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Trends in learning and teaching of geometry: The case of the Geometry and its Applications Meeting

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ABSTRACT

We characterize the thematic trends of the *Geometry and its Applications Meeting*. This meeting is held periodically in Colombia, country in which our study was carried out. We used a taxonomy of key terms specific to mathematics education to code the proceedings of this meeting. The study variables are purpose, educational level, pedagogical notions, and topics. We establish the thematic trends in terms of the values of the variables. We describe their evolution over time and, using a normalization process, we compare the extent to which geometry topics are treated with respect to the other variables. The meeting has disseminated activities and curricular innovations to a lesser extent. The community that attends the meeting is focused on the theoretical development associated with geometry and on higher educational levels. The papers that address pedagogical notions focus on learning and the classroom. The topics with the highest percentage of research are geometry in three dimensions and Euclidean geometry. We suggest that the meeting should promote the dissemination of curricular innovations and give more attention to the notions of teaching, curriculum and assessment in both research and innovations. We perceive the need to address learning and teaching in preschool and primary education.

Keywords: content analysis, document production, geometry, pedagogical notions

INTRODUCTION

Bibliometric studies provide valuable information that allows us to evaluate the development of knowledge activities and, specifically, allow us to know the scientific activity of institutions (Montilla, 2012). In addition to establishing the focuses of interest in this discipline, studies of document production make it possible to characterize the communities that converge in it. The research literature highlights the need to determine the patterns of research productivity in mathematics education in order to grant it scientific status (Fernández et al., 2003).

The studies carried out on document production in mathematics education have focused on analyzing the production of doctoral programs, and collaboration in articles, journals, and some Spanish conferences (Bracho et al., 2014). These studies emphasize patterns of productivity and networks for collaboration and authorship (Jiménez-Fanjul et al., 2013; Maz et al., 2009; Maz-Machado et al., 2012; Qin & Peng, 2011; Vallejo-Ruiz et al., 2006). The main objective of these studies has been the analysis of the evolution of research in the discipline (Bracho-López et al., 2010, 2012; Maz et al., 2009; Molina et al., 2011).

The visibility of knowledge in mathematics education, as far as the Ibero-American community is concerned, tends to be similar in terms of the number of articles published in journals and the number of contributions to meetings in national and international meetings (Castro & Gómez, 2021; Molina et al., 2011). However, few studies have been developed on the knowledge dissemination media, so they merit further research (Castro & Gómez, 2020; Maz-Machado et al., 2011). In addition, few studies are identified that focus on the content of the documents, from the specificity of the phenomena and problems they address (Castro et al., 2019, 2020). The studies that address the content indicator usually do so from topics related to education or mathematics, but not from specific categories of mathematics education. The document studies published so far do not address specific topics of school mathematics such as geometry, despite being one of the most discussed topics in the Ibero-American community of mathematics educators (Castro et al., 2019).

As part of a broader work, which seeks to characterize the Ibero-American community of mathematics education, we consider relevant to describe in this article the production of this discipline in the framework of the *Geometry and its Applications Meeting–Encuentro de geometría y sus aplicaciones*, in Spanish–(Universidad Pedagógica Nacional et al., 2016) to have a global vision of its focuses of interest for two reasons. First, this meeting is a dissemination scheme with an impact on the community of mathematics educators at the international level. The meeting deals with a specific topic in mathematics that has a direct influence on the

education of mathematically competent citizens (Ministerio de Educación Nacional, 2006). The teaching and learning of geometry require studies associated with what is evaluated and what is actually taught given its importance in national and international standardized tests (Gómez, 2011). The second reason why the study we present is relevant is related to the analysis of the thematic categories of the meeting. Although studies on collaboration and methodology have been conducted in mathematics education, the thematic focuses have not been studied to a greater extent at the international level (Bracho et al., 2014).

In this study, we characterize the document production of the community that converges around the *Geometry and its Applications Meeting*, produced in the XIII to XXIV versions. We establish the thematic trends in terms of the values of the variables purpose, educational level, pedagogical notions, and topics. We determine the focus of interest, their diachronic evolution and analyze the relationships between the variable topics with the other variables.

CONCEPTUAL FRAMEWORK

We selected content analysis (Mayring, 2015) as a technique to characterize the memories of the meeting. This technique makes it possible to formulate, from certain data, valid inferences that can be applied to their context (Krippendorff, 1990). The quantification of the bibliographic characteristics of papers can be associated with the number of papers with metric terms in the title, abstract, and keywords; the number of papers with metric terms in the introduction and methodology; and the number and/or percentage of content elements that the papers address (Verdejo, 2011). We used principles of descriptive bibliometrics (Montilla, 2012) to perform the content analysis of the meeting proceedings. To do this, we defined bibliometric indicators, which allow us to measure sources of productivity that can be tabulated and give the possibility of establishing comparisons (Bueno Sánchez & Fernández Cano, 2003), such as document production and thematic categories (Montilla & Pérez, 2016).

To characterize the document production of the community that converges in the meeting, we selected a taxonomy of key terms. In this study, we rely on the taxonomy constructed by Gómez and Cañadas (2013). These authors propose a taxonomy based on a standard for the construction, format, and management of controlled vocabularies (National Information Standards Organization, 2005). The authors propose categories for purpose, educational level, curriculum theory topics, and mathematics topics. We describe these categories below.

Purpose

The purpose is associated with the purpose of the paper: research, essay, innovation, and activity.

Educational Level

The educational level focuses on the type of training of the subjects referred to in the paper: early childhood education, primary education, secondary education, middle school education, adult education, undergraduate degree, postgraduate studies, vocational training, and all educational levels.

Themes of Curricular Theory

The taxonomy is based on a conceptual framework specific to mathematics education and on a curriculum approach that seeks to address four central issues: knowledge to be taught, learning, teaching methods and assessment of learning (Rico, 1997). From this theory, the category of curriculum theory contains key terms associated with

1. education system,
2. school,
3. classroom,
4. learner,
5. teacher,
6. learning,
7. teaching,
8. assessment, and
9. curriculum.

Mathematics Topics

The taxonomy proposed by Gómez and Cañadas (2013) distinguishes the key terms that refer to mathematics education from those that refer to mathematical content. In terms of mathematical content, the authors differentiate school mathematics from higher mathematics. The category of school mathematics includes the contents of calculus, statistics, geometry, measurement, numbers, probability, and algebra. The category of higher mathematics includes the contents of algebra, analysis, combinatorics, calculus, differential equations, statistics, geometry, mathematical logic, and discrete mathematics, among others. In this study, we focus on the topics of geometry in school mathematics (ruler and compass constructions, geometric shapes, analytic geometry, geometry in three dimensions, Euclidean geometry, geometric relations, theorems, basic topology, geometric transformations, trigonometry, other topics of geometry in school mathematics, and general subjects of geometry in school mathematics) and on geometry in higher mathematics.

Table 1. Number of papers per meeting

Meeting	Number of documents
XIII	33
XIV	28
XV	41
XVI	56
XVII	28
XVIII	48
XX	56
XXI	51
XXII	35
XXIII	23
XXIV	39

Table 2. Variable values

Variable	Variable values
Purpose	Activity, essay, innovation, and research
Educational level	Preschool, primary, secondary, middle, or high school, undergraduate, postgraduate, vocational education, adult education, all levels of education, other level, and no educational level
Pedagogical notions	Educational system, educational center, classroom, student, teacher, learning, teaching, assessment, curriculum, other notions of mathematics education, and mathematics education and other disciplines
Topics	Constructions with ruler and compass, geometric shapes, analytical geometry, geometry in three dimensions, Euclidean geometry, vector geometry, geometric relations, theorems, topology, geometric transformations, trigonometry, general school geometry, general higher geometry, other geometry subject, other subjects (not geometry), and general all topics

OBJECTIVES

The general objective of the study is to characterize the document production of the XIII to XXIV proceedings of the meeting in terms of the values of the variables purpose, educational level, pedagogical notions, and topics, which respond to the categories mentioned in the conceptual framework. We characterize this document production by attending to three of its characteristics: its focus of interest, its evolution over time, and the behavior of the variable topics in relation to the values of the other variables. We establish the following specific objectives of the study.

1. To establish the focal points of interest of the meeting in terms of the values of the variables.
2. To present the evolution of the values of the variables throughout the meetings.
3. To establish the behavior of the values of the variable topics with respect to the values of the variables purpose, educational level, and pedagogical notions.
4. To identify the behavior of the values of the variables purpose, educational level, and pedagogical notions in the values of the variable topics.

METHOD

The study is exploratory and descriptive. In what follows, we describe the sources of information and the study procedures.

Sources of Information

We characterized the document production of the meeting with the proceedings of meetings XIII to XXIV, except for meeting XIX, whose proceedings are not available in open access. We worked with 438 papers. In **Table 1**, we present the number of papers that were published per meeting.

Procedures

The content analysis and bibliometric study that we conducted of the proceedings required the definition of the variables, the design of the coding instrument, the coding of all the papers, the summary of the coding, the organization of the coding results and the analysis of the coding results. We focused on the analysis of the bibliometric characteristics of the papers from the identification of key terms and their organization into categories according to the proposal of Gómez and Cañadas (2013).

The variables that we used to analyze each paper arise from the categories of the conceptual framework. The category curriculum theory is the basis for the variable pedagogical notions. We associate mathematical content with the variable topics; however, we focus on content associated primarily with geometry. In **Table 2**, we present the values of the variables in the study.

To the previous variables, we added the variable time whose values correspond to the numbering of the meetings: XIII (2002), XIV (2003), XV (2005), XVI (2006), XVII (2007), XVIII (2008), XX (2011), XXI (2013), XXII (2015), XXIII (2017), and XXIV (2019). We do not use the year of the meeting since proceedings were produced annually until the XVIII meeting and biannually from the XX meeting onwards.

Table 3. Crossing the variable purpose with the variable time

Purpose/meeting	Activity	Essay	Innovation	Research
XIII	15.2	27.3	0.0	57.6
XIV	10.7	46.4	3.6	39.3
XV	9.8	48.8	0.0	41.5
XVI	12.5	8.9	0.0	78.6
XVII	46.4	46.4	0.0	7.1
XVIII	18.8	64.6	2.1	14.6
XX	25.0	41.1	7.1	26.8
XXI	33.3	25.5	0.0	41.2
XXII	25.7	22.9	17.1	34.3
XXIII	0.0	4.3	78.3	17.4
XXIV	0.0	5.1	61.5	33.3

Note: Values are given in percentages per meeting

Table 4. Crossing the variable purpose with the variable topics

Purpose/purpose	Activity	Essay	Innovation	Research
Geometric shapes	39.40	27.30	9.10	24.20
Analytic geometry	46.70	20.00	6.60	26.70
Geometry in three dimensions	18.75	12.50	25.00	43.75
General school geometry	18.10	24.30	29.70	27.90
General upper geometry	17.70	22.80	0.00	59.50

Note: Values are given in percentages

We coded the data with values of the variables after a detailed reading of each paper. The coding team was composed of a group of coders, a coding reviewer, and a researcher in mathematics education. The coders, with undergraduate degrees in mathematics education, recorded the bibliographic information of the papers (title, abstract, authors, and year), and established their purpose (activity, essay, innovation, or research) and educational level. Subsequently, they identified the set of key terms of the paper (values for the variables pedagogical notions and topics). After the coding of a paper, the coding reviewer (with a master's degree in mathematics education) checked the validity and accuracy of the bibliographic information that was recorded, and verified that the key terms, purpose, and educational level that were assigned to the paper were appropriate. To ensure the coding process, a researcher in mathematics education randomly checked the work of the coders and the coding reviewer. Each paper in the proceedings could be associated with one or more educational levels, one or more aspects of pedagogical notions, and one or more themes, except for the value other themes (non-geometry), but could only be placed in one type of paper (purpose) and one meeting.

We recorded the results of the coding in a database system. In what follows, we describe the procedures we performed to address each of the specific objectives. In some cases, for clarity, we present examples of these procedures with data that emerged from the coding.

Focuses of interest of the meeting in terms of the values of the variables

First, we obtained descriptive values from the papers to establish the extent to which the values of each variable are treated in the proceedings of the meetings. In the case of the variables educational level and pedagogical notions, we took each value independently; that is, the sum of the percentages of the values of these variables is not 100%. The average in which a value is treated corresponds to the percentage of papers that treat it. For the variable purpose, we provide the absolute frequency of each value and the corresponding percentage. For the variables educational level, pedagogical notions, and subjects, we consider the percentage that represents the importance of the values in each variable.

Evolution of the values of the variables throughout the meetings

To analyze the evolution over time of the values of the variables in the meetings, we used cross tables of variables and took the time variable as a base. We obtained percentages of the papers of each meeting in the values of the variables purpose, educational level, pedagogical notions, and topics with respect to the variable time. In **Table 3**, we present as an example the table in which we cross the variable purpose with the variable time, and we present the percentage of papers per meeting according to the type of paper. After organizing the data in the tables in which we cross-referenced the variables, we were able to establish the evolution over time of the proportion of the values of all the variables covered in the different meetings.

Behavior of the values of the variable topics

To make comparisons between the values of the variable topics in the meeting, we cross-referenced this variable with the variables purpose, educational level, and pedagogical notions. We obtained the percentage distribution of the topics with respect to the values of each variable. As an example, in **Table 4**, we show the distribution of five of the 16 values of the variable topics in the types of papers (purpose variable).

To identify the topics that are distinguished by having the highest, one of the highest, one of the lowest or the lowest percentage of publication in the values of the variables purpose, educational level, and pedagogical notions, we carried out a process of normalization of the percentages of papers of each topic in each value of the other variables. If a topic has a normalized

Table 5. Percentage normalization of the geometric shapes theme to the values of the purpose variable

Geometric shapes	Percentage (%)	Normalized
Activity	39.39	1.00
Essay	27.27	0.60
Innovation	9.09	0.00
Research	24.24	0.50

Table 6. Behavior of the values of the subject variable at each value of the purpose variable

Geometry topics	Activity	Essay	Innovation	Research
Ruler and compass constructions	1.00	0.00	0.00	0.33
Geometric shapes	1.00	0.60	0.00	0.50
Analytic geometry	1.00	0.33	0.00	0.50
Geometry in three dimensions	0.20	0.00	0.40	1.00
Euclidean geometry	0.67	1.00	0.00	1.00
Vector geometry	0.00*	0.00*	0.00*	1.00
Geometric relations	0.44	0.22	0.00	1.00
Theorems	1.00	1.00	0.00*	0.50

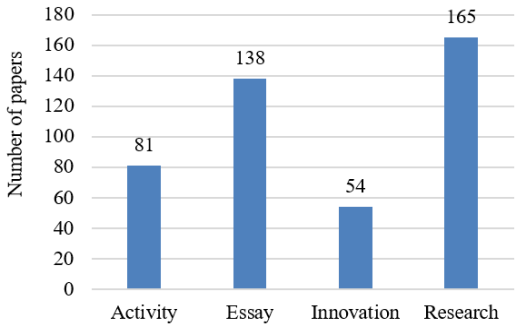

Figure 1. Number of papers by purpose

value of 1, then it has the highest level of publication. If the normalized value is 0, then it is the lowest. If it is not the highest or the lowest, we catalog its publication as one of the highest if the normalized percentage of papers in each value of the variable is greater than or equal to 0.75 and it is not the highest (Castro et al., 2019). Similarly, we categorize it as one of the lowest if the normalized percentage of papers is less than or equal to 0.25 and is not the lowest. If a topic is not addressed (the percentage is 0%), we use the symbol * to differentiate it from those topics that also have a normalized value of 0 but are addressed in some proportion. In **Table 5**, we exemplify the normalization we performed on the percentage of papers that address the topic ruler and compass constructions in the variable purpose. We performed the same exercise with the values of all the variables.

As a result of the process of normalizing the percentages of the topics in each value of the variables purpose, educational level, and pedagogical notions, we obtained a summary table that indicates the behavior of each value of the variable topics with respect to the values of the other variables. In **Table 6**, we present as an example the summary of the behavior of some topics in each value of the variable purpose. The information in **Table 6** shows, as an example, that the highest percentage of papers dealing with the topics ruler and compass constructions, geometric shapes, analytic geometry, and theorems are activities. The papers that deal with geometric relations are, to a greater extent, research.

Behavior of the values of the variables purpose, educational level, and pedagogical notions in each topic

Finally, we established in what proportion each topic is treated in the values of the variables purpose, educational level, and pedagogical notions. We used the same normalization process we presented previously to identify the level at which each topic is covered in the values of the other variables. For example, we can indicate that the activities deal more with general school geometry and other (non-geometry) topics. Essays and investigations deal to a greater extent with other topics (non-geometry).

RESULTS

As previously indicated, we analyzed 438 papers that correspond to the proceedings of the XIII to XXIV meetings, except for the XIX meeting. In what follows, we present four types of results according to the objectives of the study: focuses of interest of the meetings, evolution of the values of the variables, comparison of the geometry topics and behavior of the geometry topics.

Focuses of Interest

We identified the focuses of interest of the meeting in terms of the values of the variables. We found that most of the papers presented at this meeting are research (37.7%) and essays (31.5%), while 18.5% of the papers are activities and 12.3% correspond to innovations. **Figure 1** shows the number of papers that have been presented at the meeting, distributed in the variable purpose.

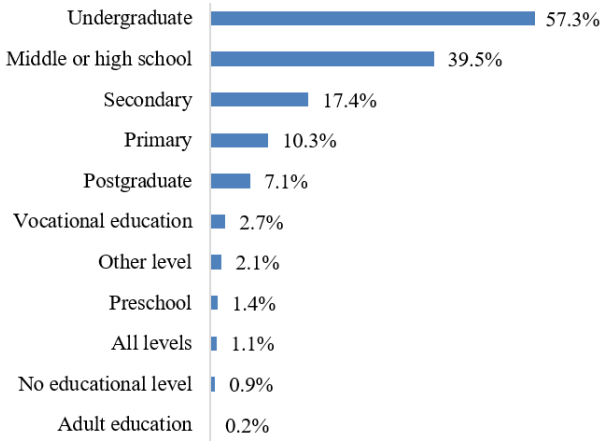

Figure 2. Relative importance of the values of the education level variable

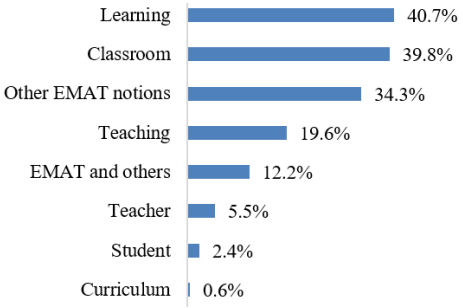

Figure 3. Relative importance of the values of the variable pedagogical notions

We determined the relative importance of each value of the educational levels in the meeting proceedings, according to the percentage of papers that address it. We found that the levels most frequently discussed in the meeting proceedings are, in order, the undergraduate and middle or high school. The difference in the proportion of papers that deal with these educational levels and those that follow (secondary, primary, and postgraduate) is considerable, as can be seen in **Figure 2**. Less than 3% of the papers deal with preschool and vocational education.

Of the 438 papers analyzed, 111 are not associated with the values of the variable pedagogical notions. We calculated the relative importance of these notions in the 327 that do address the variable. We found a remarkable interest in the notions of learning and classroom. The notion called other notions of mathematics education also has a high relative importance. Less than 20% of the meeting papers deal with the notion of teaching. The percentage of papers that deal with this notion is followed by the percentage of papers that deal with the relationship between mathematics education and other disciplines. The notions with less relative importance are curriculum, student, and teacher. We did not identify papers related to educational system, educational center, and assessment. **Figure 3** shows the relative importance of the values of the variable pedagogical notions.

The results of the analysis of the proceedings with respect to the topics show that more than 30% of the papers are specifically associated with other mathematics topics, other than geometry, for example, numbers, algebra, and measurement. Regarding this result, it is important to point out that, within the framework of the XIII to XVIII meetings, meetings I to VI of the arithmetic meeting were developed. This is followed by the percentages of papers dealing with general questions of school geometry and general questions of higher geometry. **Figure 4** shows the relative importance of the topics, organized from highest to lowest.

In relation to specific geometry content, the theme of geometric shapes stands out. This is followed, in the same proportion, by geometric relations and transformations. Geometry in three dimensions, analytic geometry and Euclidean geometry are treated, on average, in 3% of the papers. After these subjects, we find theorems and ruler and compass constructions. The topics with less relative importance are trigonometry, topology, and vector geometry.

Evolution Over Time of the Values of the Variables

We analyzed the evolution of the values of the variables purpose, educational level, pedagogical notions, and topics in the meetings XIII to XXIV. We present in **Figure 5** the behavior of these values over time.

In the variable purpose, we find that research and essay stand out from the others. Research is above the other values in the XII, XVI, XXI, and XXII meetings with percentages of 57.6%, 78.6%, 41.2% and 34.3%, respectively. Only in meeting XVII, the percentage of research was below the average (7.1%). Most of the papers in the proceedings of the XIV, XV, XVIII, and XX meetings are essays (46.4%, 48.8%, 64.6% and 41.1%). The maximum percentage of essays was presented in the XVIII meeting. From that year on, the proportion of essays has a decreasing behavior. The percentage of activities oscillates around 17.9%; in the XVII meeting, it had its peak with 46.4% (same percentage of essays). Between meetings XIII and XXII, the percentage of innovations did not exceed 20%. The production of curricular innovations has increased since the XXI meeting.

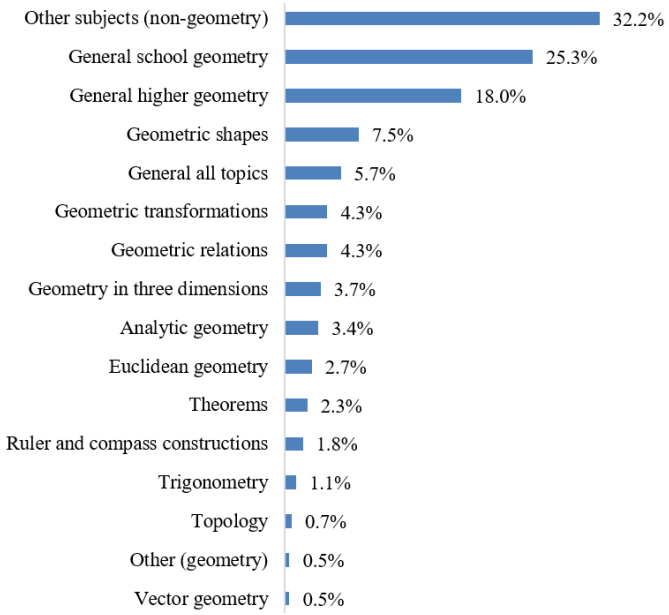

Figure 4. Relative importance of the values of the variable topics

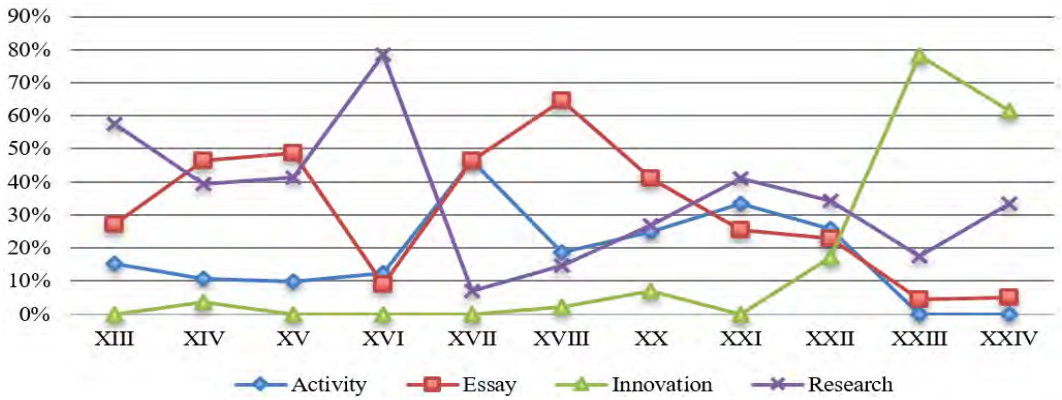

Figure 5. Behavior of the values of the variable purpose

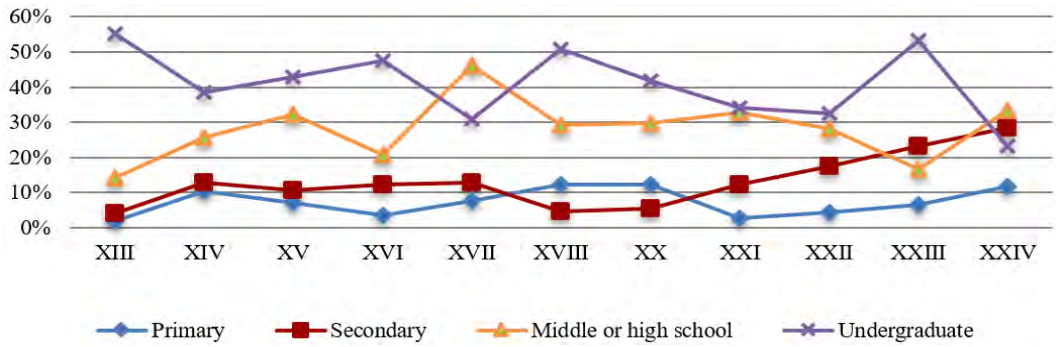

Figure 6. Behavior of the values of the purpose variable

In relation to the behavior of the values of the educational level variable, the percentage of papers associated with undergraduate and middle or high school education stands out in all meetings. **Figure 6** shows the behavior over time of the values of the educational level variable whose relative importance is greater than 10%.

The educational level of the undergraduate degree was dealt with, on average, 41% in the different meetings. This level was dealt with to a lesser extent in the XVII, XXI, XXII, and XXIV meetings. Middle or high school education does not vary considerably over time; on average, it is dealt with by 28% of the papers. In the XVII meeting, it had its highest percentage and was less than 20% in the XIII and XXIII meetings. The percentage of secondary education, except for meetings XIII, XVIII, and XX, is above 10%; its behavior is increasing from meeting XXI onwards. The proportion of primary education is between 2% and 12.5%. The other values of the educational level variable (preschool, postgraduate, vocational, and adult education) have average percentages below 5%. The preschool level has only been addressed in meetings XIV, XV, XVI, XVIII, and XXI, in minimal proportions in comparison with the other educational levels.

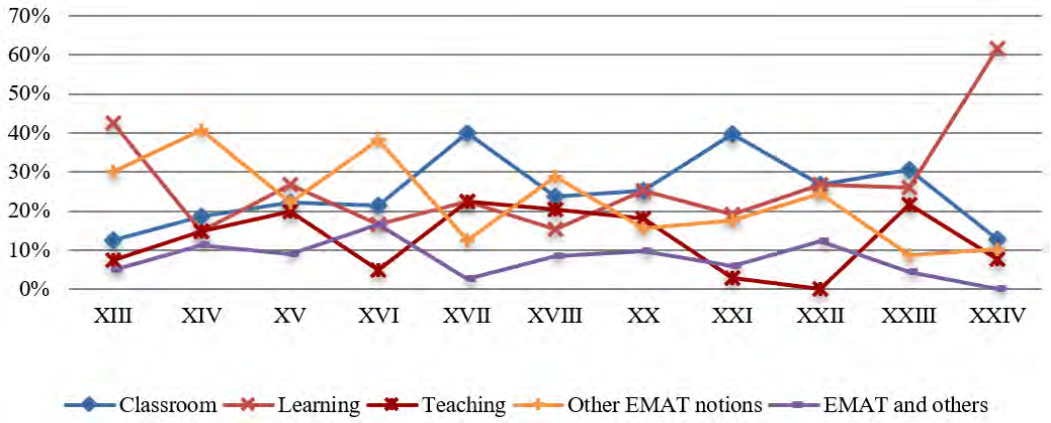

Figure 7. Behavior of the values of the variable pedagogical notions

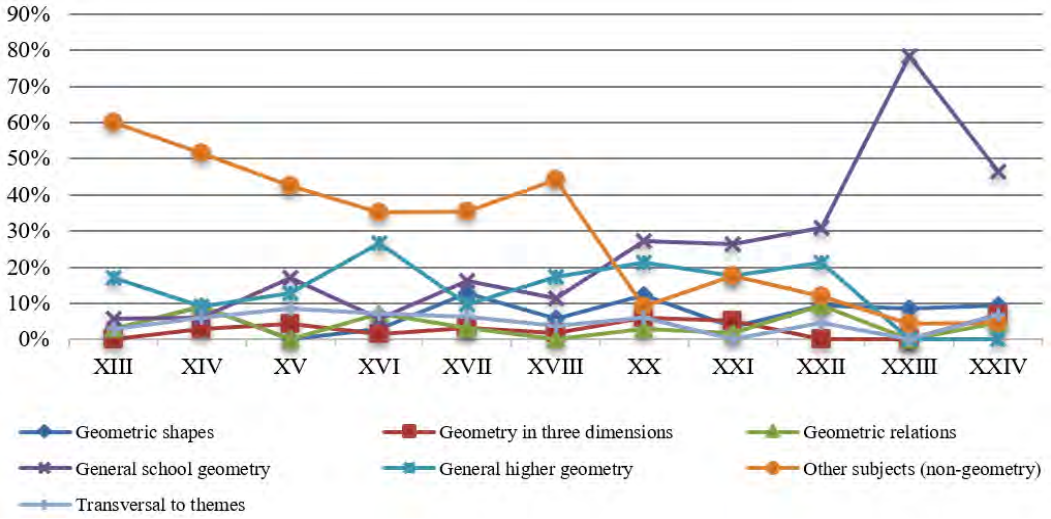

Figure 8. Behavior of the values of the variable topics

Figure 7 shows the behavior over time of the pedagogical notions with greater relative importance in the meeting.

In the pedagogical notions, learning was dealt with to a greater extent in meetings XIII and XXIV. In the other meetings, the percentage ranged between 15% and 27%. The notion of classroom had the highest percentage (close to 40%) in meetings XVII and XXI; its percentage was above 15% in the other meetings, except for meetings XIII and XXIV. The extent to which teaching was treated had notable decreases in meetings XVI and XXI; it was not addressed in meeting XXII.

The behavior of the value called other notions of mathematics education, which includes questions related to problem solving and representation systems, is not constant. For example, while in the XVI meeting this notion was treated in 38.1% of the papers, in the XVII meeting, it dropped to 12.5%. More than 25% of the papers included in the proceedings dealt with this notion in the XIII, XIV, XVI, and XVIII meetings. The relation of mathematics education with other disciplines was dealt with in meetings XIII to XXIII. The extent to which it was treated in meetings XIII, XVII, and XXIII did not exceed 5%; the percentage was nil in the XXIV meeting.

The notion of teacher was addressed to a greater extent in the XXI, XXII, and XXIII meetings (10.3%, 7.3% and 8.7%). The curriculum was only addressed in the XXI and XXIV meetings, with a minimal proportion (1.4% and 2.6%). Similarly, the notion of student was only considered in meetings XVI (2.4%), XX (3.6%), XXI (2.9%) and XXII (2.4%). We did not identify papers related to notions such as educational system, educational center, and assessment.

With respect to the topics, the emphases are distributed in two periods. In the first period, between meetings XIII to XVIII, the extent to which other (non-geometry) topics are dealt withstands out. In the second period, meetings XX to XIV, the percentage of papers associated with general questions of school geometry stands out. **Figure 8** shows the behavior of the values of the variable topics that were most regularly dealt with in the meeting.

Between the XIII and XVIII meetings, in which the arithmetic meeting was held simultaneously, more than 35% of the papers dealt with mathematical topics other than geometry. From meeting XX onwards, the percentage did not exceed 17.5% and was less than 5% in the XXIII and XXIV meetings. In contrast, the behavior of general school geometry subjects was less than 17% in meetings XIII and XVIII and its tendency increases from meeting XX onwards. The highest point was in the XXIII meeting, with 78.3%. General subjects of higher geometry were regularly addressed between meetings XIII and XXII, in an average percentage of 17% of the papers.

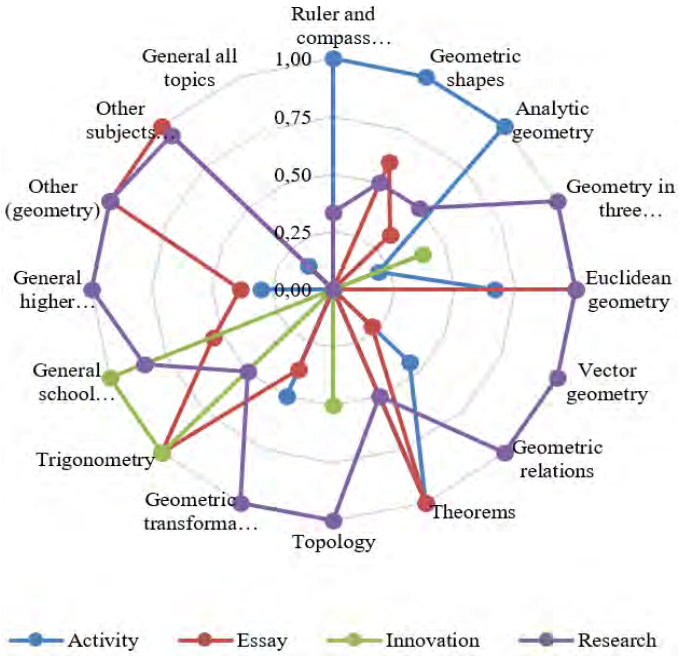

Figure 9. Behavior of the topics in values of the variable purpose

The proportion in which specific geometry topics were treated in the meetings does not show a uniform behavior. The XX and XXI meetings present greater diversity of content (nine of the 11 values of the variable corresponding to specific topics were dealt with in them). In the XX meeting, no issues related to Euclidean geometry and vector geometry were addressed, and in the XXI meeting no papers on vector geometry and topology were presented. The topic called geometric shapes was only left out of the XV meeting. Although it is a topic that was dealt with in most of the meetings, the proportion of papers that allude to it is changing; it was less than 5% in meetings XIII, XVI, and XXI. Topics such as vector geometry, topology and trigonometry were not addressed with relevance during the 11 meetings. We only identified papers associated with vector geometry in the XV meeting (4.3% of the papers), topology papers in the XV (2.1%) and XX (1.5%) meetings, and trigonometry papers in the XX (1.5%), XXI (1.8%), XXIII (4.3%) and XXIV (4.7%) meetings. The topics ruler and compass constructions, analytic geometry and theorems were dealt with in six of the 11 meetings, and Euclidean geometry in seven of them. The proportion of papers dealing with these topics per meeting did not exceed 7%.

Behaviour of Geometry Topics with Respect to Other Variables

We analyzed the behavior of the topics with respect to the values of the other variables. This allowed us to establish the topics that differ from the others by having the highest, one of the highest, one of the lowest or the lowest percentage of papers in each value of the variables purpose, educational level, and pedagogical notions.

Figure 9 shows the behavior of the topics in the variable purpose.

For the variable purpose, we conclude that the topics constructions with ruler and compass, geometric shapes, analytic geometry, and theorems are distinguished from the other topics by having the highest percentage of activities. Papers dealing with Euclidean geometry, theorems, trigonometry and other geometry topics and other topics (non-geometry) are essays to a greater extent. Trigonometry, together with general school geometry stand out as being dealt with to a greater extent in innovations. Finally, the values of the variable topics that stand out because the papers that deal with them correspond mostly to research are geometry in three dimensions, Euclidean geometry, vector geometry, geometric relations, topology, geometric transformations, general higher geometry, other geometry themes and general all topics. Vector geometry, topology, trigonometry, and other geometry topics have no related activities. There are also no essays on vector geometry and topology. We found no curricular innovations that address vector geometry, theorems, general higher geometry, and other geometry topics.

In **Figure 10**, we show the behavior of the topics in the educational level variable, according to the normalized value of their percentages, in the values primary, secondary, middle, or high school, and undergraduate, which are those of greater relative importance in the meetings.

In the crossing of the variables topics and educational level, we found that, with the exception of vector geometry, topology, general higher geometry and other themes (not geometry) and general all topics, the other topics are at the highest level of proportion of their documents associated with the middle or high school. Geometry in three dimensions, vector geometry, topology, general questions of higher geometry, other geometry topics, other topics (non-geometry) and general all topics stand out for having the highest level of papers in relation to undergraduate education. The value other topics has the highest level in postgraduate education. In addition, the following values stand out for having percentages of associated papers in the highest group: general all topics in middle or high school, and analytic geometry, Euclidean geometry, geometric relations, and trigonometry in undergraduate education. In addition, we found that all topics are at the lowest level of production in preschool, postgraduate, vocational, adult education, all levels, other level, and no level.

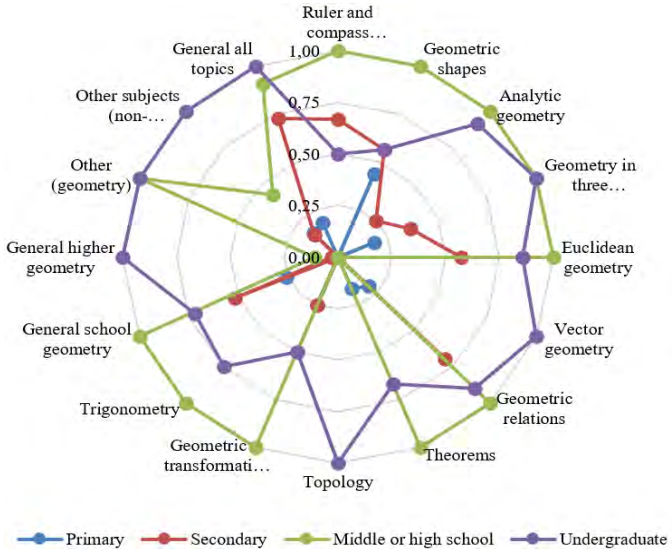

Figure 10. Behavior of the topics in values of the variable educational level

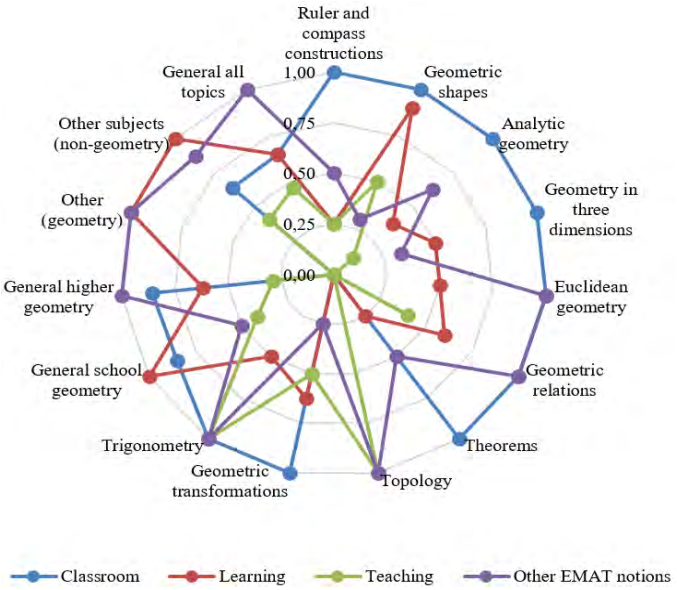

Figure 11. Behavior of the topics in the values of the variable pedagogical notions

In **Figure 11**, we present the summary of the behavior of the geometry topics in the variable pedagogical notions for the values classroom, learning, teaching, and other notions. We use in the representation the normalized values of the percentages of papers of each topic in the values of the other variables.

In the relationship between topics and pedagogical notions, we see that 11 of the 16 topics stand out for having the highest level in the notion of classroom. Also, in half of these topics, other pedagogical notions, such as representation systems, are treated in a remarkable way. The papers associated with the topic of vector geometry do not address pedagogical notions. We find that all topics have the lowest level in student, teacher, curriculum, and relation of mathematics education with other notions. Especially, curriculum is only minimally addressed in analytic geometry and in general school geometry.

Behavior of the Purpose, Educational Level, and Pedagogical Notions in the Subjects

Finally, we analyzed the behavior of the values of the variables purpose, educational level, and pedagogical notions in geometry topics. We found that the highest level of activities and innovations is related to general school geometry issues. In the same way, in the papers of secondary, middle school, vocational and adult education, and of those that deal with the notions of classroom, student, teacher, learning, teaching and curriculum, the highest percentage deals with this topic.

The essays and research, as well as papers of preschool, primary, undergraduate, and postgraduate education, and those that deal with notions student, other notions, and the relation of mathematics education with other disciplines have their highest percentage of papers related to other topics (not-geometry). The topics of ruler and compass constructions, geometric shapes, analytic geometry, geometry in three dimensions, Euclidean geometry, vector geometry, geometric relations, theorems, topology, geometric transformations, and trigonometry are addressed to a lesser extent in the values of the variables purpose, educational level, and pedagogical notions.

Table 7. Values of the variables purpose, educational level, and pedagogical notions that are distinguished from the others

Topic/value and level	General school geometry	General higher geometry	Other topics (not-geometry)
1-Highest	Activity Innovation Secondary Middle school Vocational education Adult education Classroom Student Teacher Learning Teaching Curriculum		Essay Research Preschool Primary Undergraduate Postgraduate Student Other notions EMAT and other
[0,75, 1)-One of the highest	Primary	Research Vocational education	Activity Middle or high school Learning Teaching
(0, 0.25]-One of the lowest	Preschool Postgraduate	Primary Secondary Middle or high school	Innovation Teacher
0-Lowest		Innovation Preschool Adult education Student Teacher Curriculum	Vocational education Adult education Curriculum

We present in **Table 7** a summary of the values of the variables purpose, educational level and pedagogical notions that are distinguished by having their highest and lowest levels in general school geometry, general higher geometry, and other topics (non-geometry). We restrict ourselves to these values of the variable topics because, in the crosstabs of variables, they are the ones that are most addressed in the proceedings of the meetings. We do not include in the table the values of the other variables whose normalized value of the percentage is in the interval (0.25, 0.75).

DISCUSSION

The results found in the study show that the papers included in the proceedings of the meeting are focused on research results and essays and, to a lesser extent, on curricular activities and innovations. We see that the community that converges in the meeting is more interested in the theoretical development associated with geometry than in the generation of practical proposals that can be taken to the classroom. The interest observed in the dissemination of research work compared to other types of work is consistent with what is observed in the Ibero-American community and shows the trend towards the systematic formalization of stances related to mathematics education (Castro & Gómez, 2021).

The undergraduate level of education has had the highest percentage of papers in the different meetings, followed by middle or high education. This can be explained by the nature of the research, which is developed especially by university teachers, whose objective is to study the development of concepts that they deal with in their professional work. Regarding pedagogical notions, we identified that a quarter of the papers do not address pedagogical notions. Of the papers that do, the emphasis is on topics related to learning, classroom, and other notions of mathematics education (problem solving and representation systems). Specifically, with respect to the pedagogical notion of the classroom, the papers emphasize issues associated with didactic resources, such as manipulative materials and computer resources. In the proceedings, there is no evidence of papers that deal with the notions of educational system, educational center, and assessment. In addition, we have identified a reduced interest in studies related to the curriculum (design, implementation, and evaluation).

Most of the papers are associated with general school geometry and other subjects (not geometry). Regarding the topics, we see that most of the papers deal with general school geometry and other non-geometry topics, such as numbers, algebra, and calculus. The relevance of these contents is justified by the fact that, within the framework of the XIII to XVIII meetings, versions I to VI of the arithmetic meeting were developed. These documents were also included in this study. Topics such as trigonometry, topology and vector geometry have not been studied to a greater extent, despite the high percentage of papers in undergraduate and middle or high school education. This situation evidence works opportunities for the community that converges around geometry.

In meetings XIII to XXIV, we found a regular behavior of the production of research and essays, and that the production of innovations is on the rise as we have noticed for the Ibero-American community of mathematics educators (Castro & Gómez, 2019). Between the XV and XXIII meetings, learning was regularly addressed, but we observed an important upturn in the XXIV meeting: it was addressed by more than 50% of the papers associated with pedagogical notions. This aspect may be associated with the fact that the meeting is attended by both researchers who are teacher educators and teachers in training, for whom the teaching and learning processes of geometry become their focus of study.

When analyzing the relationship of the variable topics with the other variables, it is highlighted that papers related to specific topics of geometry such as ruler and compass constructions, geometric shapes, analytic geometry, and theorems correspond to a greater extent to activities. The topics with the highest percentage of investigations are geometry in three dimensions, Euclidean geometry, vector geometry, geometric relations, topology, and geometric transformations. Except for vector geometry and topology, the specific topics of geometry have the highest percentage of papers related to the middle or high school level and to the notion of classroom. A natural result is that vector geometry, topology and general questions of higher geometry are dealt with in a higher proportion at undergraduate education.

Finally, it is noteworthy that the topics in which the values of the variables purpose, educational level and pedagogical notions have the highest percentage of associated papers are general school geometry and other topics (non-geometry). This implies that the specific topics (ruler and compass constructions, geometric shapes, analytic geometry, geometry in three dimensions, Euclidean geometry, vector geometry, geometric relations, theorems, topology, geometric transformations, and trigonometry) are addressed to a lesser extent in the other variables.

CONCLUSIONS

Geometry and its Applications Meeting is one of the few meetings that focus on specific contents of mathematics. Its periodic development has become a meeting point for the Spanish-speaking community to reflect on the development of geometry. In this study, we characterize its document production from the proceedings of the XIII to XXIV meetings in terms of the values of the variables purpose, educational level, pedagogical notions, and topics. We determine their focus of interest, the diachronic evolution of these and analyze the relationships between variables.

The results we provide highlight the focuses of interest of the community under study, but also account for issues that have not been addressed and that are susceptible to further study. Although this community stands out for the extent to which it addresses the undergraduate level of education, it also stands out for the fact that it focuses minimally on the learning and teaching of geometric notions in early education. Likewise, the relevance of the formalization of stances in this community is evidenced. The growth of curricular innovation in recent years is observed.

The results lead us to suggest that the pedagogical notions of teaching, curriculum and assessment should be addressed, both in research and in innovations. The learning and teaching at the preschool and primary school levels should also be addressed.

We consider that this study not only contributes to the community of teachers and researchers who converge in the *Geometry and its Applications Meeting*, in relation to the behavior of its bibliographic production, but also shows how the educational levels and pedagogical notions have been addressed in the study of geometry. Based on the variables we used and their values, it is possible to identify which have been the focuses of interest in this meeting and to analyze their evolution. This information allows us to characterize the mathematics education community in relation to a specific mathematical content.

This study stands out from other document studies in which the emphasis has been on the analysis of impact indicators. It makes it possible to identify thematic trends in a community based on the analysis of the content of its documentation from a controlled vocabulary specific to mathematics education. From it, there are open lines for further studies in which other values or sub values of some variables are specified. However, this work can be taken as a starting point for future research related to the study of geometry or any other subject of mathematics. It is possible to define and study sub-values of the values of the variable pedagogical notions to establish the focuses of study of mathematics education with respect to specific mathematical contents. The procedures used in this study can be replicated and adjusted in comparative studies of communities of researchers and mathematics educators, in document studies of thematic categories and in the distinction of categorical variables.

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