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A meta-synthesis of studies on the use of augmented reality in mathematics education

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ABSTRACT

Augmented reality (AR) helps three dimensional, virtual objects to be viewed, interactively, in a real-world setting. AR technology is used in many fields such as medicine, advertisement, military, industry, and increasingly in education. AR has an important role in concretizing educational platforms and achieving permanent learning. This study aims to evaluate studies on the use of AR in mathematics education through meta-synthesis. 20 research articles, two master's theses, and two PhD theses, published between 2010 and 2021, centered on the use of AR technology, utilizing either qualitative or mixed methodology were analyzed in this study. The databases used for selecting the texts including Google Scholar, EBSCO, Education Resources Information Center, Elsevier Scopus, ProQuest Dissertations and Theses Full Text, Springer Link, Taylor & Francis Online, TUBITAK ULAKBIM Dergipark, YOK Thesis Centre. The studies included in the meta-synthesis have been grouped according to their descriptive properties, research methods, data gathering tools, data analysis, the mathematics subjects they focus on, their validity and reliability, and the conclusions and suggestions they present. The similarities and differences between these studies have been compared, and in accordance with the nature of meta-synthesis they have been categorized into themes for synthesis. Parallel to technological progress, it was observed that the use of AR in mathematics education has tended to increase over the years. In conclusion, most of the studies on AR in mathematics education focus on increasing spatial ability. It was seen that the visualization and concretization aspects of AR were often used. This meta-synthesis offers suggestions for future researchers who are interested in the use of AR in mathematics education.

Keywords: augmented reality, educational technologies, mathematics education, meta-synthesis

INTRODUCTION

Technological tools, such as PCs, tablet computers, and smartphones are widely used in many aspects of our daily lives. While the development of these technologies makes our lives easier in many ways, they offer many additional benefits in the field of education. One of the technologies that especially helps concretize educational spaces and increase permanent learning, is augmented reality (AR). Azuma (1997) describes AR as a technology that converges real and virtual objects, offers real-time interaction, and records scenes in 3D. Users see virtual objects in a way that overlaps with the real world (Azuma et al., 2001), According to Billinghurst (2002), AR is the ability to install computer graphics over real world locations. AR is seen as an extension of virtual reality (VR) (Azuma, 1997; Wojciechowski & Celary, 2013). Unlike VR, AR offers an experience that interacts with the real world, and that aims to complete the real world, by refusing to create a completely fake space with only virtual objects (Höllerer & Feiner, 2004).

The AR technology was subject of research in many fields including aviation, production, medicine, tourism, entertainment, social networking and education in the beginning of the 1990s (Bower et al., 2014). While the history of AR goes back as far as the 1960s, it took until the 2000s for it to become more commonly used and researched within the field of education (Billinghurts, 2002). As technology develops, areas in which it is used change and develop as well; therefore, technological developments have benefitted the current use of AR.

In the 1990s, AR systems were implemented through a helmet, and a computer. For a while the technology continued to develop in a wearable form. In the 2010s, with the mobilization of computer technologies, AR systems also became portable. This has led to a great leap in the development and use of AR. During this time, the graphic quality, recording process, and hardware

This work is derived from the first author's master's thesis.

of AR have also gotten much better, and with the devices getting smaller and more portable, AR use naturally became more common. Dey et al. (2016) observed that after 2009 the screens mounted on users' heads gave way to hand-held devices. Similarly, Kara (2018) in their document review on the use of AR in education, observed that in the beginning of the 2000s there were very few studies, and that the number began to increase after 2009. There have been many studies aiming to identify the latest trends in AR and its usability. Wu et al. (2013) have suggested that approaching AR as a concept, rather than a technology would be beneficial to educators and researchers. According to Karakus et al. (2019), the AR concept is mostly studied in the areas of science education, mathematics education, and educational technology.

AR technology allows educators to create scenarios and add characters to the learning experience, as well as making it possible to integrate images and items into the learning space (Bower et al., 2014). AR makes it possible to view objects that are too big or small to bring into the classroom, at a reasonable cost. In their study, aiming to assess the effectiveness of AR in learning spaces, Yilmaz and Batdi (2016) found that AR had a positive effect on cognition and emotion and that it landed reality to the learning space. The literature emphasizes that AR supports and improves various pedagogic approaches, such as inquiry-based learning, game-based learning, cooperative learning, and the constructivist approach (Bower et al., 2014; Chen & Tsai, 2012; Dunleavy et al., 2009; Johnson et al., 2010; Rasimah et al., 2011). AR links information to real life situations when bringing it to the classroom, thus makes learning easier. According to the study Uygur et al. (2018) conducted with trainee teachers, AR's visualization aspect helped the trainees to understand difficult subjects with ease and to retain information permanently.

The Ministry of National Education (2018a, 2018b) curriculum for primary, secondary and high school education includes learning outcomes for improving logical and spatial thinking skills. Technology that allows visualization can be used to achieve these outcomes. AR is one example of such technologies. Many studies have mentioned that the use of AR improves spatial skills in mathematics education (Dünser et al., 2006; Gun & Atasoy, 2017; Ibili & Sahin, 2013; Lin et al., 2015; Martín-Gutiérrez et al., 2010; Ozcakir & Aydin, 2019; Salinas & Pulido, 2016).

The technologies developed in recent years and the increased use of AR in educational spaces has led to the development of various AR materials that use software such as Unity 3D and Microsoft Silverlight; and libraries such as Slartoolkit and Vuforia. These application development platforms allow many users to produce content. Unity 3D, is a software that allows users to develop AR applications without any previous programming knowledge. The AR applications with these platforms have been used as material in mathematics classes, and various studies regarding different mathematical subjects have been conducted. GeoGebra, which is a commonly used software in mathematics education, released its AR version, named "GeoGebra AR" in September 2017 (GeoGebra AR, 2021). The GeoGebra 3D calculator was update to its 5th version in July 2019, and now enables users to add AR objects to the application (GeoGebra 3D Calculator, 2021).

The compilation studies made on AR are mostly systematic examination, bibliometric analysis, meta-analysis and content analysis studies that focus on AR's advantages, uses, usability, which fields it is popular in, trends in AR research, its applications, and AR technologies (Altinpulluk, 2018; Avila-Garzon et al., 2021; Bacca Acosta et al., 2014; Dey et al., 2016; Icten & Bal, 2017; Martin et al., 2011; Onal, 2017; Ozdemir, 2017; Radu, 2014; Santos et al., 2014; Sirakaya & Sirakaya, 2018; Tekedere & Goker, 2016; Usta et al., 2016 ; Yilmaz & Batdi, 2016; Yilmaz & Goktas, 2018). In their literature review, Akkus and Ozkan (2017) have reviewed AR applications developed in the fields of mathematics and geometry, and 12 research studies on this subject. According to Akkus and Ozkan (2017), most of the studies they found were focused on the role of AR in the development of spatial intelligence in mathematics education.

The advantages of AR use in educational environments is often mentioned in the literature (Chen & Tsai, 2012; Dunleavy et al., 2009; Rasimah et al., 2011; Yuen et al., 2011). According to Garzón et al. (2019), given the apparent multiple benefits of using AR systems in educational settings, stakeholders have great opportunities to develop new and better systems that benefit all learners. In previous studies, the use of AR technologies on different groups and their effect on academic achievement were examined, but no study was found that indicates, which type and level of AR technologies would be appropriate to use (Gun & Atasoy, 2017). Although many studies have been conducted on its use in educational environments since AR use became more common, there is still need for studies that aim to discover the opportunities and qualities that AR presents (Cheng & Tsai, 2013; Wu et al., 2013). When examining studies on the use of AR in education, it was found that there is a lack of work that systematically examines the use of AR in mathematics education, what sources can be used to generate content, how AR can be integrated into the learning environment, and what AR can contribute to mathematics education. The number of studies has rapidly increased in correlation to the advancements in AR technology. With this increase, it has become necessary to comprehensively examine the AR studies conducted in the mathematics education field, and this work aims to do.

This field has been actively growing over the past decades in terms of the research and development of new technologies (Avila-Garzon et al., 2021). With this study, we aim to present a meta-synthesis of the AR studies conducted in the field of mathematics education between the years 2010-2021. The intention of this study is to present a holistic perspective of the use of AR in educational spaces, and to present a framework for future researchers. The problem statement of this work can be put as: "What kinds of studies have been done on AR in mathematics education, and kind of uses can there be for AR in mathematics education?"

The sub-problems that this study aims to answer are as such:

1. What are the descriptive properties of the studies included in the meta-synthesis (like in terms of year of release, countries where they were conducted, their sample group, and sample size)?
2. What are the objectives and significances of these studies?
3. How are these studies distributed in terms of data gathering tools?
4. How are these studies distributed in terms of methodology?

5. How are these studies distributed in terms of mathematical subjects?
6. How were the data analysis processes (data analysis, validity and reliability tests etc.) of these studies structured?
7. At which stage or in what way were AR technologies used?
8. What are the results presented in these works?

LITERATURE REVIEW

There are studies on the use of AR in classroom environments, done in many fields of education. The common use of mobile tools has made AR easily usable in class. Sirakaya and Sirakaya (2018), have stated that AR is a technology used in pretty much all levels of education. When examined, the related literature touches upon many of the advantages of AR. The majority of works were found to be in life sciences. The reason for this is thought to be the high number of AR applications designed for the fields of biology and astronomy.

There are studies in the literature that focus on AR's effects on concentration, interest, and gratification (Bujak et al., 2013; Di Serio et al., 2013); success and attitude (Ibáñez et al., 2014; Korucu et al., 2016; Ozdemir et al., 2018; Uluyol & Eryilmaz, 2014; Wojciechowski & Celary, 2013, Yuen et al., 2011). According to Yuen et al. (2011), AR technology brings experiences that are difficult to create in the real world into the classroom, motivates students, and contributes to the cooperation and interaction between the teacher and the students. In Uluyol and Eryilmaz's (2014) study, teachers' opinions about AR applications were gathered, and the findings showed that AR increased motivation. In Uygur et al.'s (2018) study with trainee teachers, the participants described AR as fun, labour-saving, and motivational.

Abdusselam and Karal (2012) have found AR to be useful in teaching abstract concepts in physics education. Similarly, Bujak et al. (2013) mentioned AR's concretizing effect in their study. Saidin et al. (2015) have said that AR has a great potential for visualization. According to Saidin et al.'s (2015) work, visualization and concretization are effective for increased learning. Dunleavy et al. (2009), put forward that AR technology can help gain the interest of students with low academic success, create a cooperative environment, and help develop problem solving skills. AR based teaching techniques are structurally student-centric and lend themselves to explorative learning. In this regard they are different from traditional classroom techniques which tend to be teacher-centric, and presentation-based (Wu et al., 2013). AR puts the student in the center of the learning experience (Bacca Acasto et al., 2014), and allows for instant feedback (Ibáñez et al., 2014). According to Wu et al. (2013), AR makes content three dimensional and thus contributes to visual learning, supports cooperative learning, and creates a bridge between formal and informal learning. In educational activities that require expensive materials, AR can help reduce expenses (Bacca Acasto et al., 2014).

In many studies, it has been stated that the use of AR in geometry education improves spatial ability (Gun & Atasoy, 2017; Isik, 2019; Ibili & Sahin, 2013; Lin et al., 2015; Martín-Gutiérrez et al., 2010; Ozcakir & Aydin, 2019; Ozcakir & Cakiroglu, 2022; Salinas & Pulido, 2016; Topraklikoglu, 2018). Cai et al. (2019) investigated the effect of AR on learning and motivation in teaching probability and statistics to students at different levels of self-efficacy. Ibili (2013), on the other hand, stated that augmented reality technology contributed to students' cognitive skills in geometry teaching. Chen (2019) investigated whether the use of AR in mathematics education affects learning, motivation and mathematics anxiety among students with high and low anxiety. He concluded that high-anxious learners performed better in algebra and geometry.

To summarize, the number of works focusing on AR increases every day. It is foreseeable that AR will be widely used in the future, due to its many favorable features, such as allowing dangerous situations to be safely simulated within the classroom, helping to concretize abstract concepts, increasing attention, motivation, interest and success in students, making hidden objects visible, making the learning experience more enjoyable, helping students learn at their own pace, and creating a sense of reality.

METHODOLOGY

The Data Gathering Process

Criteria for works to be included/excluded

For this study, the meta-synthesis method, which is a qualitative research design, was chosen for examining the AR studies in mathematics education between 2010 and 2021. In meta-synthesis, the objective is to look at the results of studies through a wider perspective and to transform these results through interpretation (Sandelowski et al., 2007). Meta-synthesis aims to discuss various works on a certain subject and interpret them by comparing their similarities and differences (Calik & Sozbilir, 2014). In the literature, meta-synthesis is described as the interpretation of qualitative findings, or findings from qualitative research, thus, Polat and Ay (2016) describe meta-synthesis as the evaluation of qualitative findings that come from qualitative or mixed research. The explicit use of both quantitative and qualitative methods in a single study, a combination commonly known as mixed methods research (Maxwell & Loomis, 2003). In accordance with this description, studies with both qualitative and mixed research designs have been included in this study.

We've examined the works that explain how the meta-synthesis method is conducted in the literature as well as looking at studies that use meta-synthesis as their research design; and have seen that most of them suggest similar process steps (Aspfors & Fransson, 2015; Noblit & Hare, 1988; Paterson & Canam, 2001; Polat & Ay, 2016; Sandelowski et al., 2007; Yildizli et al., 2018). These process steps can be itemized as such:

Table 1. AR studies in mathematics education by year

Year	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Number of works	2	2	2	6	2	8	4	4	2	20	6	1

1. **Meta-synthesis design:** This is the step in which the subject, objective, and research questions of study are determined.
2. **Literature review:** Before this step begins, the keywords related to the study and the criteria for including or excluding works are specified. Subsequently, the designated databases are reviewed according to these keywords, and data is gathered according to the inclusion/exclusion criteria.
3. **Reading and evaluating works:** In this step the studies included in the research have been thoroughly read through. The descriptive qualities of these studies have been recorded to an electronic chart, thus been readied for data analysis. This step is described in detail under the title “Data Preparation.” Each study examined for meta-synthesis has been given a number and a code (A1, A2, ..., A24).
4. **Data analysis:** Firstly, we examined the chart containing the descriptive properties of the works included in the study. Then, the categories were examined under different categories, on basis of the research questions and a separate chart was prepared for these properties (i.e., a different chart was prepared for the category of “publication date”). Thus, the studies were compared and related to each other. Each work was reviewed and coded according to the research questions. Codes with similar properties were grouped together and given appropriate themes. The data generated by this process are presented in the Results section, under corresponding titles.
5. **Synthesis:** The tables given in the Results section have been discussed and compared, and the common traits of codes under the same theme were pinpointed to reach a synthesis. The findings related to the examined works were discussed through a holistic perspective in order to present a synthesis on the use of AR in mathematics education.
6. **Reporting and presentation of the meta-synthesis:** The findings have been summarized in the Discussion and Result section and compared to similar studies written on this subject; and a detailed report of the meta-synthesis was given.

Criteria for inclusion/exclusion of the works

The suggestions presented by Noblit and Hare’s book, published in 1988, explaining how meta-ethnography, which is a synthesis of qualitative works, is conducted have been taken into consideration when determining the criteria for including or excluding works from this study (Noblit & Hare, 1988). Studies that include AR applications and practices in the field of mathematics education have been included. We took care to make sure the works included were published between 2010 and 2021. The works included in this study are qualitative or mixed design studies, which include papers published in peer reviewed, scientific publications, masters and PhD theses. Conference papers, compilations, e-books, and works that were not available as full text were not included in this study. Eight conference papers, one e-book, one compilation study, and 29 papers with quantitative research designs were excluded from this study. If both a thesis and an academic paper belonged to the same author, only one of these were chosen. Three papers were excluded because the thesis versions were more expansive. Works where the research problem(s), objective, methodology, data collection techniques, findings, discussion and results were written clearly and comprehensibly were preferable. Studies that approach augmented, and virtual reality together were excluded. Attention was paid to ensure that the chosen studies present suggestions on software development, how to use existing software, or learning methods for the application of AR in educational spaces. Experimental studies in primary, middle, and higher education and theoretical studies in the field of mathematics education were included in this study. Works that were not available in Turkish or English were excluded from this study.

Collecting data

Several national and international databases, including Google Scholar, EBSCO, Education Resources Information Center, Elsevier, Scopus, ProQuest Dissertations and Theses Full Text, Springer Link, Taylor & Francis Online, TUBITAK ULAKBIM Dergipark, YOK Thesis Center scanned; using the search phrases “augmented reality,” “augmented reality and education,” “augmented reality and mathematics education” in both English and Turkish, in order to find the works published between 2010-2021. In order to include the most recent studies these databases were rescanned, and new works were added to the data. The final date was April 2021. 402 works containing the keywords we designated in August 2020. After reading the titles and summaries of these 402 works, ones that focus on AR in mathematics education were chosen. From this selection, e-books and conference papers were excluded, leaving 57 works. According to Bondas and Hall (2007), the optimum number of works for meta-synthesis is 10-12. Because meta-synthesis is an in-depth synthesis process, most sources advise against including too many works. Including too many works, hinders the ability to make in-depth interpretations and syntheses (Calik & Sozbilir, 2014). Including all 57 works that were accessed in the previous step would go against the criteria. 29 of these works were excluded because their research design was quantitative, one because it was a compilation study, and three because they were the paper and thesis versions of the same research. This meta-synthesis was conducted with 24 works that fit the criteria for inclusion. The works included are listed in **Appendix A** (**Table A1**).

There is an apparent increase in the number of works on AR in the field of mathematics between the years, 2010-2021. The distribution of relevant works in the literature according to years is shown in **Table 1.** The works in this table include quantitative, qualitative and mixed research designs. In accordance with the nature of meta-synthesis, this study was conducted with works that either have qualitative or mixed research designs.

Table 2. Data gathering form

Properties of the introduction and methodology of the works								
Code	Publication date	Country	Research design	Sample group	Sample size	Focus/subject of the research	Data collection tools	Data analysis/validity-reliability
Properties of the findings and results of the works								
Software used for AR design		Mathematics subject			Conclusions		Recommendations	

Table 3. Themes and categories

Theme	Category
Objectives	The effect of AR
	Contribution to education/developing AR content
	AR experience
	Skill development
	Use of AR in special education
	Use of AR in teaching concepts
	The benefits and limitations of AR
Data collection tools	Interview
	Questionnaire-test-scale
	Observation
	Documents
	Other

Other themes (Research methods, Mathematics subjects, Data analysis, Reliability-Validity, Software for AR design, Conclusions, Recommendations)

Data Preparation

The researchers read the works chosen to be included in the study in full and took notes. The 24 works were listed alphabetically and given codes from A1 to A24.

To prepare the works for the data analysis process, and collect the data from **Table 2**, a form was created on an electronic chart program. To summarize the descriptive qualities of the 24 works included in the meta-synthesis, each work was examined in full, and each quality was listed in a separate column of the chart. This chart consists of two different sections. The qualities of the introduction and methodology are summarized in the first section. The second section contains properties that are related to the findings and results of the studies. **Table 2** shows the form used to gather and summarize the data.

Data Analysis

Content analysis was used to analyze the data in this research. The codes and themes of this meta-synthesis were shaped according to the research questions and the findings. The process used for the analysis of the data includes all of the steps summarized in the methodology section of this study.

Coding

After the works included in this meta-synthesis were recorded in the data collection form, the chart that was formed was re-examined in order to find the similarities and differences between the works, and themes were specified accordingly. The works were reexamined under these themes and works with similar properties were grouped together under various categories (**Table 3**). The themes in this study are objective, method, data collection tools, data analysis, validity-reliability, conclusions, and recommendations.

Validity and Reliability of the Study

A clear and understandably stated data analysis and coding process increases validity and reliability (Calik & Sozbilir, 2014). The research questions, research objective, data collection and analysis process has been expressed clearly. Before the data coding began, the publication years, participants, objectives, data collecting tools, samples, countries, research designs of the selected works were presented in a chart, and the criteria used to include and exclude data was stated clearly. When specifying themes and categories, for the research, experts were consulted. In order to increase the reliability of the study, direct quotes were taken from the selected works.

Polat and Ay’s (2016) recommendations were taken into consideration to increase the validity and reliability of this study. For validity, the data collection process was explained, and criteria was specified for inclusion and exclusion of data. A detailed explanation of why certain works was excluded from this study was presented. The descriptive qualities of the included works were presented in a chart. The data analysis and designation of shared themes were explained clearly.

For the reliability of the study, when generating themes and codes, the data was checked with the authors; and discussions continued until cohesion was reached between codes. The works were re-read for each research question. These readings allowed us to re-check the existing codes and themes and add to them when necessary. With this step, we tried to ensure consistency; that if the study were to be conducted at a different time, under similar conditions similar results will be found.

Table 4. Distribution of publication years for works included in the meta-synthesis

Year	Works	Frequency
2010	A16	1
2012	A23	1
2013	A3, A9, A21	3
2015	A4, A5, A7, A14, A20	5
2016	A13, A15, A22	3
2017	A18	1
2018	A24	1
2019	A8, A10, A12, A19	4
2020	A1, A6, A11, A17	4
2021	A2	1

Table 5. Distribution of works included in the meta-synthesis by country

Countries	Works	Frequency
Turkey	A1, A2, A8, A9, A18, A19, A24	7
Mexico	A4, A7, A15, A20, A21, A22	6
USA	A3, A10, A11	3
Spain	A6, A16	2
Portugal	A5	1
Poland	A17	1
Ukraine	A12	1
Taiwan	A14	1
Korea	A13	1
Sweden	A23	1

FINDINGS

This section touches upon the descriptive qualities, research objectives, research methods, data collection tools, mathematics subjects, data analysis, validity-reliability, AR design software, results, and recommendations presented in works chosen for this study.

The Descriptive Properties of the Studies Included in the Meta-Synthesis

In this section, the included works are examined according to their years of publication, countries they were conducted in, sample groups and sizes.

Distribution of Works according to Year of Publication

With mobile devices becoming more commonly used in the 2010s, AR technology has also become more commonly used and studies on the subject can be seen to have increased accordingly. **Table 4** shows that the most studies were published in 2015. The table does not show a steady increase in numbers of publications with each year. The reason for this is thought to be the exclusion of quantitative works.

Distribution of Included Works According to Country

Table 5 shows the distribution of countries the works included in this study were conducted in. The exclusion of works that are not in the English or Turkish language has created limitations. Works published in their original language, which is not one of these two were excluded from this study. Therefore, it would not be possible to conclude that Turkey is the country in which the most studies were published on this subject as the table indicates.

Distribution of the Works According to Sample Group

Table 6 shows the distribution of chosen works according to sample group. The group in which the most studies were conducted appears to be bachelor’s degree level education. Within the studies conducted with bachelor’s students, the highest number belongs to the field of engineering. No data was found on studies conducted with preschool or high school level students. For primary school, no studies focusing on grades 1, 2, and 4 were found. Four studies were found for special education, eight for primary and middle school, and nine for bachelor’s level. The sample level was not specified for the work coded A22. A3 and A12 have a theoretical basis and do not involve a sample group. While A6 was conducted with middle school students, the class levels were not specified.

Distribution of Works According to Sample Size

Most of the works included in this study (f=8) have sample sizes of 1-20 participants. The works coded A3, A6, and A12 are theoretically based, and do not include any practices using participants within educational environments. While A7, A17, A20, and A21 contain practices applied in educational environments, the sample sizes were not mentioned in these works (**Table 7**).

Table 6. Distribution of the works included in the meta-synthesis according to sample group

Sample/group type	Class level/age	Research code
Special education	5th grade	A8
	8th grade	A11
	10th grade	A10
	Students requiring visual aid	A17
Primary school	3rd grade	A1
	5th grade	A13
Middle school	6th grade	A9
	7th grade	A18, A24
	8th grade	A14
	Age 15	A23
	—	A6
Bachelor's	Faculty/department	
	Education	A2, A19
	Mathematics	A15
	Architecture and industrial design	A7
	Engineering	A4, A5, A16, A20, A21
Unspecified/theoretical study		A3, A12, A22

Table 7. Distribution of works included in the meta-synthesis according to sample size

Number of participants	Code	Frequency
1-20	A2, A5, A8, A10, A11, A15, A18, A23	8
20-40	A1, A21	2
40-60	A4, A16, A19, A24	4
60-80	A13, A14	2
80-100	A9	1
Unspecified	A7, A17, A20, A22	4
Theoretical work	A3, A6, A12	3

The Objectives of Included Works

As **Table 8** shows, the objectives of the works included in this study were examined and grouped under seven categories. These categories are the effect of AR contribution to education/developing AR content, AR experience, developing skills, use of AR in special education, use of AR when teaching concepts, and the benefits and limitations of AR. Works that have a similar objective have been grouped under the same category. The secondary objectives, if prominent have been coded as sub-titles. The works included in these categories, and the commonalities between these works have been explained below.

The effect of AR

The categories shown in **Table 8** were formed when the works on AR use in mathematics education, were examined. According to the table, the studies coded A1, A9, A14, and A24 research the effect of AR on mathematics education according to certain variables. A24 additionally suggests that using AR for teaching geometric objects will have beneficial effects on students, in terms of success and structuring knowledge. A1, puts forward that AR is an effective aid for difficulties encountered when learning symmetry. Also A1, proposes that the national literature on AR supported geometry education tend to be limited to solid objects and the appearance of objects from different perspectives; and that this study on symmetry will contribute to the field. This work researches the effect of mobile AR on student success in learning symmetry. A9, with the reasoning that other studies merely focus on the technical aspect of AR, examines the effects of AR use on student success, and attitude in geometry education. The findings reached in A9 are thought to have the potential for giving ideas for AR developers and help them produce content that can meet educational needs. A24 suggests that before new AR applications are developed, the benefits of AR for learning should be thoroughly researched, and the needs of this field should be specified. Through these two works (A9, A24) the conclusion can be reached that there is a need for studies that focus on the educational aspect of AR. Based on the effect of AR and visually-based education on students, A14, researches the relationship between mathematics success and spatial skills in educational spaces where AR is integrated into the educational activities.

Developing AR content

The studies coded A5 and A12 focus on the developmental aspect of AR, aiming to improve the learning process of students. These two works examine existing applications and AR developing tools in order to contribute to the use of new technologies that increase the quality of education. Three of the studies in this category emphasize visualization in geometry (A6, A12, and A21). A12 suggests that using AR along with dynamic mathematics software (GeoGebra etc.) can significantly increase visualization levels in mathematics. A12 also claims to be the first study to approach the problems caused by AR use in mathematics education. A6 aims to develop new technologies that will make the teaching-learning process easier, and to teach how to use these new technologies. To this end, an AR application was developed with the aim of visualizing geometry through multi-dimensional shapes. In A15 an AR application was developed in order to design an impressive technological tool for improving. In A21, an AR application that includes mathematical objects and allows student interaction.

Table 8. The objectives of the works included in the meta-synthesis

Categories	Objectives	Code
The effect of AR	The effect of AR on success	A1, A9, A14
	The effect of AR on changing attitude	A9, A24
	AR's effect of cognitive learning	A9
	AR's effect on self-sufficiency	A19
	AR's effect on learning	A24
Contribution to education/ Developing AR content	Researching the developmental role of AR in education (whether it works, how it works)	A5
	Developing methodology for teaching mathematics through the use of new technologies; encouraging active learning through the opportunities AR presents	A12 A22
	Providing visualization in 3D geometry/Increasing visualization levels and skills for mathematics	A6, A12, A21
	Developing AR content to provide ease for education/Producing didactic innovative sources and analyzing applications for mathematics education in STEM/Analyzing the properties and qualities of AR technology/Creating a learning environment where mathematics is more comprehensible (visual and concretized) through the use of AR	A6, A12, A15, A21, A22
	Presenting framework for developing a free AR application	A4
AR experience	Experiencing AR in the classroom /Examining previous experiences/Students' use and acceptance of AR /Consultation /AR's positive role in structuralizing knowledge	A1, A4, A9, A14, A19, A24
Developing skills	Developing rationalization skills	A2
	Developing spatial awareness /Developing spatial analysis skills /Increasing spatial skills/Encouraging spatial visualization	A7, A14, A16, A18, A20, A22, A24
Use of AR in special education	The effect of AR in the education of students with special learning difficulties/Presenting a practical framework for AR use in the education of disabled students/Independent mathematics education for visually challenged students	A8, A10, A11, A17
Use of AR when teaching concepts	Supporting game-based education environments with AR for learning concepts/using AR in learning concepts/Design of AR activities with a purpose to assist in understanding the connection between proportionality and geometric similarity	A13, A15, A23 A2
The benefits & limitations of AR	Presenting a theoretical basis for the benefits and limitations of the AR experience	A3

AR experience

A9 and A14 researches the usability of AR in education; A1 and A24, the opinions of students regarding their AR experiences; A4, how AR can aide the teaching-learning process and the reactions of students toward AR.

Developing skills

The works A7, A14, A16, A18, A20, A22, and A24 use AR applications for purposes such as developing students' spatial awareness, developing spatial analysis skills, increasing spatial skills, and encouraging spatial visualization. These studies research the subjects of geometrical objects, and the appearance of these objects from different angles in relation to the development of spatial skills. According to A20, spatial skills are not constant, and can be developed through dynamic interactions, such as AR. This skill can be increased through enhanced new technologies. In A16 and A20, AR technology is used to increase engineering students' spatial reasoning skills in class. In A2, the aim is to develop trainee teachers' proportional reasoning skills. A8, aims to better the understanding of two-dimensional representations of three-dimensional objects. A mobile device was developed to this end.

Use of AR in special education

A8 and A11 researches the effect of AR on students with learning difficulties, and A17 on students who are visually challenged. Because mathematical formulas are mostly inaccessible to students who are either partially or fully visually impaired; A17, suggests a method to enable students to learn independently from textbooks, in a creative and interactive way. A10 provides a framework for those who want to use AR as an educational strategy for visually challenged students.

Use of AR in teaching concepts

In the works A13, A15, and A23; AR applications are used in order to develop a teaching technique for the education of certain concepts. These studies enable students to interact with both real and virtual objects in order to teach those basic geometric concepts (A13), Euclid vectors in physics and mathematics (A15), and the concept of measure (A23).

The benefits and limitations of AR use

While many of the works included in the study touch upon the advantages and disadvantages of AR. Since, A3 is the only work that has the objective of explaining the benefits and limitations of AR use, it is the only work that falls under this category.

The Data Collecting Tools Used in Works Included

In **Table 9**, the findings regarding the data collection tools of the works included in the meta-synthesis have been categorized as interview, questionnaire-test-scale, observation, documents, and unspecified. When the works under "interview" are examined, it can be seen that *in-person interviews* are often used. Under the questionnaire-test-scale category, mixed method research was seen to be included, and pretest-posttest practices were seen often for works with mixed research design. Among

Table 9. The data collecting tools used in works included

Categories	Data collecting tools	Code of work
Interview	Student interview form	A1, A19
	In-person interview	A4, A5, A8, A9, A13, A18
	Focus group	A14
	Audio recordings	A18, A23
Questionnaire-test-scale	Geometry achievement test	A9
	Achievement test	A1, A8
	Pen and paper test	A2, A14
	Spatial ability test/Mental rotation test (MRT)/Differential ability test (DAT-5: SR)	A14, A24/A16/A16
	Mixed method questionnaire	A13
	Information collection questionnaire	A4
	Open ended questions	A13, A15, A21
	Mobile AR questionnaire	A25
	Questionnaire by interview	A5
	Van Hiele geometric thinking test	A9
	Attitude toward geometry test	A24
	Attitude toward mathematics test	A9
	Attitude scale for AR applications	A24
	Satisfaction questionnaire	A16
	Motivation questionnaire	A25
	Mathematics anxiety scale	A25
	System usability scale	A14, A15
	Self-sufficiency regarding technology use scale	A19
	Pretest-posttest	A11, A18, A24, A25
	Social validity questionnaire	A11
Observation	Video recordings	A2, A8, A11, A20, A21, A23
	Observation	A4, A5, A10, A13, A14, A15, A17, A18
Documents	Databases (collection/theoretical)	A3, A12
	Data recording table	A8
	Student studies through booklets	A8, A18
	Data obtained from screen captures of AR applications	A18
	Application control list	A10, A11
Other	Unspecified	A6, A22

the works shown in **Table 9**, some use more than one data collection tool. In A14, A16 and A24, tests were used to evaluate spatial skills. A6 and A22 do not mention data collection tools.

In the interviews, the students were asked questions about their AR experiences. Also, the thoughts students have on the AR application used in the practice were researched in the examples given below. Some examples to interview questions are as follows:

> "Do you think that square prism teachings supported with augmented reality material helps you to maintain attention on the subject?" "Has learning about square prisms with the support of augmented reality material enabled you to visualize objects in 3D?" "What are your thoughts on your AR supported education process?" (A8).

> "If I were to ask you to describe your augmented reality experience with three words, what would you mention?" "What do you think of the augmented reality application we have used?" "What do you think about classes being taught with augmented reality?" "How has the class been taught with augmented reality benefit you other than learning the course subject?" "How were your interactions within the classroom throughout the practices?" (A9).

> "What do you think about AR technology?" "What could be the advantages and disadvantages of having AR in the classes?" (A24).

Research Methods of Included Works

In **Table 10**, the works included in the meta-synthesis have been categorized according to their research methods. A1, A9, A13, A19, and A24 have opted for mixed methods that utilize both qualitative and quantitative methods. A5 and A23 have specified their method as design-based research; and A18 as educational design research in their respective methodology sections. These two research designs include some of the same research steps.

For A11, A15, A16, A20, and A22, the research method is "unspecified." While A11 and A16 include both qualitative and quantitative data, the research design for these works are not openly stated. While A15 does not fully state a research design, an AR application was developed in order to teach a specific mathematical concept, and this application was evaluated. In A20, an AR application was designed, and a pilot study was conducted, yet the research method was not clearly stated. Quotations from the methodology sections of these studies have been given as examples below:

Table 10. Research methods of the works included in the meta-synthesis

Research method	Work Code	Frequency
Mixed (qualitative and quantitative) method	A1, A9, A13, A19, A24	5
Design based research/educational design research	A5, A18, A23	3
Single subject research method	A8	1
Qualitative research	A17	1
Participatory action research	A14	1
Case study	A2, A4, A21	3
Theoretical study	A7, A12	2
Theorization	A3	1
Didactic study (developing teaching material)	A6	1
Unspecified	A10, A11, A15, A16, A20, A22	6

Table 11. Distribution of works included in the meta-synthesis according to mathematics subjects

Mathematics field/subject		Work code	Frequency
Geometry	Geometry	A13	1
	Geometrical objects	A6, A8, A9, A14, A12, A19	6
	The appearance of objects from different angles	A16, A18, A19, A24	4
	Conics (analytical geometry)	A7, A22	2
	Euclidian vectors	A15	1
	Similarity	A2	1
	Symmetry	A1	1
Numbers and operations, algebra, measurement, probability	Length measurements	A23	1
	Functions	A21	1
	Second degree equations	A4, A17	2
	Parabolas	A22	1
	Probability	A12	1
	Analysis (calculus)	A5, A12, A20, A21	4
	Basic mathematics	A3	1
	Integers	A11	1
	Four operation skills	A10	1

> "… this study has been conducted accordingly to the mixed research method. For the quantitative part of the research, pretest-posttest quasi-experimental design was used without a control group; and for the qualitative part basic interpretive design was used" (A1).
>
> "Because the trainee teachers receive education as a single class in the higher education institution where this study has been conducted, this study has been designed as a single-group study. Since there is no control group to compare the quantitative data collected for this study, the finding has been supported with qualitative data in order to increase the validity of the data. … this study has been designed with explanatory mixed method" (A19).
>
> "Because this research is about designing and developing a mobile AR interface, and SPATIAL-AR tool-set which includes a series of spatial tasks, in order to develop 7th grade students' spatial skills, the research methodologies proposed by Nieveen and Folmer had been taken into consideration while designing this research" (A18).

Distribution of Works According to Mathematics Subjects

For the learning fields show in **Table 11**, the divisions were based on the ones in the middle and high school curriculums in Turkey. Most of the works approach geometry subjects, such as geometrical objects, the appearance of objects from different angles, symmetry, conics, similarity. For A7, A12, A13, A15, and A22, geometry/analytical geometry; for A6, A8, A9, A14, A12, and A19, geometrical objects; for A16, A18, A19, and A24, the appearance of objects from different angles; for A1 symmetry, and for A2 similarity are the chosen subjects. A5, A20, and A21 have focused on higher education level analysis subjects. In A21, software was developed for the 3D modeling of functions. A fewer number of studies focus on subjects other than geometry, such as numbers and operations, algebra, measurements and probability.

A8 and A24, the mathematics subjects were designated by conducting needs-based analysis, through "designating needs, analyzing" which is the first step of developing designs. In A1, since AR studies on subjects utilizing 3D models, such as the appearance of objects from different angles and volume, through the interviews conducted it was concluded that supporting the teaching of symmetry with AR would be more meaningful and effective.

Data Analysis and Validity-Reliability in Included Works

Data analysis

In A10, while an educational strategy was being practiced with the students, the operators filled in a prepared control list in order to do task analysis. Similarly, in A11, while the study was being conducted, every step completed by the participants was added to the control form in percentages. A9 puts forward that the collected data was categorized into themes through descriptive

Table 12. Data analysis of the works Included in the meta-synthesis

Data Analysis	Work code
Content analysis/frequency	A2, A19, A24
Descriptive analysis	A9
Purposive analysis	A23
Documenting the data collected through interviews, observations, audio and video recordings in text form	A2, A14, A18, A20, A21
Documental analysis; inferential and descriptive statistical analysis	A5
Qualitative analysis through grounded theory	A14
Task analysis through a checklist, percentage calculations	A10, A11
Graphical analysis method	A8
Shapiro-Wilk normalcy test/significance test	A1, A19
t-test	A1, A9, A14, A24
Two factor ANOVA, Mann-Whitney U test	A9
Pearson correlation analysis	A14
Quantitative data (standard deviation, median, usability and learnability points)	A5, A13
Unspecified	A3, A4, A6, A7, A12, A22

analysis. In A19, the data obtained through interviews was divided into themes, categories and codes which were presented in a table displaying the percentages and frequencies of these codes. In A2 and A24, the data from video recordings and pen and paper tests have been examined using content analysis. Some parts of the video recordings were transcribed, and this data was included in the work. Since A1, A9, A13, A19, and A24 all have mixed design research methods, separate analyses were conducted for the qualitative and quantitative data. In A8 the data was collected through video recordings. The correct and incorrect reactions of students during the education process were identified, and the "correct reaction percentage calculation formula" was utilized. The data collected during education sessions was processed in graphics and examined through graphical analysis. In A10 and A11, the data collected through observation have been recorded using a checklist, and the data was presented in percentages. In A23, for purposive analysis, the data obtained during education practices was transcribed, and presented as direct quotes. It was observed that the purposive analysis in A23 and the content analysis in A2 are similar methods. There was no explanation of how data was analyzed in the works coded A3, A4, A6, A7, A12, and A22 (**Table 12**).

In A14, in order to analyze the research question *"Can AR supported programs develop students' spatial skills?"* the pretest-posttest scores of three groups with different academic success rates were compared, and it was seen that AR supported education had a positive effect on students with low academic success. In the same study, it was researched whether mathematical and spatial success is related, using Pearson correlation analysis. The focus group interview data analysis for A14:

> "The participant observation revealed that the students were joyful when they were instructed on how they would use the system. The students were also seen to have significant interest and anticipation towards the lesson. The interviews and impressions of the instructor also shows that the students found the system very interesting. After the experiment was finished, focus group interviews were conducted. The open coding was done based on grounded theory. The focus group interviews were analyzed using qualitative analysis according with grounded theory, and the four following structures were obtained: system usability, performance by oneself, motivation toward work, and expected effects" (A14).

Reliability and validity

In this section we have examined whether the reliability and validity analysis were done in the works included, and to what extent the concepts of reliability and validity were included.

Reliability: When the works included in this study are examined in terms of their reliability, it was seen that A24 used the Miles and Huberman (1994) formula, A1 the kappa formula, and A8 the "calculated via [Consensus/(Consensus+Dissensus)×100]" (p. 69) to calculate the reliability between coders. In A9 and A24 the Cronbach's alpha coefficients were calculated; in A9 and A18 expert opinions were taken into consideration, in A14 and A14 reliability was calculated, but the method was unspecified; for A9 it was stated that a pilot study was conducted in order to assure reliability, and the KR-20 reliability coefficient was calculated for the test developed within the research (**Table 13**).

In some studies, while the data analysis process was supported by using varied data collection tools, supporting findings with direct quotes, or consulting experts for the data collection process, these practices were not openly expressed to be in order to increase the reliability of the study. In A19, through t-test analysis, the difference between the pretest and posttests conducted to measure students' perception of self-sufficiency in technological integration; and the confidence interval was calculated as 95%. Yet there was no mention of an implementation to increase the reliability of this study. As for A5, the questionnaire, which is the data collecting tool, has been used after expert evaluation, and all the steps of this study which utilizes design-based research design have been explained in detail. In A2, which uses multiple data collection tools, the video recordings were transcribed, and the data from the pen-and-paper tests have been coded in the electronic chart in detail. For A23 the data collection process was explained and analyzed in detail, the practice sessions were recorded as video and audio and this data has been kept.

For A14 and A15, the "system usability scale", developed by the same person, was used to evaluate the systems designed in these works. The AR system designed for teaching vectors in A15 was found adequate according to its usability and learnability scores. We could find no explanation of how reliability was achieved for the works A1, A2, A3, A4, A5, A6, A7, A10, A12, A15, A20, A21, A22 and A23; whereas some works have clearly stated the measures taken to ensure reliability. Direct quotes from studies that mention the factors that ensure reliability are given below:

Table 13. The reliability and validity of works included in the meta-synthesis

Reliability	Work code
Calculating reliability coefficient between observers, the kappa point	A8, A11
Application reliability form	A8
Calculating reliability between coders	A24
Cronbach's alpha reliability coefficient	A9, A24
KR-20 reliability coefficient	A9
Calculating scale/test reliability	A14, A15
Triangulation strategy	A18
Peer/colleague examination, expert opinion	A18, A9
Pilot study	A9
Unspecified	A1, A2, A3, A4, A5, A6, A7, A10, A12, A13, A19, A20, A21, A22, A23
Validity	
Social validity form/questionnaire	A8, A11
Triangulation design/strategy	A14
Pearson correlation analysis	A14
Using scales with pre-calculated validity	A24
Expert opinion	A9, A11
Content validity of scale items	A9
The average and standard deviation calculations for Likert scale data	A13
Direct quotes	A13
Pilot study for content validity	A19
Supporting quantitative data through qualitative data	A19
Unspecified	A1, A2, A3, A4, A5, A6, A7, A10, A12, A15, A20, A21, A22, A23

> "…multiple data sources were used in this study for the data analysis procedure. Peer/colleague review, in order to examine the same data set and determine whether the results seem consistent. A mathematics education expert has assisted with the data analysis and coding procedures" (A18).

> "The results of this study are consistent with the results of similar studies which focus on the effectiveness and validity of AR in teaching certain skills" (A11).

Validity: When the works included in the meta-synthesis were examined in terms of validity, it was seen that that the scale used as a data collection tool in A24 was developed by someone else, and the validity study was already conducted for this scale; in A8 and A11 a social validity questionnaire/form was used; for A9, A11, and A18, expert opinions were consulted to ensure validity; in A13 qualitative data was presented as direct quotes, and average and standard deviation was calculated; for A14 and A18, the triangulation strategy/approach was used in order to obtain more information about the data. No information was found on validity in works A1, A2, A3, A4, A5, A6, A7, A10, A12, A15, A20, A21, A22, and A23 (**Table 13**).

The measures taken to ensure validity in A18 have been described in detail under titles such as "internal validity," "external validity," and similar strategies were used for reliability and validity, and multiple data collection tools were used in order to increase reliability and validity. In A19, qualitative data was used in order to increase the validity of the study. During the design stage of the application used in the practices of this study, a practice was done with middle school students in order to test the usability and content validity of the design. A9 and A19 have supported quantitative data with qualitative data in order to increase validity. Examples about validity are given below:

> "The validity evaluation of the AR applications attitude scale use in order to assess attitude toward AR applications has been done…" (A24).

The AR Technologies Used in Works Included in the Meta-Synthesis

Although Unity 3D is a game development platform, due to the AR add-on this platform has become suitable for developing AR content. The AR practices/content in works A2, A4, A8, A13, A18, A19, and A24 have been designed with Unity 3D and sourcing was done through the Vuforia Library. Unity 3D is a free software which has more advanced, paid versions. SketchUp 3D which is a software used for 3D modelling has produced an AR supported version. In A7 the SketchUp 3D software has been used with AR Happens, BuildAR and Aumentaty Author add-ons (**Table 14**).

In A10 AR was used as a video recording tool. The students used videos that could be viewed with AR technologies as guides in order to answer the study questions posed to them. In A24, HP Reveal was used prior to the design development stage. Because of the problems encountered during the pilot study, the AR events were re-designed using the Unity 3D platform for the main study. The advantages of Unity 3D were expressed as *"The 3D models adhere well to the target images, the 3D models work in a flexible and compatible way with the AR add-on, the separate target images of all the activities can be worked through a single. apk file, and internet connection is not needed during the activities."* in A24.

Unlike the other design-based studies, in A12, material was prepared by producing objects on *GeoGebra AR* and it was proposed that *GeoGebra AR* could be used in teaching subjects such as advanced mathematics, probability, analysis, and analytic geometry. Material that can help students' spatial thinking abilities was developed in the studies coded as A7, A8, A9, A14, A16,

Table 14. The AR technologies used in works included in the meta-synthesis

The software development tools used for AR design or technology used for AR	Work codes
Unity 3D software with Vuforia SDK	A2, A4, A8, A13, A18, A19, A24
HP Reveal (previously named Aurasma)	A1, A10, A11, A24
Silverlight, Slartoolkit	A9
GeoGebra AR, GeoGebra 3D Calculator	A12
MicrosoftTM Kinect Sensor ve Xbox 360 Game Console	A15
SketchUp, AR Happens, buildAR and Aumentaty Author	A7
Brainstorm eStudio, HUMANAR library	A16
Technical details unspecified	A3, A5, A6, A14, A20, A17, A21, A22, A23

A18, A19, A20, A22, and A24. The technical properties of the AR contents have not been mentioned in A3, A5, A6, A14, A17, A20, A21, A22, and A23.

The Conclusions of the Works Included in the Meta-Synthesis

The conclusions of the works suggest that AR is interesting/attention getting (A3, A8, A9, A21), fun (A8, A9, A19, A24), beneficial/useful (A24), interactive (A9), effective (A3, A10, A24), practical/useful (A13, A14, A15), portable (A21), and cost effective (A16) tool. Some works emphasize that AR supports individual learning. AR has allowed students to learn independently and allowed students to manage their own learning process (A10); it has given independency to their learning (A6); has enabled the education process to become individualized, the application to be understood without instructions and the students to work on their own (A1, A12, A17, A18, A21) (**Table 15**). According to A23, AR encourages mathematical questioning, and can be used in inquiry-based mathematical education. Another significant conclusion is that AR helps increase spatial intelligence/spatial visualization in students. In the studies coded A6, A14, A16, A18, and A20 these skills have been titled as, spatial ability, spatial visualization, spatial intelligence, spatial analysis, spatial sense, and spatial skill. With the designed AR tool, students have been able to understand two dimensional images, and mentally visualize three dimensional objects shown to them in 2D. The AR tool can also be used to make up for the differences between students with varying spatial skill levels and enable students to increase their spatial abilities (A18). A2 has reached the conclusion that AR contributes to reasoning skills; A1, A3, A6, A15, and A22 that AR contributes to visualization; and A2, A3, A6, A8, and A15 that AR makes concept learning easier for students. A1, A8 and A9 emphasize that AR helps to concretize the course content and mathematical concepts. A9 has pointed out that AR is beneficial for preventing misconceptions. A3 emphasizes that because AR makes information suitable for the students' attention, it is an effective learning tool. A7 points out that AR requires sufficient software knowledge in order to generate content in computer environments. A16, points out that because AR requires basic and low-cost equipment such as computers and cameras, these activities can be carried out in a cost-effective way. A12 states that the use of the GeoGebra 3D Calculator, which includes AR, for mathematics education in STEM, can help solve the problem of individualization, which is one of the issues in STEM activities. Because spatial objects such as mathematical formulas and graphics are components which hinder individual mathematics learning for students with partial or complete visual impairments, A17 has developed a multi sensual AR tool which reorganizes the contents of textbooks and worksheets in order to make these contents accessible for visually impaired students; and this tool has gained acceptance from the users. A24 concludes that AR increases students' creativity by presenting them with an enriched learning environment. In A19, during the application done with trainee teachers, the participants have expressed that they would like to use this tool in their classes because it raises the motivation levels of their students.

Some of the studies have also touched upon the negative aspects of using AR in educational environments. Some examples are the potential of AR to increase technological dependence (A8), causing distraction for the students (A24), causing eye-related health issues (A24), the batteries of mobile devices used for AR running out too quickly (A24) and students not sharing the mobile devices with each other (A1).

DISCUSSION, CONCLUCION, AND SUGGESTIONS

The conclusions drawn from the analyses of the current state of studies on the use of AR in mathematics education are presented in this section.

Results According to the Descriptive Properties of Studies Included in the Meta-Synthesis

When the descriptive properties of the works included in this study were examined, it was seen that the highest number of works between 2010 and 2021 was published in 2015. According to **Table 4**, there was an increase in the number of AR studies in 2013. This result supports the findings of Sirakaya and Sirakaya (2018) that qualitative research design AR studies have increased after 2013. When the number of quantitative studies that have not been included in this study is also considered, it can be seen that the number of studies has tended to increase with the years. This is concurrent with the findings in the literature that suggest that the number of works has increased as mobile devices have become more common (Bilinghurst, 2002; Yilmaz & Batdi, 2016). Because quantitative works have not been included in this study, the number of works is seen to increase with the years.

The distribution of the works examined shows that most studies were conducted in Turkey, Mexico and the U.S. In their examination of AR studies in the field of education, Kara (2018) found that Taiwan, Spain, and the U.S. are the top three countries in terms of the number of studies. Similarly, according to the bibliometric analysis results by Karakus et al. (2019), the leading countries in augmented reality in education are Spain and Taiwan. Turkey, Canada, and Mexico follow this order. Altinpulluk (2018)

Table 15. The conclusions of the works included in the meta-synthesis

Categories	Conclusions regarding AR	Work code
The properties of the AR tool	Being interesting/attention getting	A3, A8, A9, A21
	Being fun	A8, A9, A19, A24
	Being beneficial/useful	A24
	Being effective	A3, A10, A24
	Being interactive	A9
	Being cost effective	A16
	Being practical/usable	A13, A14, A15
	The tool for AR being portable	A21
	Creating/enhancing visualization	A1, A3, A6, A15, A22
AR's contribution to the mathematics field	Contribution to developing reasoning skills	A2
	Contribution to the development of spatial intelligence skills/spatial visualization	A6, A14, A16, A18, A20
	Contribution to mathematical questioning skills	A23
	Concretization of lesson content/concepts	A1, A8, A9
	Making learning concepts easier/reducing cognitive effort when learning concepts	A2, A3, A6, A8, A15
	Helping to reduce conceptual fallacies	A9
AR's contribution to learning	Positive contribution to sense of self-sufficiency	A19
	Positive effect on self-evaluation	A1
	Contribution to individual/independent learning, individualizing learning	A1, A6, A10, A12, A18, A21
	Encouraging cooperative learning / enabling peer support	A1, A19
	Supporting in-depth learning	A9
	Saving time / enabling lesson content to be learned in less time	A9
	Increasing learning speed	A9
	Increasing success	A1
	Increasing attitude	A9, A14, A15, A16, A24
	Increasing motivation	A6, A9, A12, A13
AR's contribution to the learning environment	Increasing communication	A19
	Increasing active participation in the classroom	A1, A19
	Multi-sensory learning environment for visually challenged students	A17
	Enriched learning environment	A3, A16, A23
	Enabling three-dimensional animation	A8
	Enabling a natural interaction with virtual objects	A19, A21
	Creating a sense of reality	A19, A21
The negative aspects of AR	The potential for increasing technological dependence	A8
	The mobile devices used for AR not being shared by the students	A1
	Causing distraction for the students	A24
	The mobile devices used for AR running out of power frequently	A24
	Causing health issues for the eyes	A24

says that a significant number of the studies on AR come from the Asian continent. Although the findings of this meta-synthesis have been similar to the findings of Karakus et al. (2019), they have not been similar to the findings of Altinpulluk (2018) and Kara (2018). The cause for this difference is thought to be the exclusion of quantitative studies in this meta-synthesis. Only works that are available in English or Turkish were selected for the data collection process, and any works with a different language were excluded. Therefore, although Turkey comes first in the distribution in terms of the number of published works, this cannot lead to the conclusion that most studies on AR have been conducted in Turkey overall.

A significant number of studies (33%) have been conducted with 1-20 participants. In terms of sample level, most studies (f=9) were conducted with bachelor's level students. Because increasing spatial skills, geometrical objects, and analysis are subjects that are suitable for producing class content related to AR, it is thought that there are more studies related to these subjects. There are eight studies that focus on primary and middle school levels. Because the whole of primary school, and the first years of middle school (ages 7-11) fall into Piaget's concrete operations period (Piaget, 1964; Sutherland, 1992), it is pointed out that AR studies done with students at this level will lend concretization when learning concepts (Sirakaya & Sirakaya, 2018). Many studies touch upon AR technology making concretization easier (Abdusselam & Karal, 2012; Bujak et al., 2013; Saidin et al., 2015). Even though these class levels are in their concrete operations period and suitable for AR use, no works conducted with 1st, 2nd, and 4th-grade students were found. Four of the works included in the meta-synthesis are in the field of special education. These studies focus on the needs of students with visual impairments, special learning deficiencies, and mental challenges. The studies in special education are important in terms of stating the needs and deficiencies in this field. AR is effective in the teaching of concepts, visualization of three-dimensional objects, and the concretization of abstract concepts in mathematics education. Because AR technology is able to stimulate multiple senses (Azuma et al., 2001) it is suitable for the use of students with special needs (Kellems et al., 2019, 2020; McMahon et al., 2016; Mikulowski & Brzostek-Pawlowska, 2020; Isik, 2019; Zainuddin et al., 2010).

Results of the Objectives of the Included Studies

When the works were examined according to their research objectives and significance. Seven themes were found. These are "effect of AR," "contribution to education/developing AR content," "AR experience," "developing skills," "use of AR in special education," "use of AR when teaching concepts," and "benefits and limitations of AR. "Most of the studies fall under the "contribution to education/developing AR content" and the "developing skills" categories. Generally, the objective of the works is to achieve visualization in mathematics and geometry education, create an innovative system that will make education easier, create an understandable (visual and concrete) educational environment, and develop spatial skills. It is often emphasized in the literature that using AR in education will aid visualization, concretize abstract concepts, and make teaching easier (Abdusselam & Karal, 2012; Bujak et al., 2013; Gun & Atasoy, 2017; Martín-Gutiérrez et al., 2010; Ozcakir & Cakiroglu, 2022; Saidin et al., 2015; Wu et al., 2013).

Distribution of Studies in Terms of Data Collection Tools

When the works included in the study were examined in terms of the data collection techniques they use, most of the works were seen to use techniques that fall under the titles of "Observation" and "Interview." In most studies, the AR application has been experienced in a classroom environment. In order to collect data on the students' experiences, interviews were conducted with the students, and the practices where they use the AR applications have been recorded via video. When asking the students about their opinions on the activities they participated in, it was seen that the questions tend to focus on what their experience with AR was like, and what their opinions on AR applications are.

Distribution of Studies in Terms of Methodology

When the works included in the meta-synthesis were examined in terms of their research methodology, it was seen that most studies used mixed-design research methods. Altinpulluk (2018), says that 80% (f=58) of the studies done in the field of education have either quantitative or mixed research designs. There are three works in this study designed as "design-based research/educational design research." Design-based studies differ from studies that do not focus on design problems, such as ethnography and empirical studies. Design-based studies merge the educational application and conceptual studies together (Kuzu et al., 2011). Also, because these works explain the design process in detail and mention the difficulties and setbacks encountered in the design stages, and are evaluated in the educational environment, they are able to serve as guides for other researchers who are interested in the field. Therefore, design-based research conducted in order to create AR applications will be beneficial both to the development of AR technology and help guide researchers who would like to work in this field.

Results for the Distribution of Studies in Terms of Mathematical Subjects

When the mathematics subjects used in the works were examined, it was seen that 13 were in the field of geometry, such as the appearance of objects from various angles, geometrical objects, and measuring length. These subjects that require more visual material are thought to be the reason for this. When looking at the early studies on AR in mathematics education, it is seen that geometry is the main subject of focus. One of the first works in the field has been done by Kaufmann et al. (2000), in which an AR tool was designed for geometrical structures. Like Kaufmann et al. (2000), many studies on the use of AR in mathematics education, aim to develop spatial skills. The works examined in this study indicate that research was conducted on mathematical subjects that are bachelor's degree level and in the field of engineering. Few studies have been conducted on AR's use in primary school education. AR is a technology that lends itself to improving primary school students' reasoning, mathematical literacy, numerical and operational skills. It's entertaining, concretizing, and visualizing aspects may be useful for capturing the interest of students and developing these skills. Similarity, symmetry, functions, integers, and second-degree equations are among the subjects included in the works. This shows that various mathematical subjects are suitable for developing AR content and that AR can be used in mathematics education. In summary, the subjects in these works were often chosen because of the need for visualization or concretization in mathematics class. Therefore, researchers more often developed material that would be beneficial for improving students' spatial skills.

Results of the Data Analysis Process

When the methods used for data analysis in the works were examined, both qualitative and quantitative data analysis processes were followed in the five mixed design researches. Most of the works utilized video recordings and asked the participants open-ended questions when collecting qualitative data. The video recordings were transcribed and along with the data from the pen-and-paper tests examined using content analysis. While not all of the 10 studies clearly state that they have used content analysis, the fact that the data was transcribed, and then examined under certain categories and themes leads us to think that content analysis was used. In some studies, standard deviation, medians, usability, and learnability scores were calculated using quantitative data. For the analysis of quantitative data, methods such as the t-test, Pearson correlation analysis, two-factor ANOVA, the Mann-Whitney U test, and the Shapiro-Wilk test for normalcy/significant meaning were used.

Expert opinions, reliability calculation between coders for analyzing qualitative findings, Chronbach's alpha reliability coefficient calculation, pilot studies, KR-2O coefficient calculation, describing the steps one by one and in detail, are among the ways in which the works included in the meta-synthesis have aimed to achieve reliability. When the works were examined in terms of their validity, it was seen that qualitative data has been presented with direct quotes, scales which's reliability had been previously checked have been used, the content validity of the data collection tools was examined, and the averages and standard deviations of the data were calculated, expert opinions were consulted, and the triangulation strategy (research methods, data sources, researcher) was used. 12 of the studies have not mentioned any steps taken for reliability and validity, which shows that 50% of the works included are insufficient in terms of reliability and validity.

Conclusions on the Stage or How AR Technologies are Used

The most common software used to generate AR content in the works included in this study has been Unity 3D. In comparison with other software, Unity 3D has advantages such as offering an adaptable workspace, offline use, and access to open-source codes that are often updated. The Vuforia Library is used to create 3D models in Unity 3D. In some works, HP Reveal (previously named Aurasma) was used to add 3D digital objects to real-world images. HP Reveal is often preferred due to its practicality and ease of use (Altiok, 2020). In their thesis, Ozcakir (2017) has stated that the GeoGebra 5 software visualizes three-dimensional geometric objects and allows for these to be seen via screen through two-dimensional layers; and that therefore using this software in educational environments may lead to concept errors. In 2021, with the AR add-on, it has become possible to view virtual 3D objects in naturalistic environments in GeoGebra 3D. Due to the continuous development of technology, we will be coming across various new software for creating AR content in future studies. Due to these developments, it is seen that AR will be used more commonly in educational environments.

Results of Studies Included in Meta-Synthesis

When the conclusions of the works included in the study are examined, the prominent properties of AR are that it is interesting/attention-getting, fun, and interactive. AR is an effective, beneficial, cost-effective, portable, practical, and usable tool for educational environments, which animates 3D objects, enables interaction between virtual and natural objects, provides an enriched learning environment, and creates a sense of reality. According to the conclusions of the works in this study, the contributions of AR to mathematics education are that it improves spatial abilities and questioning skills. The works examined indicate that visualization practices are important for developing spatial skills, and AR is an effective tool for visualization. That AR lessens cognitive effort when learning concepts, concretizes course content and concepts, and helps prevent concept errors are among the conclusions reached. Another significant conclusion is that AR individualizes the learning environment and enables independent learning. These studies have also touched upon the negative aspects of AR, such as the batteries of mobile devices running out too soon, mobile device use causing issues with eye health, and distracting students during class.

We did not come across a comprehensive study on the accessibility and availability of AR applications when reviewing the literature. Therefore, answering questions such as “Which operational system does it work on?”, “Which software(s) was it written with?”, “Which platform is it accessed through?” regarding AR applications would be beneficial to the literature.

With the AR add-on designed for the 2019 version of GeoGebra 3D, it has become possible to carry over three-dimensional objects into real-world environments. No studies that used materials prepared with the GeoGebra 3D AR add-on were found in the literature. Compared to game development engines such as Unity 3D, which require technical know-how in order to develop AR content, the GeoGebra software is easier to use, and AR course material can be prepared using this software.

There is a need for AR research to develop primary students’ skills, such as mathematical literacy, counting and basic operations, and reasoning. Similarly, because there are no qualitative studies conducted with preschool and high school level participants, these sample groups have the potential for future AR research. AR content can be generated for high-ability and special needs students, and these students’ education can be supported through AR applications. Since AR is commonly used in fields such as production, construction, technique, the use of AR in vocational education is a potential area of research.

According to the literature, it has been reported that AR supports mathematical inquiry and can be used in inquiry-based mathematics education. In the literature, it is emphasized that AR contributes to various pedagogical approaches such as game-based learning and cooperative learning. However, few studies have been found that associate AR with pedagogical approaches. Thus, how AR supports pedagogical approaches could be a potential issue.

While it is said that AR use in the field of education has become more common, this should not only be of interest to education researchers. In addition to education research, in order to increase the use of AR in educational environments, it would be beneficial for teachers to be given in-service training. Guidebooks and introductory content can be produced in order to help teachers actively use AR in their classes. There is a potential for projects and science fairs to help introduce AR and its uses in education.

According to the literature, studies about the use of AR in mathematics education are concentrated in the field of geometry, and there is a need for contemporary studies in other fields of mathematics. This study has reached the conclusion that suitable AR content can be generated for many areas of mathematics. In light of this conclusion, qualitative and design-based AR research can be conducted in various fields of mathematics.

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APPENDIX A

Code List of Studies Included in Meta-Synthesis

Table A1. List of studies included in meta-synthesis

Research Code	Work
A1	Altiok (2020)
A2	Arican and Ozcakir (2021)
A3	Bujak et al. (2013)
A4	Barraza Castillo et al. (2015)
A5	Coimbra et al. (2015)
A6	Fernández-Enríquez and Delgado-Martín (2020)
A7	González (2015)
A8	Isik (2019)
A9	Ibili (2013)
A10	Kellems et al. (2019)
A11	Kellems et al. (2020)
A12	Kramarenko et al. (2019)
A13	Laine et al. (2016)
A14	Lin et al. (2015)
A15	Martin-Gonzalez et al. (2016)
A16	Martín-Gutiérrez et al. (2010)
A17	Mikułowski and Brzostek-Pawłowska (2020)
A18	Ozcakir (2017)
A19	Ozcakir and Aydin (2019)
A20	Quintero et al. (2015)
A21	Salinas et al. (2013)
A22	Salinas and Pulido (2016)
A23	Sollervall (2012)
A24	Topraklikoglu (2018)