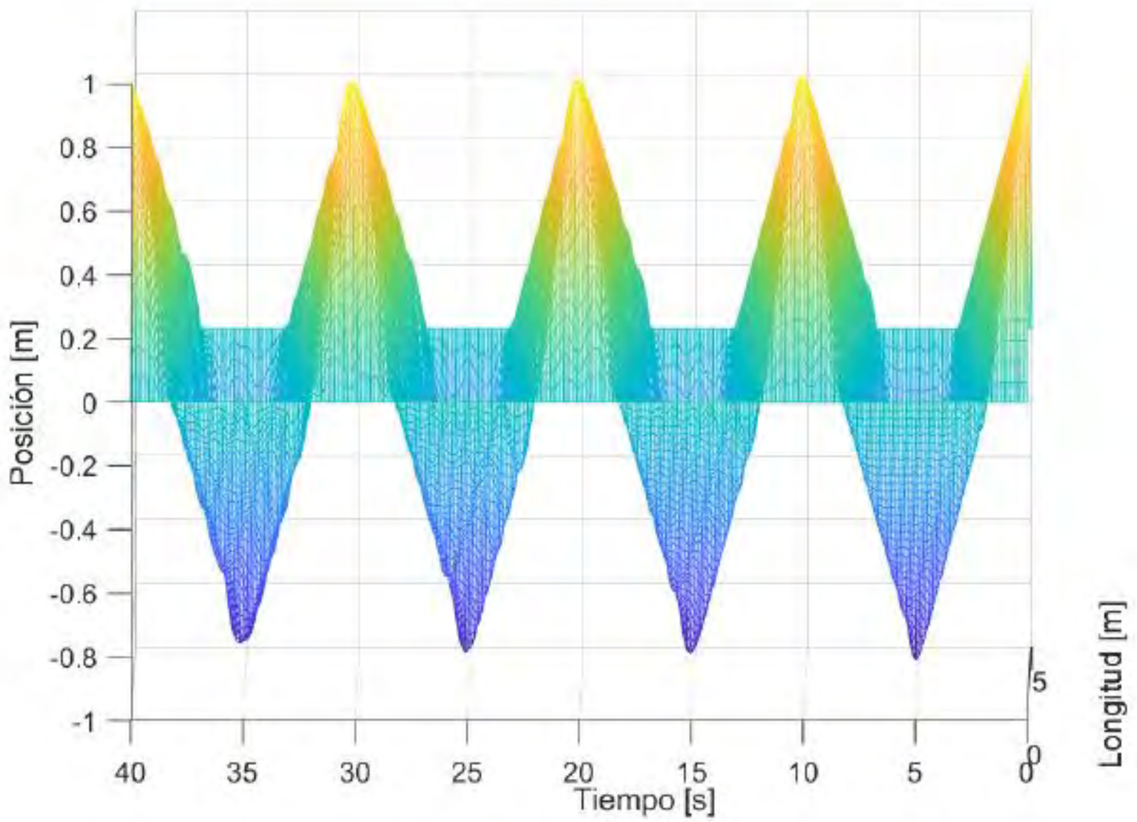

Figure 6. Position of EDP onda (own source)

4. Conclusions and Discussions

- In this project it was possible to observe that Matlab software is used for research and to solve practical problems in engineering and mathematics, with great emphasis on signal processing and control applications.

- With the programs carried out and with the results obtained, we can see that these equations can be implemented in engineering problems where ordinary differential equations and partial differential equations must be solved.

- With the graphs obtained, it can be seen that both the heat equation, the wave equation, and Laplace's show their maximum and minimum points. These motivate the study of the existence of non-trivial radial solutions.

- Didactic situations should be used in projects in which the student can put into practice the theory seen in class.

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