EXAMINING INTEGERS-BASED MATH GAME CREATED BY A FOURTH GRADE STUDENT

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Abstract

An integer-based mathematical game was the main subject of the study. The aim of this study was to analyse the mathematical content of the game which was developed by a fourth-grade student called Esin, and to examine the intuitive learning that develops through this game. The game was based on the addition of the positive or negative numbers won in each round. The integer is regarded as an abstract topic and is taught in secondary school. It is also regarded as a topic that students have difficulty in learning. However, in the case examined by this research, a fourth grader created an integer addition game using her own intuitive understandings and played it with three of her friends. This research was a case study focusing on the analysis of the game. The study group of the research was Esin and her three friends who played the game. Data was gathered in two stages. In the first stage Esin played the game with researchers and in the second stage Esin and her friends played the game together. All the games were recorded, and the observation form was used to record data. Descriptive analysis, which consists of two codes (cardinal or ordinal meaning), was carried out in data analysis. As a result of the study, it was seen that, Esin had defined negative and positive numbers over the idea of win / lose. Students were able to use both the cardinal and ordinal meanings of the number while playing the game. Cardinal meaning was usually established through the neutralization relation. Ordinal meaning was mostly used over positioning the number relative to zero. This research contributes to the literature on students' learning of integers at an early age.
Keywords: *cardinal meaning, educational games, intuitive learning, negative numbers, ordinal meaning, primary school students*

Introduction

Learning begins in social life. Concepts such as numbers, counting, comparing numbers develop through our daily life experiences before starting formal education. In other words, our experiences in everyday life form the basis of intuitive learning. This intuitive learning is sometimes seen in children's games. So, games can provide opportunities for children to develop various mathematical skills.

Children use the skills and knowledge they have accumulated so far while playing, and they can transform their knowledge into another form of information within the game (Bennet et al., 1997). Children who use their current knowledge while playing games can also make

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connections between the information schemes they have, thus reinforcing the knowledge they have learned (De Holton et al., 2001). In addition to these comments on the relationship between game, knowledge and learning, Faulkner (1995) argued that game is a process based on real life experiences in which the foundations of mathematical thinking are laid.

Game is an art of learning (Yıldız, 1997) or it can be considered as a learning tool used by every child. The game provides active learning environments in which children participate voluntarily. Students both have fun and learn while playing games. Also repeating the game strengthens learning and via games, students become more interested in the subject to be learned. Games, as a teaching tool, contribute to doing mathematics enjoyable, encouraging students to work individually or in groups, and the reinforcement of knowledge (Uğurel & Moralı, 2008).

Games can therefore be created in accordance with the curriculum, just like other teaching materials, and become a component of instructional activities. Educational games are used for this purpose in groups of a wide range of ages from children to adults (e.g., Amalia et al., 2018; Kim et al., Norton, & Samur, 2017; Rossiou, & Papadakis, 2007). Apart from the games designed by educators, the games developed by children in the course of daily life can also be educational.

The game, which is the subject of this research, was designed by a fourth grader by using her own life experiences, not by educators. The game was based on the intuitive learning of its designer Esin and the addition of negative and positive numbers. Esin was only in the fourth grade and had not yet learned integers when she created this game.

Natural Numbers to Integers

Learning about the concept of counting and numbers starts within the children's own culture and social environment (Butterworth, 2005). Games play an important role in this learning. Children sometimes count by skipping some numbers, though by imitating someone. And sometimes they compare the number of toys with each other's in the game by using the "more or less" relation. As they get older, the games they play and the level of mathematical knowledge in these games change as well.

Throughout our lives, we encounter situations where numbers are used for different purposes. Numbers can be categorized as nominal, formal, cardinal, and ordinal according to their way of use. Nominal numbers are mostly used for labelling / naming and do not have a quantitative meaning (Haylock & Cockburn, 2013). For example, the number on the bus 220 is a label, and our cell phone numbers are denominations of us. In the formal meaning of the numbers, numbers are used in an abstract context and students algebraically approach numbers, they do generalizations from what they already know (Bishop et al., 2014). For example, a child who encounters the expression -3 - - 3 may deduct that "when a number is subtracted by itself, the remaining would be zero" without any knowledge on operations with negative numbers. This generalization is related to the formal entity of the number. In the cardinal meaning of a number on the other hand, a relation is constructed between the number and the quantity. It is creating a relation between the last number used while counting objects in a group and the quantity of that group (Gelman, & Gallistel, 1986). Ordinal value/meaning is related to the concept of ordering, and it means that students for example perceive the set of integers as in an order, as positional and use numbers over ordinal or positional relations (such as -3 is a number greater than -4 and smaller than -2) rather than the quantity they express (Bishop et al., 2014).

All these meanings constitute the concept of numbers as a whole. In order to accomplish meaningful learning, these meanings must be learned together. For example, for a child who only understands the concept of numbers over cardinal meaning, the number -4 cannot be the number of elements of any set. This makes it difficult to understand the negative numbers. Understanding the numbers only as the quantity of a group of objects creates an effect that

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makes it difficult to learn the integer set (Fischbein, 1987; Otten, 2009). Here it is important to know the different meanings of numbers. For example, in teaching of integers, it is necessary to use activities including the ordinal meaning of the number. There are studies suggesting that such activities facilitate learning negative numbers and that this teaching can be realized even at a young age (Beswick, 2011; Bishop et al., 2014; Hativa, & Cohen, 1995; Sfard, 2007; Wilcox, 2008). Hence, it is important to develop an understanding of all meanings of numbers, especially cardinal and ordinal meanings, in teaching integers.

In a teaching process, where cardinal and ordinal meanings are taken into consideration jointly, it is crucial to employ the number line as a model in addition to the manipulatives. When children learn natural numbers in primary school, they create a number line in their minds that includes the ordering of natural numbers. How this number line can be expanded to cover negative numbers is crucial. To help students to move from using a mental number line only for positive integers to using a mental number line including both positive and negative integers, Vosniadou and Brewer (1992) offered a conceptual framework. In their framework, there exist conceptual information about how children define and think about concepts. This framework has three categories—initial, synthetic, and formal models—that might be used to explain students' mental representations. Bofferding (2014) defined this conceptualization of Vosniadou and Brewer on integers as follows: Students who have "initial mental models" for numbers may use positive integer rules to respond to inquiries regarding negative integers (e.g., students ignore the negative signs and order negative numbers as if they were positive). While allowing for numbers less than zero to exist, the "synthetic mental model" upholds the idea that numbers further along in the counting sequence from zero have bigger worth (e.g., since -6 is farther from zero than -3, it is bigger than -3). The number sequence is extended in a "formal mental model" of an integer, with the negative sign coming before the numbers that are ordered to the left of zero and the numerals symmetric about zero (e.g., -3 is greater than -6 because it is closer to 0 than -6).

Peled (1991), who examined the mental models of students in the learning process of integers, defined four understanding levels. At the first level of understanding integers, students are familiar with the symmetrical ordering of numbers around zero as well as the order of all integers. Students can add positive numbers to any integer at the second level; add or subtract two positive or two negative numbers at the third level; and add or subtract any two integers at the fourth level. Considering the studies of Peled (1991) and Vosniadou and Brewer (1992) together, it is expected that students first perceive negative and positive numbers. Then they can order these numbers correctly (it means they have a formal mental model) and finally they can perform four operations step by step (second level to fourth level).

Despite this conceptual framework for students' learning about integers, negative numbers are still one of the known issues that students have difficulty in learning (Beswick, 2011; Bryant et al., 2020; Erdem et al., 2015; Kilpatrick et al., 2001; Van de Walle, 2013; Vlassis, 2008). On the other hand, some studies also reveal that primary school students have been aware of negative numbers, they could order integers, and perform some basic operations (Bishop et al., 2014; Bofferding, 2010; Bofferding, 2014; Cengiz et al., 2018). It is remarkable that a subject that students have difficulty in learning in secondary school can be done in primary school. Examining students' mental models and intuitive learnings of these subjects provide important data on students' learning processes. Studies on integers with younger age groups are crucial for this reason. Students' intuitive learning constitutes their pre-knowledge. The teaching procedure can be better planned with the help of these pre-learnings. In this study, students' intuitive learnings that they gained through the game were examined.

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Research Aim

The game developed by fourth grader Esin was the focus of this study. The researchers were not involved in the game's development. Esin and her friends, who had been playing the game together for a while, were discovered by one of the researchers. The development of a game on a subject that secondary school students had a difficulty in learning by primary school students was an interesting case. Additionally, fourth graders in this game were somehow adding negative and positive integers. It was thought that the mathematical content of the game and the mental models of the students while doing the addition would reveal crucial information about the students' learning processes. For this reason, the aim of this study was to analyse the mathematical content of the game and to examine the intuitive learning that develops through the game. Therefore, this study sought to answer the following research questions:

1. What was the mathematical content of this game developed by Esin?
2. What did the students learn about integers and addition in integers through the game?

In other studies, with younger age groups (Bishop et al., 2014; Bofferding, 2010; Bofferding, 2014; Cengiz et al., 2018; Wilcox, 2008) students were asked questions that usually included integers. Students' learning was examined through their answers to these questions. In this study, unlike the others, a situation that exists outside the intervention of researchers was analysed. The study is original in this regard.

Research Methodology

General Background

This research was designed as qualitative research. It was a case study focusing on the analysis of the game developed by Esin. Case studies are a type of research in which the researcher collects detailed and in-depth information through multiple sources of data such as observation and interviewing about a current limited situation in real life (Creswell, 2012). The situation can be an event, a person or a small community, and this reality is investigated in detail through the case study (Merriam, 1988). In this study, the game, namely the mathematical content of the game and game-based learning, was the case of the research. Throughout the research the game was named as "Let's put here a number with minus sign”. This was a phrase that students often used in the game.

Study Group

In this research, the game "Let's put here a number with minus sign” is considered as a case. The person who created this game and her three friends who played this game with her constituted the study group of this research. Four female fourth graders who created and took part in the game were chosen as the study group, and a purposive sample was created. These four students had been attending a primary school in Ankara, Turkey, together with kids from middle-class families. Families of all four participants were well-educated, and at least one parent held a university degree. The families of the participants gave their permission for the study, and the participants were informed about the research. Also, the names of all participants were changed, and new names were given to them. Throughout the research, the participants were named as Esin, Buse, Nazlı and Esra. According to their grades and teachers’ view, these participants had different success in mathematics course. Esin and Nazlı were described by their teachers as successful students, while Buse was a student at the intermediate level. Esra, on the other hand, was defined as a student who had difficulties in mathematics lessons.

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Instrument and Procedures

This research was carried out in 2018. The data collection process included the following stages:

Stage 1: First, Esin was asked to explain to the researchers the game and how to play it. Esin and the three researchers then played the game together. In the meantime, the researchers constantly asked Esin how to proceed in the game and she was told to explain the mathematical content required by the game to them. The game was recorded as a video. Afterwards, this video was watched, and two researchers filled out The Observation Form 1 (see at App.1) separately. Then, the two researchers compared the observation forms they filled in and completed the missing notes. The observation form consisted of some questions about the game, and some empty spaces to write the operations, justification of the operations and the duration. For the observation form, the expert judgment of two academicians was sought.

Stage 2: After a week, Esin and her friends played the game together. In this second game, the researchers focused on the information that Esin and her friends learned through the game. While playing the students also explained the reason of the operations to the researchers. Each player explained what she was doing and how she was thinking while playing the game. This game was also recorded as a video. In the video analysis, The Observation Form 2 (see at App.1) was used. For the observation form, the expert judgment of two academicians was sought.

Data Analysis

Game videos were watched and analysed by two of the researchers. Coding was performed according to which meaning of the numbers is used. The participants' expressions were coded over whether their expressions include the meaning of cardinal or ordinal value. So descriptive analysis, which consists of two codes (cardinal or ordinal meaning), was carried out in data analysis. The coding was done separately and then the encodings were compared for reliability of the research. The consistency of encodings was first at 80% according to Miles and Huberman's (1994) formula, and then with the inclusion of the third researcher the inconsistencies were discussed. After this discussion, the coding consistency was achieved and on the different coded portions, a decision was made jointly. Direct citations from the discussions were used when reporting the findings, this strengthened the validity of the findings.

Research Results

Mathematical Content of the Game "Let's put here a number with minus sign"

The game required a coloured pencil for each player and a blank paper. The game could be played with at least two players. First, each player chose one corner of the paper as their starting point and named it. As it can be seen from Figure 1, the starting points in the game designated by researchers and Esin were named as "Sun Forest", "Rose Garden", "Starlight" and "Heart Balloon" from bottom to top.

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Figure 1
The Image of a Finished Game

Each player then randomly placed numbers between 0 and 10 on the paper. "If we want to make it harder, we could use numbers up to 20" said Esin while the researchers were writing down the numbers. "Now put a minus or a plus sign next to the numbers" she added after that. Meanwhile, Esin explained the meaning of plus and minus signs as follows:

Researcher 1: Why do we put minus and plus signs?

Esin: You lose points with minus and gain them with plus.

Researcher 2: Well, should we write down the number first and then put the plus or minus sign next to it? [Referring to 3- or 3+ notation]

Esin: No, it should be like minus 3 and plus 3. So, first sign then number.

As it can be understood from this dialogue, Esin defined positive and negative numbers in the game over the cardinal meaning of the number (by establishing a relationship between number and quantity), associating the positive state with the gain and the negative state with the loss. She also correctly used symbolic representation of negative and positive states of the numbers like in the example of "minus 3, plus 3". In Turkish, there is only one word for "negative" or "minus", and only one word for "positive" or "plus". Therefore, the expressions of these students who had not yet learned the negative and positive numbers were translated by using "minus" and "plus".

Afterwards Esin told how to play the game as follows; "We start drawing a line from the starting point while our eyes are closed and randomly stop at somewhere. I'll get the point where I stop. Then we add this new point that we acquired to our previous point. At the end of the game, the player who gathers the highest points wins." In these sentences, Esin described a game based on the addition of positive and negative numbers. In order to calculate the points earned in the game, the numbers that we arrive to or get close to must be added. These numbers can be positive or negative. So, how does Esin performs addition with negative numbers?

The second round started after each participant recorded the number of points they initially reached. In the second round of the game, adding negative and positive numbers began. Since the researchers didn't know how to add integers in this game, Esin did all the calculations for them. Esin explained what each player was supposed to do after reaching the second number in the following:

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Researcher 1:	(-1) + 0	Esin: "You add 0, nothing changes, you are still minus 1."
Researcher 2:	(+1) + (+2)	Esin: "You were plus 1, adding plus 2 you become plus 3."
Esin:	(-2) + (-8)	Esin: "It is minus 10, because we've added the minuses, and when we add the minuses, our number with minus becomes greater. ... we now have a greater number with minus."
Researcher 3:	(-10) + (-2)	Esin: "You became minus 12. As you have more minuses."

As it can be understood from her explanations, in the second round of the game, Esin provided a justification for the sum of two negative numbers, which she had not yet learned at school. Here Esin explained negative numbers with respect to their cardinal value and she described -10 as "we now have a greater number with minus". What she actually means here is that the amount of that player's loss becomes greater quantitatively. At first, from her expressions of "our number with minus becomes greater" and "we have a greater number with minus" it can be thought that Esin had a wrong perception about comparing the magnitude of negative number, but it was observed later in the game that she also developed a correct perception about ordering integers.

As the game continued, both numbers and the difficulty of operations increased. Later in the game, some of the operations that Esin explained are as follows:

(+8) + (-6)	Esin: "I become plus 2. When I reach minus 6 my points drop, I lose 6 points. After that I am left with plus 2."
(+3) + (-7)	Esin: "when you reach minus 7, after adding points, you become minus 4. First, we ***subtract*** 3 from 7. When we subtract 3 from 7, the plus sign is reset. What is left is 4. And that becomes minus 4 because of the minuses."

Here Esin made her expressions with respect to the logic of elimination or neutralization of plus and minus signs in two-color counters, meaning the cardinal value. It was observed that Esin also used the concept of "subtraction" when describing the addition process in integers. For example, when calculating (+3) + (-7) Esin who used the expression "First we'll subtract 3 from 7" established a subtraction logic from the neutralization relation between minus and plus values. However, it was also observed from the dialogues later in the game, along with cardinal value of negative and positive numbers, Esin was also able to construct an ordinal (ordering) relation between these numbers. For example, the researcher 2 and Esin had the following conversation for (-12) + (+4):

Esin: You were at minus 12. It will still be ***more*** in the negative but not yet 0. You couldn't pass zero. Since you get +4, we will find it by subtracting 4 from 12. You became minus 8.
Researcher 2: You said it will be more, but what is more?
Esin: Your minuses will still be more; you couldn't reach to zero. You are still in loss.
Researcher 2: Why did you subtract 4 from 12?
Esin: I have subtracted 4 from 12 because I got ***more*** points, it should have been higher. You get higher in numbers with minus by subtracting.

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In this dialogue, Esin used the word “more" for twice but with two different meanings. At the beginning of the dialogue by saying “it will still be more” what she meant is that the researcher is still in loss, comparing her points with zero. In her later expressions she used a correct ordering relation between negative and positive numbers. She defined the gain by taking +4 as getting higher from -12 and she considered -8 as a "higher" number compared to -12. The word "more" that she used for the second time on the other hand has an ordering logic, different than the first. While sharing information about the addition in integers, Esin showed in her own words that she had a correct reasoning relating to the ordering in integers. Esin's explanation of (-1) + (+1) also followed an ordinal approach; "We add minus 1 with plus 1, you became 0. As you were minus 1 and we added 1 to it. When we add 1, the point of minus 1 gets higher and it becomes 0". Esin defined the greater numbers in integers by the concept of getting "higher".

At the end of the game, Esin answered the question of "who won?" by making a correct ordering in integers. This time a smaller number was defined by the word "less".

Researcher 1: I'm at -10, Researcher 2 is at 0. Which of us has a higher score, who won?

Esin: Researcher 2.

Researcher 1: Why?

Esin: Because she is zero, you're less than zero.

Esin's explanations about the game showed that this game was based on the cardinal and the ordinal meaning of the integers. Throughout this game, Esin could use these two meanings of integers together. For example, while describing the relationship between being greater and smaller in integers she referred to number's ordinal meaning while using the concept of "higher” and referred to its cardinal meaning while using the concept of "less".

Learning through the Game

A week after the meeting where the game was learned, this time friends of Esin with whom she played this game for a while joined her. Esin, Nazlı, Esra and Buse were classmates. They were students at a primary school where children of middle socio-economic class families are educated, in Turkey's capital Ankara. They performed differently in mathematics and this difference could easily be observed while they were performing addition operations in the game. For example, each student did the addition at a different speed. During the game children were able to discuss on operations that they did, object to each other and remind some information to each other. So, the game was played in its natural flow.

All four of these friends were asked to explain the operations they performed with their reasons while playing the game. It is observed that with the help of the game children had different levels of improvements in their understanding relating to cardinal and ordinal meaning of the integers. However, their terminology was sometimes mathematically incorrect. Even though they used an incorrect expression, they actually meant something different. For example, when Nazlı arrived at -5 while she had +2 points, she declared that her score was now -3 and explained it with the following words: “because when you subtract it from plus two, what remains is 3”. Similarly, when Esra had +1 point and she arrived at -3, she declared that her score became -2 and she said, "I subtracted plus 1 from minus 3, since it is 3 which is minus here, I subtracted plus 1”. Here neither Nazlı nor Esra meant the operations of +2 - (-5) and -3 - (+1). The concept of “subtraction” that they used in their statements actually described the process of neutralization of the pluses and the minuses through the relation of cardinal value. As the sum of the points they gathered, they used the remaining value after this neutralization process.

Another misuse was caused by the similarity of the number signs and the operational symbols. When she arrived at -1 with -2 points, Esra who was a bit slow from others while doing operations said, "because I had two minuses and I got another one, I add them when they are same operations". Esra used the idea of putting together the same-coloured counters in this operation. Since she was accustomed to use + and - symbols while performing operations, she used incorrectly the word "operation" at the end of the sentence while talking about negative values.

Buse, who had arrived at +2 with -8 points at this round, explained how she performed addition operation by saying she had "lessened" her minus points with plus points she'd just got. There is a cardinal meaning in her explanation, too.

Buse: -8, adding +2 I became -6.

Researcher 1: How did you become -6, Buse?

Buse: Arrived at +2, we need to lessen them [meaning her points] by 2 which makes -6.

There were few moments in the game when they had incorrect results, or they didn't make the right explanations. In such moments, children were able to object to each other and discuss it together. The discussion about the total score of one of the children sometimes was brought into question the rules of the game. Sometimes it had been observed that these discussions improved children's learning. For example, in one of the rounds, Buse and Esra got confused at first by the points Esin got in her turn and they objected to the operation she performed. During the discussion (after the rule of the game was reminded) Buse made a comment on the relation of ordinal value (ordering) in addition to cardinal value of integers. At the beginning of this round, Esin had -1 point and she arrived at -5. Then;

Esin: I became minus 6.

Researcher 1: How did you find the result?

Buse: Nooo, you got -5, you should become -4.

Esin: No that is correct, if I add minus 5 to minus 1, it's minus 6.

Esra: Why are you adding them?

Buse: You should add if it is plus, but you got minus.

Researcher 1: What was the rule of your game, Esin? How do you find the total score?

Esin: We need to add the numbers.

Buse: Right Esra, it will be -6, -6 is more, now which one is more; minus 1 or zero? Minus 1 is more.

Researcher 3: What does it mean more? Which one of these numbers is lesser or greater?

Buse: Now I'm minus 4, minus 6 is more than me. Because it is actually lower at the end. Because it's more due to the state of decrease. Now Esin has decreased even more. Esin was right.

Buse, who initially objected to the result, was aware of the fact that Esin lost more by arriving at -5. Children sometimes get confused about which operation to use because of the signs in front of the numbers. In this example, immediately after being reminded that the score would be calculated by adding the points they got, Buse realized her mistake and took the discussion to a new level. In this discussion, Buse used the word "more" in a similar context to Esin's by saying: "Now I'm minus 4, minus 6 is more than me. Because it is actually lower at the end". Even though this sentence looks like self-contradicting, in fact it has both cardinal and ordinal value relation of numbers in it and therefore it is true. She simply didn't use the proper mathematical terminology in her explanation. However, it should be noted that these kids are in fourth grade.

Buse presented a win-lose argument by claiming that Esin, who had minus points, lost "more" by reaching another minus point by saying "because it is more due to the state of decrease". Here, Buse used the word "more" through its cardinal meaning. And by saying

"Because it is actually lower at the end" she made an ordinal reasoning and said that Esin had the lowest value of points when compared to others.

The terms children used were sometimes incorrect and sometimes they had different meanings. Despite this, they could explain us the addition of integers which they had developed through game. These expressions showed that their knowledge about integers was not limited to cardinal value, but they could also establish an ordinal relationship between numbers. In addition to Buse's statements mentioned above, in another round of the game Nazlı first miscalculated her score and then realized her mistake and made the following statement: (Nazlı had -3 points, then she arrived at +2)

Nazlı: It becomes +1.

…

Nazlı: No, it is wrong. [looks back at the paper] it is -1.

Researcher 2: Why did you change it? Which one is true? I don't get it.

Nazlı: Look, if it was a number greater than minus 3, say 4, it would pass over plus side. But since it was less than minus 3, it couldn't pass over that side.

With a number greater than -3, Nazlı meant numbers like +4, +5. According to her, being greater than -3 meant that if it was added, the result would reach a level above zero and being less than -3 meant that if it was added, it couldn't make it pass or reach zero. Thus, her expression of "pass over plus" had an ordinal meaning.

The dialogues in this game also showed that Esin and her friends were able to use both the cardinal and ordinal meanings of the number when adding integers. Sometimes they confused the sign of the number with the sign of the operation, and sometimes they did not use mathematical terms appropriately. Nevertheless, they were able to create a reasoning for adding negative and positive integers as fourth graders.

Discussion

In this research a game created by fourth-year student Esin on a subject she had not yet studied was analysed. The game, named "Lets' put here a minus sign", was based on adding negative and positive numbers. The concept of negative numbers is abstract for primary school students. It is difficult for a student who learns the concept of number primarily through cardinal value, to associate the quantity of objects with negative numbers. Students can understand negative numbers by using win-lose relationships or the idea of debt over the meaning of cardinal value. On the other hand, when teaching negative numbers, the number line, whether it be horizontal or vertical, the thermometer, or the depths below sea level can all be used as examples of ordinal meaning. Despite all these examples that are used to embody the concept of negative number, integer is one of the subjects that students have difficulty in concretizing (Kilhamn, 2008). Pound (2008) pointed out that games encourage children to think abstractly as well as supporting mathematical development. The game that was the focus of this study was thought to have these two effects on students: thinking abstractly and supporting mathematical development.

Negative numbers exist in our daily life practices like the negative temperatures listed on a thermometer and the children are intuitively aware of these numbers. Many studies conducted with young children showed that they had intuitive understanding of integers, or the concept of integer could be taught to them before secondary school (Bescwick, 2011; Bishop et al., 2014; Cengiz et al., 2018; Hativa & Cohen, 1995; Sfard, 2007; Wilcox, 2008). The game analysed in this study reveals that a fourth-grade student Esin has intuitively grasped negative numbers. Esin used states of being plus and minus (in numbers) which she defined through win / lose situation in her game, and she designed a way to perform addition of integers. This intuitive knowledge of hers created the game, and this game promoted some new learnings on integers in her friends she played the game with. By the help of this game Esin, Nazlı, Esra and Buse could perform addition of integers when they were just 4th graders (in Turkey this subject is taught in 7th grade).

Although integers are taught in the secondary school, some studies show that younger students can understand negative numbers, sorting integers, and even adding and subtracting integers (Beswick, 2011; Bishop et al., 2014; Bofferding, 2010). Bofferding (2010) interviewed second grade students about addition and subtraction of integers and examined how they defined the term "minus sing". According to this research almost half of the students, even though their terminology was sometimes mathematically incorrect, were willing to explore more meanings of the minus sign. Although they used incorrect terminology in their expressions as in this article, students were able to develop an idea about a subject they did not learn yet, and according to the researcher they thought comparably better to the students who had had integer instruction. Similarly, in the case study with Violet (Bishop et al., 2014), a second-year student, Violet had shown that those young students already have an intuitive understanding of negative numbers before receiving formal training. The order-based (ordinality) approach that Violet used in addition and subtraction was similar to the reasoning of the students in this study. In this research, students used both the ordinal and cardinal meanings of the number while adding integers and explaining how to add them.

In this research cardinal meaning was usually established through the neutralization relation. Ordinal meaning was mostly used over positioning the number relative to zero. For example, when Esin explained the operation (+3) + (-7), she used a subtraction logic from the neutralization relation between minus and plus values when she said, "when you reach minus 7, … you become minus 4. First, we subtract 3 from 7. When we subtract 3 from 7, the plus sign is reset. What is left is 4. And that becomes minus 4 because of the minuses”. But when she explained the operation (-1) + (+1) she used an ordinal approach when she said, “When we add 1, the point of minus 1 gets higher and it becomes 0”. When students learn natural numbers in primary school, they deal more with the cardinal meaning of the number. Fischbein (1987) emphasized that understanding the number only through its cardinal meaning would make it difficult to learn negative numbers. Therefore, in order to accomplish meaningful learning, these meanings must be learned and used together. As in this research, some other studies have shown that primary school students could also use ordinal meaning. Violet, who was a second-grade student, was one of them (Bishop et al., 2014). According to Bishop and his colleagues Violet had intuitive ideas about integers before the school-based instruction and she could use order-based approach at integer additions. She used a counting strategy up to zero while doing (-9) + 5 operation. Another second-grade student Robbie also used ordinal meaning while explaining (-4) + 2 = -2 operations as “Negative two… it’s just uh, going up, so negative two … is more” (Bofferding, 2010). At this operation according to Robbie adding meant “going up”. All these studies (also including this research) had revealed that students could learn about integers even before secondary school.

In the primary school, mental models for integers can be developed in students. The results of this study also revealed this. The students' statements regarding "who won" after the game and their narratives about negative numbers showed that they possessed a “formal mental model” (Bofferding, 2014). They were able to position negative and positive numbers relative to zero. Even though they did operations at various speeds, they were all generally capable of doing addition with negative numbers. Some of the number pairs reached in the game, also revealed examples that students could add any two integers (e.g., positive, and negative) that was the fourth level of Peled (1991) on integer addition. Of course, this intuitive learning was not enough for learning integers. But this level of learning, which students developed through the game, is important. For Esin, who created the game, included this learning into intuitive knowledge. What needs to be done is to carry this intuitive learning, which students develop in their daily lives, further through school learning.

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This research also revealed various results in terms of terminology use. Most of the participants employed their own terms, and even though these expressions occasionally had mathematical errors, they reached the correct conclusion mathematically. The learning that students develop outside of their school learning creates their own terminology that is fed by their lives. In this research, the students explained the large number, for example, with “getting higher”. Sometimes, even though they incorrectly use certain terms, they give those terms a new meaning. For example, sometimes they used the term “subtraction” not for the operation, for describing the process of neutralization of the pluses. Similar to this study, there are other studies in which students create new and their own concepts for that subject area (like Bishop et al., 2014, Maher, & Martino, 1996). These studies usually include topics that students have not yet learned in the classroom. For example, Maher and Martino (1996) examined the development of an idea of proof of a first-year student named Stephanie in their 5-year longitudinal study. In this research, Stephanie used new terms such as "cousin", "opposite of cousin" while describing her intuitive ideas. Although students expressed negative numbers correctly in this research, some students could invent new notations for "negative". For example in a similar another research, a first grader Nola invented a new notation for numbers below zero, she called them the “something” (Bishop et al, 2014). She counted numbers as something 1, something 2, something 3 and wrote them S1, S2 and S3. Language and terminology are crucial components of learning. Examining language and terminology in research on students' learning will provide important data on learning and thinking.

Conclusions and Implications

In this research, intuitive learning about integers developed by students before secondary school was examined. This learning developed through a game, which was intended by one of the participants of the research, Esin. Unlike other studies with primary school students, an existing and discovered situation was analysed in this study. Researchers did not intervene to the study group, the game and the learning created by the game were examined over the existing situation.

When the intuitive learning through the game "Let's put here a number with minus sign" was examined, it had been found that:

- The students defined the state of being negative with losing points and the state of being positive with winning points.
- The final score of the game was calculated by adding every negative or positive points gathered in each round.
- The students were able to use both the cardinal and ordinal meaning of the number while performing addition.

According to the results of the study, fourth-grade students, who were playing this game, could understand negative numbers, add negative and positive numbers, and determine the larger number in integers. They used both cardinal and ordinal meaning of the numbers in all this process. These findings are important in terms of the pre-knowledge that students have about integers in primary school. Curriculum designers should take this pre-knowledge into account, and also intuitive learning / knowledge should be taken into account in the curriculum development process. It is possible to re-evaluate the best time to begin teaching integers in light of this and related studies. Also, this game or any other game to be developed with similar content can be used in teaching integers. Another study might focus on how this game will be used in the classroom, and the effect of teaching with the game in the classroom can be analysed. Teaching via games should be prioritized in subjects that students struggle with. According to this research, through this game:

- Children had illustrated how to use mathematics in real life.
- Doing math had become a game of friends which proved to be fun.
- Learning was achieved in an environment of peer communication.

The advantages of game-based learning, that also was observed in this research, may help to overcome some learning difficulties. In this research, it was observed that a subject, which was difficult for secondary school students, was learned in a fun way through the game by primary school students.

Declaration of Interest

The authors declare no competing interest.

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Appendix 1

Observation Forms

Observation Form 1: Mathematical Content of the Game
How many people was the game played with?
How was the game played?
What were the rules of the game?
What were the meanings of the positive and negative numbers in the game?

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First Round			
Operations [like (-3) + (+5)]	Duration (like 12'30'')	The justification of the operation or dialogue	Code (Cardinal or ordinal)

Second Round (same as the third and the other rounds)			
Operations [like (-3) + (+5)]	Duration (like 12'30'')	The justification of the operation or dialogue	Code (Cardinal or ordinal)

How Esin explained "Who won the game and why"?

Observation Form 2: Learning Through the Game			
Operations [like (-3) + (+5)]	Duration (like 12'30'')	Student name, her justification of the operation or dialogue	Code (Cardinal or ordinal)

The dialogue about "Who won the game and why"?

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