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The sequence of algebraic problem-solving paths: Evidence from structure sense of Indonesian student

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Abstract

The algebraic structure is one of the axiomatic mathematical materials that consists of definitions and theorems. Learning algebraic structure will facilitate the development of logical reasoning, hence facilitating the study of other aspects of axiomatic mathematics. Even with this, several researchers say a lack of algebraic structure sense is a source of difficulty in acquiring algebraic structures. This study aims to examine a pattern of sequences of problem-solving paths in algebra, which is an illustration of learners' algebraic structure sense so that it can be utilized to enhance the ability to solve problems involving algebraic structure. This study employed a qualitative descriptive approach. Students who have received abstract algebra courses were chosen to serve as subjects. The instruments include tests based on algebraic structure sense, questionnaires, and interviews. This study reveals the sequence of paths used by students in the structure sense process for group materials, i.e., path of construction–analogy (constructing known mathematical properties or objects, then analogizing unknown mathematical properties or objects), path of analogy–abstraction (analogizing an unknown mathematical property or object with consideration of the initial knowledge, then abstracting a new definition), path of abstraction-construction (abstracting the definition of the extraction of a known mathematical structure or object, then constructing a new mathematical structure or object), and path of formal-construction (constructing the structure of known and unknown mathematical properties or objects through the logical deduction of a familiar definition). In general, the student's structure sense path for solving problems of group material begins with construction, followed by analogy, abstraction, and formal construction. Based on these findings, it is suggested that there is a way for lecturers to observe how students develop algebraic concepts, particularly group material, so that they can employ the appropriate strategy while teaching group concepts in the future.

Keywords: Algebraic Structure Sense, Group, Mathematics, Path Sequence

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Algebra is one of the branches of mathematics that is still considered difficult by students viewed from their way of solving algebraic problems (Schüler-Meyer, 2017; Novotná & Hoch, 2008; Wasserman, 2017). Accordingly, Souminen (2018) and Wilburne and Long (2010) state that one of the students' difficulties in solving algebraic problems is their inability to connect basic algebra concepts with the algebra at the college level. This issue arises because some algebraic materials are abstract and difficult to describe mathematically (ÇETIN, 2021). On the other hand, algebra is the key to learning other branches of mathematics (Bone et al., 2021; Ralston et al., 2018). Furthermore, Kamacı (2021) explain that one of the algebraic studies is algebraic structure. This algebraic structure is one of the axiomatic mathematical materials, and because it instills logical reasoning, it will be very useful for learning more difficult aspects of mathematics (Usiskin, 2001). Algebra is therefore one of the subjects that students find challenging, even though it is necessary for mastering more complicated mathematical studies.

Widodo et al. (2020), Nu (2019), and Hwang et al. (2019) explain that the difficulties experienced by students in solving algebraic structure problems can be traced based on the sequence of algebraic problem-solving paths. The sequence of algebraic problem-solving paths indicates the student's structure sense ability (Taban & Cadorna, 2018). In line with this, Hoch and Dreyfus (2010) explain that this structure sense is used to analyze the use of algebraic techniques learned earlier by students. Furthermore, Hwang et al. (2019) and Novotná and Hoch (2008) explain that learning designed with the characteristics of student structure sense can facilitate cognitive processes, thus the students' cognitive development can be optimally developed. Therefore, the use of student characteristics based on structure sense can be used as a foundation for determining effective learning strategies in algebra courses. The characteristics of students' abilities in this study were based on the structure sense category (SSE-1, SSE-2, SSE-3, SSE-4, SSP-1, SSP-2, SSP-3, SSP4, and SSP-5).

Hoch (2003) suggests that structure sense is a collection of abilities, separate from manipulative ability, which enables students to make better use of previously learned algebraic techniques. Accordingly, Hoch and Dreyfus (2010) argue that structure sense is the ability to see an algebraic expression or sentence as an entity, recognize an algebraic expression or sentence as a previously met structure, divide an entity into sub-structures, recognize mutual connections between structures, recognize which manipulations it is possible to perform, and recognize which manipulations it is useful to perform. Furthermore, Novotná and Hoch (2008) explain that the students' structure sense ability can be reflected in the sequence of solving algebraic problems. Therefore, the inability of students in constructing the structure of mathematical properties or objects can be judged by how well they can solve algebraic problems.

The importance of the structure sense process in the thinking scheme helps the construction of components of a scheme connected in a network of mathematical connection capabilities, as well as the growth of taste or sensitivity to the structure of properties or mathematical objects or structure sense through habituation (Junarti et al., 2019). The structure of mathematical properties or objects formed through the process of habituation at the stage of the path which includes extraction, analogy, and construction, to the process of abstraction can build the ability of mathematical connections (Junarti et al., 2020). There are at least three paths (Junarti et al., 2020) that can be traversed in the stages of extraction, analogy, construction, and abstraction in mathematical thinking (Novotná & Hoch, 2008). The first path is extracting known structures to form the basis of definitions, from which abstract concepts are constructed in a general context. The second path is extracting properties from known structures leading to generalizations and then to definitions. The third path is constructing the concepts through the logical deduction of the student definition. Thus, this study aims to help students develop their structure sense and find alternative ways to help them solve problems in group materials. One of the theoretical urgencies of this research is to develop a path developed by Novotná et al. (2006) (a discovery path to facilitate how students construct concepts that Novotna has not mentioned). The problem was that some students did not use the path set by Novotna during the pre-survey. Furthermore, the problems that must be investigated are related to the sequence of paths, namely, to find alternative sequences that can be used to improve the ability to solve algebraic structure problems. While the urgency of the application can be seen after the lecturer knows how the sequence of paths is constructed, the lecturer can provide appropriate learning according to the path followed by students. The objective of this study was to describe how students structure sense sequences in learning the structure of group material.

From the above description, it is critical to conduct studies on how the path sequence of the student's structure sense in learning the structure in the group material. This is critical so that in the future, lecturers can implement the appropriate learning method when presenting structures to group materials, allowing students to better understand the material given.

METHODS

Research Procedures

This qualitative descriptive research aims to describe how the sequence of the student's structure sense path in learning the structure in the group material. The process of recognizing structure sense, dealing with group prerequisite materials, which includes sets, binary operations, and their properties by using three paths as trials have been conducted in the previous study (Junarti et al., 2020). According to Novotná et al. (2006), the reference of structure sense used to develop pathways in group concepts is using the following identifications.

a. SSE: (Structure Sense as Applied to Elements of Sets and the Notion of Binary Operation)
   A student understands the structure sense if s/he can
   (1) (SSE-1) describe elements of the set as objects to be manipulated, or understand closed properties
   (2) (SSE-2) describe binary operations in known structures
   (3) (SSE-3) describe binary operations in "unknown" structures
   (4) (SSE-4) describe similarities and differences of shapes and define operations (formulas, tables, others)

b. SSP: (Structure Sense as Applied to the Property of Binary Operations)
   A student understands the structure sense on the property of binary operations if s/he can
   (1) SSP-1: (a) describe the identity elements by definition; (b) describe the inverse element by definition
   (2) SSP-2: describe the relationship between identity elements and inverse elements
   (3) SSP-3: (a) relate commutative properties to describe identity elements; (b) relate commutative properties to describe inverse elements
   (4) SSP-4: (a) describe the quantitative sequence of writing identity elements as well as inverse elements; (b) maintain the quality of the sequence of describing identity elements and inverse elements
   (5) SSP-5: mention the knowledge of identity elements and inverse elements spontaneously

This study identified the stages of structure sense introduction in group materials by adopting three paths from Novotná et al. (in Oktac, 2016, p. 311) and one additional path to the structure sense process that occurred in the student's thinking. In addition, Table 1 shows the results of previous studies focusing on path analysis study.

Explanation of V_A, V_B, and D in Table 1: V represents a property or an object, A represents a familiar structure, B represents a non-familiar structure, and D represents a formal definition (Novotná et al., 2006). The two approaches presented in Table 1 by Simpson and Stehliková (2006) are: (1) through the definition of concepts given first with the expectation that students will observe examples as different examples of general definitions, and (2) through an example-based approach with the intent of generalizing. While one additional path is one of the paths that students might take to get at the process of logical deduction, students can also arrive at the process by other paths. This analysis uncovered this approach as a potential theoretical explanation. Following is a description of the $D \rightarrow V_A \rightarrow V_B$ process design for the preparation of group materials using the Structure Sense process.

Table 1. Previous Studies of Structure Sense Process Framework Theory

Research by	Structure Sense Process Path Type for Groups			
	$V_A \rightarrow D \rightarrow V_B$	$V_A \rightarrow V_B \rightarrow D$	$D \rightarrow V_A, V_B$	$D \rightarrow V_A \rightarrow V_B$
Dubinsky et al. (1994)	√			
Harel and Tall (1991)		√		
Stehlíková (2004)			√	
Simpson and Stehlíková (2006)		√	√	
Novotná et al. (2006)	√	√	√	
Oktac (2016)	√	√	√	
Additional path				√

Figure 1 explains the design of the structure sense process for the group material, which starts as follows.

1) Specific considerations from the results of previous research on structure sense and the processes or stages of recognizing structure sense in abstract algebra, for example, from studies by Linchevski and Livneh (1999), Hoch (2003), and so on.
2) Then reviewing previous research specifically comparing the process path of structure sense from the study of Harel and Tall (1991), Dubinsky et al. (1994), Stehlíková (2004), Novotná et al. (2006), Oktac (2016), and Junarti et al. (2020) that the stages carried out by students when carrying out the concept construction process on binary operations, models for groups, which involve binary operations, sets, and coordination through axioms that are much more complex.
3) Furthermore, from the results of the literature review process in points 1 and 2 the results of the structure sense process path for the group are obtained as a form of a theoretical formulation consisting of 3 types called the structure sense process draft path as a framework in research.
4) The next stage is the expert validation process related to the structure sense process path in group material in the form of content and construct validation, as well as field trials (pre-research), followed by task-based interviews with research subjects.
5) Based on the process in step 4) above, a revision of the structure sense path is obtained for group material consisting of 4 types called the structure sense process path improvement.
6) Next, the structure sense process path for the revised group material is validated by sources from subjects whose answers are the same and method validation through interviews and group material test work results.

Finally, the theoretical hypothesis about the four paths of the structure sense process for group material is obtained.

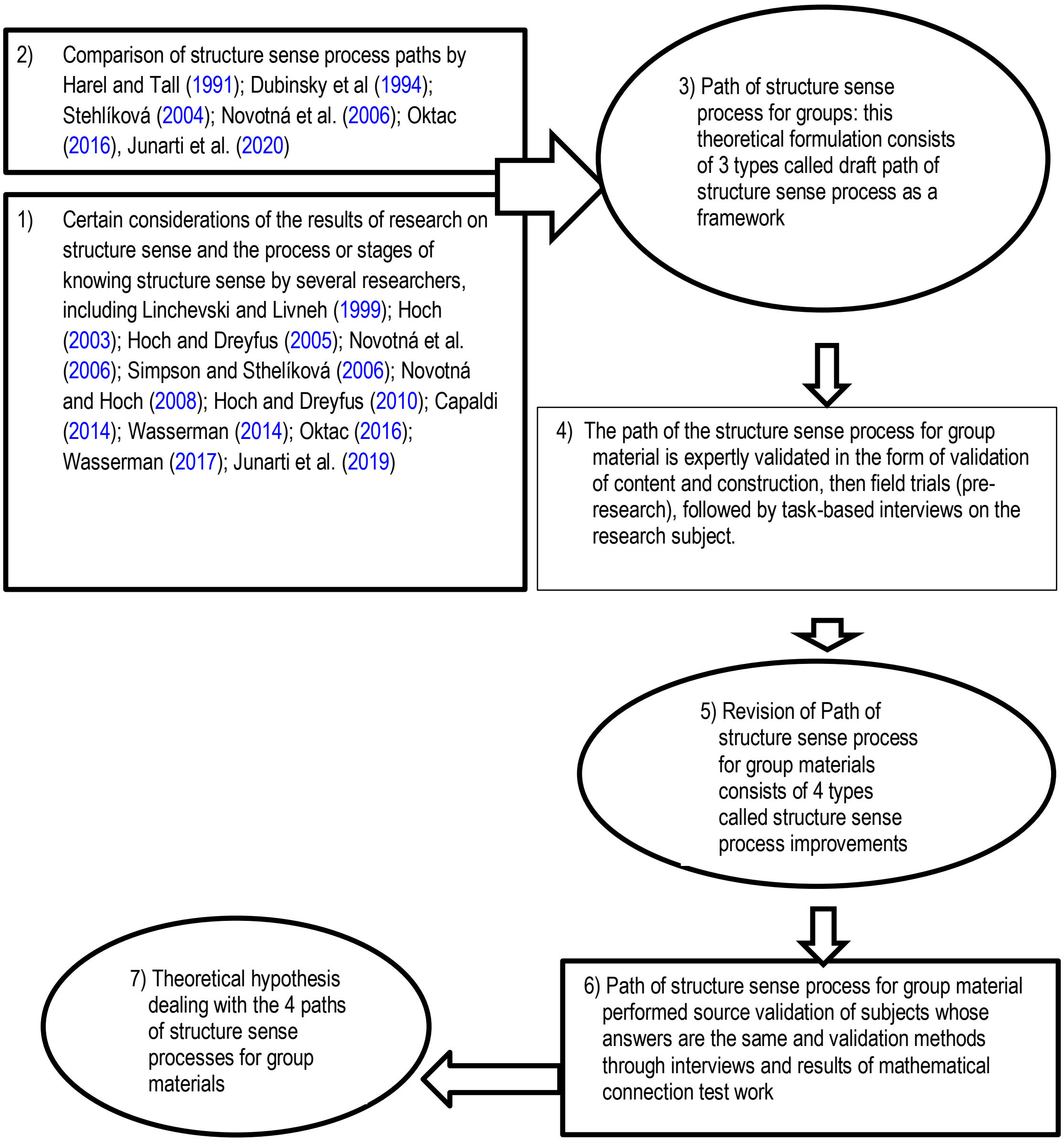

Figure 1. Design of Path Drafting of Structure Sense Process for Group Material

Notes:

▭ : a process of path analysis from literature revie

◯ : the beginning or end of the analysis results process

Subject

Respondents in this study were fifth-semester students who had taken an abstract algebra I course. It was studied in abstract algebra courses because it is a course consisting of complex structures, and based on the two authors' previous studies, there is a tendency for students to be unable to recognize the structure of set elements because the structure of the elements is not in the form of numbers, so it is done numerically (Junarti et al., 2019), as well as the tendency for students not to be able to abstract definitions to construct structures of properties or mathematical objects that are well known and newly known (Junarti et al., 2020). Then, for the determination of research subjects, it was considered that those who attended abstract algebra I course, those who performed individual tasks, took examinations, completed questionnaires about paths, were able to communicate, and were willing to serve as research subjects.

The subjects were chosen based on four types of paths, three paths proposed by Novotná et al. (2006) and Oktac (2016) and one path as a test track (additional path). References for the four paths: three paths by Novotná et al. (2006) and Oktac (2016). One path is a development path that is still in the form of a hypothesis, so further testing is still needed. Four experts have tested the validations, constructs, and empirical tests of these four paths on students of mathematics education study programs in the previous. Subject was chosen using a fixed comparison of two subjects from each of the four path types (Cresswell, 2017).

Prospective subjects in this study were students who participated in the abstract algebra course I in the fifth semester of the Mathematics Education Study Program at IKIP PGRI Bojonegoro Indonesia, with as many as 32 students. Then, the 32 students selected 26 students who attended lectures regularly, worked on individual assignments, answered test questions, and answered questionnaires according to the path indicators. Finally, two students were taken per track with the criteria of having complete answers, a good level of communication, and the willingness of prospective subjects to be interviewed Furthermore, the results of the questionnaire work were sorted (reduced) based on the same answers and according to the four types of paths used by the subjects so that the tendency of the subjects to use path-1 and path-2 was obtained, namely M-7, M-8, M-10, and M-15; the tendency of subjects to use path-3 namely M-19 and M-21, the tendency of subjects to use path-4 namely M-2, M-4, M-12, M-13, M-17, M-20, M-23, and M-26. For each of these subjects, two subjects were selected from each trend of the path used and who were able to communicate and were willing to be interviewed to become respondents, so six subjects were selected (M-8, M-15, M-2, M-12, M -19, and M-21). Because no completeness of the work was found from the test questions covering nine categories (SS and SSP) in subject M-7, the subjects representing the path-1 and path-2 groups were subjects M-8, M-10, and M-15.

Furthermore, the communication skills of the three subjects, M-8, M-10, and M-15, were studied in the good category, then the willingness to be interviewed by these three subjects; there was one subject, M-10, who was not willing to be a respondent. While the subjects who entered represented path 3, the same process was carried out, indicating that both subjects had communication skills and were willing to be interviewed. Furthermore, for subjects who could represent path 4, the same process was carried out, selecting M-2 and M-12 who were willing to be interviewed and had good communication skills.

These four paths started from reviewing the previous research literature on the needs of students in constructing concepts in binary operations. These group models involve many binary operations, sets, and coordination through much more complex axioms. The description of the three paths adapted from Novotná et al. (2006) and Oktac (2016) is a reference for describing the fourth path draft. First path Description: Students can extract familiar mathematical structures or objects to abstract definitions in constructing new/ unknown mathematical structures or objects (Oktac, 2016). Second path description: Students can extract known mathematical property structures or objects through analogy to construct new or unknown mathematical structures or objects, they can only abstract definitions (Oktac, 2016). Third path description: Students can construct structures of known and unknown mathematical properties or objects through logical deductions from definitions known to students (Oktac, 2016). The fourth path draft description: Students can construct the structure of a known mathematical property or object through a logical deduction from the definition, then can only analogize the structure of a new or unknown mathematical property or object. The three paths and one draft path were carried out in the process of compiling the instrument grid, which contains the formulation of predictive indicators for each trait and mathematical object from the nine categories of structure sense. In addition, expert validation tests were carried out in terms of content and constructs, and empirical tests were carried out on mathematics education study program students of IKIP PGRI Bojonegoro in the previous class.

To obtain valid data using the fixed comparison method, a minimum of 2 subjects were selected for each path to represent the path. In this fixed comparison analysis method, the researchers constantly compared one data with another and then compared categories with other categories. The background for selecting subjects in each path was adjusted according to the level of completeness of the answers, communication, and the prospective subjects' willingness to be interviewed.

Each path was interpreted through student questionnaire work, carried out when students worked on three test questions on group material (through the formulation of predictive indicators for each of the nine structure sense categories). Path-1 was used when students chose the available options in the questionnaire using the path-1 scheme ($V_A \longrightarrow D \longrightarrow V_B$). At the same time, students worked on group material test questions from nine categories (SSE-1, SSE-2, SSE-3, SSE-4, SSP-1, SSP-2, SSP-4, SSP-5). Similarly, apply the same calculations to paths 2, 3, and 4.

Data Collection

Task-based instruments, questionnaires, interviews, and description tests were employed in this study. The role of the test instrument on group material is to explore the abilities of the nine SSE and SSP categories, which simultaneously facilitate the description of which types of pathways lead students in constructing the structure of their mathematical properties or objects. The role of the questionnaire instrument about paths is to categorize the types of students go through when carrying out their thought processes in solving group material problems (which contain the nine SSE and SSP categories). The role of the interview is to clarify and deepen the results of the test instrument and the questionnaire results as a form of method triangulation and, at the same time, source triangulation between two subjects in the same path category. Qualitative analysis was used to analyze data (Cresswell, 2017). The questionnaire about paths consisted of 22 questions with four choice alternatives covering four paths. The test instrument was prepared by adapting the descriptions of the nine SSE and SSP categories, then distinguishing three forms of set presentation (sets written in the form of membership requirements, number sets, and bounded sets which are presented symbolically), then combined with standard, non-standard, and certain defined binary operations. Furthermore, the questionnaire instrument was prepared by adapting the descriptions of Novotna's three paths based on the previous study (Junarti et al., 2020). At the same time, the interview guidelines were prepared based on the needs contained in the test instrument and questionnaire. For the instrument test, expert validation was carried out both in terms of content and constructs and empirical validation.

The question of group material test consisted of three items of description questions and was arranged based on prediction indicators of structure sense process in three distinct set forms, i.e., $= \{x \mid x = 2k + \sqrt{p}, k \in \mathbb{Z}, p \in \mathbb{N}\}$, $S = \{e, p, q, r\}$, and $\mathbb{N}$, which are the set of natural numbers. Furthermore, it is chosen on different binary operations, such as the standard addition operation, the "$\oplus$" operation defined on "$x \oplus y = x + y - 5$" $\forall x, y \in \mathbb{N}$, and the binary operation "$\circ$" defined in Figure 2.

∘	e	p	q	r
e	e	p	q	r
p	p	q	r	e
q	q	r	e	p
r	r	e	p	q

Figure 2. Cayley table on item 3

The explanation in Figure 2 is the Cayley Table for the definition of the binary operation "∘" in item 3. It shows the path flow preparation chart used as part of the structure sense process. The figure shows the stages of study that went into making the structure sense process path. Starting with the stages of comparing the structure sense process path from different expert studies, the theoretical formulation can be used as a framework for the structure sense process path based on the results of previous research studies. Also, experts validate the path's content, the path is tested in the field, and task-based interviews are done. Based on the results of the validation, the tests, and the interviews, the path was changed to four paths that were revised. Then, there four paths were validated. Source validation was done based on the subject whose answers are the same, validation methods were done through interviews and test work results. In the last step, the four paths of the structure-sense process in the group material were used to come up with a theory.

Data collection was carried out separately with each instrument which was carried out collectively. The instruments used in this study were questionnaires about paths, tests about groups, and interviews. Meanwhile, the collection of data about paths was carried out through collective path questionnaires. Next, the path of the questionnaire work was described for each student following the number of questionnaire questions that describe each of the nine categories of structure sense from the three group material questions. Furthermore, the tendency of the pathways of each student to be selected was sorted to describe each of the four types of pathways. Furthermore, for each of the four types of paths, two subjects were selected using the same path to represent research subjects. Furthermore, each research subject must trace their work and test results to obtain data about the concept construction process of group material (using nine categories of structure sense). Then to deepen or to confirm the data, the interview with six research subjects was conducted.

Before the questionnaire was carried out, students were given instructions regarding work instructions and four alternative choices involving four paths. In the instructions, an imperative sentence is given "Write your thoughts on each question by placing a tick (√) in the column of the answer sheet that fits the following criteria." Furthermore, in the instructions, four alternative path options are presented in the form of a schematic and description. For example, in the choice "a): $V_A \xrightarrow[Abstraksi]{} D \xrightarrow[Konstruksi]{} V_B$ In working on the problem, I think of examples/structures of properties/mathematical objects that are already known (which already exist) to be able to abstract the basic definition, to be able to construct structures of properties/mathematical objects that are newly recognized". For example, in the choice "b): $V_A \xrightarrow[Analogi]{} V_B \xrightarrow[Abstraksi]{} D$ In working on the problem, I think of examples/structures of properties/mathematical objects that are already known as an analogy in constructing structures of properties/new objects, only then can I get to know the definition." The instructions also explain the symbols used in the four-line scheme. In the following, an example of a questionnaire is presented in point 1, point 2, and the instructions for doing it are as follows.

Question 1: I can describe the structure of the set elements in the form of a structure that I am already familiar with because what I was thinking at that time was ....

Question 2: I can describe the elements of the set that represent the set written with membership conditions because what I thought at that time was ....

Questions:

1) It is known that $Q = \{x \mid x = 2k + \sqrt{p},\ k \in \mathbb{Z},\ p \in \mathbb{N}\}$ where $\mathbb{Z}$ is the set of integers and $\mathbb{N}$ is the set of natural numbers, showing that Q with the standard addition operation is a group!
2) Given: $x \oplus y = x + y - 5\ ; \forall x, y \in \mathbb{N}$, $\mathbb{N}$ is the set of natural numbers.
   a. Does N with the binary operation "$\oplus$" satisfy the closed property? Explain!
   b. Does N with the binary operation "$\oplus$" satisfy the associative property? Explain!
   c. Does N with the binary operation "$\oplus$" have an identity element? Explain!
   d. Does N with the binary operation "$\oplus$" have an inverse element? Explain!
   e. Is N with the binary operation "$\oplus$" a group? Give a reason!
3) 3. Given: $S = \{e, p, q, r\}$ and the binary operation "$\circ$" which is defined in the following Cayley Table:

Cayley Table

$\circ$	e	p	q	r
e	e	p	q	r
p	p	q	r	e
q	q	r	e	p
r	r	e	p	q

Prove that the set S with the binary operation "$\circ$" forms a group!

Each of the nine SSE and SSP categories formulated its predictive indicators on the test item grid. For example, it is said to refer to SSE-1 (description: recognize the elements of the set as objects to be manipulated/ or recognize closed properties) if, for example, in answer to part of item 3 (Take any $x, y \in$

$\mathbb{N}$, with x ≥ 3 and y ≥ 3 and so on) then the prediction indicator formula is a) "Because, can describe $, y \in \mathbb{N}$ and the elements chosen x ≥ 3 and y ≥ 3 represent the set $\mathbb{N}$", b) then "Because, it can describe $x, y \in Q$ and those elements $x = 2k + \sqrt{p}$ and $y = 2m + \sqrt{p}$, with $, m \in \mathbb{Z}$, $p \in \mathbb{N}$ is representing a known set", c) then "Because, it can describe $e, p, q, r \in S$ and the elements are representative of the known set.

RESULTS AND DISCUSSION

According to the findings of a three-path trial of the structure sense process, students prefer to know the structure sense in the group prerequisite material through the second path, followed by the first path, and finally the third path (Junarti et al., 2020). Based on 22 questionnaires regarding path selection in familiarity with 9 categories of structural sense, 23% selected the second path, 4% selected the first path, 9% selected the third path, 35% selected other alternatives that repeated examples of questions, and 29% abstained. In the form of problem examples, students continue to rely on the structure of known mathematical characteristics or objects. Students have a propensity to be unable to abstractly characterize the structure of previously known and newly discovered mathematical qualities or objects.

Table 2 provides an overview of the findings of questionnaires on the path taken by students who are familiar with the structural sense of group concepts.

Table 2. Distribution of the number of students by Path Type and Structure Sense Category

Structure Sense Category	Test Item	Path Type and Number of Students				
		Path 1	Path 2	Path 3	Path 4	Abstain
SSE-1	(1)	6	7	1	4	8
	(2)	4	7	4	5	6
	(3)	4	3	4	9	6
SSE-2	(1)	3	6	1	10	6
SSE-3	(2)	1	4	3	12	6
	(3)	1	4	4	11	6
SSE-4	(3)	-	6	2	11	7
SSP-1	(1)	-	6	2	11	7
	(2)	2	3	1	14	6
	(3)	1	1	3	14	7
SSP-2	(1)	-	4	1	14	7
	(2)	-	6	-	13	7
	(3)	-	3	3	12	8
SSP-3	(1)	-	3	3	13	7
	(2)	-	4	2	13	7
	(3)	-	6	3	10	7
SSP-4	(1)	1	5	3	10	7
	(2)	4	8	-	10	4
	(3)	2	2	3	14	5
SSP-5	(1)	1	2	4	11	8
	(2)	2	11	-	8	5
	(3)	1	2	2	16	5
		33	90	49	245	142

Based on the distribution shown in Table 2, path 4 has been utilized 245 times by students to learn about the nine categories of group concept structure sense. Path 2 accommodates up to 90 students, Path 3 up to 49 students, and path 1 up to 33 students, in that sequence. This distribution demonstrates that the subject makes extensive use of path 4. Path 3 should be employed in mathematics, even though path 4 is frequently chosen by students. This indicates that students have begun to develop the ability to make logical inferences (i.e., the logical deduction stage). Path 4 (construction to the analogy), Path 2 (analogy to abstraction), Path 1 (abstraction to construction), and Path 3 (construction to the analogy) are used to understand the subject's work (formal construction). In general, the sequence of the student sense structure process for comprehending the group material is as different as the following: constructions, analogies, abstractions, and formal constructions.

The Analysis of the Work of Subjects Who Often Utilize Path 4

Subjects M-2 and M-12 fall into the category of those who are more likely to utilize path 4. The results of the questionnaire, group material test and results of interviews with M-2 and M-12 are summarized in the following section.

Figures 3a and 3b show the results of the path questionnaire from the subjects of M-2 and M-12, respectively, with a summary of the response distribution.

SSE-1 : J-4, J-4, J-3
SSE-2 : J-4
SSE-3 : J-4, J-4
SSE-4 : J-3
SSP-1 : J-3, J-4, J-4
SSP-2 : J-4, J-4, J-4
SSP-3 : J-4, J-4, J-2
SSP-4 : J-4, J-4, J-4
SSP-5 : J-4, J-4, J-4

Figure 3a. Path Distribution of M-2

SSE-1 : J-3, J-4, J-4
SSE-2 : J-2
SSE-3 : J-3, J-3
SSE-4 : J-2
SSP-1 : J-2, J-3, J-4
SSP-2 : J-4, J-2, J-4
SSP-3 : J-4, J-2, J-4
SSP-4 : J-4, J-4, J-4
SSP-5 : J-4, J-4, J-4

Figure 3b. Path Distribution of M-12

Note:
J: represents the path; J-1: represents path-1; J-2: represents path-2; J-3 represents path-3; J-4: represents path-4

Explanation of Figure 3a and Figure 3b is the distribution of paths from the results of questionnaire work for subjects M-2 and M-12 when carrying out the work process on the group material test questions given. Furthermore, in Figure 3a and Figure 3b the abbreviations SSE-1, SSE-2, SSE-3, SSE-4, SSP-1, SSP-2, SSP-3, SSP-4, and SSP-5 are explained as follows.

Based on the distribution of pathways taken by subjects M-2 and M-12, both tended to take the same path, notably path-4. Subject M-2 used path-3 (3 times), path-4 (18 times), and path-2 (once) out of 22 questions. Subject M-12 utilized path-2 (5 times), path-4 (12 times), and path-3 (4 times). Path-1 is not used by M-2 or M-12. This demonstrates their proclivity to carry out the process of structure sense identification by extracting existing structures, abstracting definitions, and then creating novel structures. The results of both subjects' surveys were validated by the results of test work related to the structure sense they tread on.

Items 1 and 3 were correctly answered by both M-2 and M-12 in their test results. In the case of item 2, both make mistakes, resulting in the screenshot that highlights a problem with how something was done. M-2 and M-12 subject test results are shown in Figures 4a and 4b in the following section.

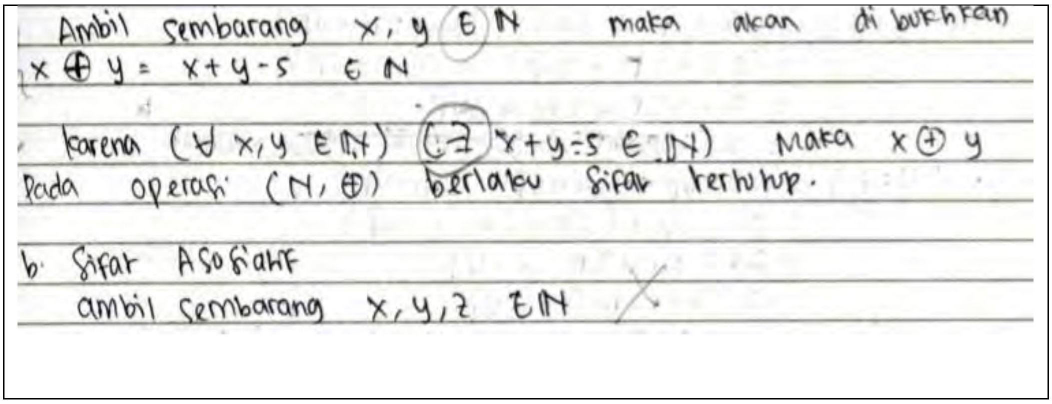
Ambil sembarang $x, y \in \mathbb{N}$ maka akan di buktikan
$x \oplus y = x + y - 5 \in \mathbb{N}$
karena $(\forall x, y \in \mathbb{N})$ $(\exists x + y - 5 \in \mathbb{N})$ Maka $x \oplus y$
Pada operasi $(\mathbb{N}, \oplus)$ berlaku sifat tertutup.
b. Sifat Asosiatif
ambil sembarang $x, y, z \in \mathbb{N}$

Figure 4a. Answer of M-2

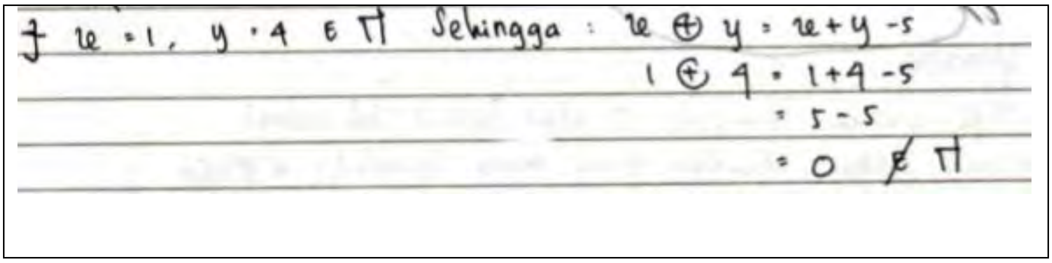
$\exists\ x = 1,\ y = 4 \in \mathbb{N}$ Sehingga : $x \oplus y = x + y - 5$
$1 \oplus 4 = 1 + 4 - 5$
$= 5 - 5$
$= 0 \notin \mathbb{N}$

Figure 4b. Answer of M-12

Figure 4a and Figure 4b are group material test answers from subjects M-2 and M-12 in item 2, which use the same path (namely path-4) in recognizing set elements (SSE-1) and use different paths in the same category, namely in SSE-3.

According to the findings on item 2, both M-2 and M-12 subjects familiarize themselves with the set element through the same path. M-2 and M-12 subjects utilize path-4 when recognizing the structure sense of SSE-1 and paths 4 and 3 when familiar with SSE-3. These distinct paths are employed to identify non-standard binary operations. Both subjects in Figures 4a and 4b have erroneous answers. So that Figures 4a and 4b show students failing to use path-4. Students whose habits in developing the structure of mathematical properties/objects still rely on examples of similar problems to understand abstract definitions are influenced by the various ways of understanding structure sense. The two subjects of their works are unable to pick the set elements that meet $\mathbb{N}$ when the subject realizes the structure of the $\mathbb{N}$ set elements connected with non-standard binary operations expressed in the formula "$x \oplus y = x + y - 5$". Both subjects demonstrated that the picked element did not reflect the alignment of the set components that meet the binary operational structure; hence, both subjects failed to use path-4 in the recognition process.

The following interview excerpt shows the results of triangulation methods, which were subsequently validated by interviews. (R = Researcher)

Interview excerpt with M-2:

R : Why do you use path-4 to learn about binary components and operations at item 2?
M-2 : Because I assumed path-4 was complete when I had the definition of a binary operation, ma'am.
R : What comes to mind when you consider the meaning of binary operations? (Pointing to binary operations on question 2)

M-2 : Every time I take an element from the set of natural numbers, the result will always be in the set of natural numbers.
R : Why do you think so sure about this?
M2 : Yes, Ma'am, I am certain of it.
R : What if the chosen element has $x = 1$ *and* $y = 1$ *and, or if* $x = 2\ and\ y = 2$, *or* $x = 2$ *and* $y = 4$?
M-2 : Yes, indeed. Why didn't I think so?

Interview excerpt with M-12:

R : Why do you use path-4 and path-2 when you are familiar with binary elements and operations regarding item 2?
M-12 : Because I saw the definition of a binary operation in the right place.
R : What comes to mind when you consider the meaning of binary operations?
M-12 : I take the elements of the set of natural numbers, I choose 1 and 4, then the result is 0, even though 0 is not an element of the set of natural numbers
R : Why do you think so sure about this?
M-12 : Because one element is missing, it may be deduced that the natural number does not include everything.
R : What if the chosen element has $x = 1$ *and* $y = 5$ *and, or if* $x = 5\ and\ y = 1$, *or* $x = 3$ *and* $y = 3$?
M-12 : oh yeah. I now understand
R : Ok, thank you, then.

Based on the excerpt of the interview results, both subjects showed the same error in item 2, namely in choosing a set element whose characteristics should be limited, but after the interview, it turned out that the two subjects had different reasons. The inability of both subjects is equally in choosing set elements that are not considered with the enforceability of their binary operations. Thus the initial conclusion of the work of the two subjects in getting to know structure sense (1) equally has not been able to abstract the definition of binary operations that are not standard properly; (2) the nonconformity of the path used with the subject's understanding of point 2 in the process of knowing structure sense in the categories of SSE-1, SSE-3, SSP-1, SSP-2, SSP-3, SSP-4 and SSP-5; (3) failure to work on the question of point 2 is not caused by a path error, but the structure sense of the set element associated with the binary operation is not well known; (4) both subjects are equally dominant with a tendency to use the 4-line in the process of knowing the structure sense; (5) path-4 used by both subjects in getting to know structure sense, tending to be from familiar or unfamiliar structures; (6) path-3 and path-2 used by both subjects in getting to know structure sense tend to be from familiar or unfamiliar structures; (7) path-1 is not used at all by either subject, it indicates the subject is trying not to extract known structures in abstracting definitions.

The Analysis of the Work of Subjects Who Often Utilize Path 2

Figures 5a and 5b show a summary of the distribution of answers to the findings of the path questionnaire work from M-8 and M-15 subjects. They show the distribution of the paths used by subjects M-8 and M-15 through the results of their questionnaire work when carrying out the work process on the three items on the group material test given. Based on the path distribution depicted in Figures 5a and 5b, subject M-8 tended to select path-2 (12 times out of 22 questions), path-4 (9 times), path-1 (once), and path-3 not at all. While the subject M-15 tended to choose path-2 (11 times), path-4 (6 times), path-1 (5 times), and path-3 was not chosen at all.

Both subjects, M-8 and M-15, responded to questions 1 and 3 are both true and false. Regarding item 2, both subjects tended to provide numerous incorrect answers. In this section, answers with several errors will be offered to illustrate the tendency of structure sense that cannot be gleaned from the path used to learn structure sense.

SSE-1 : J-2, J-2, J-4
SSE-2 : J-4
SSE-3 : J-4, J-1
SSE-4 : J-4
SSP-1 : J-2, J-2, J-4
SSP-2 : J-2, J-2, J-4
SSP-3 : J-4, J-2, J-2
SSP-4 : J-2, J-2, J-4
SSP-5 : J-2, J-2, J-4

Figure 5a. Path Distribution of M-8

SSE-1 : J-1 J-1, J-1
SSE-2 : J-1
SSE-3 : J-4, J-2
SSE-4 : J-4
SSP-1 : J-4, J-1, J-4
SSP-2 : J-2, J-2, J-2
SSP-3 : J-4, J-2, J-4
SSP-4 : J-2, J-2, J-2
SSP-5 : J-2, J-2, J-2

Figure 5b. Path Distribution of M-15

Note:
J: represents the path; J-1: represents path-1; J-2: represents path-2; J-3 represents path-3; J-4: represents path-4

In addition, the findings of the path questionnaire depicted in Figures 5a and 5b will be examined in conjunction with the test results of both individuals because of triangulation techniques. Both subjects were exposed to incorrect test task fragments to identify categories of structure sense that could be learned. The following are screenshots showing the work of subjects M-8 and M-15 on question number 2.

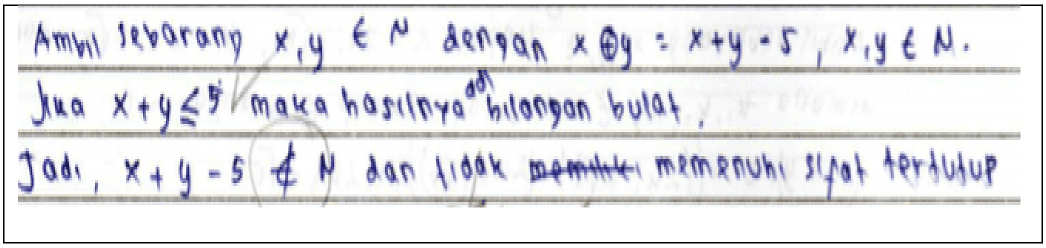
Ambil sebarang x,y ∈ N dengan x ⊕ y = x+y-5, x,y ∈ N.
Jika x+y ≤ 5 maka hasilnya adl bilangan bulat.
Jadi, x+y-5 ∉ N dan tidak ~~memiliki~~ memenuhi sifat tertutup

Figure 6a. Answer of M-8

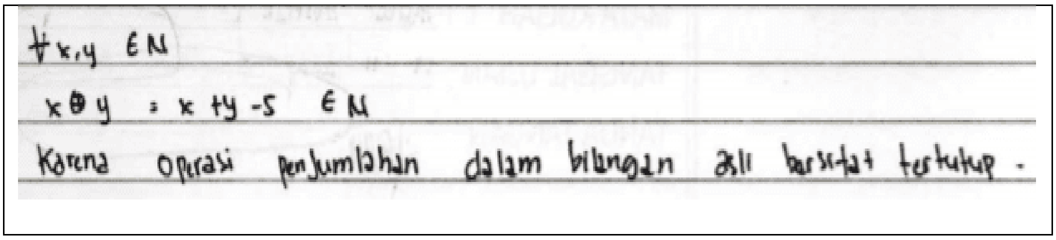
∀x,y ∈ N
x ⊕ y = x+y-5 ∈ N
Karena operasi penjumlahan dalam bilangan asli bersifat tertutup.

Figure 6b. Answer of M-15

Figure 6a and Figure 6b are parts of group material test work from subjects M-8 and M-15, who made mistakes in proving the closed nature of item 2 by using the same path, namely path-4. The test results of both subjects were wrong while answering questions about closed nature. Both subjects use the same path-4 to learn about non-standard binary operations, which indicates that both have attempted the abstraction process while familiar with the definition of binary operations but have not yet reached the correct answer. M-8 uses path 2 to become familiar with set elements, while M-15 uses path 1. This implies that both subjects with the chosen path still have a dependency by analogy from the same or similar problem example. At the nature of the proximity, the selected element is constrained, such as "$x + y \leq 5$", causing $x \oplus y = x + y - 5 \notin \mathbb{N}$.

While the subject M-15 employs path-1 to demonstrate the nature of closeness by selecting two elements as $x, y \in \mathbb{N}$ without being confined to criteria, it is directly concluded that the results of the operation are contained in N since the closed nature is met. Different faults with different paths also suggest that both subjects are not caused by the path employed, but that knowing the subject is not followed by the introduction of the correct structure. The paths taken by both subjects are consistent with the stages of mathematical concept understanding, although the subjects do not have a thorough understanding of structure sense in SSE-1, SSE-2, SSE-3, SSE-4, SSP-1, SSP-2, SSP-3, SSP-4, and SSP-5, particularly in question number 2.

The results of the two aforementioned subjects will be confirmed by reviewing the following interview excerpt.

Interview excerpts with M-8:

R : Why do you use path-2 to learn about the set element at item 2?
M-8 : Because I remember the example, but I am a little bit forgot about it, ma'am.
R : What about the definition of this binary operation "$x \oplus y = x + y - 5$"?
M-8 : What I think is that if the result of x and y is less than or equal to five, the result of the operation is not contained in the set of the natural number, ma'am.
R : What about elements that are more than five?
M-8 : I didn't think about it, ma'am.
R : So, how do we get the selected elements into the set of natural numbers?
M-8 : Honestly, I didn't think about it.

Interview excerpts with M-15:

R : Why do you use path-1 to learn about the set element at item 2?
M-15 : As far as I know, the example in the prerequisite material with a binary operation definition is similar to that, so I chose path-1.
R : Are you sure if the operation's result is contained in the set of Natural numbers?
M-15 : Yes, of course.
R : What if the chosen element has x=1 and y=1? Does it meet the natural numbers?
M-15 : No, it doesn't. Oh, I see, it means that I am wrong.
R : Do you know your mistake?
M-15 : Yes, I do.

The findings of the interviews with both subjects revealed that the path taken on the questionnaire aligned with their answers to the interview questions and that the rationale corresponds to how test questions are answered. However, the subject's inaccuracy in response owing to the inability to recognize the set element structure associated with the binary operational structure is not common. Each subject perceives the selected set element in only one chosen part for a distinct reason. While another part is never considered globally or thoroughly. Therefore, the selected set element's alignment cannot accurately reflect the results of its operation.

The task of both M-8 and M-15 subjects on using paths to recognize structure sense has led to five conclusions: 1) Neither M-8 nor M-15 subjects have been able to abstract definitions logically or with path-3; 2) Both M-8 and M-15 subjects still use analogies to figure out structure sense because they both need examples of the same problem structure; 3) Both M-8 and M-15 subjects tend to use path-2 when figuring out whether a structure sense is familiar or not; 4) Path-2 is often used to understand the natural structure of the identity element, the inverse element, and their interrelations; and 5) Both subjects tend to use familiar or unfamiliar structures on Path-1 to learn about structure sense.

The Analysis of the Work of Subjects Who Often Utilize Path 1

Path-1 is the process of tracing structure sense by extracting a familiar mathematical structure or object to abstract definitions in constructing new or unknown mathematical trait structures or objects (or VA →D →VB). Based on Table 2, the summary of the path used by students shows a tendency to use path-1 when the subject knows SSE-1 on item 1 (6 times), item 2 (4 times), and item 3 (4 times). Then path-1 is also used in recognizing SSP-4 in item 2 (4 times). The number of lane-1 choices less than 4 times was used to spread out in recognizing SSE-2, SSE-3, and SSP-1.

Based on Figures 5a and 5b, the summary of the type of path selected by S-8 and S-15 in using path-1 (5 times by the S-15) and (1 time by the subject S-8). S-15 subjects use line-1 when familiar with structure sense category SSE-1 (in the form of questions 1, 2, and 3), SSP-2 (in the form of question 2), SSP-1 (in the form of question 2), and SSP-3. Path-1 is used by S-15 subjects when familiar with structure sense SSE-1 questions 1, 2, and 3. S-15 tends to use path-1 when familiar with any element in the set $Q = \{x \mid x = 2k + \sqrt{p}, k \in \mathbb{Z}, p \in \mathbb{N}\}, and\ S = \{e, p, q, r\}\mathbb{N}$. S-15 in recognizing the set element $Q, S, \mathbb{N}$, was through extracting familiar structures first through the examples in the book to form the basis of the group definition. In addition, path-1 is used when the subjects are familiar with SSE-2, SSE-3, SSP-1 (questions 2 and 3), SSP-4, and SSP-5. SSP-2 was used in dealing with the structure of the relationship between the identity element and the inverse element. While SSP-3 is used by S-15 subjects when connecting commutative properties with identity elements and connecting commutative properties with inverse elements in question 2.

Subject M-15 uses path-1, indicating the subject still has an analogy from an example of the same or similar problem to the nature of the closeness, with the selected elements in the group $(Q, +)$, (and $S, \circ)$, (and $(\mathbb{N}, \oplus)$. The subject of M-15 uses path-1 to demonstrate the nature of the closure by selecting two such elements without $x, y \in \mathbb{N}$ being restricted, then directly inferred to meet the closed nature meaning that the results of the operation are contained in $\mathbb{N}$. Path-1 used by both S-8 and S-15 subjects in familiarity with structure sense tends to be from familiar or unfamiliar structures when determining set elements in the process of proving closed nature.

The Analysis of the Work of Subjects Who Often Utilize Path 3

Based on the path distribution in Figure 3a, subject M19 indicates that all paths are used in familiar with structure sense and have a tendency to use path-3. The distribution of the path used is dominated by path-3 (11 times from 22 questions), path-4 (5 times), path-2 (4 times), and path-1 (2 times). Path-3 used subject M-19 indicates that the subject abstracts the definition logically by logical deduction through understanding the definition of the set of visible elements that the selected element meets its definition structure. It is done when it is related to the structure sense category when familiar with SSE-1 (for questions 1 and 3), SSE-3 (for items 2 and 3), SSE-4 (for items 3), SSP-1 (for question 3), SSP-2 (for question 3), SSP-3 (for questions of points 1 and 3), SSP-4 (for questions of points 1 and 3), and SSE-5 (for question 1).

SSE-1 : J-3, J-1, J-3
SSE-2 : J-1
SSE-3 : J-2,J-3
SSE-4 : J-3
SSP-1 : J-4, J-4, J-3
SSP-2 : J-4, J-2, J-3
SSP-3 : J-3, J-4, J-3
SSP-4 : J-3, J-2, J-3
SSP-5 : J-3, J-2, J-4

Description: J-i= j-th line

Figure 7a. Path Distribution of M-19

SSE-1 : J-3, J-3, J-4
SSE-2 : J-3
SSE-3 : J-3, J-4
SSE-4 : J-4
SSP-1 : J-3, J-3, J-3
SSP-2 : J-3, J-4, J-4
SSP-3 : J-3, J-3, J-3
SSP-4 : J-3, J-1, J-3
SSP-5 : J-3, J-2, J-3

Figure 7b. Path Distribution of M-21

Note:
J: represents the path; J-1: represents path-1; J-2: represents path-2; J-3 represents path-3; J-4: represents path-4

In Figure 7a and Figure 7b, the path distribution of the results of the questionnaire work from subjects M-19 and M-21 to the stages carried out when working on the group material test given. Almost like Subject M-19's tendency to employ path 3 in the logical deduction process to learn about the structure of sense, subject M-21 does the same. A majority of M-21 (as shown in Figure 3b) took path-3 on an average of 15 out of 22 questions. With path-3, M-21 behaves similarly to M-19 subjects. To put it another way, the SSE-1, SSE-2, SSE-3, and SSE-4 groups of M-21 participants were likely to use logical deduction procedures in a common structure meaning. Path 4 appears five times in the process of learning about SSE-1, SSE-3, SSE-4, and SSP-2, but it is only utilized once. When learning about SSP-4 and SSP-5, path-1 is used once, and path-2 is used once. The following is a screenshot showing the work of subjects M-19 on the group material test.

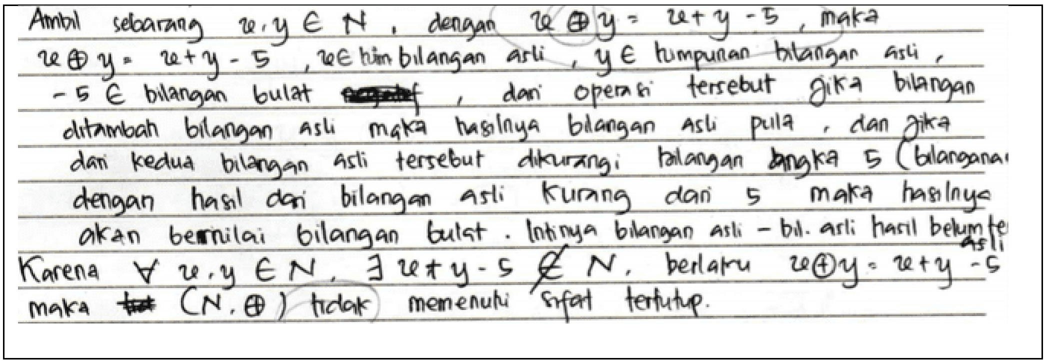
Ambil sebarang $x, y \in N$, dengan $x \oplus y = x + y - 5$, maka
$x \oplus y = x + y - 5$, $x \in$ him bilangan asli, $y \in$ himpunan bilangan asli,
$-5 \in$ bilangan bulat, dari operasi tersebut jika bilangan
ditambah bilangan asli maka hasilnya bilangan asli pula, dan jika
dari kedua bilangan asli tersebut dikurangi bilangan angka 5 (bilangan a
dengan hasil dari bilangan asli kurang dari 5 maka hasilnya
akan bernilai bilangan bulat. Intinya bilangan asli – bil. asli hasil belum te asli
Karena $\forall x, y \in N$, $\exists x + y - 5 \notin N$, berlaku $x \oplus y = x + y - 5$
maka $(N, \oplus)$ tidak memenuhi sifat tertutup.

Figure 8a. Answer of M-19

The following is a screenshot showing the work of subjects M-21 of structure sense.

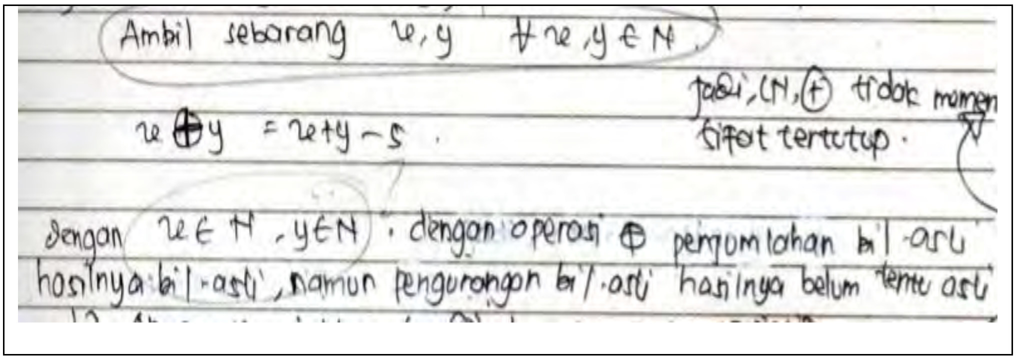

Figure 8b. Answer of M-21

Figure 8a and Figure 8b are parts of group material test work from subjects M-19 and M-21 by different paths used in the same item (item 2) and the same structure sense category (namely SSE-1): in describing the elements of a set-in closed properties).

According to the findings of the research of the work of subjects M-19 and M-21 in Figure 8a and Figure 8b, the difference in pathways employed is that subject M-19 uses path-3 to recognize the set element in question 2, whereas subject M-21 uses path-4 to detect the set element in question 3. On the matter of question 2, whereas the subject M-19 utilizes path-2 in recognizing the results of binary operations, the subject M-21 employs path-3. Subject M-21 uses path-3 and path-4 in familiarity-inverse elements while subject M-19 uses path-4 and path-2 on question 2. Thus, if the concept structure is familiar, path-3 is used by the subjects in the structure sense process.

The result of the answer to the questionnaire fulfilled by M-19 shows that when he performs the non-standard operation of the two set elements, he uses path-2. Path-2 is used i.e., by extracting the structure of properties or mathematical objects of known binary operations through analogies to construct the structure of new or unknown mathematical properties or objects on non-standard binary operations "⊕", subject M-19 fails to be able to abstract the definition " $x \oplus y = x + y - 5$". The subject is not able to capture the definition of the binary operation to meet the results of the operation contained in the set. This failure is because the subject never knows the structure of the selected set element, necessarily any x, the set element of the natural number is more than 3, or if x = 1, then y = 5, or if y = 1 then x = 5, or if x = 2 then y = 4, or if x = 4 then y = 2. The failure of the M-19 in abstracting this non-standard definition of binary operations from questionnaires and tests will be confirmed through interviews. Interview excerpt with M-19:

R : Why do you often use path-2 to recognize the structure when working on question 2 of abstract algebra?
M-19 : Yes, I know from the tasks given, for about 6 times. That's why I know the definition, ma'am.
R : Why did you fail in recognizing the structure of the non-standard binary operation in question 2?
M-19 : Which one, ma'am?
R : This one (Pointing at the result of work "⊕")
M-19 : I'm still confused about its operating results. If the elements I choose are started from 1 and 2, the results are not contained in the set $\mathbb{N}$.

Based on the results of the interview, it shows that the subject of M-19 used path-2 and he answered that he understands the definition, but the work shows that the results of the operation did not meet the set of natural numbers. The work of the M-19 subject is in accordance with the answer of the questionnaire who selects path-2 to extract the structure of a known mathematical property or object through analogy in constructing the structure of a new or unknown mathematical property or object, then can only abstract the definition.

Based on questionnaires and test work of the subject M-21 there is a conformity of results, that the subject has difficulty in operating the two elements of the selected set. Furthermore, the results of this conformity were confirmed through interviews. The results of the interview showed that specifically for question 2, the subject expressed difficulty in understanding the structure of the definition in non-standard operations. Here is an excerpt of his interview. Interview excerpt with M-21:

R : Why did you use path-3 in recognizing the set element in question 2?
M-21 : I saw the operation is on the left side, ma'am.
R : What did you think about the left side operation?
M-21 : If x and y are natural numbers starting from 1, the results will be integers. So, the conclusion is the results are not contained in natural numbers.
R : Okey, didn't you think about the next set element?
M-21 : No, I didn't.
R : Why?
M-21 : Because the natural numbers is always started from 1, then after the trial I found that I didn't meet, so that I concluded so.

The results of the interview excerpt with the subject of M-21 have not been able to recognize the structure of binary operations is not standard, although it has tried to abstract the definition of binary operations using path-3, it is still not able to help construct structures that are not yet known by analogy. The obstacle is done because the subject takes the element only one element at the beginning to represent x and y, so the alignment of the element taken has not met the condition of closure in the binary operation.

Based on the results of the study of the work of subject M-19 and subject M-21 show the difference in the path used, namely subject M-19 uses path-3 to recognize the set element in question 2, while subject M-21 uses path-4 to get to know the element of question 3. While subject M-19 uses path-2 in recognizing the results of binary operations not standard in question 2, subject M-21 uses path-3. When subject M-19 on question 2 uses path-4 and path-2 in familiarity-inverse elements, subject M-21 uses path-3 and path-4.

The difference in path selection in knowing the structure sense of the two subjects from this high learning independence group was determined based on the familiar structure of each subject. However, both M-19 and M-21 subjects tend to use the same path, path-3, in the structure sense process. The tendency of path-3 that both subjects use in the structure sense process if the concept structure is familiar.

Briefly based on the results of task analysis, analysis of questionnaires, analysis of interview results, and results of source validity tests as well as the results of method validity tests, it is obtained that the path chosen by the majority of all subjects is path-4. Thus, all subjects have a tendency not to be able to abstract definitions to logical deduction thinking. So, the conclusion is that the subjects tend to not be able to abstract the definition of binary operations of non-standard forms and the definition of the scope of the set of natural numbers well for question 2. The dependence of the subject is still visible when working on the form of a problem that is not often encountered, then the subject will have difficulty. In other words, the subjects still have a very high dependence on the same examples available.

The sequences of the path that is often used by the subject are path-4, path-2, path-1, and path-3, but to be able to get used to the introduction of structure sense in group materials using path-3 leads to logical deduction thinking. For subjects who use path-3, there is still found to be a dependency on examples of questions that are specific or similar. The sequence of these paths in this study describes the stages that students take based on the level of student understanding (student cognitive ability) in understanding structure sense in group material. Not all students use all paths, but sometimes students use all paths depending on students' cognitive abilities and previous understanding abilities of prerequisite material.

The results of this study show that the tendency of students to abstract definitions still depends on examples of problems to construct the structure of newly known mathematical properties/objects. Although not all students use all paths, some students have reached the third path which is the path that is expected to occur in thinking on group materials (logical deduction). The stages of the path to arrive at the logical deduction path or the third path that is before constructing the structure of new known mathematical properties/objects are passed by the analogy of known structures, although students have been able to construct structures known as additional paths to help students who will achieve logical deduction.

The structure sense process in this study is the path used in thinking during recognizing 9 categories of structure sense. Four paths are used as a framework for the search of structure sense processes by adapting three paths from the development results of Novotná et al. (2006) and one path as an additional path in this study. The description of the structure sense process path in the group concept is described based on the development of Oktac (2016) description of Novotná et al. (2006) three paths developed into 4 paths presented in Table 3.

Table 3. The Description of Four Paths in Structure Sense Process

Type of Paths	Paths	Indicators Description adapted from Novotná et al. (in Oktac, 2016, p. 311)
Path-1	$V_A \rightarrow D \rightarrow V_B$ (*Abstraction*, *Construction*)	Students can extract familiar mathematical structures or objects to abstract definitions in constructing new/ unknown mathematical structures or objects.
Path -2	$V_A \rightarrow V_B \rightarrow D$ (*Analogy*, *Abstraction*)	Students can extract known mathematical property structures or objects through analogy to construct new or unknown mathematical structures or objects, they can only abstract definitions.
Path-3	$D \rightarrow V_A, V_B$ (*Formal Construction*)	Students can construct structures of known and unknown mathematical properties or objects through logical deductions from definitions known to students.
Path-4	$D \rightarrow V_A \rightarrow V_B$ (*Construction*, *Analogy*)	Students can construct the structure of a known mathematical property or object through a logical deduction from the definition, then can only analogize the structure of a new or unknown mathematical property or object.

Notes:
V : property or object
A index : the known structure
B index : unknown structure
D : definition
$\rightarrow$: the next stage

The tendency shows the sequence of the types of paths that are widely used by students in getting to know the 9 categories of group concept structure sense, namely path 4, path 2, path 1, and path 3. This distribution shows that the subject mostly uses path-4. Path 4 used by students means that it shows the tendency of students to have begun to be able to think logical deductions, although it should be used in mathematics is path 3 (i.e., logical deduction stages). Here is a profile of the sequence of the student's structure sense path in getting to know the structure in the group material based on the results of this study.

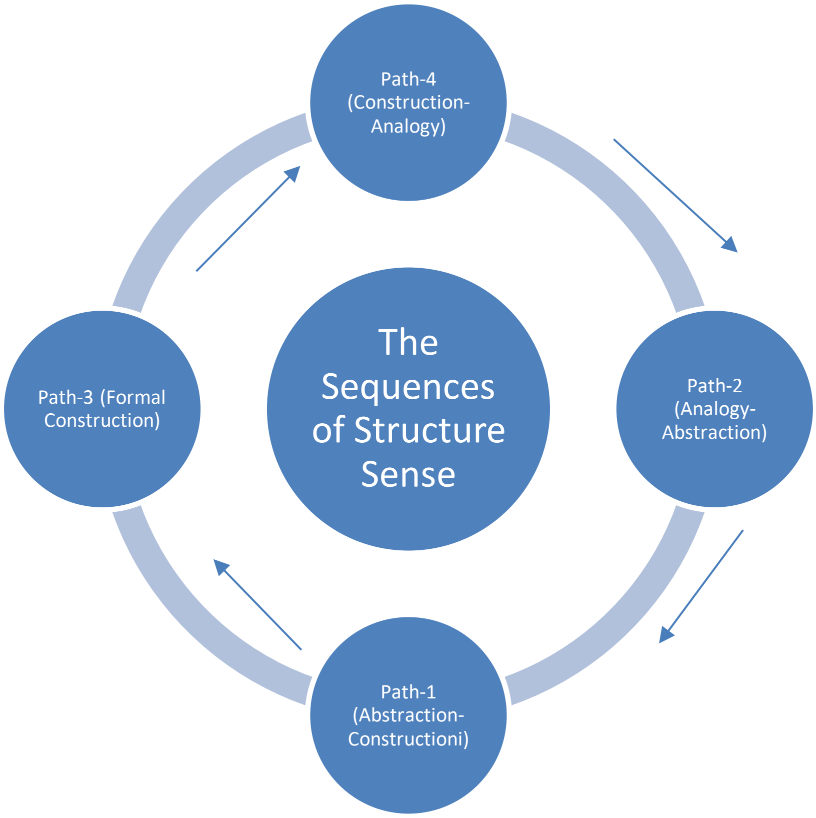

Figure 9. The Sequences of Structure Sense Process

Figure 9 explains the sequence of structure sense pathways carried out by students at the stage of understanding group material. There are sequential processes in each path, that is, students start with the initial construction that is owned then continue the process of analogy, and abstraction, and finally students can make construction formally. For formal construction on one particular part of the material, this will be used as the basis for the initial construction of the next material, then the process runs by analogy, abstraction to formal construction again, and so on. For example, when the student wants to construct an initial understanding of group materials, then he performs the process of analogy and abstraction until he can carry out a formal construction process on the group material. Why does the above cycle after formal construction remain circular? This is because when the student has understood the formal construction of group materials, it will be used as an initial provision and initial construction to understand the next material, then he does analogies and abstractions until finally, he can carry out a formal construction process in understanding the next material, and the process goes onwards. This process is described in Figure 10.

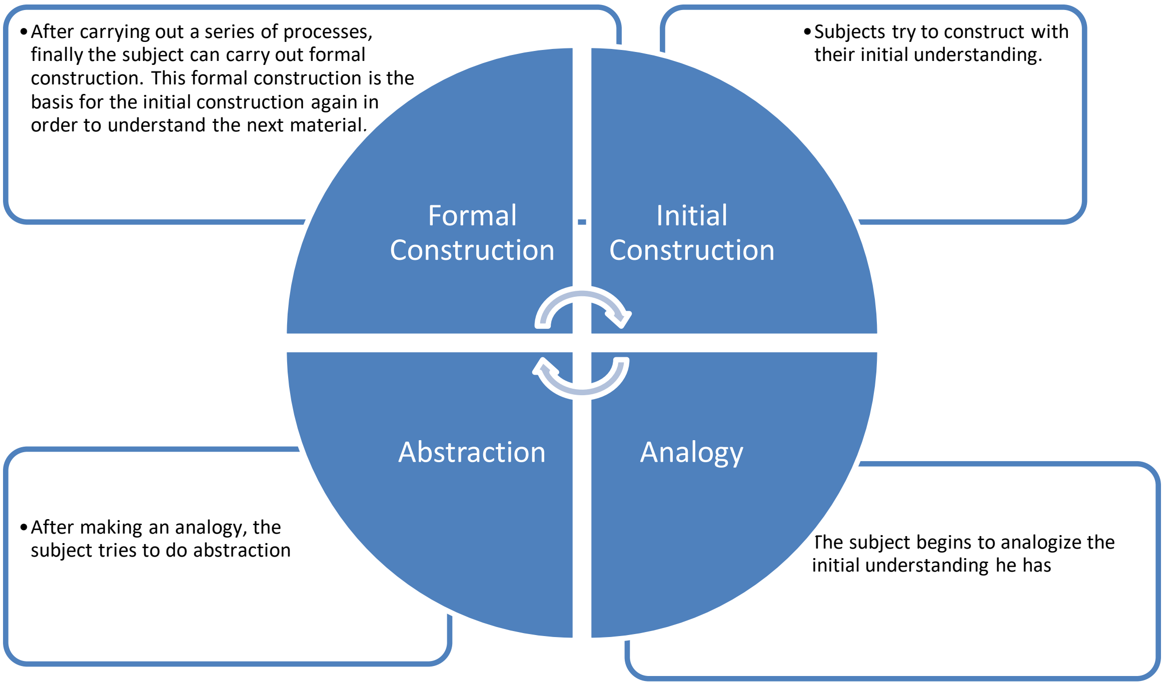

Figure 10. Construction Process in Structure Sense Process

Figure 10 explains the construction process in the structure sense path, including 1) Formal construction is the basis for carrying out the initial construction again to understand the following material: 2) Subjects try to construct with their initial understanding; 3) The subject begins to make an analogy of his initial understanding; 4) After making an analogy, the subject tries to do abstraction. Then, when it comes to abstraction, it can turn back to the formal stage, and so on.

Schneider (2021) explores those cognitive structures (i.e., mental schemes and models) provide meaning and organization to events and enable humans to move beyond the information provided. Bruner (1966) went on to say that to improve student understanding, teaching theory necessitates structured knowledge, and a suitable strategy for organizing that knowledge should result in simplification, the generation of new propositions, and increased manipulation of information structures. One of the information structures suggested by Novotná et al. (2006) and Oktac (2016) in recognizing the structure of mathematical properties/objects has at least three paths in learning mathematics, namely abstraction-construction, analogy-abstraction, and construction. Additional paths are discovered, such as analogy construction, which still employs examples while developing an unknown structure, and this step is the one that will lead to logical deduction.

Based on the findings of the study, the paths used by students in identifying structure sense fluctuate, implying that all paths are taken, but students prefer path-4, which is taken 245 times. Furthermore, path-2 is used up to 90 times, while path-3 is used up to 49 times, and then path-1 is used up to 33 times

Path-4 is a discovery path that is described as a process that begins with establishing the structure of known mathematical properties or objects through logical deduction from definitions, followed by the ability to analyze the structure of new or unknown mathematical properties or things. The analogy process in path-4 is employed to aid in the building of an unknown structure, however, it does not include an abstraction process and instead employs a logical deduction process. The logical deduction technique used in this path is a must for students learning advanced mathematics. Path-4 can also aid in the training of students to understand structure sense through precise and rapid stages of students' reliance on the structure of known mathematical properties/objects. It is envisaged that students will no longer rely on question examples, which can occasionally confuse students' grasp of mathematical structures. This implies that students who do not fully comprehend the definition will struggle to develop a newly discovered structure (Junarti et al., 2019).

Path-2 is utilized 90 times when recognizing 9 structure sense categories. This path-2 stage corresponds to the stages described by Stehlikova (in Novotná et al., 2006, p. 255), Simpson and Stehliková (2006), and Mason (2009). Students who know particular arithmetic structures can gradually progress from dependence on new structures on regular arithmetic to independence. Gradual independence is defined here as having reached the point of abstracting the concept by analogies from known structures. Path-2 is likewise an inductive stage, beginning with known structures in a specific form and progressing to a broad form or definition. Meanwhile, Mason (2009) compares path-2 to the term shift from specialized to structural understanding. This is also consistent with Simpson and Stehliková's (2006) study, which found that when students succeed in working through examples to think abstractly about mathematical structures, they go through a series of shifting steps. Furthermore, the structural characteristics that will serve as the foundation for the abstraction are subsequently linked in a formal and general way with the specific example (Simpson & Stehliková, 2006).

While path-3 is utilized by students to learn structure sense by generating abstract concepts through logical deduction from definitions, the stages of path-3 correspond to the stages of Harel and Tall (1991). The abstraction process happens when the subject focuses on specific qualities of a particular object and then considers these properties separately from the original. Whereas the process of constructing an abstract concept by logical deduction from the definition is known as formal abstraction, the first of these processes is known as a formal abstraction when it takes the form of a new concept by selecting the generative properties of one or more specific situations (Harel & Tall, 1991).

Path-1, which students use when recognizing structure sense in the structure sense category, is recognizing the structure of set elements, which begins with extracting the structure of known mathematical properties/objects through examples of similar problems, and then students can abstract their definitions and construct unknown structures. Such stages correspond to the paths of Dubinsky et al. (1994) and Novotná et al. (2006). This method is intended to aid in the process of recognizing the structure of non-standard binary operations, which must also consider the associated set elements.

Individual knowledge of the group concept should encompass an understanding of numerous mathematical properties and the independent construction of particular examples, such as groups of undefined elements and binary operations that meet axioms (Dubinsky et al., 1994). Examined in terms of the group material test results, the subject continued to make errors in recognizing the structure of mathematical properties/objects, particularly in non-standard binary operations, as the subject tended to encounter obstacles due to the lack of appropriate examples of questions in student books. Student obstacles produced by algorithmic interconnections are a result of inadequate conceptual knowledge (Dewi et al., 2021).

This study shows that subjects' inability to answer questions about which they had no prior knowledge made them unable to design new structures using only the restricted resources of their existing knowledge. The mental structure is based on this constrained framework (Bintoro et al., 2021). There are at least three paths where the various paths are distributed in accordance with the three paths proposed by Novotná et al. (2006) and Oktac (2016). There is only one, path-4, which is the path that results from the development of the three existing paths as theoretical hypotheses that still need to be tested. Novotna et al. (2006) have at least three ways to understand the structure, and Oktac (2016) later states that there are at least three ways to understand a familiar and unfamiliar structure. Furthermore, Oktac (2016) explains that this model constructs the concept of binary operations, models for groups, which involve binary operations, sets and coordination through much more complex axioms. At the same time, this research has answered that four interrelated paths exist. The significance of the findings of Path 4 is to facilitate students in constructing conceptual structures through analogies. Based on the findings of this study, it is suggested that the most recent knowledge in the field of mathematics education be used to create a sequence of pathways for students' structure sense in knowing the structure of group material as an alternative to making it easier for students to think logically during the formal construction of group material.

CONCLUSION

At the initial construction stage, the process carried out by the subject follows path-4, namely the construction process, which is continued by analogy. Then the process is continued on path-2, namely the analogy to the abstraction process. Finally, for the following sequence, path-1 is to continue the abstraction process with advanced construction to produce formal constructions in path-3. The following is a description of each path.

Path-4 is the finding path used by the subject as a process that begins with constructing the structure of known mathematical properties or objects in the form of the same or similar examples through their definitions, then by analogizing the shape of the examples the subject will be able to recognize the structure of the properties of mathematical objects. new or unknown. Path-4 is the path that results from the development of the three existing paths as theoretical hypotheses that still need to be tested.

After that, Path-2 is an inductive stage, starting from known structures in a special form to a general form or definition. The shift from specific to structural understanding indicates the shifting steps that the subject goes through when succeeding in working through examples to think abstractly about the structure of mathematical properties or objects. The structural aspects that will be the basis of the abstraction are then related in a formal and general form to the specific example.

Then, path-1, Path-1 used by the subject when recognizing structure sense in the category of recognizing the structure of set elements begins with extracting the structure of properties/mathematical objects that are already known through examples of similar questions, then the new subject can abstract the definition, and then construct structures that have not been identified.

The last sequence is, Path-3 is used by the subject when recognizing the category of structure sense by going through the process of constructing abstract concepts through logical deduction from definitions. The process of abstraction occurs when the subject focuses attention on certain properties of a given object and then considers these properties separately from the original. The next process is the subject can construct the structure of properties or mathematical objects that are already known and unknown.

The limitation of this study is that the existence of path-4 is a path that can help lead the subject to a logical deduction process, but the subject's dependence on the same and similar examples of questions cannot be left out. Suggestions for further research are further studies on the sequence of students' structure sense paths in recognizing the structure of group material by developing more diverse instruments. Will the research results show the same sequence of paths with the development of multiple instruments? This can be studied more deeply so that the sequence of students' structure sense paths in recognizing the structure of group material becomes patent to make it easier for students to think logically deductively in the formal construction process on group material.

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REFERENCES

Bintoro, H. S., Sukestiyarno, Y. L., Mulyono, & Walid. (2021). The spatial thinking process of the field-independent students based on action-process-object-schema theory. *European Journal of Educational Research*, *10*(4), 1807-1823. https://doi.org/10.12973/eu-jer.10.4.1807

Bone, E., Bouck, E., & Witmer, S. (2021). Evidence-based systematic review of literature on algebra instruction and interventions for students with learning disabilities. *Learning Disabilities*, *19*(1), 1–22.

Bruner, J. S. (1966). Toward a theory of instruction (Vol. 59). Harvard University Press.

Capaldi, M. (2014). Non-traditional methods of teaching abstract algebra. *PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies*, *24*(1), 12-24. https://doi.org/10.1080/10511970.2013.821427

ÇETIN, Ö. F. (2021). The importance of algebra teaching; daily life variables and number systems corresponding to these variables. *International Journal of New Trends in Arts, Sports & Science Education*, *10*(5), 297–315.

Cresswell, J.W. (2017). *Research Design.* Sage

Dewi, I. L. K., Zaenuri, Dwijanto, & Mulyono. (2021). Identification of mathematics prospective teachers' conceptual understanding in determining solutions of linear equation systems. *European Journal of Educational Research*, *10*(3), 1157-1170. https://doi.org/10.12973/eu-jer.10.3.1157

Dubinsky, E., Dautermann, J., Leron, U., & Zazkis, R. (1994). On learning fundamental concepts of group theory. *Educational Studies in Mathematics, 27*(3), 267-305. https://doi.org/10.1007/BF01273732

Harel, G., & Tall, D. (1991). The general, the abstract, and the generic in advanced mathematics. *For the Learning of Mathematics, 11*(1), 38-42. https://www.jstor.org/stable/40248005

Hoch, M. (2003). Structure sense. *The 3rd Conference of the European Researchers in Mathematics Education*.

Hoch, M., & Dreyfus, T. (2005). Students' difficulties with applying a familiar formula in an unfamiliar context. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*. Melbourne, Australia: PME. Vol. 3, 145-152.

Hoch, M., & Dreyfus, T. (2010). Developing katy's algebraic structure sense. *Proceedings of CERME 6*, 529–538.

Hwang, J., Riccomini, P. J., & Morano, S. (2019). Examination of cognitive processes in effective algebra problem-solving interventions for secondary students with learning disabilities. *Learning Disabilities*, *17*(2), 205–222.

Junarti, Sukestiyarno, Y. L., Mulyono, & Dwidayati, N. K. (2019). The profile of structure sense in abstract algebra instruction in an Indonesian mathematics education. *European Journal of Educational Research*, *8*(4), 1081–1091. https://doi.org/10.12973/eu-jer.8.4.1081

Junarti, Sukestiyarno, Y. L., Mulyono, & Dwidayati, N. K. (2020). The process of structure sense of group prerequisite material: A case in indonesian context. *European Journal of Educational Research*, *9*(3), 1047–1061. https://doi.org/10.12973/EU-JER.9.3.1047

Kamacı, H. (2021). Linear Diophantine fuzzy algebraic structures. *Journal of Ambient Intelligence and Humanized Computing, 12*(11), 10353-10373. https://doi.org/10.1007/s12652-020-02826-x

Linchevski, L., & Livneh, D. (1999). Structure sense: The relationship between algebraic and numerical contexts. *Educational Studies in Mathematics, 40*(2), 173-196. https://doi.org/10.1023/A:1003606308064

Mason, T., Stephens M., & Watson A. (2009). Appreciating mathematical structure for all. *Mathematics Education Research Journal, 21*(2), 10-32. https://files.eric.ed.gov/fulltext/EJ883866.pdf

Novotná, J., & Hoch, M. (2008). How structure sense for algebraic expressions or equations is related to structure sense for abstract algebra. *Mathematics Education Research Journal*, *20*(2), 93–104. https://doi.org/10.1007/BF03217479

Novotná, J., Stehlíková, N., & Hoch, M. (2006). Structure sense for university algebra. In J. Novotná, H. Moraová, M. Krátká, & N. Stehlíková (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education*. Prague, Czech Republic: PME. Vol. 4, 249-256.

Nu, A. T. (2019). Algebra thinking process on vocational school students in completing line problems. *KOLOKIUM Jurnal Pendidikan Luar Sekolah*, *7*(2), 75–87. https://doi.org/10.24036/kolokium-pls.v7i2.350

Oktac, A. (2016). Abstract algebra learning: Mental structures, definitions, examples, proofs and structure sense. *Annales De Didactique Et De Sciences Cognitives*, *21*, 297 -316.

Ralston, N. C., Li, M., & Taylor, C. (2018). The development and initial validation of an assessment of algebraic thinking for students in the elementary grades. *Educational Assessment*, *23*(3), 211–227. https://doi.org/10.1080/10627197.2018.1483191

Schneider, S., Beege, M., Nebel, S., Schnaubert, L., & Rey, G. D. (2021). The cognitive-affective-social theory of learning in digital environments (CASTLE). *Educational Psychology Review*, *34*, 1-38. https://doi.org/10.1007/s10648-021-09626-5

Schüler-Meyer, A. (2017). Students' development of structure sense for the distributive law. *Educational Studies in Mathematics*, *96*(1), 17-32. https://doi.org/10.1007/s10649-017-9765-4

Simpson, A., & Stehlikova, N. (2006). Apprehending mathematical structure: A case study of coming to understand a commutative ring. *Educational Studies in Mathematics, 61*(3), 347-371. https://doi.org/10.1007/s10649-006-1300-y

Souminen, A. L. (2018). Abstract algebra and secondary school mathematics connections as discussed by mathematicians and mathematics educators. In *Connecting Abstract Algebra to Secondary Mathematics, for Secondary Mathematics Teachers.* Springer.

Stehlíková, N. (2004). *Structural Understanding in Advanced Mathematical Thinking*. Prague: Charles University in Prague, Faculty of Education.

Taban, J. G., & Cadorna, E. A. (2018). Structure sense in algebraic expressions and equations of groups of students. *Journal of Educational and Human Resource Development*, *6*, 140–154.

Usiskin, Z. (2001). Teachers' mathematics: A collection of content deserving to be a field. *Mathematics Teacher*, *6*(1), 86–98.

Wasserman, N. H. (2014). Introducing algebraic structures through solving equations: Vertical content knowledge for k-12 mathematics teachers. *PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate, 24*(3), 191–214. http://doi.org/10.1080/10511970. 2013.857374

Wasserman, N. H. (2017). Exploring how understandings from abstract algebra can influence the teaching of structure in early algebra. *Mathematics Teacher Education and Development*, *19*(2), 81–103.

Widodo, S. A., Irfan, M., Trisniawati, T., Hidayat, W., Perbowo, K. S., Noto, M. S., & Prahmana, R. C. I. (2020). Process of algebra problem-solving in formal student. *Journal of Physics: Conference Series*, *1657*(1), 012092. https://doi.org/10.1088/1742-6596/1657/1/012092

Wilburne, J. M., & Long, M. (2010). Secondary pre-service teachers' content knowledge for state assessments: Implications for mathematics education programs. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, *1*(January), 1–13.