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Developing Awareness Around Language Practices in the Elementary Bilingual Mathematics Classroom

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This study contributes to efforts to characterize teaching that is responsive to children's mathematical ideas and linguistic repertoire. Building on translanguaging, defined in this article as a pedagogical practice that facilitates students' expression of their understanding using their own language practices, and on the literature surrounding children's mathematical thinking, we present an example of a one-on-one interview and of the circulating portion of a mathematics class from a second-grade classroom. We use these examples to foreground instructional practices, for researchers and practitioners, that highlight a shift from a simplified view of conveying mathematics as instruction in symbology and formal manipulation to a more academically ample discussion of perspectives that investigate critically both mathematical concepts and their modes of transmission, which involve language practices, that are crucial for educating bilingual children.

KEYWORDS: bilingual education, elementary, mathematics, translanguaging

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Dual language programs have become popular across the United States as a promising bilingual alternative to English-only and transitional instructional models (Martínez et al., 2015). However, this alternative has been shown to promote bilingualism through a policy of language separation that discourages students and teachers from using both of their languages during instruction (Martin–Beltrán, 2010; Palmer & Martínez, 2013). This language separation in the classroom occurs by content area, teacher, or time (e.g., on a weekly or daily basis; Hamman-Ortiz, 2019). In a review of the literature on the debate surrounding language separation, Hamman-Ortiz (2019) described how those in favor of language separation argue that "minoritized languages need a safe space to thrive and, thus, must be 'protected' from the infiltration of the majority language" (p. 388). In the same review, Hamman-Ortiz (2019) noted that language separation in the classroom perpetuates already existing societal language imbalances by encouraging learners to draw on features from the majority language during class time allocated to the minority language (see also Ballinger et al., 2017). In addition, Hamman-Ortiz (2019) stated that students enrolled in programs promoting language separation often report a preference for the majority language. In many cases, students use the majority language more often when interacting with peers (Hamman-Ortiz, 2019). As a result, language separation often results in instruction that favors or promotes the use of the majority language.

Language separation is especially restrictive during mathematics instruction due to widespread perspectives on language and mathematics teaching and learning. A consensus among mathematicians is that, because the concepts of mathematics are universal, their language of expression should be irrelevant. As history has borne out, the concepts of mathematics aim to transcend differences of language, for they purport to be the distillation of certain laws of the human mind (Lager, 2006; Moschkovich, 2007; Mosqueda, 2010; Stillwell, 2010). As a consequence, the concepts of mathematics are independent of the language used for their expression, but a common practice among instructors is to spuriously conclude that the language of instruction is, as a consequence, irrelevant to the teaching of mathematics. Bilingual contexts demand a rethinking of this assumption: for children learning a new language, requiring expression of mathematical concepts in the new language can impose a cognitive burden that undercuts the learning of the very mathematical concepts under study (Bossé et al., 2019). Additionally, as is often the case with emergent bilingual children who are policed by "symbolic language borders" in their schooling (Valdés, 2017), language separation is touted as the means to achieve the *academic language* needed for mathematics achievement.

Our work counters these misconceptions and offers two examples from one dual-language classroom showcasing the work of an experienced bilingual teacher who recognizes what other researchers have proposed as a deeper and more powerful way of understanding the language practices of bilingual children, called *translanguaging* (García, 2009), defined here as a pedagogical practice that facilitates students' expression of their understanding using their own language practices. In this classroom, translanguaging afforded learning opportunities that would not have arisen if the children had needed to limit their language practices to conform to an English-only model (Maldonado Rodríguez et al., 2020).

Importantly, this classroom's mathematics instruction was based on Children's Mathematical Thinking (CMT; Carpenter et al., 1989). Children's Mathematical Thinking-based instruction includes a body of teaching moves designed to elicit, as directly as possible, mathematical concepts as represented in a given child's own mind. Central to CMT-based instruction is that the teacher avoid imposing their own understanding of those same concepts, hint at correctness, or impart any other preconceptions that might unfaithfully render the child's own understanding. Teachers employing this methodology consequently design instruction by building on these elicitations of the child's own understanding, rather than centering the teacher's personal understanding in instruction. In this classroom, the children's mathematical thinking was therefore expressed without limits or constraints towards an expected solution strategy or answer. Children were encouraged to use a range of strategies to express their mathematical understanding, even when their ideas were incomplete or apparently not yet correct, because the underlying mathematical relationships within these strategies could be used as building blocks for extending everyone's understanding (Celedón-Pattichis & Turner, 2012; Turner & Celedón-Pattichis, 2011).

With these two tenets in mind—using the strategy of translanguaging and employing CMT-based instruction—we will draw on the literature and the examples from the classroom to propose a rethinking of the strict separation of languages during mathematics teaching. The two examples we present in this article seek to answer the question, "How might instructional practices displace the focus on monolingualism in the mathematics classroom to emphasize children's mathematical ideas?"

Few examples exist as points of reference where proper use of standardized language is subordinated to the task of eliciting children's unfiltered expressions of their mathematical understanding (Valencia Mazzanti & Allexsaht-Snider, 2018). By deemphasizing linguistic form, and even language choice, and allowing a freer bilingual mode of discourse, the resulting environment facilitates the teachers' efforts to capture children's ideas in their entirety before the strictures of "proper" language use have the chance to impede their expressions in mid-flow. For instance, in a study with a group of 5- and 6-year-old Spanish-English bilingual Latinx children, Valencia Mazzanti and Allexsaht-Snider (2018) identified how the different representations (e.g., phonological and orthographic) used by the Latinx children in the study provided avenues for learning that supported the children as they explored quantities through representations in different languages. Through this work, the children developed an awareness of how numbers, words, and sequences are used in counting.

Authors' Positionality

An explanation to the readers of who we are will add context to the dialogues we analyzed, reflected on, and acted upon in this article. We are three Spanish-English speaking Latina educators. Maldonado Rodríguez and Krause (henceforth MR&K) conducted all interviews for this study and collected all data from Ms. Adams's classrooms. Krause conducted the interview with Hugo presented here. Ms. Adams, the teacher in this study, is second author. Our work together goes back more than a decade, beginning when Ms. Adams was a novice teacher. Since then, MR&K have learned with and from her. The three of us have also collaborated on different research projects with a focus on children's mathematical thinking. Our work together has always centered on foregrounding the voice and richness of bilingual children's ideas in the mathematics classroom. Much like the children that we worked with in this study, the three of us have existed as border crossers throughout our learning and academic experiences (Anzaldúa, 2012). We view our research as relational (Patel, 2016) and thus position the children not as lacking or missing something, but rather as people from whom we have something to learn. As a teacher, Ms. Adams has taught in defiance of language separation policies and emphasized bilingualism as the norm (García et al., 2016). In her classrooms, she offers opportunities to disrupt deficit narratives about emergent bilingual children's mathematical capabilities.

Bilingualism and Bilingual Practices: Review of Terminology

Recent estimates suggest that at least half of the world's population is bilingual (Grosjean, 2010). Bilingualism is so widespread throughout the world that it can arguably be considered more normal than monolingualism. In the United States alone, an estimated 60 million people (21% of the population) age 5 and over spoke a language other than English at home in 2011 (Ryan, 2013). According to a report from the U.S. Census Bureau (2019), 61.4% of people who speak a language other than English in the United States speak Spanish, amounting to roughly 42 million people. According to a report from the National Center for Education Statistics (2018), in the fall of 2015, Spanish was the home language of 3.7 million English Language Learners. This number represented 77.1% of all English Language Learners in K–12 classrooms.

The historical context of U.S. education policy around language reveals a deficit-oriented stance towards this sizeable group of children, with a traditional overemphasis on mastery of English and no acknowledgement of the resources they possess (Gándara & Orfield, 2010; MacDonald, 2004). In a recounting of historical events, Hickey (2016) described policies from as early as 1754 where Native

American youth were sent to schools "to learn English and to be stripped of their indigenous home languages" (p.16). In this same retelling of events, Hickey noted that the Bilingual Education Act of 1968 was a key element in recognizing the needs of children whose first language was not English. However, there is still a great deal of variation with regard to the meaning of bilingual education (Hakuta, 1987). *Bilingual education* was defined by the Bilingual Education Act as follows:

> A program of instruction, designed for children of limited English proficiency in elementary or secondary schools, in which, with respect to the years of study to which the program is applicable . . . there is instruction given in, and study of, English, and, *to the extent necessary to allow a child to achieve competence in the English language* [emphasis added], the native language of the child of limited English proficiency, and such instruction is given with appreciation for the cultural heritage of such children, and of other children in American society, and with respect to elementary and secondary school instruction, such instruction shall, to the extent necessary, be in all courses or subjects of study which will allow a child to progress effectively through the educational system. (Cubillos, 1988, p.10)

The italicized phrase is critical. In essence, it says that other languages will be permitted only insofar as they support the learning of English.

Later, in 2002, when No Child Left Behind (NCLB) passed, it had a significant impact on the Bilingual Education Act and bilingual education in the United States, mainly because of its emphasis on high-stakes testing. After NCLB, the Bilingual Education Act was renamed the English Language Acquisition, Language Enhancement, and Academic Achievement Act (NCLB, 2002). Though the act still permits state and local educators to choose instructional methods and to employ bilingual methods, the accountability requirements further underline the fact that the primary objective of public educators continues to be English acquisition, and not bilingualism. See for instance the first purpose of the English Language Acquisition, Language Enhancement, and Academic Achievement Act outlined under NCLB (2002):

> (1) to help ensure that children who are limited English proficient, including immigrant children and youth, attain English proficiency, develop high levels of academic attainment in English, and meet the same challenging State academic content and student academic achievement standards as all children are expected to meet. (p. 115)

In the 2016–2017 school year, Texas, where this study was conducted, reported having 5,343,834 students, of which 18.8% were enrolled in bilingual and English Language Learning programs (Swaby, 2017). At the same time, the Texas Education Code requires the following for bilingual education and English as a second language program content and method of instruction:

> (a) A bilingual education program established by a school district shall be a full-time program of dual-language instruction that provides for learning basic skills in the primary language of the students enrolled in the program and for carefully structured and sequenced mastery of English language skills. A program of instruction in English as a second language established by a school district shall be a program of intensive instruction in English from teachers trained in recognizing and dealing with language differences. (Texas Education Agency, 2017, p. 112)

It is important to point out that these goals for bilingual education do not mention content area learning. Rather, the goals make language acquisition and basic skills the entire focus of bilingual children's learning, inherently limiting their ability to develop critical content area skills. In addition to this, school districts nationwide face major challenges to implementing bilingual programs on a large scale; among the challenges are the current politics around bilingualism and the shortage of qualified bilingual teachers (Gándara & Escamilla, 2017).

Given the variety of definitions and terminology used in and around language and language practices in bilingual classrooms, we found it important that we define the terminology we use in the analysis of the work presented here.

The Terms Bilingual and Bilingualism

In the present work, we use the following definition of what it means to be bilingual: "Bilinguals are those who use two or more languages (or dialects) in their everyday lives" (Grosjean, 2010, p. 4). It is worth noting that this definition (1) focuses on use, not fluency; (2) includes dialects; and (3) includes two or more named languages. We opted for this definition as it is ample, it allows for dialects to be included (which is important given the variations in the Spanish and English language based on the culture of the speaker), and it deemphasizes the focus on fluency (which pertains to a point we will convey regarding mathematics instruction). We also use this definition because it allows us to talk more amply about the population of children we serve in the United States while also acknowledging that a more restrictive definition of bilingualism could apply to an appreciable portion of children in schools across the United States.

The Terms Code-Switching and Borrowing

Two practices often observed in the speech of bilingual individuals are *code-switching* and *borrowing*. Code-switching is the "alternate use of two languages" (Grosjean, 2010, p. 51), while borrowing is "the integration of one language into another" (Grosjean, 2010, p. 58). There are two basic types of borrowing. The most common is when the speaker uses a *loanword*, that is, the speaker takes a word from one language and uses it with its original meaning in the context of speaking another language, perhaps adapting its sounds and forms slightly to conform to the native style of the language being spoken. Take for example the word "*pocicle*" in Spanish ("popsicle" in English). The Spanish word is used to represent exactly the same dessert as the English term, though the second "p" is lost in the Spanish rendering because of the rarity of the sound combination "ps" in Spanish.

By contrast, the borrowing known as a *loanshift* takes a word in the language being spoken but uses it with a meaning that more properly approximates the sense of a clearly related word in another language. An example of loanshifting is the word "*esmoquin*" in Spanish, which means "a tuxedo or dinner jacket." This is a borrowing of the English word "smoking" into Spanish. The sense of the borrowing derives from the English phrase "smoking jacket" (a jacket worn only while smoking) and refers to a jacket's function. The loanshifting of *esmoquin* within Spanish expanded the term to refer to jackets of a similar fashion so that it now refers to a particular jacket's style rather than its function.

When bilingual individuals code-switch, they are speaking in one externally identifiable language, say English, and for a moment shift to another language, such as Spanish (e.g., "I went upstairs—*sólo por un momentico*—to check on the baby"). Grosjean (2010) highlighted several negative attitudes encountered with respect to code-switching. For instance, some monolingual individuals reported that code-switching can create "an unpleasant mixture of languages" or lead to a form of "semilingualism," suggesting that a bilingual's knowledge of a particular named language might be rendered somehow deficient by virtue of regular recourse to an alternate language (Grosjean, 2010, p. 52). However, Grosjean also explained some of the motivations for bilingual individuals to code-switch. In particular, a bilingual speaker may feel that some concepts or notions are better, more easily, or more economically expressed in another language. If the interlocutor also speaks that other language, then the bilingual speaker might employ that other language to express that particular concept in a way that more faithfully represents the intended sense. Grosjean further pointed out how code-switching can fill a perceived linguistic need, for example, where the language currently spoken lacks certain technical terminology or a robust vocabulary for a particular body of understanding. For instance, the recent research project Decolonise Science is translating more than 180 scientific papers into African languages such as isiZulu, Northern Sotho, Hausa, Yoruba, Luganda and Amharic (Masakhane, n.d.; Wild, 2021). In part, the motivation for the study derives from the paucity of native scientific terminology in these languages, which can often lack clear, specific, or conventional terms for originally imported concepts such as dinosaurs, viruses, bacteria, etc. These linguistic lacunae can have major consequences for teaching and learning in local languages within Africa (Masakhane, n.d.; Wild, 2021).

Both code-switching and borrowing rely on a perspective of bilingual language use that emphasizes practices that appear to cross the boundaries between named languages (García et al., 2016). However, the internal perspective of a bilingual speaker is just as important. As speakers are developing their use and understanding of language, they encounter gaps—be they of the language as such or of their current learning of it—and must navigate across fuzzy boundaries between named languages to meet the needs of particular communicative contexts. As they communicate, they are constantly drawing on all their linguistic resources in ways that navigate across the fuzzy boundaries between named languages while attending to the needs of particular communicative contexts. When learning language, sometimes these navigations lead to challenges and innovations.

The Term Translanguaging

Translanguaging is the flexible use of linguistic resources across various everyday contexts (García & Wei, 2014). Otheguy et al. (2015) defined translanguaging as "the deployment of a speaker's full linguistic repertoire without regard for watchful adherence to the socially and politically defined boundaries of named (and usually national and state) languages" (p. 283). Naturally, this intersects with the notion of code-switching. As mentioned above, Grosjean (2010) cited two common reasons for code-switching: 1) ease of expressing an idea in another language, or 2) meeting a linguistic need (i.e., a perceived deficit). It is worth noting, however, that Grosjean's description asserts no particular model for how language is represented in the bilingual mind. Perhaps the bilingual holds in mind two languages kept completely distinct, or perhaps the bilingual holds in mind a unified linguistic apparatus of which named languages appear merely as facets or aspects.

Some researchers, like García and Wei (2014), assert that when bilingual individuals engage in translanguaging, they are not alternating between two languages but rather are utilizing features from their single, encompassing linguistic system. In this paper, we work with a narrower concept of translanguaging: we view translanguaging as a *pedagogical practice* that allows code-switching when this facilitates a focus on, and an expression of, the concepts being learned in the classroom. That is, translanguaging emphasizes content while deemphasizing strict adherence to a target language, allowing learners to express their grasp of ideas by the linguistic means they find most suitable in the moment. Translanguaging, from this perspective, need not assert a particular model of the cognitive representation of language within the bilingual mind (and thereby remain relevant even should neuroscience and linguistics refine or update their models of how language works in the brain). Rather, it emerges as a powerful pedagogical technique that allows bilingual students to be bilingual when classroom discourse is focused on topics other than language.

The rising prevalence of the term translanguaging in the field of education seems to derive in part from a deemphasis on the particular systematics of the linguistic data characterizing certain bilingual practices. In fact, use of the term is a response to an increased emphasis on the social constructs that spurred these practices, as well as on the world view that these practices derive from and in turn facilitate or enable. Translanguaging challenges and reframes. We, as educators, must attend to language as a social construct and the implications this social role can have in mathematics and in bilingual classrooms.

Situating in the Literature

Teaching Mathematics in Bilingual Settings

Research focusing on the teaching and learning of mathematics in the bilingual classroom has identified three common tensions for teaching: (1) the tension between using formal or informal language, (2) the tension between using children's home languages and the language of the school, and (3) the tension between teaching mathematics and teaching language (Adler, 2002). Prior to Adler's (2002) study, Moschkovich (1999) and Khisty (1995) investigated teachers' practices in promoting bilingual children's participation and engagement during mathematical discussion. Khisty (1995) found that those teachers who appeared more effective tended to pay attention to the interaction between language and mathematics content. When the educators in this study focused on language, they highlighted specific vocabulary that arises in both Spanish and English that could cause ambiguity, hindering children's understanding of mathematics concepts. In contrast, Moschkovich (1999) found in her study that teachers who appeared more effective focused mostly on mathematics and placed less weight on correcting language or teaching vocabulary.

Most researchers now recognize that focusing on language when teaching mathematics does not simply mean starting with vocabulary. Learning and doing mathematics includes mathematical ways of talking, arguing, and explaining (Barwell, 2009); these are complicated topics in a single language, all the more so in bilingual settings. The contravening tendencies described in the studies above, therefore, might come as little surprise given the complexity of the teaching practices involved. Although research studies on mathematics teaching, mathematical attainment, and bilingualism are fairly common and have reported some connection between proficiency in a second language and mathematical attainment (Bialystok, 2018; Henry et al., 2014; Lager, 2010; Martiniello, 2010; Shannon & Milian, 2002), they are far from straightforward. It remains unclear whether differences in mathematical attainment relate to language, culture, economic or social factors, or a combination of all of these (Martiniello, 2010).

Other studies hint at a positive relationship between bilingualism and mathematical attainment. For instance, Clarkson (2007) has suggested that bilingualism allows children to think more efficiently when reasoning about mathematics. He has found evidence that young bilingual children, relative to their monolingual counterparts, show greater cognitive flexibility and creativity, as well as improved problem-solving abilities in mathematics. Moschkovich (2007) has also suggested that bilingual children have an enhanced capacity to reason about mathematics problems. In particular, bilingual children can effectively identify the information relevant for solving problems and ignore less important information.

Research on translanguaging, and in particular research on translanguaging in the mathematics classroom, has further clarified the relationship between bilingualism and mathematical understanding. This derives from research investigating how rethinking language practices in the mathematics classroom can support students' full language practices while also providing a foundation for understanding mathematics (DiNapoli & Morales, 2020; Maldonado Rodríguez et al., 2020). For instance, Maldonado Rodríguez et al. (2020) presented a study from a bilingual classroom where a child provided a wrong answer for a particular mathematics problem. Importantly, the child provided the answer in Spanish. The particular episode was used by the teacher to not only open the space for mathematical discussion to understand why the wrong answer would make sense mathematically, but also the teacher used this answer to build on mathematical language together. The authors of the study argued that allowing the child to share his idea in Spanish provided a space not only for the child but for the entire class to use language as a tool through which mathematics was understood.

Despite the positive results of this relationship between bilingualism and mathematical attainment, teachers continue to use language practices in instruction that cultivate *dominant language practices*, defined as "standardized ways of speaking, listening, reading, and writing that are often referred to as 'Standard American English' or 'academic English'" (Martínez & Martínez, 2019, p. 234). These practices are often used even among linguistically diverse learners, which frequently excludes children's cultural and linguistic knowledge in favor of monolingual English instruction (Martínez-Álvarez, 2017). Such tendencies introduce the possibility of children seeing their own culture and language as incorrect, inappropriate, and in need of remediation (Kohli, 2014; Paris, 2012). In addition, these standardized practices in the mathematics classroom tend to obscure bilingual children's actual grasp of mathematics (Barwell, 2009). Language practices directed toward a standard can implicitly or explicitly downgrade other forms of mathematical expressions (Barwell, 2009). For instance, by focusing on "correctness" of vocabulary or grammar use, a teacher might foreground linguistic styles and understanding precisely when understanding a mathematical idea should take the foreground, letting idiosyncrasies of linguistic expression take a temporary back seat. Naturally, such idiosyncrasies of expression might mask underlying confusion, which only further probing can elucidate, but the change in mindset itself is meaningful. In particular, a focus on such idiosyncrasies can be detrimental to children, and in extreme cases potentially mix with racialization, that is, with "the extension of racial meaning to a previously racially unclassified relationship, social practice, or group" (Omi & Winant, 2014, p. 111). Rosa (2016) in fact argued that dominant perceptions of language can serve to delegitimize the language of racialized groups while furthering their racialization. Because of this, we argue that in order to truly center children's mathematical thinking, we must also take care to counter ossified notions of what it means to speak mathematically.

Translanguaging in Teaching Mathematics

Our work in the mathematics classroom is rooted in the long-standing research on children's mathematical thinking (Carpenter et al., 1989; Fennema et al., 1996; Jacobs et al., 2007) and situated in the translanguaging work of García and Wei (2014) and Otheguy et al. (2015). In the context of teaching mathematics using children's mathematical thinking, teachers must have a deep awareness of what their students know or can intuit in order to support them as they connect their current mathematical understanding to concepts of standardized mathematics. Research has linked teachers' understanding of children's mathematical thinking to productive changes in teachers' knowledge and beliefs, classroom practices, and student learning (Carpenter et al., 1996; Carpenter et al., 1989; Fennema et al., 1996). In these studies, researchers have paid particular attention to the development of children's problem-solving strategies, common misconceptions, as well as frameworks for understanding problem structures. Other studies built upon and further established the effectiveness of instruction informed by knowledge of children's mathematical thinking for children from diverse racial, socioeconomic, cultural, and linguistic backgrounds (Adams, 2018; Dominguez, 2011; Dominguez & Adams, 2013; Turner & Celedón-Pattichis, 2011; Villaseñor & Kepner, 1993). We connect the translanguaging practices of building on children's ways of knowing with this main tenet of research on children's mathematical thinking because it can leverage children's intuitive and informal ideas as the basis for instruction.

During mathematics instruction, this means that children need to express their ideas in ways that make sense to them. When bilingual children are forced to accommodate their language to English-only instruction, they run a greater risk of misrepresenting their mathematical thoughts because they must reformulate their ideas in order to share them (Krause & Colegrove, 2020; Moschkovich, 1999, 2007, 2012, 2015; Turner & Celedón-Pattichis, 2011; Turner et al., 2013). This could contribute to a teacher's inability to access a true representation of what the child is thinking, which may in turn allow potentially brilliant ideas to be interpreted as incorrect or incomplete. A critical aspect of being an effective mathematics teacher for linguistically diverse children is developing knowledge, dispositions, and practices that support building on children's mathematical thinking, as well as on their cultural, linguistic, and community-based knowledge (Adams, 2018; Dominguez, 2011; Dominguez & Adams, 2013; Turner & Celedón-Pattichis, 2011; Villaseñor & Kepner, 1993). Representing ideas through language requires intellectual work. Transforming ideas in order for them to be expressed in English only—when they were not initially conceived as such—forces a step that could potentially misrepresent the ideas themselves. Through this lens of bilingual mathematics instruction, we see children as agents capable of expanding their sense of what they know and can do mathematically.

Children's Mathematical Thinking

Ideally, when students learn mathematics, they learn ways of thinking that go beyond a collection of disconnected procedures for carrying out calculations. Within this context, children learn how to generate mathematical ideas, how to express these ideas (in any way that makes sense to them), and how to explain these ideas and those of others (Carpenter et al., 2015; Franke et al., 2001). More than three decades of research on children's mathematical thinking has shown that elementary school children are capable of engaging in this type of mathematical learning, but often they are not given the opportunity to do so (Campbell et al., 1998; Carpenter et al., 1996; Carpenter et al., 2015; Carpenter et al., 1989; Empson, 2014; Empson et al., 2020; Empson et al., 2006; Empson & Levi, 2011; Franke et al., 2001; Jacobs et al., 2019; Jacobs et al., 2007).

When children are invited to solve problems on the basis of what makes sense to them, they use a variety of informal strategies driven by their experiences and the world around them. These strategies have been well documented by research (Empson et al., 2020; Empson & Levi, 2011). For example, when children start learning fractions, they tend to partition items into pieces, repeatedly halving and then distributing the resulting pieces. As a result, halves and fourths tend to be familiar pieces for children. Additionally, in these early strategies children tend to distribute wholes without considering the number of people sharing (Hackenberg & Lee, 2015). Similar strategies for solving problems with whole numbers have been documented in the work of Franke et al. (2001), Jacobs and Ambrose (2008), and Jacobs et al. (2007), among others. Children can at times also use procedures and conventions for solving problems. What is important is to ensure during mathematics instruction that children use such procedures and conventions because their understanding of mathematics has reached a level of fluency in which such operations are routine and they do not need to decompose their strategies into simpler computations. The notion of fluency here is important, both mathematically and linguistically.

Methods

Our focus in this study was on describing how one bilingual elementary teacher emphasized bilingual children's mathematical ideas by rejecting district policies that required mathematics to be taught entirely in English. By decentering the policies that sought to standardize her and her students' language practices, all classroom community members were able to freely express their mathematical ideas. Data were drawn from interviews with the students in this teacher's classroom and observations of this teacher's mathematics instruction.

Setting and Participants

The work we present in this paper comes from the second author's classroom when she was teaching second grade. In the present work, we identify how she established a series of tasks and routines that created the space necessary for constant interaction among herself and her students.

The Bilingual Classroom of Ms. Adams. The tasks and routines in Ms. Adams's classroom entailed an expectation of collaboration and discussion that was centered on the children's mathematical ideas. An example of these activities is the instructional practice of circulating, or monitoring children's ideas in preparation for whole-group instruction (Stein et al., 2008). Researchers such as Jacobs and Empson (2016) have suggested that teachers devote a greater share of their instructional time to circulating, because when they do so, they are provided opportunities to respond to their students' mathematical thinking. Furthermore, in order for children to share their mathematical ideas, they enact their linguistic repertoire (Gutiérrez & Rogoff, 2003). In Ms. Adams's class, this repertoire was not restricted to the use of a single language. While circulating, Ms. Adams's own linguistic repertoire had to respond flexibly to her students' repertoire during conversations around problem solving.

In Ms. Adams's class, the children could expect that they would be permitted to share their mathematical ideas in whatever form and language they emerge, whether in English, in Spanish, or in both. Contrary to the conventional emphasis on the mastery of English and the general lack of interest in the particular knowledge and resources bilingual children possess (Gándara et al., 2010), Ms. Adams embraced dynamic language practices in her classroom and was mindful to foreground the mathematical ideas of her students.

We focused on the teaching of Ms. Adams in a bilingual class with 23 children. This classroom in a major Texas city served children with a variety of language resources and Latinx backgrounds. One child identified as biracial, while the other 22 identified as Latinx with ties to Mexico, Puerto Rico, El Salvador, and Honduras. Bilingualism was the norm in this classroom, with most children coming from bilingual home environments, while one child spoke an additional language, Otomí.

Two of the authors, MR&K, followed Ms. Adams and the same group of children for two consecutive years, from second to third grade. Ms. Adams approached her role as an educator with the goal of empowering children to use their full potential as bilingual citizens of the world. At the time of the study, Ms. Adams had been an elementary school teacher for 7 years, teaching in bilingual classrooms from Grades 2 to 5. During all her years of teaching, she has been connected with the exploration of children's mathematical thinking. Ms. Adams possesses extensive knowledge of children's mathematical thinking; focuses on children's thinking in all her teaching, not just mathematics; and helps other teachers learn about children's thinking and its role in instruction (Adams, 2018; Adams & Busey, 2017). In her class, children's thinking is valued and visible during problem solving. Rather than demonstrating strategies herself, Ms. Adams encourages children to generate and use strategies that make sense to them, and she routinely elicits and builds on their ideas. Children thus learn from one another, because they are expected to explain and justify each other's thinking as a part of what it means to do and know mathematics in Ms. Adams's classroom.

Data Sources

The work we present in this article comes from one-on-one interviews conducted by the first and third authors and an excerpt from a circulating portion of Ms. Adams's class when children were working independently. The interviews were conducted using guidelines for clinical interviews in which the goal of the interview is not to guide a child to the correct answer, but to ensure that the interviewer understands the mathematical reasoning of the child's strategy (Ginsburg, 1997). Questioning often focused on particular mathematical relationships noted in children's strategies (Jacobs & Empson, 2016) or on extending questions designed to push children's mathematical thinking further (Jacobs & Ambrose, 2008). Both sources of data were collected near the end of the school year in the second year we worked with Ms. Adams. All interviews and lessons were video recorded, and field notes were taken.

Interviews. The interviews took place over the course of a single week. Our plan was to interview every child in the classroom, but time constraints and occasional student absences prevented us from doing so. The first and third authors conducted 11 one-on-one interviews, each lasting about 30 minutes. We took turns when conducting the interviews. Sometimes the first author conducted the interview, and the third author was responsible for the video camera; at other times the roles were reversed.

Each child solved at least one story problem and at least one equation. Some of the children solved equal sharing problems, and some solved multiple groups problems. The equations included operations with whole and rational numbers. The

Appendix includes a sample of some of the problems and equations the children were asked to solve during the interviews. The interview problem we describe in the present study is a multiple groups problem.

A typical curriculum traditionally introduces equivalent fractions, followed by addition and subtraction of fractions (Empson & Levi, 2011). We depart from this sequence and follow the recommendations of Empson and Levi (2011), starting with a focus on children learning first how to create and name fractional quantities with equal sharing problems and then continuing with multiple groups problems. Empson and Levi defined multiple groups problems as word problems involving a whole number of equal groups of fractional amounts. This problem type allows children to engage in making connections foundational to understanding equivalency and operations (i.e., addition, subtraction, multiplication, and division) with fractions.

Individual Problem Solving—Circulating. Before the lessons we observed were taught by Ms. Adams, the three of us met and discussed the tasks Ms. Adams was going to use for each lesson. We came into Ms. Adams's classroom understanding why she selected the tasks and what goals she had set for instruction. We simply followed her with the video camera and recorded the flow of the lessons. We focused on the circulating portion of the lesson because it offered a view into the individual and small group conversations that occurred before whole group discussion of strategies. For the circulating component of the lesson we analyze here, students were solving the following equation: 65 - 38 = ____.

Both of the mathematics problems presented in this article are considered high-level cognitive demand tasks. In a high-level cognitive demand task, children's attention is placed on making connections, analyzing information, and drawing conclusions (Van de Walle et al., 2013). High-level cognitive demand tasks are nonroutine tasks that engage children in productive struggle and challenge them to make connections to concepts and other relevant knowledge (Van de Walle et al., 2013).

Analysis

Our analysis of the two examples we present here involved not only a consideration of what the teacher and researchers said and did but also of the situation, including what the children said and did. We looked for conceptual breaks in the conversation to determine our unit of analysis (Jacobs & Morita, 2002). Sometimes the unit consisted of a teacher's single comment or question, and at other times it included a linked sequence of comments and questions because teachers often need to persist to support or extend children's thinking.

We started by transcribing the children's interviews. Then we developed a provisional list of codes. This list of codes came mainly from our proposed two tenets above, namely use of translanguaging and employing CMT-based instruction. For instance, an initial code was "language use." Here we were identifying all instances where children used English or Spanish to express their ideas. Another was "use borrowing". Here we were trying to identify instances in which children decided to borrow a word from either English or Spanish. In this list we also included other general aspects of how the child expressed his or her ideas, for example, "used notation to represent his or her mathematical thinking." Once we had completed the list of initial codes, we used a randomly selected sample of four child interviews to code. Two of the authors, MR&K, double-coded this sample for reliability.

After this round of coding, we shortened our list and identified a second list of possible codes, following the coding methodology described by Saldaña (2015). Then we randomly selected another set of interviews. This second sample was also coded by MR&K, who then met to discuss and resolve discrepancies. Through this process, MR&K developed a list of codes that were related to one another in a coherent way and aligned with the research question. For example, we coded for translanguaging by individually identifying instances of code-switching and borrowing, because these can serve as evidence of the presence of translanguaging. We used observable instances of code-switching and borrowing to serve as externally visible evidence that children and teachers were free to deploy their linguistic repertoire strategically. We also coded as translanguaging instances where speakers might remain within the boundaries of one named language but showed an inventiveness to meet their communicative needs in one language though influenced by their knowledge of another language. This is evident in an example from the interview we analyze in this paper. These codes allow us to present to the reader examples from the mathematics classroom linked to what is being defined in the literature as the language practices of bilingual individuals. At the same time, we needed to study how these practices interplayed with the instructional decisions made by Ms. Adams when teaching mathematics.

We completed all coding processes by hand. However, we used MAXQDA 2020, a software package for qualitative and mixed methods research analysis (VERBI Software, 2020), to watch the videos and add analytic memos during the first phase of coding.

After finalizing the coding scheme, we randomly selected the interview with Hugo, an 8-year-old Mexican American boy in Ms. Adams's classroom, for the purpose of the work we present here and separately coded it. Through his interview, we were able to identify and distinguish the language practices of bilingual students (i.e., borrowing, code-switching, and translanguaging) described above at the same time that we were able to identify the mathematical ideas expressed by Hugo. Our purpose was to highlight the mathematical content in the conversations and the mathematical understanding represented through the connections the child makes when these practices occur. We additionally randomly selected and coded an interaction between Ms. Adams and Gabriel, another bilingual student of Ms. Adams, during the circulating portion of a lesson, identifying and analyzing similar behaviors and practices presented by Hugo. In these conversations, we tracked the linguistic practices described above with links to specific examples so that the research team could discuss the appropriateness of these categorizations. These categorizations were brought to the teacher to check their appropriateness.

Findings

The following section presents our analysis of the one-on-one interview with Hugo and circulating time with Ms. Adams. In our analysis of Hugo's language choices during the one-on-one interview, we reveal the complex process of conveying mathematical ideas in bilingual contexts. At the same time, we see that there are at least two levers which we can use to manage the complexity effectively and efficiently in the moment: one is the lever of bilingualism itself, where a bilingual teacher will more likely follow a child's bilingual expressions in real time without a great need for probing and rephrasing; the other is the willingness to table issues of language while trying to understand the content being expressed. In analyzing Ms. Adams's circulating time in the classroom, there are several instances that require pausing and paying close attention to the mathematical ideas being discussed (e.g., the ideas behind regrouping or balancing an equation). Below we attempt to unpack them.

Hugo's Interview

Hugo, a Mexican American child from a bilingual home who is not labeled an English Language Learner, was given the multiple groups problem described in Figure 1. The problem was written in both languages when presented to Hugo.

Un chef está preparando 6 ensaladas de frutas. Cada ensalada usa ½ manzana. ¿Cuántas manzanas necesitará el chef para hacer las ensaladas de frutas? [A chef is making 6 fruit salads. Each salad has ½ apple. How many apples would he need to make the fruit salads?]

Figure 1. Multiple Groups Problem

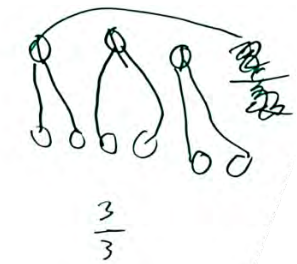

Figure 2. Hugo's Work for 6 Fruit Salads Each Containing ½ an Apple.

Hugo's drawn solution is shown in Figure 2. From our initial interpretation of Hugo's work, we regarded his notation of the solution as incorrect. When we asked him to explain what 3/3 meant in his solution, he responded as follows:

> Hugo: Este [apuntando al 3 en el numerador] son las manzanas. Oh no! Esto está mal. Es dos-tres porque hay 3 manzanas y están esplitadas en doses.
>
> [This one [pointing at the 3 in the numerator] is the apples. Oh no! This is incorrect. It is two-three because there are 3 apples and they are *esplitadas* in twos.]

If we adopt Hugo's perspective, the notation makes sense. The 2 in the numerator represents the pieces into which each apple is split. The 3 in the denominator represents the total number of apples needed. In his explanation, we see that he is borrowing from English to say "*e-split-adas*" (literally "split-ed," with "split" adopted from English and modified with the initial "*e-*" to fit the phonology of Spanish), which makes sense to us in the context of his explanation. Another noticeable feature of his explanation is his use of "*doses*" (the plural of "*dos*").

As Hugo was providing his explanation, we noted these features in his speech and continued working with him:

> Krause: Yo vi que partiste las manzanas y colocaste un pedazo de manzana en cada ensalada. ¿Por qué partiste las manzanas así?
>
> Hugo: Porque hay 6 de éstas [apuntando a las ensaladas] y nada más necesitas tres, porque las puedes cortar en un medio. Y cada uno puede agarrar un medio de cada manzana.
>
> [Krause: I saw you split the apples and add a piece of apple to each salad. Why did you split the apples that way?
>
> Hugo: Because there are 6 of these [pointing at the salads] and you only need three, because you can cut them in a half. And each one can get a half of each apple.]

Notice that the conversation with Hugo focused on his strategy and not on his use of language. However, Krause did not use Hugo's words, instead opting for "*partir*" to refer to the action Hugo was referring to earlier as "*esplitar*." Later, Hugo used "*cortar*" when referring to the same process. Likewise, he employed "*medio*" instead of "*doses.*" In both cases, we can see that Hugo's use of language was not the result of lacking more vocabulary but rather a representation of his in-the-moment thinking. In this interaction between Hugo and Krause, translanguaging allowed Krause to let go of expecting conventional fractional language and instead identify and understand Hugo's mathematical thinking. Krause provided the space for Hugo to freely express his thinking while simply responding by choosing different words in Spanish. Towards the end of the explanation, we notice that Hugo decided to use "*medio*" rather than his initial choice of "*doses.*" Both Hugo and Krause were interacting naturally, using the language that made sense to both of them for the topic at hand. As this went on, Krause was able to see more and more of Hugo's understanding and hear more of what he knows.

We chose to share the interview from Hugo in this paper because in this article we invite both researchers and teachers to consider their own practices with bilingual students. Students like Hugo come from bilingual households but are not labeled English Language Learners. Yet, as bilingual individuals, they accommodate their language choices to meet the context they are in (Adams, 2015). That means our language choices can help shape the students' own choices. As a result, this lends particular importance to how we respond to the features of their language repertoire. Hugo knew the researcher interviewing him, watched her engage with his teacher, and had a sense of who she was, which influenced his choices to respond to her in Spanish. He considered both that she spoke English and that she was speaking in Spanish and so perhaps concluded that she preferred speaking in Spanish. In other words, students will make language choices that accommodate us just as much as we make choices to accommodate them.

Gabriel's Individual Problem Solving (Ms. Adams's Circulating)

Children in Ms. Adams's classroom were allowed to use any of the resources available in the classroom to solve the problems. In the example we share below, Gabriel, a Mexican American child from a bilingual home who is labeled an English Language Learner, was using a set of Unifix cubes. The children were tasked with solving a subtraction problem using their own strategies. As the traditional subtraction algorithm—arranging differences vertically, subtracting in the ones position, borrowing from the tens, etc.—is not taught in this student-centered classroom, a variety of strategies was observed across the classroom. As he was solving the equation 65 - 38 = ___, Ms. Adams approached him and asked the following:

Ms. Adams: Okay, Gabriel, what's up?

Gabriel: Just making tens to make it to 60.

Ms. Adams: Okay, can I help you? How can I help you? Can I make something for you? Gabriel: You can make 20 tens and I can make some. You can make 10 tens and I can make 10 tens.

…

Ms. Adams**:** ¿Tienes 65? [Do you have 65?]

Gabriel: That makes no sense.

Ms. Adams: What?

Gabriel: If it's 65 minus 38, 'cuz you're taking away 38 and there's only 5 in the 65, and then… And if you're taking away 38 and then you, um, take away the 8 from the 30, then you take away 5, there's gonna be 2 more in the 8.

Ms. Adams: So, ¿lo que te está como atorando es la idea de quitarle 8 cuando sólo hay 5 en las unidades? [So, what is bothering you is the idea of taking away 8 when you only have 5 in the units?]

Gabriel: Hmm…

Ms. Adams: Oh, Okay. Pues ¿cómo lo haríamos? ¿Hay algún otro lugar donde le podríamos quitar? [Oh, Okay. So how can we do it? Is there some other place where we can take it away from?]

Gabriel: [Nods in agreement.]

We note how Ms. Adams arrived with an attitude of support and allowed Gabriel to lead the conversation. She was familiar with the children in the class and their ways of explaining their ideas, so she understood that when Gabriel asked for "20 tens," he was talking about making 20 with tens. As a result, Ms. Adams was able to infer Gabriel's thinking from the strategy he was using to solve the problem and did not focus on ensuring that he meant 20 and not 200. That is, Ms. Adams's familiarity with Gabriel's linguistic habits allowed her to recognize "20 tens" as one of Gabriel's linguistic symbols for the correct mathematical concept at issue, regardless of the imprecision of the linguistic expression itself.

This recognition allowed Ms. Adams to postpone linguistic interruptions (waiting until the concept is grasped before focusing on its linguistic form) in order not to derail Gabriel's mathematical train of thought. She was also able to use that inference to identify the part of the equation that he was having difficulty with. Ms. Adams intentionally created the space for Gabriel to talk while she carefully listened to him. She articulated the issue at hand and provided scaffolds for his next steps in problem solving. She was also careful not to correct Gabriel's language around making tens early on in their interaction, an interruption that would have taken the focus away from his strategy, and instead kept the focus on what he felt he needed in order to accomplish the mathematical task at hand. Although she may embed conversations about what "20 tens" might really be during another day's discussion, during this interaction her focus followed Gabriel's direction.

Another teaching move that we can highlight from this excerpt is the opportunity to change fluidly from one language to the other. While Gabriel provided his explanation completely in English, Ms. Adams responded completely in Spanish. Taking on a more individualized approach, which shows knowledge of the children, their language preferences, and personalities, Ms. Adams decided how to respond based on what she knew about Gabriel. During the coding process, Ms. Adams shared that she knew he would be able to understand if she questioned him in Spanish, whereas other children in the class might have required a different approach. Ms. Adams made sure that Gabriel was capable of comprehending the mathematical content they were discussing in both languages. She acknowledged and promoted Gabriel's bilingualism while providing a space that gave each child the right to choose a language to express their mathematical ideas. Once Gabriel recognized he could take 8 away from 10, Ms. Adams continued as follows:

> Ms. Adams: Ah! Okay, vamos a quitarle este 8. [Okay, we'll take away this 8.]
>
> Gabriel: two, four, six, eight.
>
> Ms. Adams: Okay, ¿qué nos quedó? [Okay, what do we have left?]
>
> Gabriel: Ten, twenty, thirty, forty, fifty. Fifty. Fifty, fifty-five, fifty-six, fifty-seven. Fifty-seven.
>
> Ms. Adams: Okay, diez, veinte, treinta, cuarenta, cincuenta, cincuenta y uno, cincuenta y dos, cincuenta y tres, cincuenta y cuatro, cincuenta y cinco, cincuenta y seis, cincuenta y siete. So, lo que nosotros encontramos es que, para quitarle el ocho, tuvimos que entrarnos a uno de los dieces. ¿Qué te faltó quitar? Porque sólo le quitaste 8. [Okay, 10, 20, 30, 40, 50, 51, 52, 53, 54, 55, 56, 57. So, what we found is that, to take away the 8, we had to enter one of the tens. What was left for you to take away? Because you have only taken away 8.]
>
> Gabriel: 30...
>
> M. Adams: ¿Ves una manera más fácil de quitar 30? [Do you see an easy way to take away 30]
>
> Gabriel: Should I just take away these? [He points at a stack of 3 tens he had.]
>
> Ms. Adams: That sounds easier, right?
>
> Gabriel: Yeah.
>
> Ms. Adams: How many did you take away?
>
> Gabriel: 30. The answer is twenty-seven.

Ms. Adams continued to support Gabriel to make sure he understood how to subtract the units that were problematic for him. She also continued to provide the language support described up to this point. However, throughout the conversation Gabriel was the one solving the problem, while Ms. Adams took advantage of her knowledge of Gabriel's work and abilities to ask questions that guided him through his own understanding. The mathematical details in Gabriel's strategy are important to note. He took away 8 from 10, and then took away 30 more from the 5 tens that remained. The flexibility in his thinking and his number sense (e.g., seeing 65 as a group of 6 tens and 5 ones) allowed him to see how to take away 8 from 10 rather than from 5, which he had reported as difficult for him. Once this difficulty was noticed and addressed by Ms. Adams, she continued to talk to Gabriel. In this excerpt, she proposed to go and look at the equation (62 - 38 = ___) they were solving earlier as a class.

> Ms. Adams: ¿Hay alguna relación entre 24 y 27? So, tú me habías dicho: 62 más 3 es 65. ¿Qué es 24, si le sumamos otros 3? [Is there a relationship between 24 and 27? So, you had told me: 62 plus 3 is 65. What is 24, if we add 3?]
>
> Gabriel: 27.
>
> Ms. Adams: 27. So, si hubiéramos ido con tu idea de hacer como una balanza entre lo que teníamos y lo que acabamos de hacer… Tú dijiste, aquí le sumamos 3. Pués aquí también le podemos sumar 3. Y eso te dio la respuesta que sacaste, ¿no? [So, if we had gone with the idea of making a balance between what we had and what we just did… You said, here we add 3. And so here too we can add 3. And that gave you the answer you got, right?]

Ms. Adams was able to identify an opportunity to go back to the initial ideas shared by Gabriel. He had initially thought of 62 – 38 =___ and was thinking that if he added 3 to 62, he would get 65, but then he got stuck and could not continue. Ms. Adams saw the opportunity to extend his thinking, going back to his initial idea after he had solved the problem. In Figure 3, we see how Ms. Adams used the idea of a scale to balance both sides of the equation Gabriel was thinking about. Ms. Adams was attempting to help Gabriel realize that the problem he actually solved, 62 – 38 =___, was nearly the problem he wanted to solve, 65 – 38 =___. This provides an example of how children can scaffold themselves into a solution to the original problem when given work they have done on a similar problem.

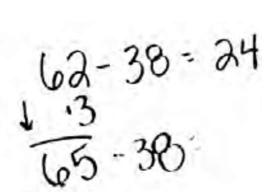

Figure 3. Strategy Shared by Gabriel When Solving the Initial Problem

As noted earlier, Gabriel was labeled an English Language Learner. Just as Hugo, Gabriel came from a bilingual home. Ms. Adams knew that his family chose a dual language program because they wanted Gabriel to maintain and grow both languages and that he comfortably interacted in both throughout the day. Translanguaging pedagogy (García & Kleifgen, 2010) often involves alternating between input in one named language and output in another. Ms. Adams had been reading about and attempting to include these pedagogical practices in her bilingual teaching repertoire. So, when Gabriel responded in English without hesitation and Ms. Adams continued to speak only in Spanish, this resulted in a classic alternation between named languages across input and output.

Discussion

This work seeks to foreground examples from a dual language classroom that allowed us to identify key aspects of how bilingual individuals interact in the context of teaching and learning mathematics in the elementary classroom. These examples not only allowed us specifically to identify practices of bilingual individuals described in bilingual research but also helped us notice two main aspects of how a bilingual teacher teaches mathematics in the bilingual classroom while minimizing the tensions described by Adler (2002) and commonly found in previous research. Specifically, we see here how the teacher foments bilingualism in a way that accepts expressions of mathematical understanding regardless of their language of expression. At the same time, she foregrounds mathematics in the instant a student expresses understanding, awaiting another moment to model other ways of expressing such understanding in one or the other of the languages of the classroom. Although we cannot use these two aspects to generalize about bilingual classroom practices, we can use them as a point of departure for promoting future research that helps the field to make spaces for teaching and learning content without sacrificing bilingualism. Below, we discuss how Ms. Adams allowed for a freer bilingual mode of discourse, language choice, and deemphasis of linguistic form as we attempt to describe a bilingual mathematics classroom that displaces monolingualism and emphasizes children's mathematical ideas.

Children Have Uninhibited Conversations With the Teacher

The present work focused on a part of a class lesson in which the teacher circulated and engaged in one-on-one conversations with children during problem solving. In our examples, Ms. Adams engaged in these conversations by responding to individual children's mathematical thinking and used them as a platform for developing an even deeper understanding of the mathematical concepts. Several points in the circulation portion of the class must be analyzed in slow motion. For instance, Ms. Adams's knowledge of the children in the class, the knowledge provided by her bilingualism, and her ability to listen carefully to the children's ideas facilitated uninterrupted communication with Gabriel, even when his initial response of "20 tens" was "mathematically" not what he needed to use for solving the problem. We cannot of course be certain what would have been the outcome of stopping Gabriel, correcting him, and moving on with the conversation. However, we suspect that by not stopping him and by allowing the conversation to continue, Ms. Adams deemphasized linguistic form in order to create a space for a fluid sharing of mathematical ideas. Ms. Adams could then use these ideas to teach the target content of the lesson. We also surmise that this deemphasizing of linguistic form creates a freer bilingual mode of discourse. For instance, we can highlight the fluidity with which both teacher and child communicate in two languages as if they were only one, which aligns with what García and Kleifgen (2010) have defined as dynamic bilingualism. The mathematical ideas were not inhibited and bilingualism was promoted. At the same time, we can identify in this excerpt that these interactions have the characteristics of a bilingual classroom, where the children will naturally switch languages and the teacher is able to attend to this constant variability in usage.

Creating these opportunities and making them widely available to children in the mathematics classroom have the potential to promote children's acquisition of English while maintaining, and hopefully improving, their abilities in any other languages they bring to the classroom (García, 2009). At the same time, they continue to build on their mathematics understanding. Translanguaging, in this case, allowed for children's natural bilingualism to further their mathematical learning. The benefit of the approach demonstrated here lies in its ability to support their use of both their languages and to help instill in them an understanding that neither school nor mathematics need impose a strict adherence to monolingualism. In this way, as a nation, we could maximize the efforts to promote a more cohesive program for preparing bilingual learners and respond to language diversity not with a bias shaped by political, social, and economic forces but rather by a systematic idea about language itself.

Using One Language

In the interview we shared in this analysis, we can see how productive it was to avoid the rigid use of only one standardized language in order to maintain mathematical engagement and to further develop the children's understanding. Hugo's engagement in a linguistic practice that is normal for bilingual students was made possible because Krause afforded him the space to share his mathematical ideas in their purest form, that is, in the form in which they naturally emerged. Because MR&K followed Ms. Adams's class (with the same children) for two consecutive years, they knew Ms. Adams's instructional practices and the children in the class well. Importantly, MR&K knew they could afford, and in fact knew it was expected of them, to give Hugo the space to express his ideas in their purest form because that was the way in which Ms. Adams had taught MR&K. Hugo solved the problem on his own, and it was not until he was asked to articulate his mathematical thinking (where he had been allowed to draw on his mathematical repertoires—those mathematical practices that he can apply and use when solving problems) that he had to draw upon his full linguistic repertoire in order to share his process aloud with the interviewer. Sharing his thinking was not only to the benefit of the researcher, however. It was necessary for Hugo to share his thinking because, at first glance, the symbolic notation he had written was incorrect, even though he had a valid strategy.

The process of engaging in discussion around Hugo's thinking made clear the depth of his understanding of the problem and the logic of his invented notation. This ensures clearer pathways for future instruction and better recognition of children's capacities. Drawing on his full linguistic repertoire allowed Hugo to make himself and his mathematics recognizable and legitimate.

García and Wei (2014) suggested that "translanguaging refers to *new* language practices that make visible the complexity of language exchanges among people with different histories, and releases histories and understanding that had been buried within fixed language identities constrained by nation-states (p. 21)." In this case, Hugo and Krause entered into their exchange with certain histories and institutional labels. Translanguaging is what allowed for them to engage in an exchange that never sacrificed complexity and that resulted in new practices, including new mathematical practices. Hugo was given the space to use complex language practices in order to communicate something new (his invented notation). That space, in part achieved by ceding the emphasis on standardized formalisms of both mathematical and linguistic expression while maintaining a focus on mathematical content, allowed his natural bilingual abilities to convey his understanding in a manner recognizable to the instructor. This is an example of how translanguaging in the mathematics classroom can serve a powerful mathematical purpose.

Krause in this case was able to identify what Hugo knew and understood, which then could be used to make instructional decisions suited to his own understanding. This is consistent with what other researchers have found working in bilingual classrooms. For example, Moschkovich (1999) found that bilingual teachers are more effective when they keep their focus on mathematical ideas, regardless of how they are expressed. Imposing an unnatural formality of language may stifle children's natural interest in mathematics. Language separation policies carry with them the risk of marginalizing and denigrating bilingual children's everyday translanguaging practices (Martínez et al., 2015). Mathematics classrooms are situated within wider sociolinguistic contexts, and language use becomes more than an instrument to teach mathematics (Barwell, 2009). Rather, it becomes a venue for promoting bilingualism as a norm and for equally valuing the use of a home language.

Our goal with our work is to engage teachers and teacher educators in respectful—yet critical—dialogue around the complex nature of everyday bilingualism in the mathematics classroom. We recognize that teachers are often promoting policies that are imposed schoolwide or even districtwide and have not been given the space to question or challenge their utility. The present work provides evidence of an actual bilingual mathematics classroom where a teacher and her students engaged in translanguaging, a perfectly normal and natural mode of bilingualism (Martínez et al., 2015). What happens in language instruction is not the main point we are trying to convey with our work. Our focus is on how mathematics instruction can be used to better understand children's ideas about mathematics and how children's ability to hear their language as it is naturally expressed allows for more aware and conscious language teaching.

In this article, we presented two examples of how a bilingual elementary teacher taught mathematics to her bilingual students. We set out to highlight these examples as a way to answer what seem to be common questions of practice: "How?" and, more importantly in the context of teaching mathematics in the bilingual classroom, "What does it look like?" Were we to provide a "recipe" for teaching mathematics in bilingual classrooms, we would necessarily overlook numerous aspects of the complex interaction among language, culture, and learning. In both examples discussed in this article, we can see how institutional labels mask the complexity of children's language repertoires. Hugo, who is not labeled an English Language Learner, sustained a mathematical interaction in Spanish with a bilingual researcher, sharing his in-the-moment mathematical thinking by flexibly deploying his linguistic repertoire. Gabriel, who is labeled an English Language Learner, shared his thinking in English while his teacher responded in Spanish, with their conversation easily transcending the boundaries of named languages (Wei & Ho, 2018). Decision making on the part of Ms. Adams required knowing both children, their families, and their stories, details that cannot be assumed away or simplified for fast takeaways.

These examples highlight how a deemphasis on the formalities of mathematical expression, both in their symbolic and linguistic form, can help teachers attain in the moment a truer glimpse into a child's understanding of mathematical content. At the same time, such an environment encourages linguistic and social practices that serve to strengthen the child's bilingual abilities and identities. Earlier we mentioned the three common tensions of teaching in bilingual contexts: (1) between using formal or informal language, (2) between using children's home language and the language of the school, and (3) between teaching mathematics and teaching language. In the examples we shared, language was ultimately not a cause of tension but rather a tool for foregrounding mathematical ideas. In the case of Hugo, the "imperfection" of his language use was essentially inconsequential to the interaction, whose primary goal was to foreground the mathematical idea. Hugo's new way of expressing his mathematical idea was a fantastically apt means for communicating his understanding of mathematics. His teacher's ability to capture this understanding and build more complex mathematical ideas on top of it is exactly the kind of instructional skill that bilingual teachers need to develop in order to support bilingual children's deep understanding of mathematics.

The examples we presented here are meant to challenge some common assumptions regarding translanguaging. Translanguaging does not require the abandonment of language goals; rather, it requires intentionality and thoughtfulness. If we want a performance in Spanish, we should consider how we will respond to students' lexical creativity. If we recognize students' bilingualism regardless of labels, we may play with conversations where input and output vary across named languages. Translanguaging and bilingualism are not monoliths or implemented simply; they are as complex and dynamic as people themselves. These examples represent moments in our data where language surprised us, as both researchers and practitioners.

A Final Word

In this article, we identified characteristics of a teaching practice for capturing and encouraging teaching that is responsive to children's mathematical ideas and linguistic repertoire. However, we recognize that the caring and respectful stance evident throughout Ms. Adams's teaching and her experience teaching is not enough. Her knowledge of Spanish and her own bilingualism are key components of the work she does in her classroom. As mathematics teacher educators, we have the responsibility to prepare more teachers with the same skills as Ms. Adams, and by promoting bilingualism in the schools we are developing a generation of bilingual citizens that may eventually become teachers themselves. Research has also provided evidence that bilingualism has benefits that extend beyond the ability to communicate in multiple languages (Kroll & Dussias, 2017). For example, greater intercultural awareness and open-mindedness (Byram, 1997) as well as increased access to post-secondary education (Kroll & Dussias, 2017) are a few examples of what we can accomplish if we focus our efforts on promoting and maintaining bilingualism.

We agree that being bilingual is in fact an advantage, and we promote the development of a bilingual teaching practice in schools. However, as Welch (2015) straightforwardly stated, "teachers need not be paralyzed by their own monolingualism" (p.93). Within their own teaching practice, teachers have the option of assuming the role of both expert and learner (Fránquiz & Reyes, 1998). Knowing their individual students and their strengths, and building their own teaching practice around these resources, not only allows teachers to encourage children to make connections and use cultural and linguistic resources (Welch, 2015) but can also help teachers plan lessons that are relevant to the children in their classroom at the same time that the teacher herself is engaged in developing her own learning.

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Appendix

Sample Problems and Equations Used During Interviews

Join Result Unkown (larger numbers)

Héctor tiene ___ películas. Hay una oferta de películas en la tienda y compra ___ más. ¿Cuántas películas tiene Héctor ahora?

Hector has ___ movies. There is a sale on movies in the store, and he buys ___ more. How many movies does Hector have now?

(30, 23) (86, 25) (127, 34)

Multiplication

La clase de segundo grado se está organizando para una fiesta del fin de año. Una de las mamás compra ___ cajas de peras para la fiesta. Cada caja tiene ___ peras. ¿Cuántas peras hay en total?

The second-grade class is getting organized for an end-of-the-year party. One of the moms buys ___ boxes of pears for the party. Each box has ___ pears. How many pears are there in all?

(4, 12) (6, 18) (8, 24)

Separate Change Unknown

Hay ___ niños jugando afuera en el recreo. Unos de los niños regresan a los salones. Ahora hay ___ niños jugando afuera en el recreo. ¿Cuántos niños se metieron a los salones?

There are ___ children playing outside at recess. Some of the children return to the classrooms. Now there are ___ children playing outside at recess. How many children entered the classrooms?

(43, 20) (51, 29) (125, 75)

Equations

1 = ½ + ____

½ + ½ + ½ = ____