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A cross-national comparison of fourth and eighth grade students' understanding of fraction magnitude

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ABSTRACT

Without a conceptual understanding of fraction magnitude, students have difficulties understanding more advanced mathematics in high school and beyond. The purpose of this study is to highlight 4th and 8th grade students' misconceptions in fraction magnitude using the Trends in International Mathematics and Science Study-2015 data. The study is informed by the theory of numerical development that focuses on the understanding of the magnitude of numbers and the recognition of the location of numbers on a number line. The results indicate that fraction magnitude understanding is a challenge at the 8th grade. Implications for higher education are elaborated.

Keywords: fractions, large scale studies, misconceptions

INTRODUCTION

Success in advanced mathematics relies heavily on a strong foundational knowledge of mathematics. For example, if students do not have the requisite knowledge in numbers and operations, they will have difficulties learning algebra (Holmes et al., 2013; Rhine et al., 2018). Fractions, in particular, is one of the mathematics topics that students have difficulties understanding in elementary and high school (Hecht & Vagi, 2010; Kloosterman, 2010). The transition from the mastery of whole number operations to fractions and the resulting solutions is a challenge for many students (Fazio & Siegler, 2011; Siegler & Pyke, 2013). Further, the several procedures for fraction operations and lack of conceptual understanding of the topic compound the difficulty in understanding this topic (Siegler & Pyke, 2013).

The challenges in students' understanding of particular concepts lead to students' errors, mistakes or misconceptions when learning fractions. Most studies emphasize the importance of teachers diagnosing students' understandings as a step to improve on their teaching and supporting students learning (e.g., Holmes et al., 2013; Mack, 1995; Ozkan & Bal, 2016). This study uses the Trends in International Mathematics and Science Study (TIMSS) 2015 to examine the patterns of errors and misconceptions from 4th and 8th grade students' responses cross-nationally in selected fraction items in the mathematics content knowledge test.

LEARNING MATHEMATICS

In teaching mathematics, errors, mistakes, and misconceptions are used interchangeably without careful consideration of their meanings and implications. Li and Li (2008) noted that students' learning difficulties are usually presented in the form of errors. Rhine et al. (2018) posited that students make errors when they overgeneralize correct rules. Further, they stated that misconceptions occur when students' understandings are in conflict with accepted meanings, whereas mistakes can result from a faulty set of rules (Rhine et al., 2018). Similarly, Perso (1992) stated that incorrect answers result from strategies or rules with correct beginnings which are then based on distorted or misinterpretations of correct procedures.

Students' misconceptions in learning mathematics originate from their previous experiences or the nature of the present learning situation (Green et al., 2008). In their extensive review of literature, Rhine et al. (2018) found the nature of students' learning experiences can lead to misconceptions. For example, when students learn that mathematics is made up of a set of rules, they may make mistakes by using the rules inappropriately or even develop and use a faulty set of rules. Further, some students incorrectly adapt the rules learned when faced with a new situation or make errors when executing the procedure (Rhine et al., 2018). These errors and mistakes lead to misconceptions in their learning.

Misconceptions that vary depending on the content and student(s) grade level should be analyzed. Common misconceptions that children make in the elementary levels become foundational as they progress through their educational career (Durkin & Rittle-Johnson, 2015; Holmes et al., 2013; Li & Li, 2008; Mohyuddin & Khalil, 2016; Saenz-Ludlow, 1994). Although making errors and having misconceptions are considered positive, there is a need for teachers to analyze and understand them. More importantly, teachers should build a repertoire of the errors, mistakes, and misconceptions common in particular topics and mathematical processes in order to support students' successful understanding of mathematics. Additionally, teachers need to analyze if the students' incorrect responses are misconceptions, mistakes or errors (Holmes et al., 2013). To further understand the challenges that students have in learning particular mathematics topics, this study focuses on a cross-national view of students' understanding of fraction magnitude at the 4th and 8th grade level.

Students' Misconceptions in Understanding Fraction Concepts

Misconceptions have been identified in fractions understanding among elementary and secondary school students. Aliustaoglu et al. (2018) stated that the concept of fractions is one of the most difficult for students to learn. The reason for the difficulty in understanding fractions lies in the fact that many whole number properties do not hold for fractions (Fazio & Siegler, 2011). In particular, the difficulties experienced in learning fractions that then lead to misconceptions are due to faulty understandings, overgeneralizing rules learned in earlier grades, and inability to apply fraction concepts (Fazio & Siegler, 2011). For example, when performing arithmetic operations, many students tend to treat the numerator and denominator as unrelated whole numbers (Fazio & Siegler, 2011; Siegler & Pyke, 2013). Another common misconception is using rules and operations for a different operation to solve a given arithmetic task (Fazio & Siegler, 2011). Also, using either idiosyncratic strategies or misunderstanding mixed numbers leads to misconceptions (Fazio & Siegler, 2011; Siegler & Pyke, 2013). In sum, these studies confirm that overdependence of rules in learning fractions limits students understanding and often leads to errors, mistakes and misconceptions.

Procedural knowledge built from conceptual understanding of fractions is key to the success in learning fractions. A comparison of 6th grade and 8th grade students classified as high achieving and low achieving found that low achieving students tended to use whole number strategies in fraction problems in both grades, whereas the high achieving students used whole number operations in 6th grade but not in the 8th grade (Siegler & Pyke, 2012). This finding points to the lack of a conceptual understanding of fractions as numbers at the different grade levels. Bailey et al. (2015) compared procedural fluency and conceptual understanding of fractions and found that procedural fluency was the ability to seamlessly use the given procedures to solve fraction arithmetic operations, whereas a conceptual understanding of fractions was understanding the properties of fraction magnitude and the reasons behind the procedural principles and notations used in the expression of fractions. In sum, conceptual understanding of fraction operations underlies the difficulties that lead to misconceptions (Bailey et al., 2015). It is important to note that difficulties in learning fractions is of global concern (Izsák, 2008). Therefore, a study of students' understanding of fractions at different grade levels cross-nationally can further shed light to policy makers, teacher educators, curriculum developers, and other education stakeholders on whether the efforts at improving learning of this concept need further discussions and interventions.

THEORETICAL FRAMEWORK

The theory of numerical development proposes that

> "numerical development is a process of progressively broadening the class of numbers that are understood to possess magnitudes and learning the functions that connect the increasingly broad varied set of numbers to their magnitudes" (Siegler et al., 2011, p. 274).

This theory focuses on the understanding of the magnitude of numbers and the recognition that the location of the numbers can be represented in a number line. In this study we investigate 4th and 8th graders mental images of the magnitude of the numbers based on the percentage of those selecting the incorrect value for the selected questions and what the selections imply about their thinking of fraction magnitude. The questions guiding the study is:

1. What is the pattern of misconceptions related to fraction magnitude among 4th and 8th grade students in participating countries in the TIMSS 2015?
2. What do these misconceptions reveal about students' understanding of fraction magnitude?

METHODS

Sample

We used the TIMSS 2015 released items to analyze the misconceptions that 4th and 8th grade students have in fraction magnitude understanding. We selected items identified as fractions, decimals or proportional reasoning in the content area of numbers and operations. We then descriptively analyzed the items to examine the student responses in the multiple-choice items. We considered the percentage of students' correct and incorrect responses on the items and selected the items with the highest percentage of incorrect responses. From these analyses we identified two items: one for the 4th grade level and one in the 8th grade level.

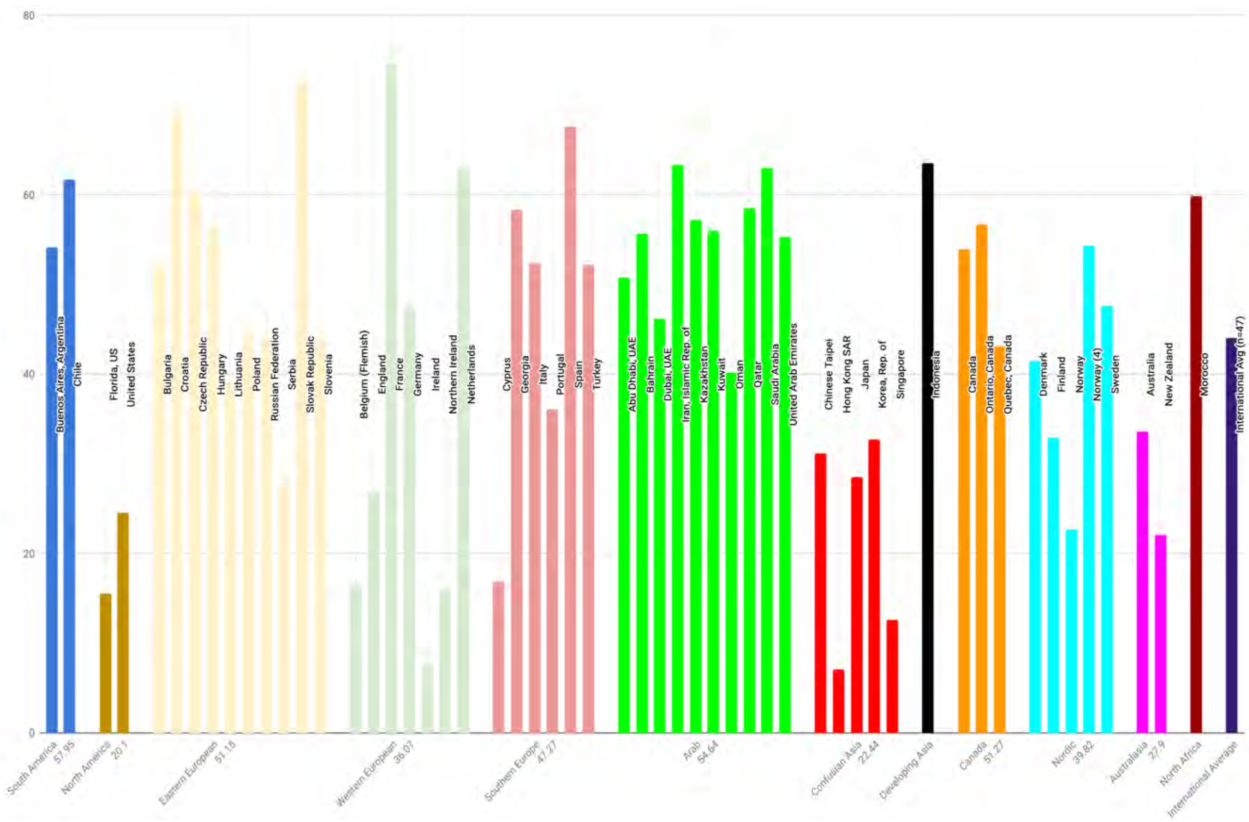

Figure 1. Graph of selection of 1/5 across the participating countries

4th grade: Which is the largest fraction (1/2, 1/3, 1/4, or 1/5)?

8th grade: What is the number closest in size to 3/4 [0.34, 0.43, 0.74, or 0.79]?

Analysis

We examined the responses of the 4th and 8th grade students on fraction items across all the participating countries and found that students have misconceptions in their understanding of fractions comparison. The 4th grade question item asked: Which is the largest fraction among 1/2, 1/3, 1/4, and 1/5?

Across the participating counties 44.33% of the students selected 1/5 as the largest fraction. The distribution for the selection across the participating countries of 1/5 is shown on **Figure 1**. From the distribution of percentage responses across the participating countries, we noted that the countries in Southern America (57.95%), Arab countries (54.64%), Canada (51.27%), and Eastern Europe (51.15%) had the highest percentage of students that selected 1/5 as the largest fraction. Confucian Asia (22.44%) and North America (20.1%) had the lowest percentage of 4th grade students who selected this item. Also, South America had the largest percentage of students who selected 1/5 as the largest fraction (from the choices of 1/2, 1/3, 1/4, and 1/5). This is the initial understanding of fraction magnitude in which the question required students' cognitive level of *knowing*. It is probable that students may have not considered a fraction as a number on its own right but perceived it as two numbers and then selected the number that is largest. Such a misconception could have occurred if students are overgeneralizing from whole numbers to rational numbers. That is, they understand whole numbers but need more support to understand how the fractions fall within the real number system.

The 8th grade item asked: What is the number closest in size to 3/4 [0.34, 0.43, 0.74, or 0.79]?

The cognitive domain for this item is on *knowing*. In this question students need the knowledge of the connections between fractions and decimals and magnitude of these numbers. Further, they need to have knowledge of place value in decimal fractions, and representations of the decimals in a continuum. Although 3/4 is an anchor number that students tend to use to compare the magnitude of numbers, the responses to this item across the participating countries indicate that the development of fraction magnitude is still a challenge at the 8th grade level cross-nationally. Also, the connections between decimal fractions and fractions seems to be problematic in particular countries. **Figure 2** provides a summary of the percentage of students' responses.

In clusters of the participating countries, the average percentage indicated that students' understanding of the connections between fraction and decimals is weak. Particularly, the participating countries in Sub-Saharan Africa, developing Asia, and South America had more than 35% of the students selecting 0.34 to be closest to 3/4. To select 0.34 as their answer indicates that the students have a challenge connecting decimals to fractions. The results indicate that at the 4th grade level students' misconceptions in fraction magnitude is higher than in the 8th grade. Further, students in particular countries seem to have misconceptions in fraction magnitude at both levels.

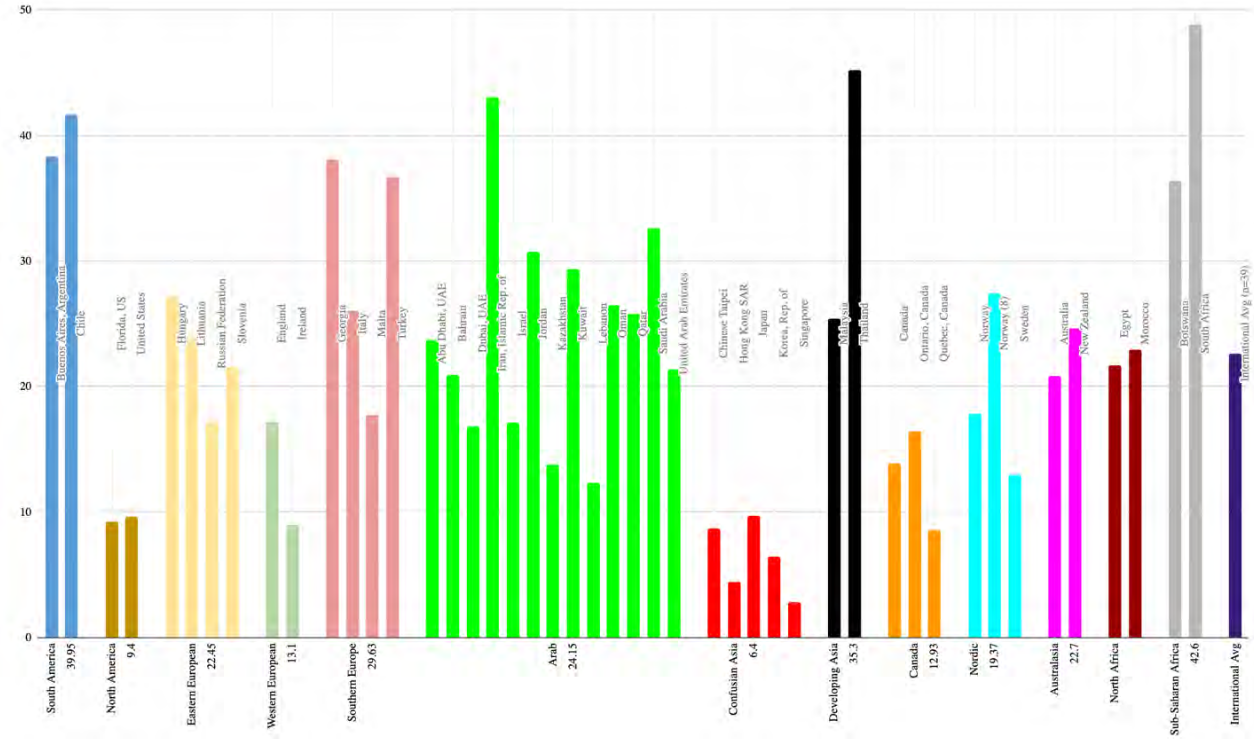

Figure 2. Graph of the percentage of students across participating countries that selected 0.34 as being closest to 3/4

At 8th grade 40% of the students in the South American countries selected 0.34 to be closest to 3/4. A closer look shows that the 8th grade students in Chile had even more students selecting 0.34 (above 40%) when compared to Argentina. Further, the symbols for decimals and fractions were ignored and the students paid attention to the similarity of the numbers 3 and 4. These students extended the rule of whole number magnitudes to decimals and fractions and considered the whole numbers without taking into consideration the symbolic representations of the two numbers.

CONCLUSIONS

The purpose of this study was to examine the 4th and 8th grade students' misconceptions on fraction magnitude in fraction items from the TIMSS 2015 mathematics questionnaire. These results suggest that students may have missed opportunities to learn based on the instruction they received. Beckmann (2017) provides five strategies students use to solve problems that involve fraction magnitude. These methods are converting to decimals, using common denominators, cross-multiplication, using common numerators, and reasoning about benchmarks to compare fractions. Considering that 1/4, 1/2, and 3/4 are benchmark fractions, the results suggest that either these students may not have knowledge of the magnitude of benchmark fractions, or they have not had adequate opportunities to engage with different representations of fractions. It could also indicate that the teachers focus more on procedures without emphasizing conceptual understanding. Perhaps the teachers tend to teach how they understand and were taught fractions.

Zhang et al. (2015b) emphasized that an over reliance of the area model representation of fractions leads to a shallow understanding of this concept. Students gain a more in-depth experience and conceptual understanding of fractions when the instruction includes a multi-model approach (Zhang et al., 2015a). Instruction in which students have opportunities to represent fractions using an area model, measurement model, set model, and number lines broaden the understanding of fractions (Beckmann, 2017). The use of number lines to represent fractions and whole numbers provide a complete picture of real numbers. Also, increased opportunities in which students engage in activities where the partitioning of the number line requires students to add more partitions in solving the problem would support students to have a deeper understanding of fraction comparison. In sum, opportunities in which teachers emphasize fractions part of a whole with partitioning and iterating units using multiple representations could support a deeper understanding of fraction comparison. More importantly, a measurement model representation using a number line may have been either missed on unclear during the learning experience of students in particular countries. That is, the education stakeholders in Chile and Argentina in South America, Thailand in developing Asia, Botswana and South Africa in Sub-Saharan Africa, and Iran need to analyze the curriculum and teachers' pedagogy when teaching fractions so as to understand the gaps that could be leading to students' misconceptions as they develop understanding of fraction magnitude.

Some countries showed a significant drop in students' misconceptions with fraction magnitude at the 8th grade level. This finding suggests that students' understanding of fraction magnitude could also be related to differences in curriculum organization. In particular, fraction magnitude may have been emphasized after the 4th grade in particular countries. For example, in Canada, at the 4th grade level, 51.27% of the students selected 1/5 as the largest fraction, whereas only 12.93% of the 8th grade students selected 0.34 as closest to 3/4. Similar patterns of a significant decrease in selection of these responses were seen in the Arab countries, Southern Europe, and Eastern Europe.

Lastly, the participating countries in Africa had a high percentage of students with misconceptions in their understanding of fraction magnitude. For example, 60% of the students at the 4th grade selected 1/5 as the largest fraction. Also, at the 8th grade 42.6% of the students did not see the difference between 3/4 and 0.34. These results imply that differentiating between whole numbers, fractions and decimals is a challenge that should be addressed. Either the curriculum does not provide for opportunities for students to understand the connections or the teachers teach these concepts as isolated units.

Implications for Higher Education

The implications of these findings are important for teacher preparation programs. Most teacher preparation programs cross-nationally provide opportunities for learning math content and pedagogy (Ayieko, 2014). Blömeke and Kaiser (2014) noted that the learning of school mathematics in teacher preparation programs varied cross-nationally. However, the importance of the emphasis of school mathematics in higher education for teacher development should not be ignored. Without considerations of teacher preparation courses including school mathematics in higher education, the pre-service teachers miss opportunities to expand their knowledge on important topics such as fractions. With the inclusion of school mathematics, there is need for opportunities for pre-service teachers to learn important topics that have been identified as problematic to students. This emphasis would ensure that pre-service teachers are not limited to teaching how they were taught certain topics but instead have the requisite knowledge to teach fractions using a multi-modal approach.

Further, opportunities for higher education learning should include curriculum analysis. Across and within countries, the curriculum materials may not be universal. As such, opportunities that allow for pre-service teachers in higher education to analyze curriculum enable them to begin their professions knowing what is offered in different curriculum and ways in which they can supplement or select appropriate resources for teaching. These opportunities also allow for teachers to inform their ministries of education about the curriculum materials appropriateness.

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REFERENCES

Aliustaoglu, F., Tuna, A., & Biber, A. C. (2018). The misconceptions of sixth grade secondary school students on fractions. *International Electronic Journal of Elementary Education, 10*(5), 591-599. https://doi.org/10.26822/iejee.2018541308

Ayieko, R. A. (2014). *The influence of opportunity to learn to teach mathematics on pre-service teachers' knowledge and belief: A comparative study* [PhD dissertation, Michigan State University].

Bailey, D. H., Zhou, X., Zhang, Y., Cui, J., Fuchs, L. S., Jordan, N. C., Gersten, R., & Siegler, R. S. (2015). Development of fraction concepts and procedures in U.S. and Chinese children. *Journal of Experimental Child Psychology*, *129*, 68-83. https://doi.org/10.1016/j.jecp.2014.08.006

Beckmann, S. (2017). *Mathematics for elementary teachers with activities*. Pearson.

Blömeke, S., & Kaiser, G. (2014). Homogeneity or heterogeneity? Profiles of opportunities to learn in primary teacher education and their relationship to cultural context and outcomes. In S. Blömeke, F.-J. Hsieh, G. Kaiser, & W. H. Schmidt (Eds.), *International perspectives on teacher knowledge, beliefs and opportunities to learn* (pp. 299-325). Springer. https://doi.org/10.1007/978-94-007-6437-8_14

Durkin, K., & Rittle-Johnson, B. (2015). Diagnosing misconceptions: Revealing changing decimal fraction knowledge. *Learning and Instruction*, *37*, 21-29. https://doi.org/10.1016/j.learninstruc.2014.08.003

Fazio, L., & Siegler, R. (2011). *Teaching fractions. Educational practices series-22*. UNESCO International Bureau of Education.

Green, M., Piel, J. A., & Flowers, C. (2008). Reversing education majors' arithmetic misconceptions with short-term instruction using manipulatives. *The Journal of Educational Research*, *101*(4), 234-242. https://doi.org/10.3200/JOER.101.4.234-242

Hecht, S. A., & Vagi, K. J. (2010). Sources of group and individual differences in emerging fraction skills. *Journal of Educational Psychology*, *102*(4), 843-859. https://doi.org/10.1037/a0019824

Holmes, V.-L., Miedema, C., Nieuwkoop, L., & Haugen, N. (2013). Data-driven intervention: Correcting mathematics students' misconceptions, not mistakes. *The Mathematics Educator, 23*(1), 24-44.

Izsák, A. (2008). Mathematical knowledge for teaching fraction multiplication. *Cognition and Instruction*, *26*(1), 95-143. https://doi.org/10.1080/07370000701798529

Kloosterman, P. (2010). Mathematics skills of 17-year-olds in the United States: 1978 to 2004. *Journal for Research in Mathematics Education*, *41*(1), 20-51. https://doi.org/10.5951/jresematheduc.41.1.0020

Li, X., & Li, Y. (2008). Research on students' misconceptions to improve teaching and learning in school mathematics and science. *School Science and Mathematics, 108*(1), 4-7. https://doi.org/10.1111/j.1949-8594.2008.tb17934.x

Mack, N. K. (1995). Confounding whole-number and fraction concepts when building on informal knowledge. *Journal for Research in Mathematics Education*, *26*(5), 422-441. https://doi.org/10.2307/749431

Mohyuddin, R. G., & Khalil, U. (2016). Misconceptions of students in learning mathematics at primary level. *Bulletin of Education and Research, 38*(1), 133-162.

Ozkan, M., & Bal, A. P. (2016). Analysis of the misconceptions of 7th grade students on polygons and specific quadrilaterals. *Eurasian Journal of Educational Research*, *16*(67), 161-182. https://doi.org/10.14689/ejer.2017.67.10

Perso, T. (1992). Making the most of errors. *Australian Mathematics Teacher*, *48*(2), 12-14.

Rhine, S., Harrington, R., & Starr, C. (2018). *How students think when doing algebra*. IAP.

Saenz-Ludlow, A. (1994). Michael's fraction schemes. *Journal for Research in Mathematics Education*, *25*(1), 50-85. https://doi.org/10.2307/749292

Siegler, R. S., & Pyke, A. A. (2013). Developmental and individual differences in understanding of fractions. *Developmental Psychology*, *49*(10), 1994-2004. https://doi.org/10.1037/a0031200

Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology*, *62*(4), 273-296. https://doi.org/10.1016/j.cogpsych.2011.03.001

Zhang, X., Clements, M. A. (K.), & Ellerton, N. F. (2015a). Engaging students with multiple models of fractions. *Teaching Children Mathematics*, *22*(3), 138-147. https://doi.org/10.5951/teacchilmath.22.3.0138

Zhang, X., Clements, M. A. (K.), & Ellerton, N. F. (2015b). Enriching student concept images: Teaching and learning fractions through a multiple-embodiment approach. *Mathematics Education Research Journal*, *27*, 201-231. https://doi.org/10.1007/s13394-014-0137-4